Master's Studies

Discount-Based Promotion Strategy Optimization

Field: Advanced Analytics – Big Data
Name of course: Advanced Simulation Modelling
Warsaw School of Economics (SGH)

Author's Name and Surname & Student No.:
Yi Wu 115756
Tianshi Wei 140164
Mengzhen Shang 140028
Liyuan Cao 140029

Warsaw, 2025.06

1. Executive Summary

- This project explores how a supermarket should set the minimum daily discount during a 90-day shopping festival to maximize total profit.
- Using simulation, we tested different discount levels and analyzed their impact on profit.
- The results show that:
- The optimal minimum discount is 12.3%, leading to a maximum profit of 13,558.84 PLN.
- A discount range between 10.6% and 18% maintains at least 95% of the maximum profit, allowing flexibility in pricing.
- Very small discounts fail to boost sales, while very large discounts reduce margins unnecessarily.
- We recommend setting the minimum discount around 12–13% to balance customer demand and profitability during promotional periods.

---

2. Organization Description

This project analyzes a medium-sized supermarket chain operating in a highly competitive retail market, offering fast-moving consumer goods (FMCG), with a focus on yogurt and fresh dairy products.
To remain competitive, especially during a 90-day shopping festival, the company uses discount-based promotional strategies to boost demand. During this period, retailers are expected to apply discounts, making pricing decisions especially critical.
Yogurt is highly price-sensitive, so finding the right minimum discount level is essential — low enough to maintain margins, but high enough to attract purchases. Excessive discounting can harm profitability or train customers to wait for sales.
This issue belongs to strategic pricing and promotion planning and focuses on how discount decisions affect sales and profit.The key challenge is to define a discount strategy that maximizes profit over the promotion period by accounting for customer responsiveness to prices and promotion dynamics.

---

3. Problem Description

The company aims to determine the optimal minimum daily discount d_min to apply during a 90-day mandatory shopping festival, where competitors also offer promotional pricing. The goal is to maximize the total profit over the campaign.
The core decision variable is:

- d_min ∈ [0.01, 0.4] – the minimum discount level allowed.

Discounts beyond 40% reduce the product price below cost (5 PLN × (1 – 0.4) = 3 PLN), making the sale unprofitable.
Mathematical Representation:
The objective is to maximize total profit:

$$\mathrm{Profit_t} = (\mathrm{price_t} + \mathrm{reward}) \cdot q_t - \mathrm{cost} \cdot \mathrm{demand_t} - \mathrm{holding\_cost_t}$$

Where:

- $\text{price}_t = \text{base\_price} \times (1 - d_t)$
- q_t is the quantity sold, based on an exponential demand model with elasticity:

$q_t \sim \text{Poisson}\left(\text{base\_demand} \cdot e^{\varepsilon(d_t - 0.2)}\right)$

- The discount level is dynamically updated based on sales using a logistic function:

$$d_t = d_{\min} + \frac{d_{\max} - d_{\min}}{1 + e^{k(q_t - c)}}$$

Simulation Logic:

- On each day t ∈ [1, 90], demand is simulated using a price-sensitive exponential demand function.
- Sales are limited by available inventory.
- If units remain unsold, holding costs are incurred and next day's demand is reduced.
- The discount level d_t is updated dynamically using a logistic function based on current sales.

---

4. Analysis Results

4.1 Overview

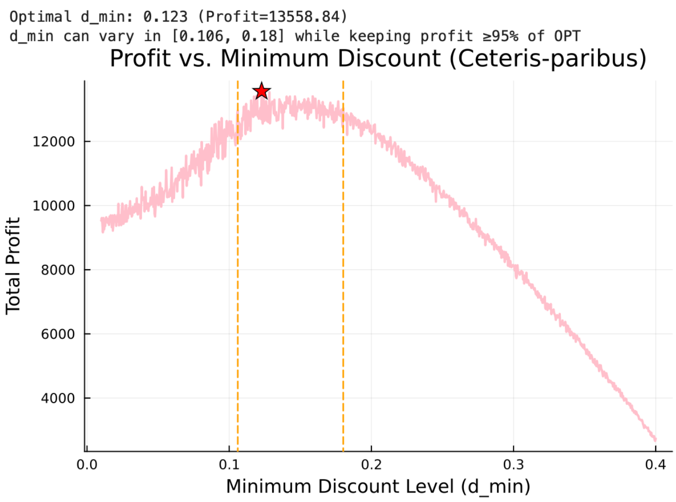

Figure 4.1.1 Profit vs. Minimum Discount Level (d_min)

The Figure 4.1.1 shows a clear nonlinear relationship:

- Profit increases with moderate discounts and peaks at d_min = 0.123, yielding a maximum profit of 13,558.84 PLN.
- Beyond this point, higher discounts reduce profit due to lower margins.
- Importantly, profit remains within 95% of the maximum when d_min lies between 0.106 and 0.18, offering a range of flexible yet effective pricing strategies.

