PROJECT 1

Mengzhen Shang 140028
Liyuan Cao 140029

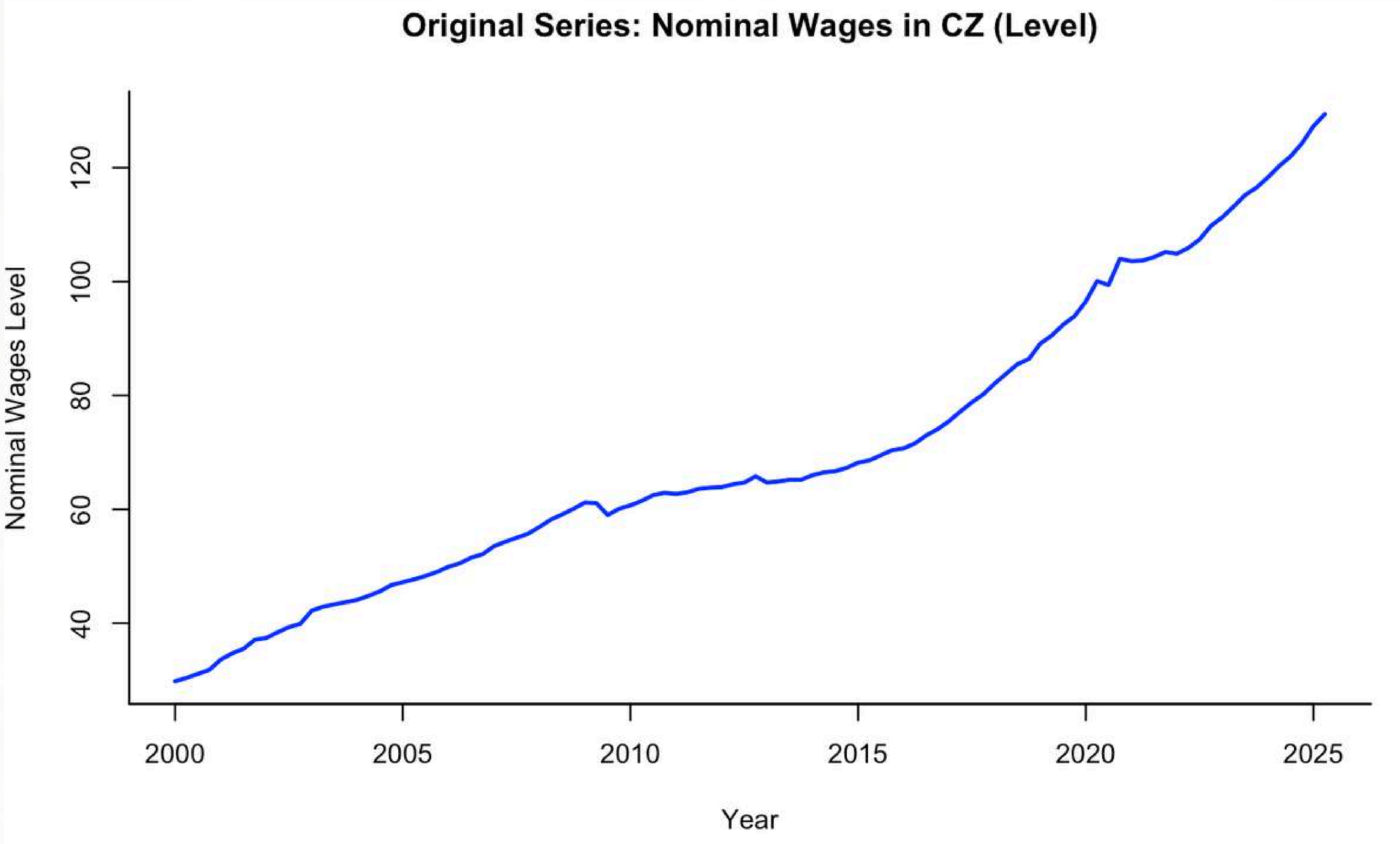
a.Variable Description & Stationarity Check

- Selected EU Country: Czech Republic (CZ)
- Core Variable (Y): Nominal Wages (W)(we also called wages later.)
- Modeling Variable (yt): Quarter-over-Quarter Percentage Change (QoQ %) of Nominal Wages.

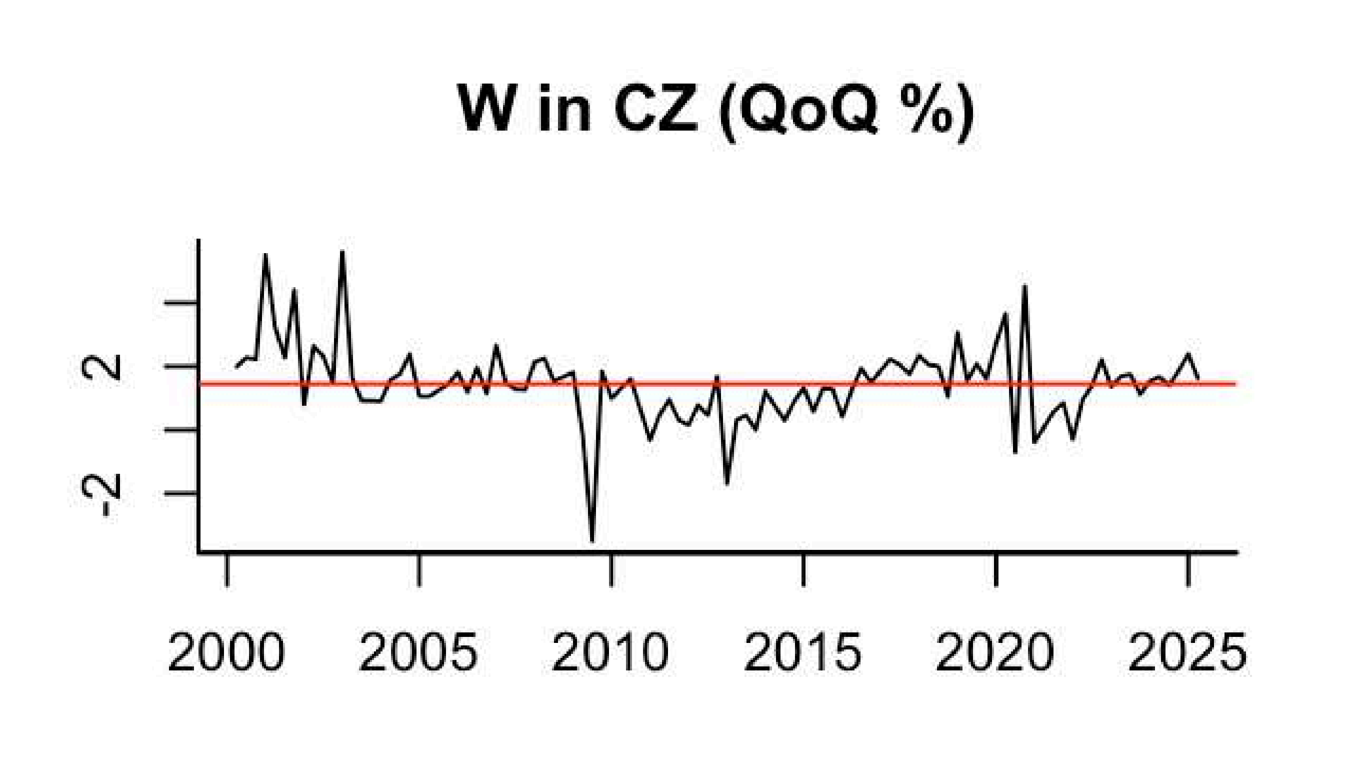
Why we use QoQ % change

Before using the Quarter-over-Quarter Percentage(QoQ%) change, let's briefly look at the Original Nominal Wage Level in CZ. As you can clearly see here, the series exhibits a deterministic and strong upward trend over time. Its mean and **variance are clearly not constant, which makes it highly non-stationary.**

Therefore, we must **transform** the data to satisfy the fundamental assumption of our ARMA and VAR models.



