Visualization of Laplace Transforms for Dynamic Systems

Prakrit Gajurel

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Contact: pgajurel111@gmail.com

1. Introduction

1 Background

Dynamic systems in fields such as physics and mathematics often require Ordinary Differential Equations(ODEs) to model them, which are usually very difficult to solve and interpret. The Laplace Transform gives a powerful technique that converts ODEs into simple algebraic equations in the frequency domain, making the solution easier. This project addresses the opportunity to allow better understanding of the transform by visually demonstrating how it operates and how it simplifies the analysis of dynamical systems.

2 Purpose

The purpose of this project is to improve conceptual understanding of the Laplace Transform by creating visual representations that show the time domain behavior of systems to their frequency domain equivalents. Using this tool, we can link mathematical theory and practical systems.

3 Scope

This project will be limited to modeling by ODEs such as first and second-order differential equations. The Visualization will be limited to standard Laplace Transforms(basically not including the inverse transform and contour integration). Numerical accuracy will depend on the software or computer limitations.

2. Objectives

- To demonstrate how Laplace Transforms convert ODEs into algebraic equations
- To visualize time-domain systems and their corresponding frequency-domain systems using plotting libraries in Python
- To support learning of Laplace Transforms through graphical methods

3. Proposed Approach

1 Overview

This project will use Python to model dynamic systems defined by ODEs and apply Laplace Transforms to them. The results will then be visualized using graphs to compare behavior in both of the domains. Libraries such as NumPy, SymPy, and Matplotlib will be used to perform the computation, plotting, and simulation.

2 Phases

- **Research and Planning**: Look over the theory of Laplace Transforms and dynamic systems, find suitable examples of such systems, plan visualization methods and outputs
- **Design and Development**: Write Python code to represent ODEs and dynamic systems, implement Laplace Transforms, and design plots for the necessary functions.
- **Testing and Optimization**: Verify solutions using known results, improve clarity of plots, and optimize code for readability and efficiency.
- **Deployment and Maintenance**: Finalize the code, prepare demonstrations and examples for presentation.

3 Key Technologies

- Python
- NumPy
- SymPy
- Matplotlib

4. SWOT Analysis

1 Strengths

- Visual approach improves understanding
- Uses widely known and open-source tools

2 Weaknesses

- Some mathematical concepts may be oversimplified heavily.
- Limited to linear systems

3 Opportunities

- Useful as an educational tool for students studying mathematics
- Has potential to be interactable

4 Threats

- Software or computational errors
- Overly complex mathematics may reduce usability

5. Expected Outcomes

This project is expected to produce a functional Python-based tool that clearly visualizes how the Laplace Transform simplifies the analysis of ODEs and dynamic systems. The users will be able to observe the behavior in both the time domain and the frequency domain, improving conceptual understanding. The project also enforces better programming and mathematical modeling skills.

6. Risk Analysis and Mitigation

- Difficulty implementing the transform - Use SymPy's built-in function
- Incorrect visual interpretation - Validate results with known solutions
- Complicated visuals - Keep plots simple and well-labeled

7. Budget Estimate

Omitted as the entirety of the project is based on open-source software.

8. Team Structure

Prakrit Gajurel - Only member in team

9. Conclusion

This project aims to make Laplace Transforms more intuitive by visualizing their application to dynamic systems and in solving ODEs. By combining mathematical theory with computational software, the project shows how complex differential equations can be transformed into simple algebraic forms. The use of Python provides a practical way to understand an important mathematical technique that has many wide applications in our world.

Appendix

1 The Laplace Transform

The Laplace Transform is a mathematical operation that converts a function of time, $f(t)$, into a function of a complex variable s. It is defined as:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

where:

- t represents time with $t \geq 0$,
- s is a complex frequency variable,
- $F(s)$ is the Laplace-domain representation of $f(t)$.

Note the inverse Laplace Transform:

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

It is useful for solving linear ordinary differential equations with given initial conditions.

2 Properties of the Laplace Transform

Linearity

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

where a and b are constants.