4.2 Interpretation and Business Relevance

This outcome confirms a key managerial insight: moderate discounts (around 12%) are most effective at stimulating demand without sacrificing margin. Very small discounts fail to boost sales significantly, while excessive discounts harm profitability.

The identified range [0.106, 0.18] allows retailers to adapt within operational limits without risking significant financial loss. This supports data-driven decision-making for pricing during competitive campaigns, aligning well with intuitive promotional strategy: discount just enough to drive volume, but no more than necessary.

---

5. Sensitivity Analysis

5.1 The impact of the minimum discount rate on total profit

5.1.1 Methodology

To examine the sensitivity of the objective function (total profit) to the minimum discount rate `d_min`, we simulated profits over 1,000 evenly spaced values of `d_min` in the range [0.01, 0.4].

5.1.2 Results Table

Row	d_min	total_profit
	Float64	Float64
1	0.122823	13558.8
2	0.128288	13535.9
3	0.119309	13417.9
4	0.123994	13405.6
5	0.146246	13401.8
6	0.158739	13401.3
7	0.142733	13398.9
8	0.14039	13398.2
9	0.117357	13395.5
10	0.118529	13391.1

Table 5.1.2.1 DataFrame

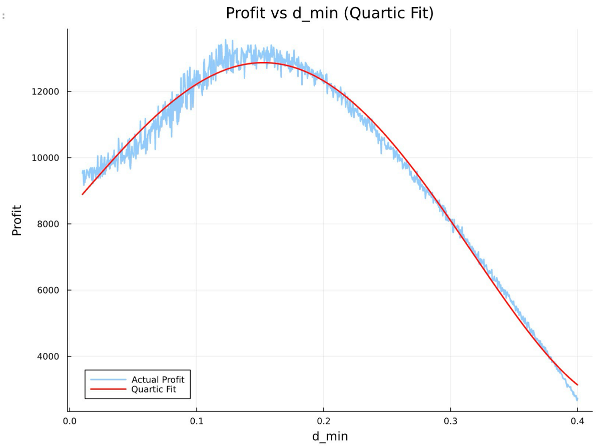

Figure 5.1.2.2 Profit vs. d_min

From Table 5.1.2.1 and Figure 5.1.2.2 we can draw the following conclusions：

Optimal Point: The fitted curve reaches its peak around d_min = 0.13, corresponding to the maximum profit of approximately 13000.

Curve Shape: The quartic fit captures a clear unimodal pattern—profit increases with d_min up to a point, then declines sharply, showing strong sensitivity beyond the optimum.

Downside Risk: Profit falls rapidly when d_min > 0.2, dropping below 10,000 at d_min = 0.27, and under 7,000 at d_min = 0.35, suggesting a steep loss in performance from over-discounting.

Stable Zone: The profit remains within 95% of the maximum (≥12,255) when d_min is in the range of approximately [0.106, 0.18], indicating a relatively robust optimal region.

5.2 The joint effect of minimum discount rate and price elasticity on profits

5.2.1 Methodology

To analyze the joint impact of minimum discount rate (d_min) and price elasticity on total profit, we conducted a comprehensive parameter sweep across 100 discount values in [0.01, 0.4] and 5 elasticity levels [1.0-3.0], employing controlled randomization (500 total simulations) to identify optimal pricing strategies.

5.2.2 Results and Visuals

Row	d_min	elasticity	total_profit
	Float64	Float64	Float64
1	0.0493939	1.0	16163.4
2	0.0533333	1.0	16130.0
3	0.0454545	1.0	16025.5
4	0.01	1.0	15997.1
5	0.0139394	1.0	15946.4
6	0.0218182	1.0	15761.0
7	0.0257576	1.0	15646.3
8	0.0730303	1.0	15584.0
9	0.0809091	1.0	15515.0
10	0.0572727	1.0	15465.0