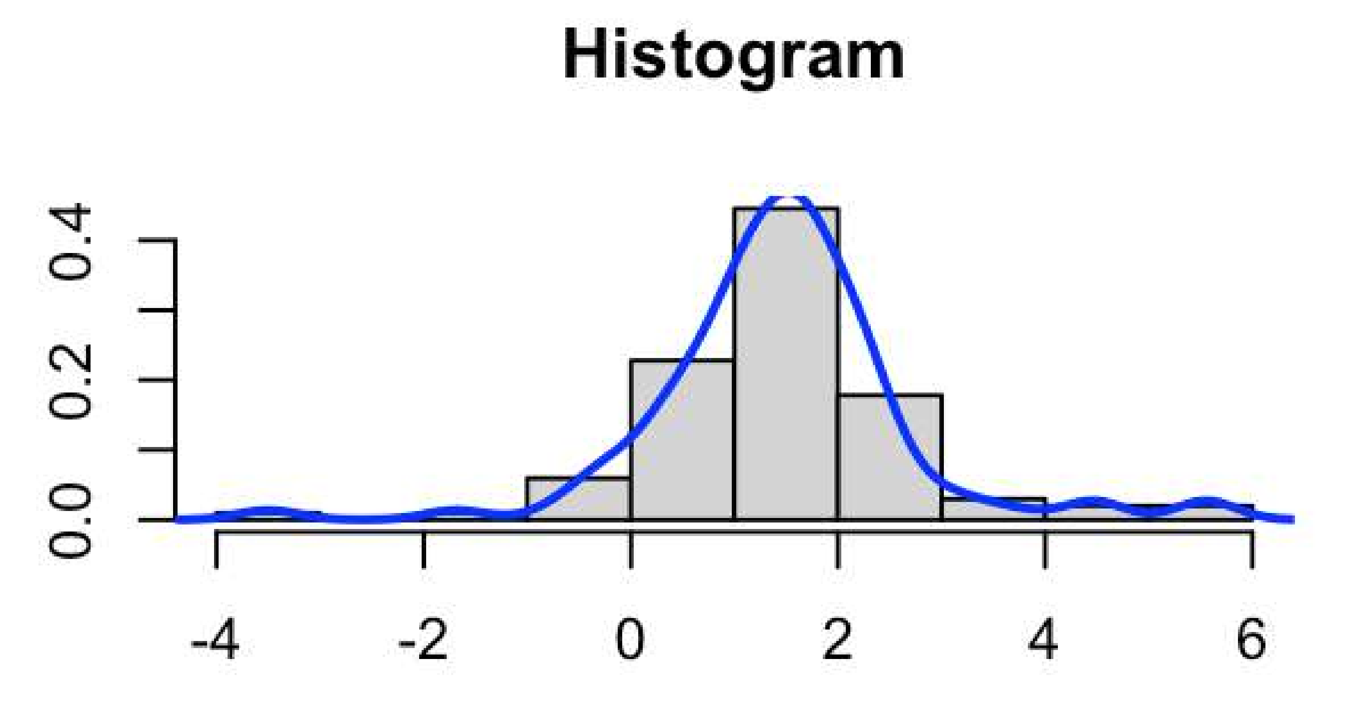
a.Variable Description & Stationarity Check



The series exhibits fluctuation around a constant mean (red line), indicating that the non-stationary trend has been successfully removed. This mean-reversion confirms the series is **visually stationary and appropriate for ARMA/VAR modeling.**

We observe periods of increased volatility, particularly during the 2008-2010 Global Financial Crisis (deep negative shock) and post-2020 (pandemic/inflation effects). This suggests the dynamics are subject to significant external economic shocks.

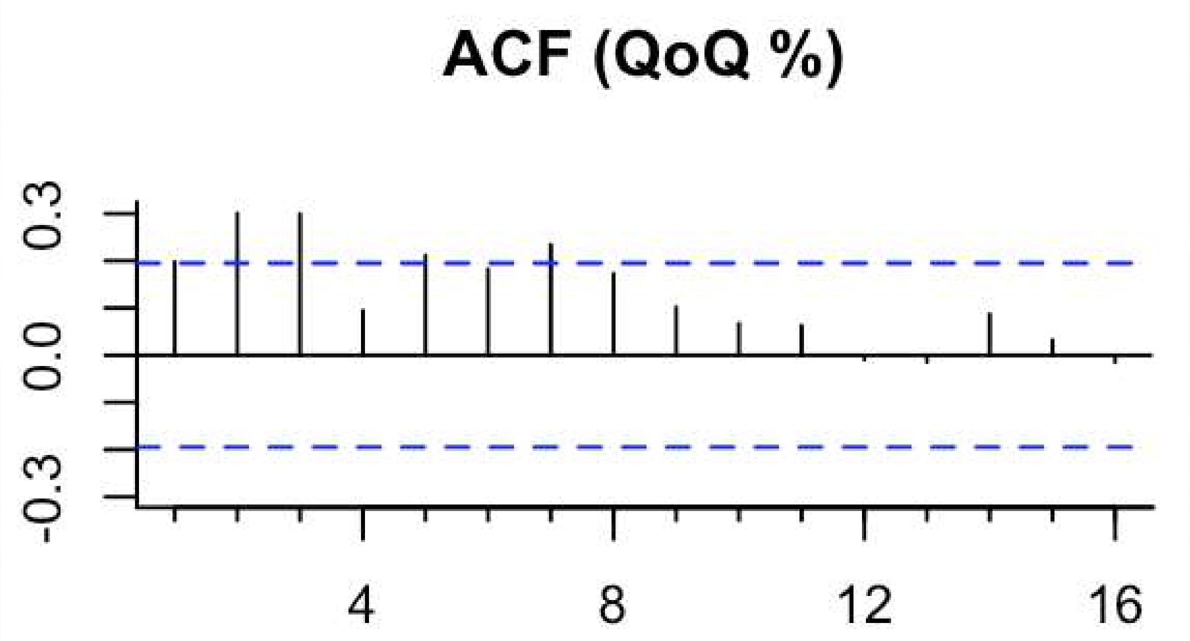
a.Variable Description & Stationarity Check

