

$$\text{Classification Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}}$$

$$\text{Classification Error} = \frac{N_{\text{miss}}}{N_{\text{total}}}$$

$$\text{Precision} = \text{PPV} = \frac{TP}{TP+FP}$$

$$\text{Recall} = \text{TPR} = \frac{TP}{TP+FN}$$

$$\text{Fallout} = \text{FPR} = \frac{FP}{FP+TN}$$

$$\text{Specificity} = \text{TNR} = \frac{TN}{TN+FP}$$

$$\text{F1-score} = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$

ROC: TPR y-axis, FPR x axis

$$\text{AUC: } P(\text{error}) = P(C_i) \cdot FP + P(C_i) \cdot FN$$

$$FP/FN = \frac{\delta - \text{mean}}{\sqrt{\text{variance}}}$$

$$\text{FLDA: } J(w) = \frac{(m_1 - m_2)^2}{\zeta_1^2 + \zeta_2^2} \rightarrow \frac{w^T S_B w}{w^T S_W w}$$

$$S_B = (m_1 - m_2)^2, S_W = \sigma_1 + \sigma_2 \text{ (within class variance)}$$

$$W_0 = \frac{-(m_1 + m_2)^T W}{2}$$

$$\text{Optimized: } S_W^{-1} S_B w = \lambda w, \text{ eigenvalues}$$

Max projection: N -1 classes

$$\text{Perceptron Error: } E_p(w, b) = -\sum_{n \in M} (w^T x_n + b) t_n$$

Stochastic Gradient Decent:

$$w^{(t+1)} \leftarrow w^{(t)} - \eta \frac{\partial E_p(w, b)}{\partial w} = w^{(t)} + \eta x_n t_n$$

$$b^{(t+1)} \leftarrow b^{(t)} - \eta \frac{\partial E_p(w, b)}{\partial b} = b^{(t)} + \eta t_n$$

Where η is the learning rate

$$\text{Logistic Regression: } \phi(x) = \frac{1}{1 + \exp(-x)}$$

Objective function: (Binary) Cross Entropy:

$$J(w, w_0) = \sum_{i=1}^N -t_i \log \phi(z_i) - (1 - t_i) \log(1 - \phi(z_i))$$

$$w^{(t+1)} \leftarrow w^{(t)} + \eta \sum_{i=1}^N (t_i - y_i) x_i$$

$$w_0^{(t+1)} \leftarrow w_0^{(t)} + \eta \sum_{i=1}^N (t_i - y_i)$$

$$\text{SoftMax: } p_k = \frac{\exp(y_k(x))}{\sum_{j=1}^K \exp(y_j(x))}$$

$$\text{where } y_k(x) = \phi(w_k^T x + b_k)$$

$$\text{Hard Margin SVM: } y(x) = w^T \phi(x) + b$$

$$\arg_{w,b} \min \frac{1}{2} \|w\|^2, t_n y(x_n) \geq 1$$

$$\text{Polynomial Kernel: } K(x, y) = (1 + \langle x, y \rangle)^d$$

$$\text{RBF kernel: } K(x, y) = e^{-\gamma \|x - y\|^2}$$

$$\text{Sigmoid kernel: } K(x, y) = \tanh(\gamma(x, y) + 1)$$

$$\text{Kernel Trick: } y(x) = \sum_{n=1}^N a_n t_n K(x, x_n) + b$$

$$\text{Constraints: } a_n \geq 0$$

$$t_n y(x_n) \geq 0$$

$$a_n (t_n y(x_n) - 1) = 0$$

Threshold:

$$b = \frac{1}{N_S} \sum_{n \in S} (t_n - \sum_{m \in S} a_m t_m K(x_n, x_m))$$

$$\text{Lagrange Multiplier: } L(w, b, a) = \frac{1}{2} \|w\|^2 - \sum_{n=1}^N a_n (t_n (w^T \phi(x_n) + b) - 1)$$

$$\text{Conditions: } w = \sum_{n=1}^N a_n t_n \phi(x_n)$$

$$0 = \sum_{n=1}^N a_n t_n$$

$$\text{Soft Margin SVM: } \arg_{w,b} \min \frac{1}{2} \|w\|^2 + C \sum_{n=1}^N \xi_n$$