Table 5.2.2.1 DataFrame

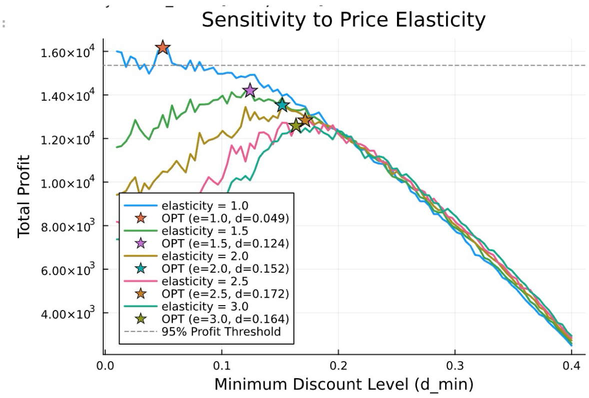

Figure 5.2.2.2 Sensitivity to Price Elasticity

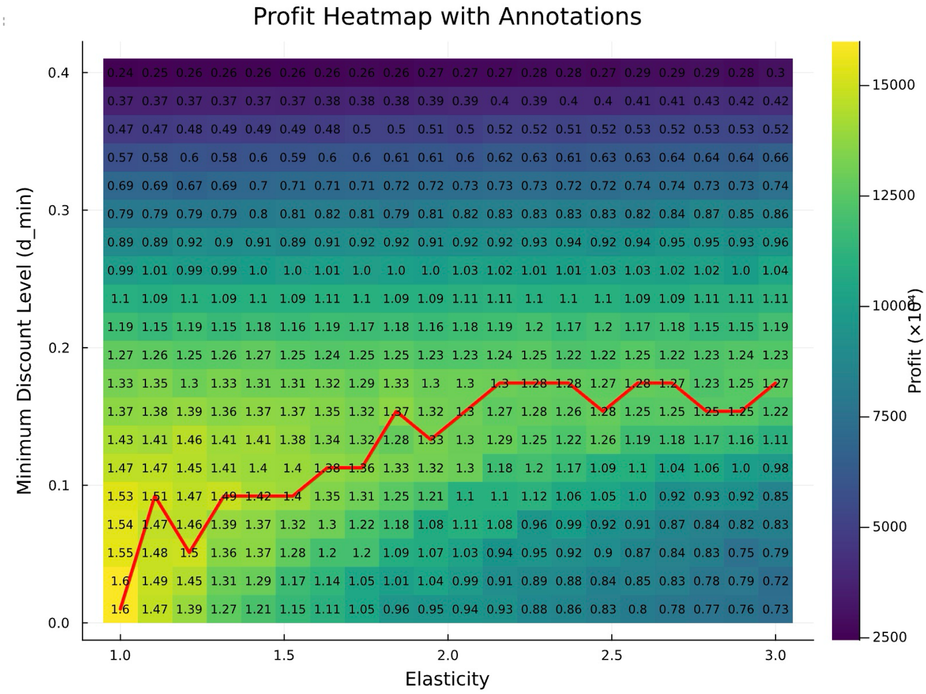

Figure 5.2.2.3 Profit Heatmap with Annotations

Optimal Solutions:

elasticity=1.0: d_min=0.049, profit=16163.39 elasticity=1.5: d_min=0.124, profit=14192.79

elasticity=2.0: d_min=0.152, profit=13524.46 elasticity=2.5: d_min=0.172, profit=12831.19

elasticity=3.0: d_min=0.164, profit=12582.5

Critical Ranges (≥95% of Max Profit):

elasticity=1.0: d_min ∈ [0.01, 0.081] elasticity=1.5: d_min ∈ [0.057, 0.152]

elasticity=2.0: d_min ∈ [0.12, 0.175] elasticity=2.5: d_min ∈ [0.132, 0.199]

elasticity=3.0: d_min ∈ [0.152, 0.211]

The analysis shows that as elasticity increases, the optimal discount level d_min also increases, while maximum profit decreases:

* At elasticity 1.0, d_min = 0.049, profit = 16,163.39
* At elasticity 3.0, d_min = 0.164, profit = 12,582.50

This represents a 22.1% decrease in profit and a 234% increase in optimal d_minfrom the lowest to the highest elasticity.

The 95% profit threshold ranges quantify the sensitivity of profit to d_min:

* elasticity 1.0: [0.01, 0.081] → width = 0.071
* elasticity 1.5: [0.057, 0.152] → width = 0.095
* elasticity 2.0: [0.120, 0.175] → width = 0.055
* elasticity 2.5: [0.132, 0.199] → width = 0.067
* elasticity 3.0: [0.152, 0.211] → width = 0.059

Although optimal d_min rises with elasticity, the 95% stability zone remains relatively narrow (approx. 0.05–0.10), indicating that profit is sensitive to deviations from the optimal value, especially in higher elasticity scenarios.