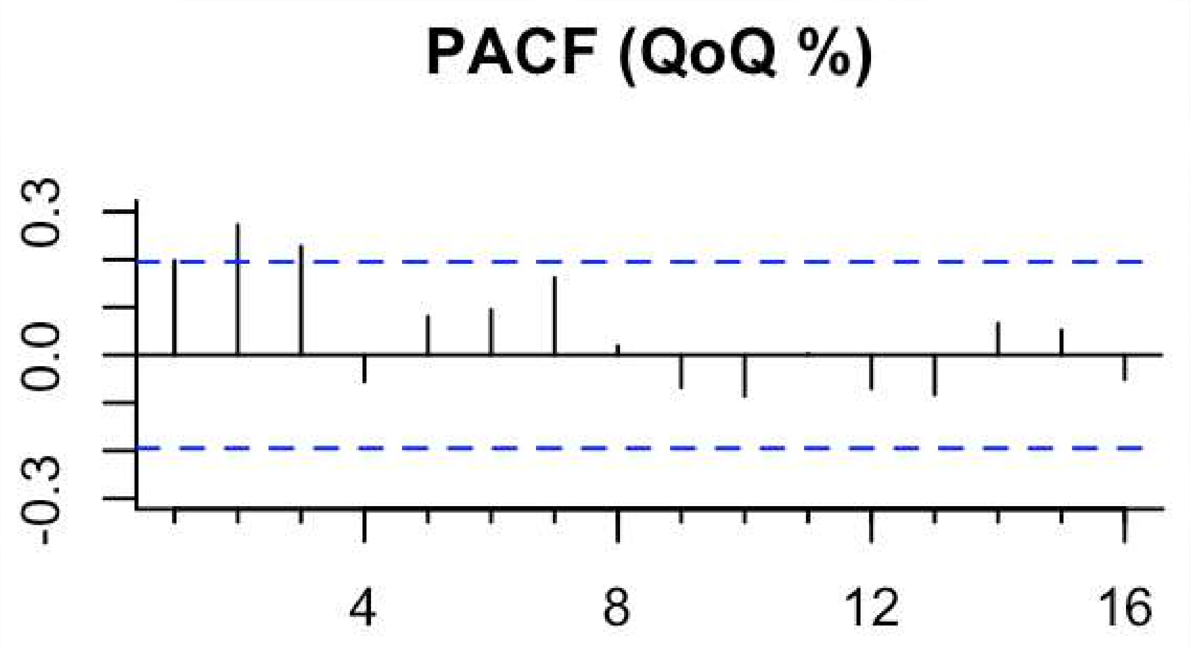


The histogram reveals the series is centered near the 1.5% mark. The distribution is largely symmetrical around this mean, with a slightly longer tail to the left (negative side), but it does not show severe skewness.

Comparing the bars to the blue density curve, the shape is generally unimodal (single peak). However, the central peak is taller and narrower than a perfect normal distribution, and the tails are somewhat heavier. This is common in financial and macroeconomic data.

a.Variable Description & Stationarity Check





The ACF plot is the decisive visual proof for stationarity. We observe that the coefficients, particularly after the initial few lags, decay rapidly towards zero and generally remain within the blue significance bounds. This swift decay is the classic hallmark of a stationary series, formally confirming our I(0) assumption.

The PACF plot is used to guide our ARMA model order. We see significant spikes at Lags 1 and 3, after which the partial correlation cuts off quickly. This pattern suggests the presence of a low-order Autoregressive (AR) component—most likely AR(1) and potentially AR(3). This provides us with our starting point for model specification in Part B.

a.Variable Description & Stationarity Check

Series	Test	H0 (Null Hypothesis)	Test Statistic	5% Critical Value	Decision at 5% Level	Final Conclusion
Y (Nominal W Level)	ADF	Unit Root (Non-Stat)	2.22	-2.88	Do Not Reject H0 (2.22>−2.88)	Non-Stationary
Y (Nominal W Level)	PP	Unit Root (Non-Stat)	3.189	-2.89	Do Not Reject H0 (3.189>−2.890)	Non-Stationary
Y (Nominal W Level)	KPSS	Stationary (Stat)	2.019	0.463	Reject H0 (2.019>0.463)	Non-Stationary
X (QoQ % Change)	ADF	Unit Root (Non-Stat)	-3.987	-2.88	Reject H0 (−3.987<−2.88)	Stationary (I(0))
X (QoQ % Change)	PP	Unit Root (Non-Stat)	-8.461	-2.89	Reject H0 (−8.461<−2.890)	Stationary (I(0))
X (QoQ % Change)	KPSS	Stationary (Stat)	0.365	0.463	Do Not Reject H0 (0.365<0.463)	Stationary (I(0))

1. As hypothesized, the Original Nominal Wage Level is clearly non-stationary. Both ADF and PP tests fail to reject the null hypothesis of a unit root, and the KPSS test strongly rejects the hypothesis of stationarity (2.019 > 0.463). This confirms that transformation was mandatory.

2. Meanwhile, the QoQ Percentage Change series is stationary.
We see strong evidence across all three tests:ADF & PP tests reject the Unit Root hypothesis (e.g. PP test stat of -8.461 is much smaller than the -2.890).The KPSS test does not reject the stationarity hypothesis (0.365 < 0.463).
Conclusion: The transformed series, X, is stationary or is I(0) and ready for ARMA and VAR modeling.

b. ARMA Model Estimation: Optimal Order Selection

Now that we have formally confirmed the stationarity of our series, we will proceed to estimate the ARMA model to capture its structure and select the optimal order.

b. ARMA Model Estimation: Optimal Order Selection

BIC (Bayesian Information Criterion)			
	ma0	ma1	ma2
ar0	335.53	337.49	335.38
ar1	336.15	329.23	332.16
ar2	333	332.07	336.69
ar3	332.16	336.38	339.9
ar4	336.6	340.74	343.81

AIC (Akaike Information Criterion)			
	ma0	ma1	ma2
ar0	330.3	329.65	324.91
ar1	328.3	318.77	319.08
ar2	322.53	319	320.99
ar3	319.09	320.69	321.59
ar4	320.91	322.44	322.89

We relied on both **BIC and AIC** criteria to select the best model.

1. **BIC Decision**: The minimum BIC value of 329.23 is clearly located at the AR(1) MA(1) specification. As BIC heavily penalizes complexity, we select this model as our primary choice.
2. **AIC Confirmation:** The AIC table strongly confirms this, also showing its absolute minimum value of 318.77 at the AR(1) MA(1) cell.

Conclusion: Both selection criteria agree. Our optimal model is ARMA(1, 1).

b. ARMA Model Estimation: Optimal Order Selection

Model: ARMA(1, 1)	Coefficient	T-Value (Implied)
AR(1)	0.9339	Highly Significant
MA(1)	-0.7779	Highly Significant
Long-Run Mean (μ)	1.58%	Significant
σ^2	1.304	

1. Significance & Structure

Both the AR(1) and MA(1) coefficients are highly statistically significant. This validates our model choice and confirms that both past growth and past errors drive current wage dynamics.

2. Strong Persistence

The AR(1) coefficient is 0.93, which is very close to 1. This signifies strong persistence: any shock to wage growth will take a considerable time to fully dissipate.

3. Long-Run Baseline

The estimated Long-Run Mean is 1.58%. This is the quarterly growth rate our forecast will converge to in the long term, serving as our stable forecasting baseline.

b. ARMA Model Estimation: Optimal Order Selection

Ljung-Box test table

Lag Horizon	LB Statistic	P-value
h=3	2.591	0.107
h=4	5.128	0.077
h=5	5.243	0.155
h=6	5.267	0.261
h=7	6.611	0.251
h=8	6.907	0.329
h=9	6.936	0.436
h=10	7.086	0.527
h=11	7.118	0.625
h=12	8.107	0.618

Conclusion

We observe that in The **Ljung-Box test table** all P-values across all tested lags are greater than 0.05%. Therefore, we Do Not Reject H(0).

Notice: Our Null Hypothesis H(0) is that the residuals are White Noise.

Model Validation

This confirms that our ARMA(1,1) model has successfully captured the entire autocorrelation structure. The model is valid and well-specified.