Slack Variable (ξ_n): $t_n y(x_n) \geq 1 - \xi_n$, on the wrong side then > 1 , correct side or on the line = 0, inside the correct margin < 1 .

$$\text{Dual Lagrangian: } L(a) = \sum_{n=1}^N a_n - \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N a_n a_m t_n t_m K(x_n, x_m) \quad b = w_0 = \frac{1}{N_M} \sum_{n \in M} (t_n - \sum_{m \in S} a_m t_m K(x_n, x_m))$$

PCA: Max var: $a_1^T R_x a_1 = \lambda_1$

Min error: $\sum_{j=m+1}^D a_j^T R_x a_j \rightarrow R_x a_j = \lambda_j a_j$

$\sum_{j=m+1}^D a_j^T R_x a_j \rightarrow R_x a_j = \lambda_j a_j$, choose the largest eigenvalue. Assume Gaussian distribution and mean centered.

Backpropagation: $\frac{\partial J}{\partial W} = \frac{\partial V}{\partial W} \cdot \frac{\partial y}{\partial V} \cdot \frac{\partial E}{\partial y} \cdot \frac{\partial J}{\partial E}$

Chain Rule:

$$J(w) = \frac{1}{2} \sum_{i=1}^N e_i^2 \rightarrow e_i(-1)\phi'(v_i)x_j$$

$$e_i = t_i - y_i$$

$$y_i = \phi(v_i), \phi \text{ is an activation func.}$$

$$v_i = w^T x_j, \text{ note that } x_j \in R^{D+1}$$

$$\begin{aligned} w_{lj}^{(t+1)} &= w_{lj}^{(t)} - \eta \frac{\partial J}{\partial w_{lj}} \\ &= w_{lj}^{(t)} + \eta e_i \phi'(v_l) x_j \end{aligned}$$

Momentum: $w^{(t+1)} = w^{(t)} + \Delta w^{(t)}$ where

$$\Delta w^{(t)} = -\eta \nabla J(w^{(t)}) + \alpha \Delta w^{(t-1)}, \alpha = 0.9$$

Nesterov's momentum: $w^{(t+1)} = w^{(t)} + \Delta w^{(t)}$ where

$$\Delta w^{(t)} = -\eta \nabla J(m^{(t)}), m^{(t)} = w^{(t)} + \alpha \Delta w^{(t-1)}$$

Activation function	$\phi(x)$	$\phi'(x) = \frac{d\phi(x)}{dx}$
Linear	x	1
Sigmoid/Logistic	$\frac{1}{1+e^{-x}}$	$\phi(x)(1-\phi(x))$
Tanh	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	$1 - \phi(x)^2$
ReLU	$\begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$	$\begin{cases} 0, & x < 0 \\ 1, & x > 0 \\ \text{undefined}, & x = 0 \end{cases}$
Leaky ReLU	$\begin{cases} 0.01x, & x \leq 0 \\ x, & x > 0 \end{cases}$	$\begin{cases} 0.01, & x < 0 \\ 1, & x > 0 \\ \text{undefined}, & x = 0 \end{cases}$
Softplus	$(1 + e^x)$	$\frac{1}{1+e^{-x}}$
ELU	$\begin{cases} \alpha(e^x - 1), & x \leq 0 \\ x, & x > 0 \end{cases}$	$\begin{cases} \alpha e^x, & x < 0 \\ 1, & x > 0 \\ 1, & x > 0 \text{ and } \alpha = 1 \end{cases}$
SELU	$\lambda \begin{cases} \alpha(e^x - 1), & x < 0 \\ x, & x \geq 0 \end{cases}$	$\lambda \begin{cases} \alpha e^x, & x < 0 \\ 1, & x \geq 0 \end{cases}$