5.2.3 4th-degree polynomial regression

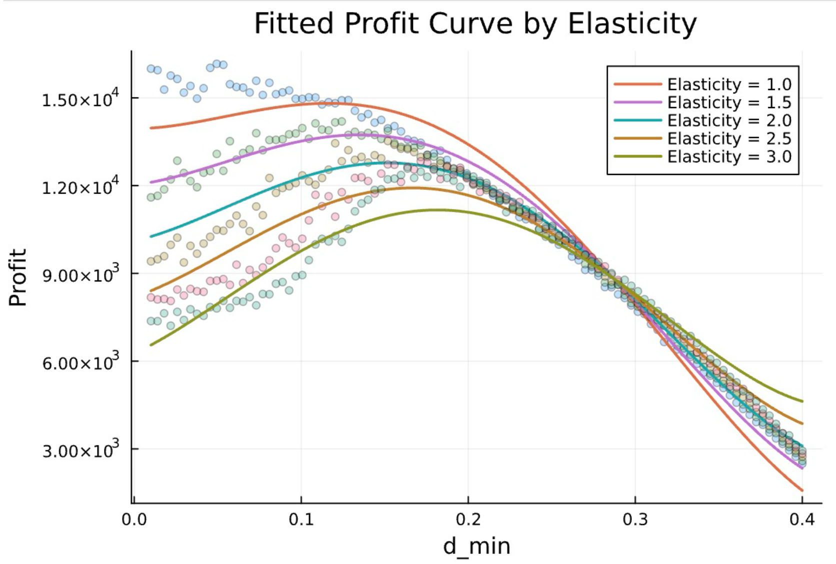

Figure 5.2.3.1 Fitted Profit Curve by Elasticity

key insights of Figure 5.2.3.1：

Peak Shifts: As elasticity increases, the profit-maximizing d_min shifts rightward, indicating that more price-sensitive consumers require deeper discounts to maximize profit: Elasticity = 1.0: peak near 0.05 Elasticity = 3.0: peak near 0.16

Profit Decline: Maximum profit decreases monotonically with higher elasticity. The top curve (elasticity = 1.0) reaches nearly 15,000, while the bottom curve (elasticity = 3.0) peaks around 12,500.

Steep Drop-Offs: All curves demonstrate a sharp decline in profit beyond the optimal d_min, especially on the right tail. This highlights the risk of over-discounting, particularly when elasticity is low.

Narrow Optima: The curvature around the peak becomes sharper as elasticity increases, suggesting higher sensitivity to small changes in d_min for more elastic markets.

5.3 The joint effect of discount rates and holding cost on profits

5.3.1 Methodology

We analyzed the joint impact of minimum discount rate (d_min) and holding cost (hold_unit) on total profit through a parameter sweep. Specifically, we tested 100 values of d_min in [0.01, 0.4] and 5 holding cost levels from 0.02 to 0.10, yielding 500 combinations.

5.3.2 Results and Visuals

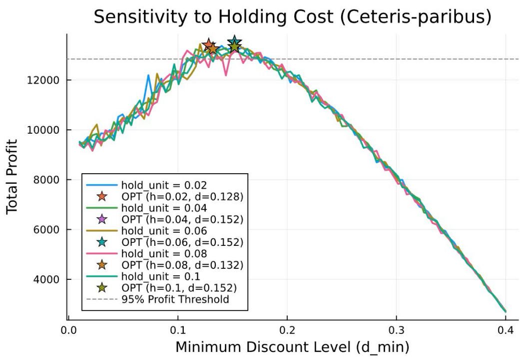

Figure 5.3.2.1

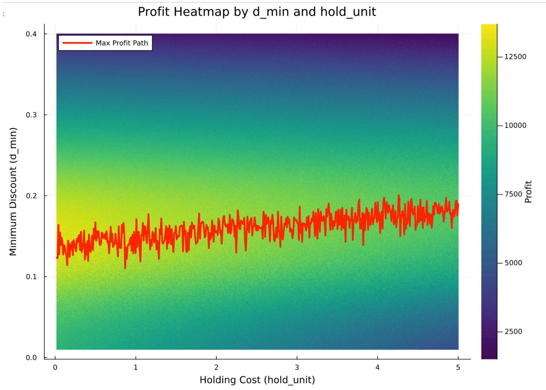

Figure 5.3.2.2

Optimal Solutions:

hold_unit=0.02: d_min=0.128, profit=13393.96 hold_unit=0.04: d_min=0.152, profit=13498.08

hold_unit=0.06: d_min=0.152, profit=13519.03 hold_unit=0.08: d_min=0.132, profit=13243.97

hold_unit=0.1: d_min=0.152, profit=13332.75

Critical Ranges (≥95% of Max Profit):

hold_unit=0.02: d_min ∈ [0.108, 0.183] hold_unit=0.04: d_min ∈ [0.108, 0.179]

hold_unit=0.06: d_min ∈ [0.12, 0.175] hold_unit=0.08: d_min ∈ [0.105, 0.195]

hold_unit=0.1: d_min ∈ [0.116, 0.183]

Observation from Figure 5.3.2.1 and Figure 5.3.2.2:

1. The optimal d_min remains relatively stable, clustering around 0.152, regardless of moderate increases in hold_unit.
2. Most ranges include the stable point d_min ≈ 0.15, showing robustness across holding costs.
3. The widest range occurs at hold_unit = 0.08, suggesting greater tolerance in decision-making under this scenario.
4. The narrowest range (0.055) appears at hold_unit = 0.06, implying a higher sensitivity to mis-specification.