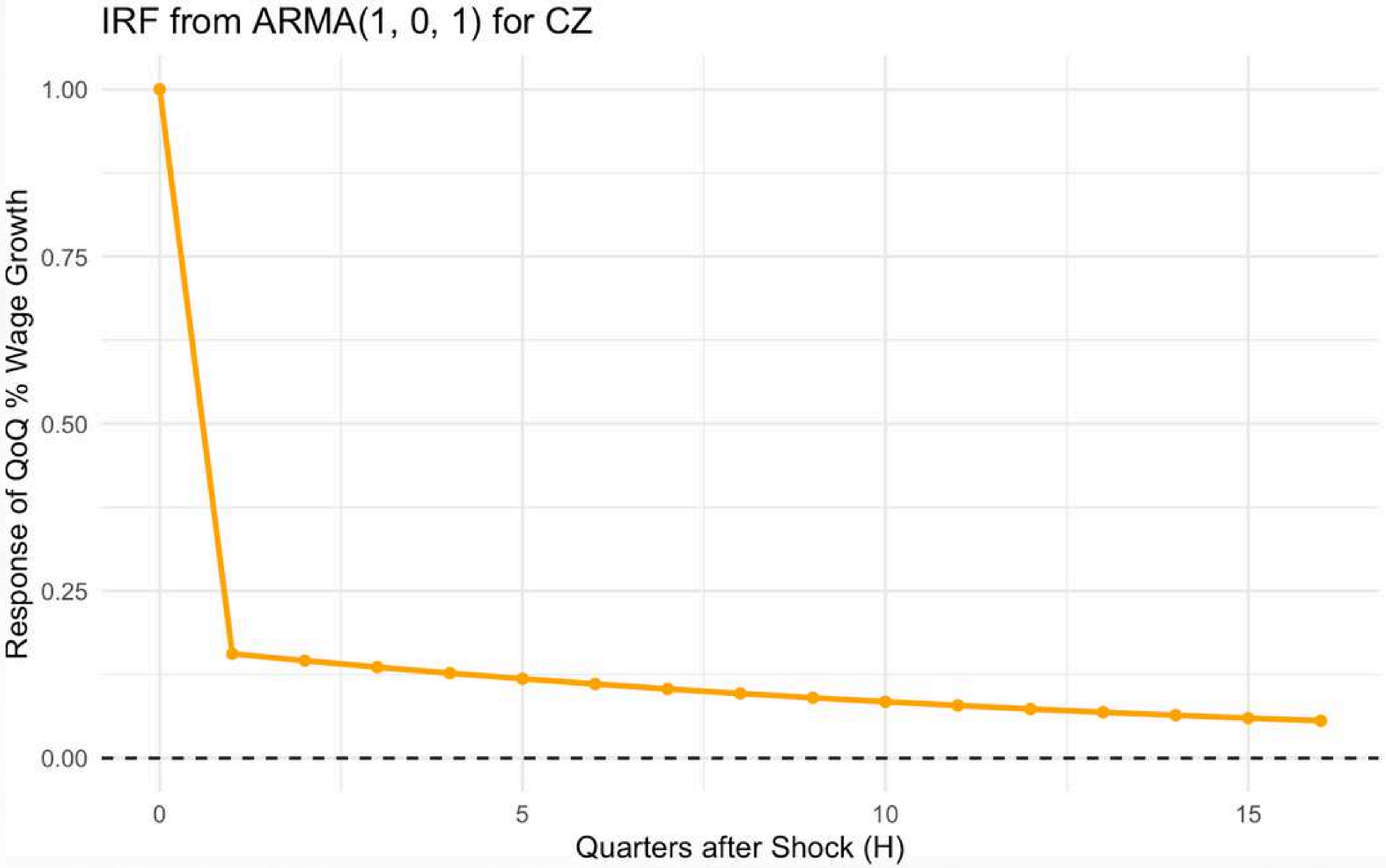
b. ARMA Model Estimation: Optimal Order Selection

1. Initial Evolution (H=0 to H=2)

The **IRF** begins at 1.00 (one-standard-deviation shock) and undergoes an extremely sharp drop in the first quarter, falling to about 0.18 by H=1. This indicates the largest portion of the shock's impact is immediately absorbed by the system.

2. Long-Term Evolution (H=3 onwards)

Following this sharp adjustment, the function transitions into a long-term, gradual decay. The curve approaches the zero baseline very slowly, confirming the model's high persistence. The shock's influence remains measurable for over 16 quarters, defining the wage growth path for the next four years.



c. Structural Impact of the Euro Area on Czech Wage Growth

The ARMA model is univariate and cannot capture external influences.
Therefore, we construct a VAR model by introducing the Euro Area variable the Euro Area Nominal Wage Growth, measured as the QoQ percentage change.

c. Structural Impact of the Euro Area on Czech Wage Growth

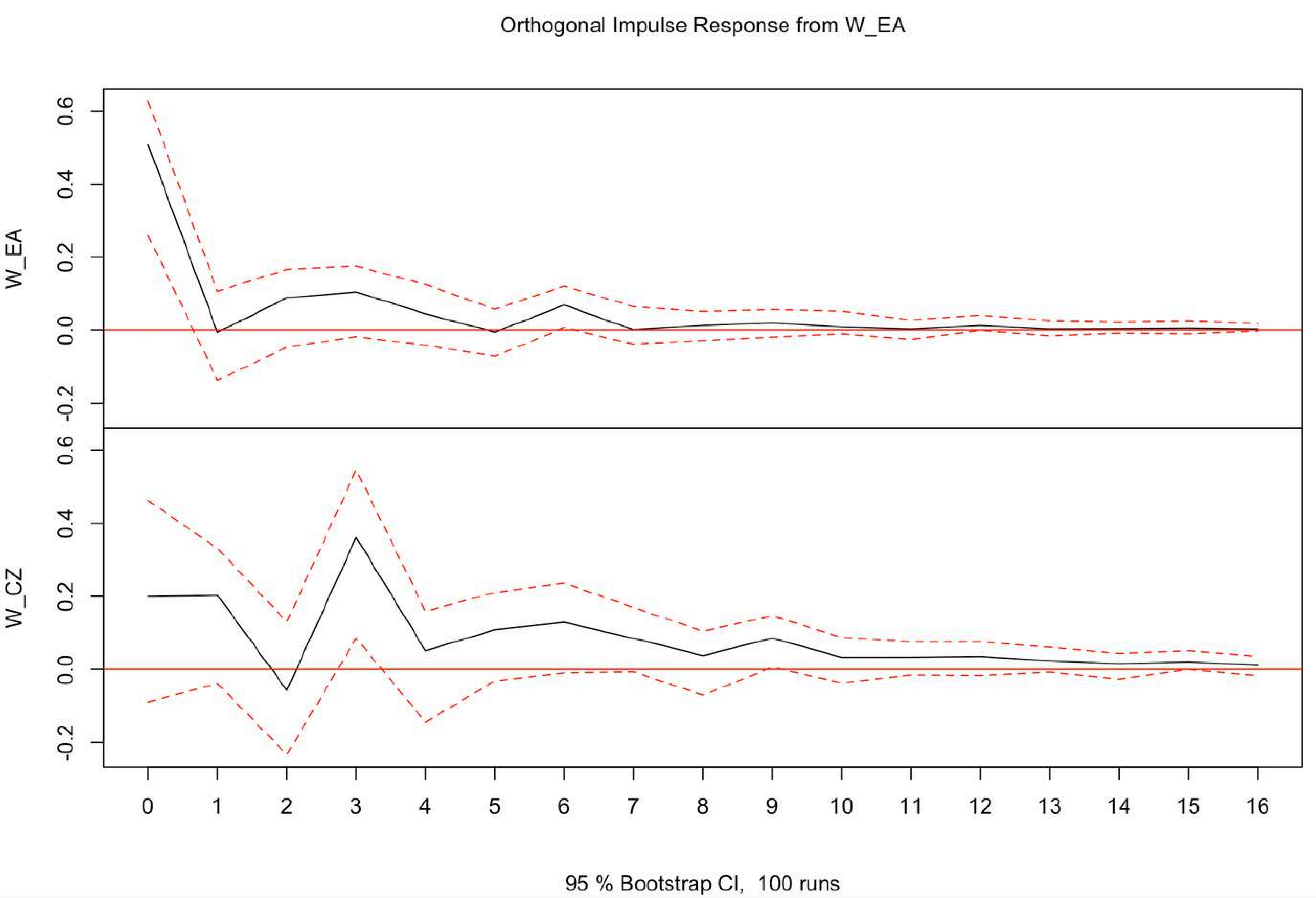
	1	2	3	4	5	6	7	8
AIC(n)	-1.0909181	-1.3463618	-1.5284461	-1.3973253	-1.3445868	-1.3590212	-1.2599868	-1.25273638
HQ(n)	-1.0073392	-1.2070637	-1.3334288	-1.1465887	-1.0381309	-0.9968461	-0.8420924	-0.77912274
SC(n)	-0.8758601	-0.9879318	-1.0266441	-0.7521512	-0.5560407	-0.4271031	-0.1846966	-0.03407422
FPE(n)	0.3359734	0.2604205	0.2174135	0.2485784	0.2632150	0.2611421	0.2909437	0.29660691