Derivative Property

If $f'(t)$ is the first derivative of $f(t)$, then:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

For the second derivative:

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

These allow differential equations to be turned into algebraic equations.

3 Example - Solving ODEs using the Laplace Transform

Consider the first-order ordinary differential equation:

$$\frac{dx}{dt} + 3x = 0$$

with the initial condition:

$$x(0) = 2$$

Taking the Laplace Transform of both sides gives:

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} + 3\mathcal{L}\{x(t)\} = 0$$

Using the derivative property:

$$(sX(s) - x(0)) + 3X(s) = 0$$

Substituting the initial condition, we get:

$$(s+3)X(s) - 2 = 0$$

Solving for $X(s)$:

$$X(s) = \frac{2}{s+3}$$

Finally, we can apply the inverse Laplace transform to this function to get,

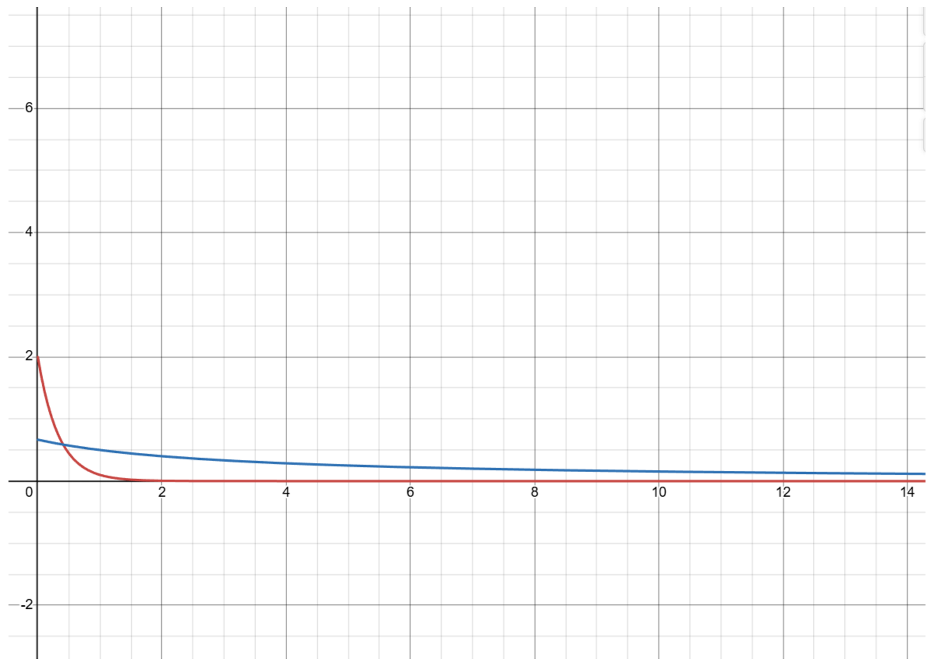
$$x(t) = \mathcal{L}^{-1}\left\{X(s)\right\} = \mathcal{L}^{-1}\left\{\frac{2}{s+3}\right\}$$

Using a Laplace Transform table, we can find the result

$$\mathcal{L}\left\{e^{-at}\right\} = \frac{1}{s+a} \implies \mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at},$$

and hence the time-domain solution is obtained as:

$$\boxed{x(t) = 2e^{-3t}}$$



Shown above is a graph plotted using Desmos(note that Desmos did NOT solve the Laplace Transform) of the functions $x(t) = 2e^{-3t}\,\{t \geq 0\}$(in red) and $X(s) = \frac{2}{s+3}\,\{s > 0\}$(in blue). Obviously, the function in blue is much easier to analyse as it is an algebraic expression, whereas the function in red is an exponential equation that is harder to analyse, requiring calculus as it comes from a differential equation. This shows how the Laplace Transform allows for transforming of an ODE into an easy-to-analyse algebraic expression while preserving the system's behavior. Analysing behavior can be especially useful in fields such as circuit analysis.