Profit Stability: The maximum profit varies mildly (range: 13,243–13,519), showing that moderate changes in holding cost do not drastically affect total profitability.

5.4 The joint impact of k value and minimum discount rate on profit

5.4.1 Methodology

To evaluate the joint impact of the minimum discount rate (d_min) and the sigmoid sensitivity parameter (k) on total profit, we conducted a parameter sweep over 100 discount values in the range [0.01, 0.4] and 7 different values of k (ranging from 0.05 to 4.0), resulting in 700 controlled simulations.

5.4.2 Results and Visuals

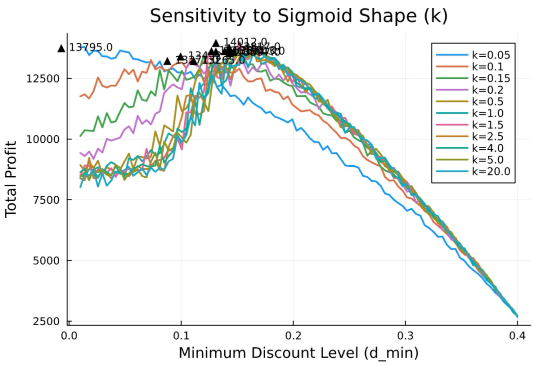

Figure 5.4.2.1

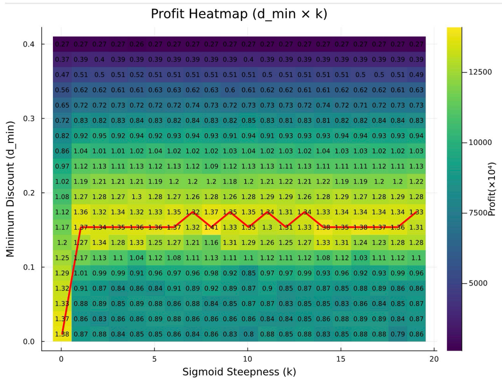

Figure 5.4.2.2

Optimal Solutions:

k=0.05: d_min=0.014, profit=13794.7 k=0.1: d_min=0.108, profit=13271.26

k=0.15: d_min=0.12, profit=13452.48 k=0.2: d_min=0.132, profit=13264.64

k=0.5: d_min=0.152, profit=13652.98 k=1.0: d_min=0.16, profit=13670.6

k=1.5: d_min=0.152, profit=14011.76 k=2.5: d_min=0.164, profit=13817.42

k=4.0: d_min=0.148, profit=13673.18 k=5.0: d_min=0.164, profit=13596.7

k=20.0: d_min=0.168, profit=13642.63

Critical Ranges (≥95% of Max Profit):

k=0.05: d_min ∈ [0.01, 0.081] k=0.1: d_min ∈ [0.049, 0.152] k=0.15: d_min ∈ [0.081, 0.175]

k=0.2: d_min ∈ [0.105, 0.195] k=0.5: d_min ∈ [0.136, 0.187] k=1.0: d_min ∈ [0.128, 0.183]

k=1.5: d_min ∈ [0.124, 0.179] k=2.5: d_min ∈ [0.152, 0.179] k=4.0: d_min ∈ [0.132, 0.187]

k=5.0: d_min ∈ [0.14, 0.191] k=20.0: d_min ∈ [0.14, 0.191]

Sensitivity Analysis Conclusion (Parameter k) based on Figure 5.4.2.1 and Figure 5.4.2.2: We analyzed how the parameter k, which controls the sensitivity of discount adjustment, affects the optimal strategy (d_min) and total profit. The results of Figure 5.4.2.2 show that when k increases from 0.05 to1.5, the optimal solution (OPT) changes significantly, as evidenced by:

1. The optimal d_min increases from 0.014 to the range of 0.152–0.164;
2. The maximum profit rises from 13,264 to 14,011;
3. The system is highly sensitive to changes in k, particularly within the range k ∈ [0.05, 1.5].

Further analysis indicates that when k increases to 2.5 or even 20.0, the optimal solution (d_min) remains nearly unchanged and the profit stabilizes. This suggests that the system becomes less responsive to discount changes in the high-k range, entering a “saturation” phase.