1. Our first step in the VAR analysis is determining the optimal lag order , p.

2. We rely primarily on the BIC (Bayesian Information Criterion). We choose **BIC because it is the most strict standard; it prefers simpler models to overly complicated ones.**

3. By inspecting the SC values, we observe the global **minimum of -1.0266 occurs unanimously at Lag 3**. Since all major criteria (AIC, HQ, and SC) agree on this point, we confidently set our optimal lag order as p=3.

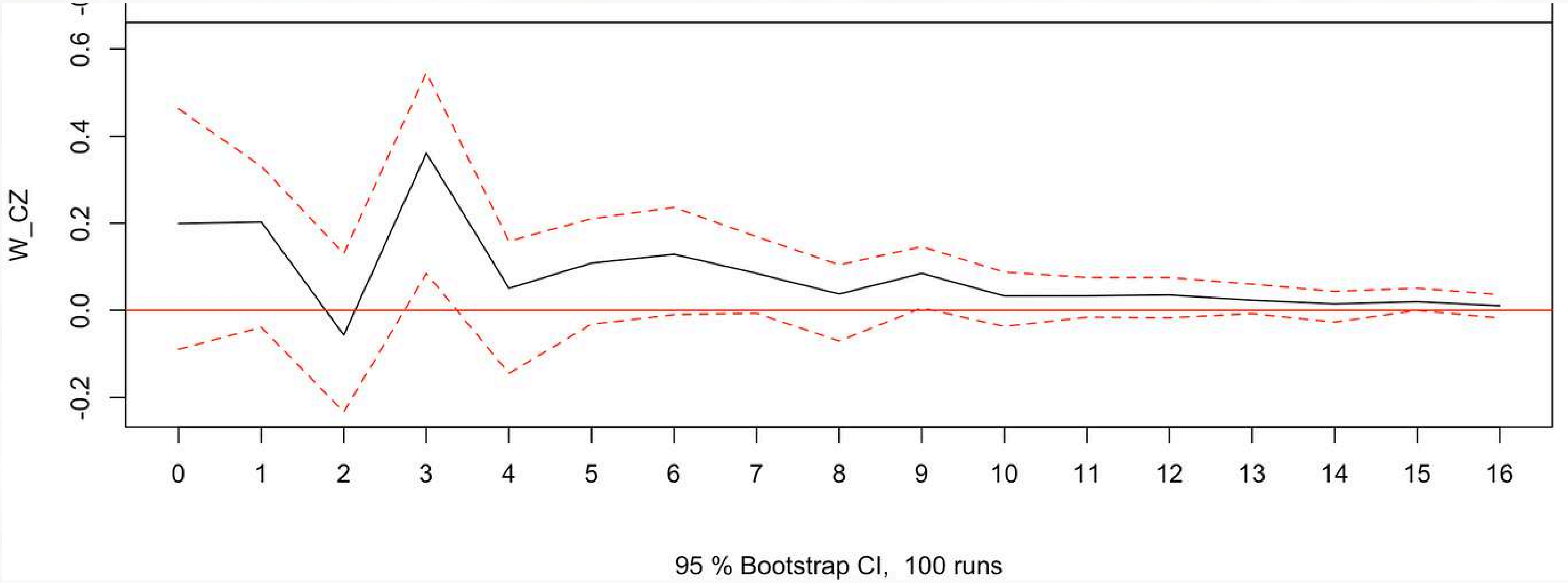
c. Structural Impact of the Euro Area on Czech Wage Growth



The top panel shows the Euro Area's response to its own unexpected shock. The impact is very strong at first, but it quickly fades after the first two quarters. This confirms the Euro Area market absorbs its own wage issues quite fast.

The bottom panel shows the most important finding: the effect that jumps from the Euro Area to Czech.

c. Structural Impact of the Euro Area on Czech Wage Growth



1. No Immediate Effect

Notice at H=0 and H=1, the red dashed lines (the confidence interval) cross the zero line. This means the immediate effect on Czech wages is not real; the market doesn't react right away. This is evidence that the shock is delayed.

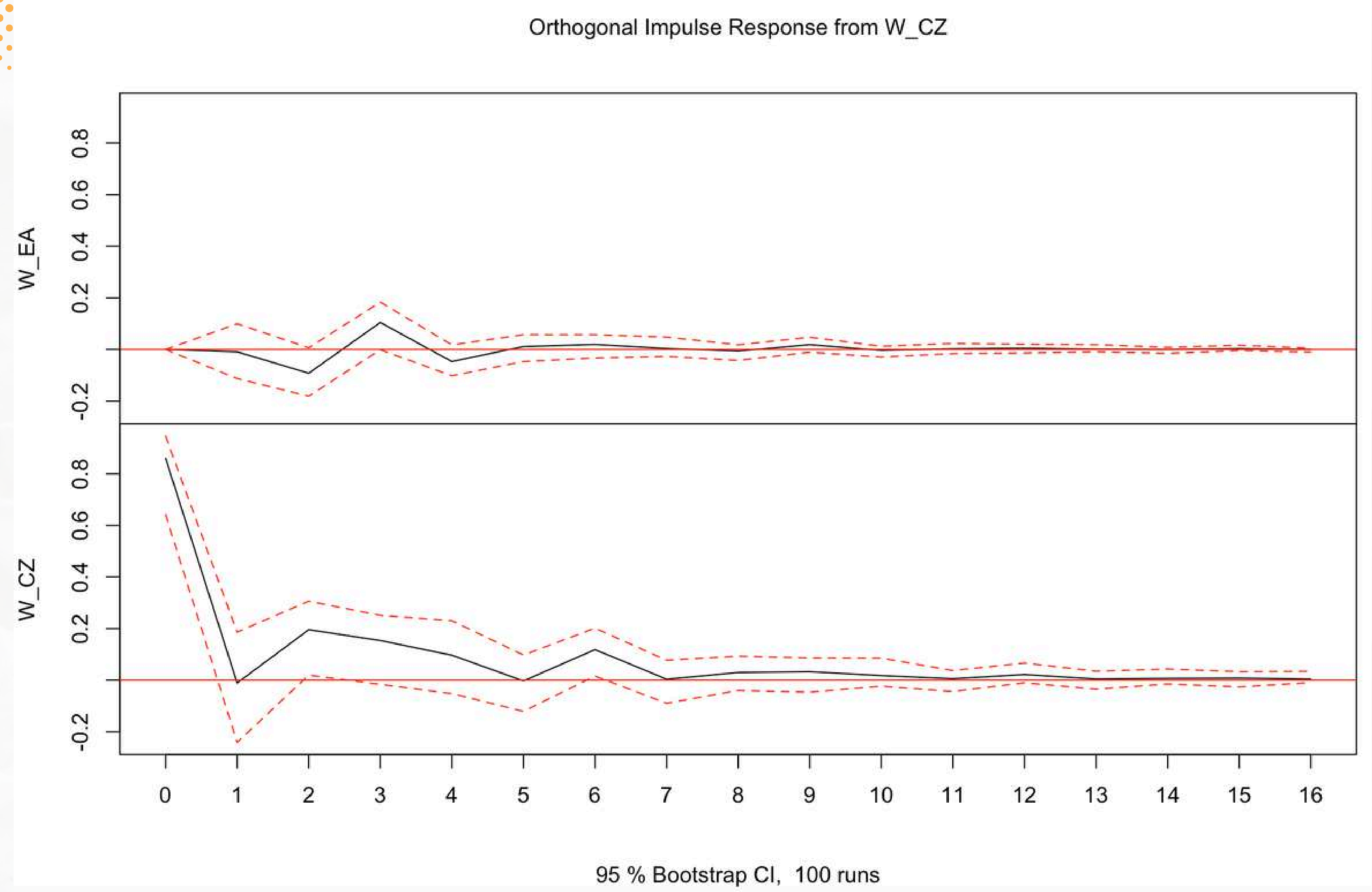
2. Peak Impact and Reality

The shock takes time to travel. The effect grows and becomes real (statistically significant) and strongest around H=3 (3 quarters later). At this point, the positive movement is clearly seen, reaching its peak near 0.40."

3. Long-Lasting Effect

After the peak, the influence lasts for a long time. The effect stays positive and lasts for many quarters (several years) before finally calming down close to the zero line. This proves the Euro Area is a sustained driver of Czech wage growth.

c. Structural Impact of the Euro Area on Czech Wage Growth



Conversely, the shock from the smaller Czech economy **shows no statistically significant feedback effect** on the Euro Area (as seen in the top panel). This result validates our structural ordering assumption.

c. Structural Impact of the Euro Area on Czech Wage Growth

$W_CZ

	W_EA	W_CZ
[1,]	0.05095939	0.9490406
[2,]	0.09847858	0.9015214
[3,]	0.09752801	0.9024720
[4,]	0.21099002	0.7890100
[5,]	0.21103269	0.7889673
[6,]	0.21994696	0.7800530
[7,]	0.22912148	0.7708785
[8,]	0.23424537	0.7657546
[9,]	0.23504875	0.7649512
[10,]	0.23989218	0.7601078
[11,]	0.24057441	0.7594256
[12,]	0.24132572	0.7586743
[13,]	0.24207402	0.7579260
[14,]	0.24244029	0.7575597
[15,]	0.24258054	0.7574195
[16,]	0.24283729	0.7571627

This table quantifies the **rising influence of the Euro Area on Czech wage growth.**

Short-Term Dominance (H=1 to H=3): In the short run, over 90% of Czech wage volatility is driven by domestic shocks CZ. The external Euro Area influence is **minimal**, only less than 10%.

Long-Term Growth: However, the external contribution **steadily rises**. In the long run, nearly one-quarter 24.3% of the Czech wage fluctuation is driven by Euro Area shocks.

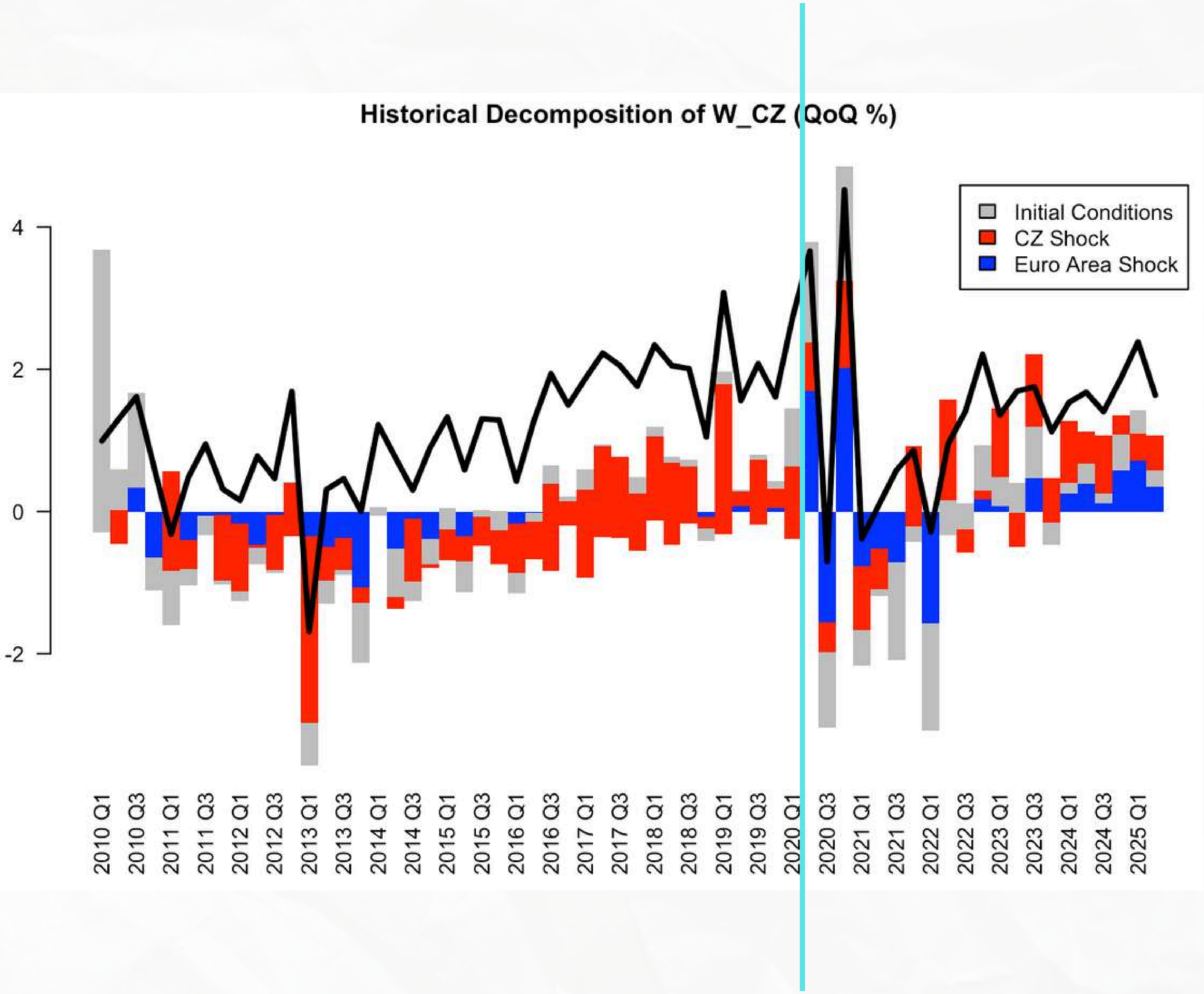
Core Conclusion: The FEVD proves the **Euro Area is the second most important driver of Czech wage volatility** in the long term, contributing almost 25% of its unexpected movements.

c. Structural Impact of the Euro Area on Czech Wage Growth

1.The Pre-Pandemic Decade (2010–2019)
For most of this decade, the changes in Czech wages were **mainly caused by internal Czech problems** (Red bars). Whether the economy was shrinking (2013) or growing fast (2018), the Euro Area influence (Blue bars) was very small and not the main reason for the wage changes.