Conclusion: A small increase in k (e.g., from 0.05 to 0.1) can already trigger a change in the optimal strategy. However, once k exceeds 2.5, the optimal solution becomes insensitive to further changes. Therefore, the impact of k on strategy optimization is nonlinear and involves a critical sensitivity range.

5.4.3 4th-degree polynomial regression

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	Upper 95%
(Intercept)	9992.07	125.298	79.75	<1e-99	9746.21	10237.9
d_min	42695.6	1154.02	37.00	<1e-99	40431.3	44960.0
d_min_sq	-1.6157e5	2634.39	-61.33	<1e-99	-1.66739e5	-1.56401e5
d_min_cu	0.0	NaN	NaN	NaN	NaN	NaN
d_min_qt	0.0	NaN	NaN	NaN	NaN	NaN
k	-2509.57	205.984	-12.18	<1e-31	-2913.74	-2105.4
k_sq	576.586	100.819	5.72	<1e-07	378.765	774.407
k_cu	-42.5514	16.6971	-2.55	0.0110	-75.3137	-9.7892
k_qt	0.967504	0.615423	1.57	0.1162	-0.240047	2.17505
dk	10958.5	755.285	14.51	<1e-43	9476.51	12440.5
dk_sq	-8762.06	2237.78	-3.92	<1e-04	-13152.9	-4371.21
d_k_sq	-1757.77	163.293	-10.76	<1e-25	-2078.18	-1437.36
dk_cu	9456.86	3576.62	2.64	0.0083	2439.01	16474.7
d_cu_k	67.251	6.65706	10.10	<1e-22	54.1889	80.3132

$R^2 = 0.9149$

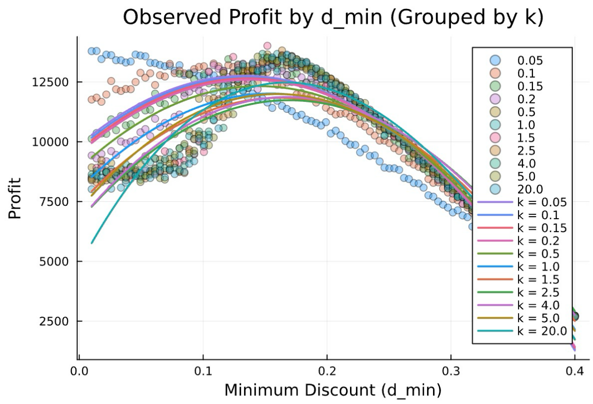

Figure 5.4.3.1

From Figure 5.4.3.1 we can draw the following conclusions：

1. Peak Shifts: Optimal Discount Shifts Rightward

The coefficient of d_min is +42,695.6 and d_min^2 is −161,570, indicating a clear single-peaked profit curve with respect to discount depth.

As k increases, the profit-maximizing d_min shifts significantly to the right:

At k = 0.05, optimal d_min = 0.014

At k = 1.5, optimal d_min ≈ 0.152

At k ≥ 5.0, it saturates in the range 0.164–0.168

Interpretation: More price-sensitive consumers (higher k) require deeper discounts to maximize profit.

2. Profit Decline: Profits Decrease as k Increases

The coefficient of k is −2,509.57 ($p < 1e{-31}$), significantly negative, indicating that:

Increasing price sensitivity tends to reduce overall profit.

This is supported by actual values:

Maximum profit drops from 14,011 (at k = 1.5) to 13,596 (at k = 5.0)

At k = 0.05, profit is 13,794, suggesting that moderate price sensitivity may yield higher profits.

3. Steep Drop-Offs: Profit Declines Rapidly Beyond the Optimum

The negative coefficients of d_min^2 (−161,570) and dk_sq (−8,762) imply that:

Profits fall sharply once d_min exceeds its optimal value, especially when k is high (e.g., k ≥ 2.5).

This is also evident in the right tail of the profit curves, highlighting the risk of over-discounting.

4. Narrow Optima: Sharper Peaks in High-k Markets

The significant interaction terms d_k_sq (−1,757.77) and d_cu_k (+67.25) show that:

In high-k markets, the profit curve becomes steeper near the optimum.

This means narrower optimal regions and less tolerance for discount deviation.

For example:

At k = 1.5, optimal d_min = 0.152,

but beyond 0.179, profits decline sharply.

5.5 The joint impact of base demand and minimum discount rate on profits

5.5.1 Methodology

To evaluate the joint impact of the minimum discount rate (d_min) and baseline market demand (base_demand) on total profit, we conducted a parameter sweep over 100 discount values in [0.01, 0.4] and 7 demand levels (60 to 180), resulting in 700 controlled simulations.