2.The Current Era (2020–2025)
The last five years are different. The big events—the 2020 slump and the recent high-wage growth—were **caused by both** the Czech (Red) and Euro Area (Blue) shocks hitting at the same time.

The Blue shock **is much more important now**. It works together with the domestic issues to make the growth faster, showing the Czech wage market is much more connected to the rest of Europe.



d. Compare the accuracy of forecasts from RW, ARMA and VAR models

Since we have already built the ARMA and VAR models, we will next compare their forecasting accuracy with the Random Walk (RW) model, using evaluation metrics based on MFE, RMSFE, the Diebold–Mariano (DM) test, and sequential forecasts.

Out-of-Sample Forecast Evaluation Methodology

Key Points:

- Forecast Horizon: Last 8 quarters (rolling window approach).
- Metric 1: MFE (Mean Forecast Error) to measure bias (under/over-estimation).
- Metric 2: RMSFE (Root Mean Squared Forecast Error) to measure overall accuracy.
- Statistical Test: Diebold-Mariano (DM) Test to verify if differences in accuracy are significant.

Candidate Models:

- RW: Random Walk with Drift (Benchmark).
- ARMA: Univariate dynamics.
- VAR: Multivariate (incorporating Euro Area wages).

Forecast Accuracy: Model Comparison (RW vs. ARMA vs. VAR)

```
[1] "=== Task d: Forecast Accuracy Metrics (with Relative RMSFE) ==="
> print(res_table)
             MFE     RMSFE Rel_RMSFE
RW   -0.03796682 0.4672451    1.0000
ARMA  0.35018161 0.4838629    1.0356
VAR  -0.17362497 0.5222954    1.1178
```

1 The Benchmark Wins

- The Random Walk (RW) with drift achieved the lowest Root Mean Squared Forecast Error (RMSFE: 0.467) and the lowest bias (MFE: -0.038).
- Using the RW model as a benchmark, the relative RMSFE of ARMA is 1.036 and that of VAR is 1.118, indicating that both models perform worse than the random walk in out-of-sample forecasting.
- This suggests that for the specific test horizon (last 2 years), the wage growth trend was dominant, and complex dynamics were secondary.

2 The Cost of Complexity

- The VAR model, while structurally rich, showed the highest forecast error (0.522).
- This indicates that adding the Euro Area variable introduced more estimation variance (noise) than predictive power during this specific volatile period.

Diebold-Mariano Test: No Significant Divergence

```
=== Task d: DM Test P-values ===
> cat("RW vs ARMA: ", dm.test(e_rw, e_arma, h=1, power=2)$p.value, "\n")
RW vs ARMA:  0.9214305
> cat("RW vs VAR:  ", dm.test(e_rw, e_var, h=1, power=2)$p.value, "\n")
RW vs VAR:   0.7817591
> cat("ARMA vs VAR:", dm.test(e_arma, e_var, h=1, power=2)$p.value, "\n")
ARMA vs VAR: 0.8286057
```

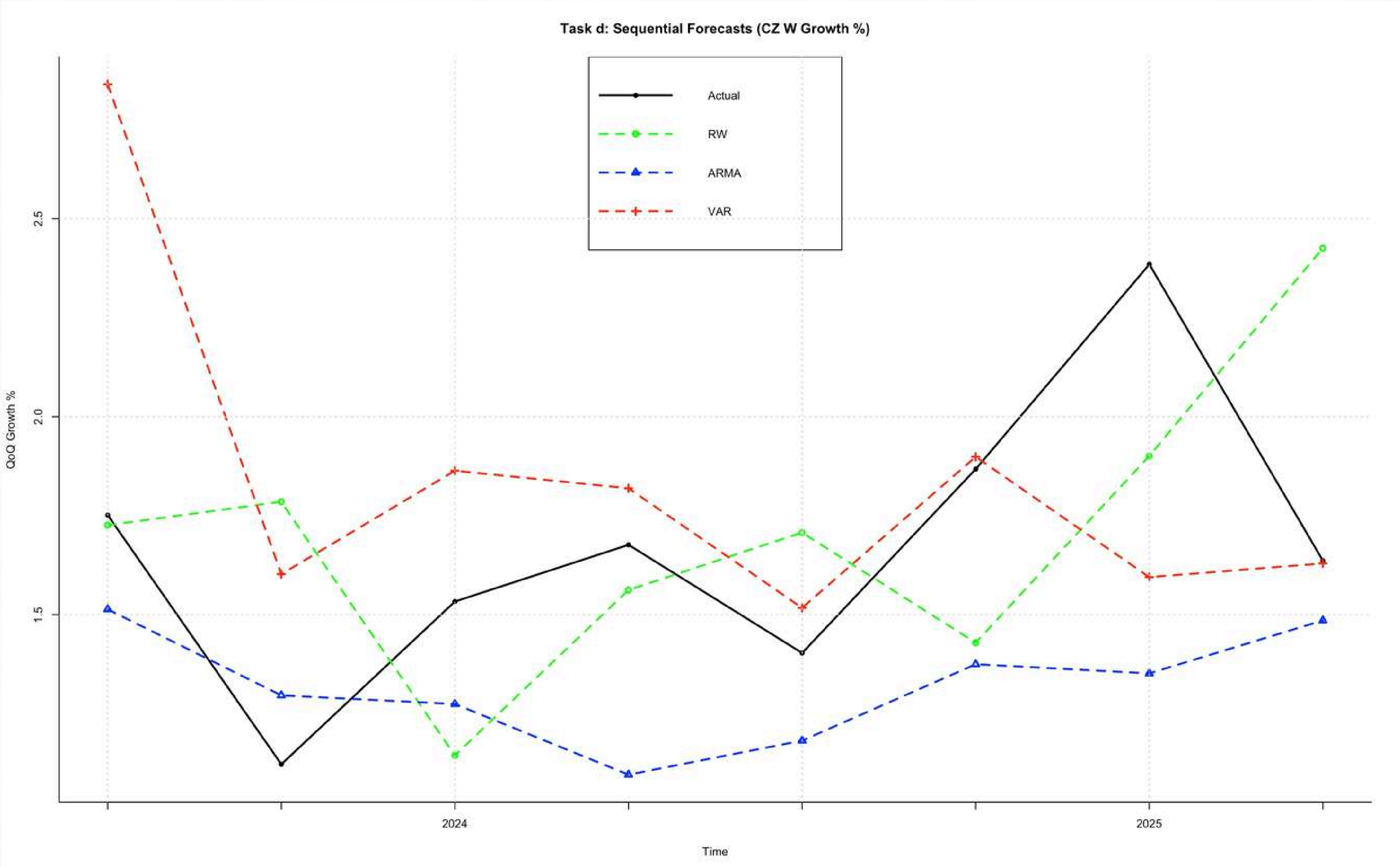
Interpretation:

- All p-values are significantly > 0.05.
- Conclusion: We cannot reject the null hypothesis of equal predictive accuracy.
- Implication: While RW is numerically superior, it is not statistically significantly better than VAR. The models are performing similarly in a statistical sense. The difference in RMSFE may be due to sample specificities rather than structural model failure.

Sequential Forecast Visualization

- Volatility: The Actual Data (Black line) shows significant volatility, swinging between 1.1% and 2.4%.
- Model Behavior:
  - RW (Green): Tracks the trend smoothly and reacts conservatively. This stability helped it achieve the lowest error.
  - ARMA (Blue): Performance was moderate. It generally followed the trend but consistently under-predicted the magnitude of the spikes compared to RW.
  - VAR (Red): Shows higher volatility. It attempts to model the turning points (dynamics) but sometimes overreacts, leading to the highest RMSFE.

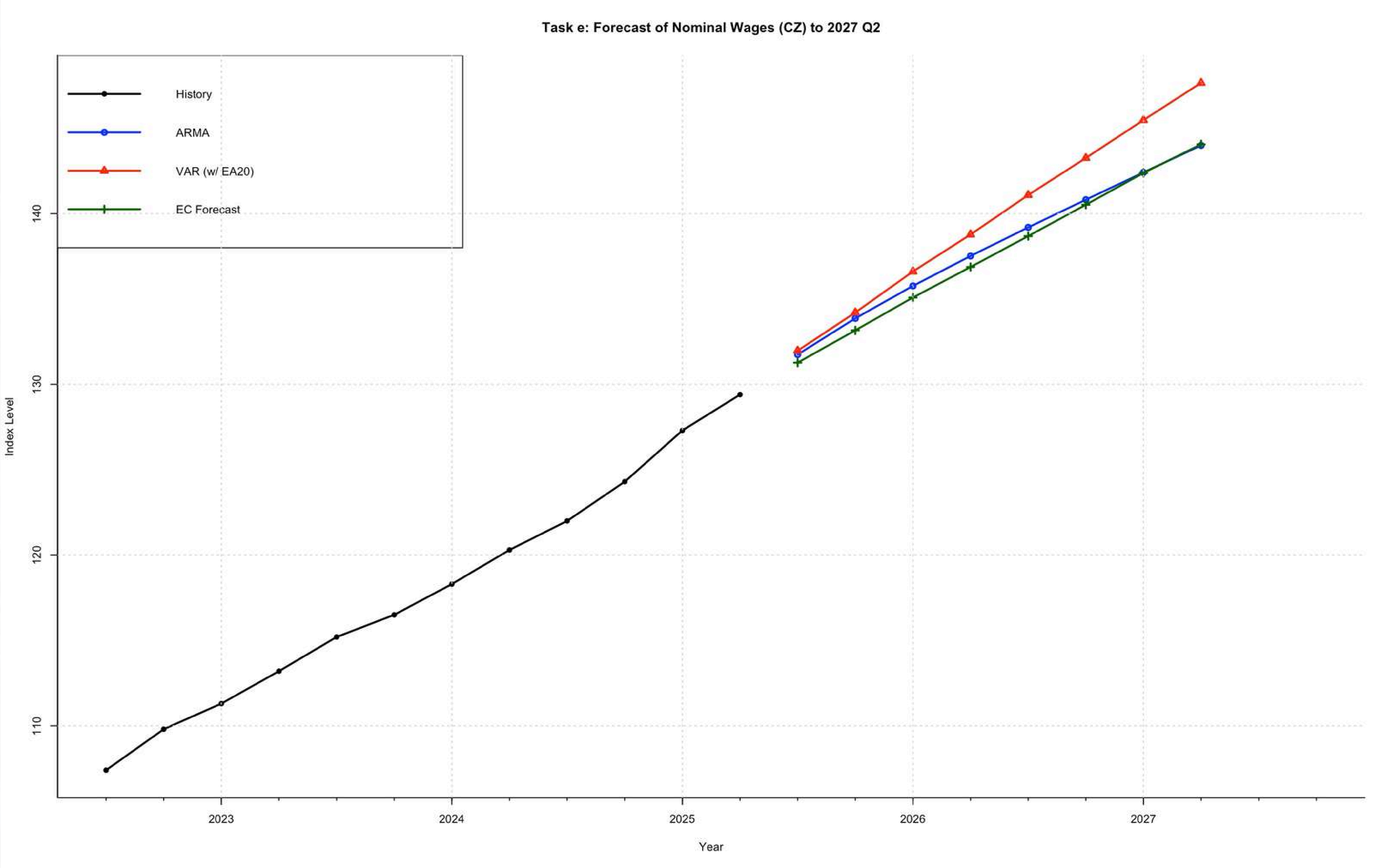
Summary: The visual confirms that while VAR and ARMA tried to capture dynamics, the "safer" trend projection of RW was superior in this specific volatile sample.



e. Future Forecasts (2025-2027)

Next, we plot the forecasts from the ARMA and VAR models for the next two years and compare them with the forecasts from the European Commission to examine whether the ARMA and VAR models can achieve professional-level forecasting performance.

Future Outlook: Diverging Paths to 2027



- Short-Term (2025): All three forecasts (ARMA, VAR, EC) are tightly aligned, predicting nominal wage levels around 133-134 by the end of 2025.
- Long-Term Divergence (2026-2027): A clear gap emerges. The VAR model (Red) becomes significantly more optimistic than the consensus.
- The Consensus: The ARMA model (Blue) and European Commission (Green) track almost identically, serving as a conservative baseline.

Strategic Analysis: Why the Divergence?

```
[1] "=== Task e: Future Forecasts Table (Levels) ==="
> print(df_future)
     Date     ARMA      VAR       EC
1 2025 Q3 131.7431 131.9660 131.2678
2 2025 Q4 133.8656 134.2026 133.1626
3 2026 Q1 135.7632 136.6072 134.9250
4 2026 Q2 137.5203 138.7754 136.7107
5 2026 Q3 139.1920 141.0851 138.5201
6 2026 Q4 140.8142 143.2547 140.3534
7 2027 Q1 142.4102 145.4745 142.0081
8 2027 Q2 143.9953 147.6550 143.6824
```

1. The "Catch-Up" Effect (VAR)
   - Our VAR model predicts the strongest growth (reaching 147.7).
   - Reasoning: As established in Part C (FEVD), the Euro Area explains ~25% of Czech wage variance in the long run. The VAR likely detects a strong "pull" from European wages that simple univariate models (ARMA) miss.
2. The Conservative Baseline (EC)
   - The European Commission (EC) forecast aligns perfectly with our ARMA model (ending ~144.0).
   - Reasoning: This likely reflects expectations of fiscal consolidation or cooling inflation—structural factors that pure time-series models may underestimate.

Economic Interpretation & Conclusion

- The "Optimism" of VAR:
- Our VAR model incorporates the Euro Area (EA20) wages.
- The strong upward forecast in VAR likely implies a catch-up effect: The model detects that wage growth in the Euro Area will pull Czech wages higher than domestic factors alone (ARMA) would suggest.
- The "Consensus" (ARMA & EC):
- The EC and ARMA are more conservative. The EC likely incorporates expected fiscal consolidation or cooling inflation in their structural models.

- Final Recommendation:
  - While RW was best for short-term accuracy (Task d), for long-term scenario planning, the VAR model provides a valuable "High Growth Scenario" driven by European integration, whereas the EC forecast serves as a baseline conservative scenario.

Thank You~