5.5.2 Results and Visuals

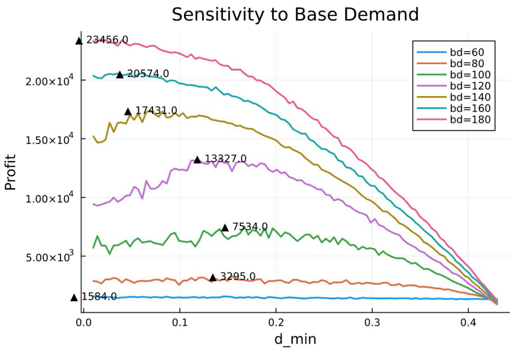

Figure 5.5.2.1

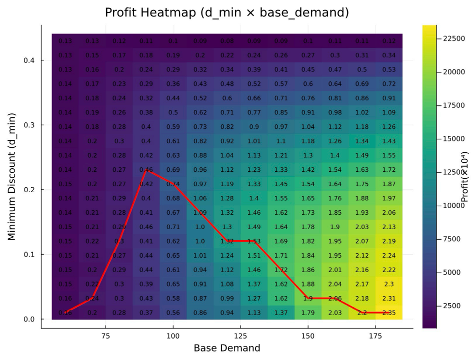

Figure 5.5.2.2

Optimal Solutions:

bd=60: d_min=0.01, profit=1583.99　　bd=80: d_min=0.154, profit=3294.83

bd=100: d_min=0.167, profit=7533.85　　bd=120: d_min=0.142, profit=13327.37

bd=140: d_min=0.069, profit=17430.81　　bd=160: d_min=0.061, profit=20573.8

bd=180: d_min=0.018, profit=23456.08

Critical Ranges (≥95% of Max Profit):

bd=60: d_min ∈ [0.01, 0.192]　　bd=80: d_min ∈ [0.031, 0.154]

bd=100: d_min ∈ [0.125, 0.197]　　bd=120: d_min ∈ [0.112, 0.188]

bd=140: d_min ∈ [0.048, 0.137]　　bd=160: d_min ∈ [0.01, 0.112]

bd=180: d_min ∈ [0.01, 0.099]

From Figure 5.5.2.1 and Figure 5.5.2.2 we can draw the following conclusions：

Sensitivity of Profit to Base Demand：

Profit increases non-linearly with base demand: From bd=60 → 80, profit increases by ~1.7k; From bd=160 → 180, profit increases by nearly 2.9k; Suggests increasing marginal returns at first, possibly flattening later.

Marginal returns may diminish: Growth slows slightly at higher demand, indicating diminishing marginal gain.

Relationship Between Optimal d_min and Base Demand：

In low-demand scenarios, aggressive discounts (d_min = 0.01) are needed to attract customers; For medium demand, moderate discounts (d_min ≈ 0.14–0.16) are optimal;

In high-demand cases, minimal discounts (d_min < 0.02) yield the best results.

This implies that in high-demand or peak seasons, pricing power can be exercised to maximize profit, whereas in low-demand periods, deeper discounts are necessary.

Critical Ranges of d_min (≥95% of Max Profit):

1. bd = 60 has the widest tolerance range: Profits remain close to optimal even if d_min varies widely; Pricing is more forgiving in low-demand markets.
2. Mid to high base demands have narrower windows: bd = 100–140 have critical ranges around 0.07–0.09; Small deviations from optimal d_min can significantly reduce profit.

3. High-demand scenarios (bd = 180) require precision: Best profits are achieved with very low d_min; Even small increases in d_min can lead to noticeable profit losses.

5.6 The joint effect of unit cost and minimum discount on profit

5.6.1 Methodology

To evaluate the joint impact of the unit cost (cost) and the minimum discount rate (d_min) on total profit, we conducted a systematic parameter sweep. Specifically, we varied d_min over 20 equally spaced values in the range [0.01, 0.4], and tested 5 different cost levels from 2.0 to 4.0 with a step of 0.5.

5.6.2 Results and Visuals

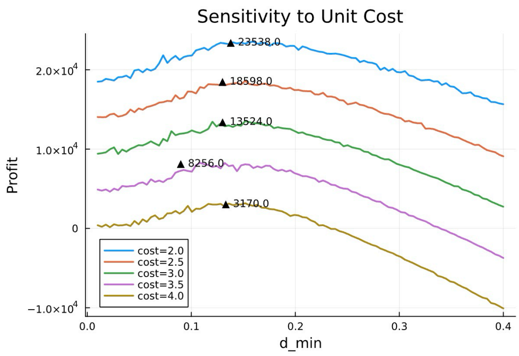

Figure 5.6.2.1

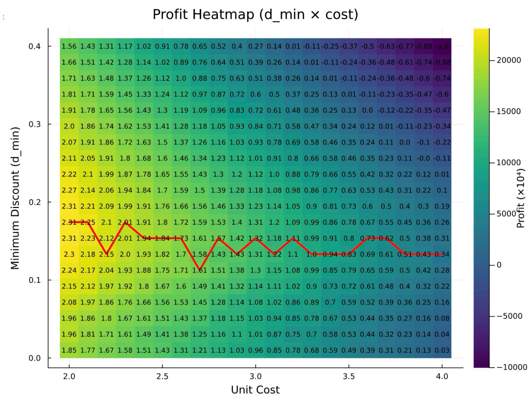

Figure 5.6.2.2

Optimal Solutions:

cost=2.0: d_min=0.16, profit=23538.48 cost=2.5: d_min=0.152, profit=18597.82

cost=3.0: d_min=0.152, profit=13524.46 cost=3.5: d_min=0.108, profit=8256.4

cost=4.0: d_min=0.152, profit=3170.2

Critical Ranges (≥95% of Max Profit):

cost=2.0: d_min ∈ [0.105, 0.219] cost=2.5: d_min ∈ [0.108, 0.195]

cost=3.0: d_min ∈ [0.12, 0.175] cost=3.5: d_min ∈ [0.105, 0.175]

cost=4.0: d_min ∈ [0.116, 0.152]

Sensitivity of Profit to Unit Cost based on Figure 5.6.2.1 and Figure 5.6.2.2:

Profit drops sharply with rising unit cost

Profit declines non-linearly as unit cost increases:

From cost = 2.0 → 2.5: Profit ↓ by 4.9k (~21%)

From 2.5 → 3.0: Profit ↓ another 5.1k (~27%)

From 3.0 → 3.5: ↓ by 5.3k (~39%)

From 3.5 → 4.0: ↓ by 5.1k (~62%)

Interpretation:

Early increases in cost reduce profit at a steady rate.

Beyond cost = 3.0, profit erosion accelerates, reflecting higher marginal loss.

At cost = 4.0, profit drops to 13% of its original value (from 23.5k to 3.2k).

Optimal d_min Strategy Adjusts to Cost Pressure:

As cost increases, the optimal discount first decreases, suggesting a strategy to protect margins. At cost = 3.5, the system switches to more aggressive discounts to stimulate demand. However, at cost = 4.0, the return to d_min = 0.152 hints at over-discounting becoming

inefficient, possibly due to negative profit impacts at high cost + deep discount.
Precision in Discount Strategy Increases with Cost：
At low cost, the profit surface is flat—many discount strategies perform well.
As cost rises, the profit surface steepens—small errors in d_min can significantly reduce profits.
Cost = 4.0 has a tight window of only 0.036, indicating maximum vulnerability to pricing errors.

---

6. Conclusions and recommendations

This simulation-based analysis explored how minimum daily discount levels influence profit over a 90-day promotional campaign. The model integrates demand uncertainty, price elasticity, inventory effects, and dynamic discount adjustments. It identifies 12.3% as the profit-maximizing minimum discount, yielding 13,558.84 PLN. Profit falls sharply beyond 18% due to margin erosion, while discounts below 10% fail to drive sufficient demand.

Regression results ($R^2 = 0.9914$) confirm a strong non-linear relationship between discount level and total profit. Moreover, the profit remains above 95% of the optimum when minimum discounts fall between 10.6% and 18%, offering a robust and flexible pricing band.

Recommendations:
1.Set the minimum discount level between 11% and 13% during similar campaigns to achieve near-optimal profit while maintaining margin control.
2.Avoid excessive discounting above 18%, as this leads to diminishing returns despite higher sales volume.

This framework demonstrates how simulation and regression-based sensitivity analysis can support tactical pricing decisions and improve profitability in competitive retail environments.

---

7. References

- Nagle T T, Hogan J, Zale J. The strategy and tactics of pricing: New international edition[M]. Routledge, 2016.
- Raju J, Zhang Z. Smart pricing: How Google, Priceline, and leading businesses use pricing innovation for profitability[M]. Pearson Prentice Hall, 2010.
- Law A M, Kelton W D, Kelton W D. Simulation modeling and analysis[M]. New York: Mcgraw-hill, 2000.
- Kopalle P K, Mela C F, Marsh L. The dynamic effect of discounting on sales: Empirical analysis and normative pricing implications[J]. Marketing Science, 1999, 18(3): 317-332.

---

8. Appendix

- Attached: Marketing strategy analysis.ipynb – includes all simulation code and figures