The key to changing variables:

The purpose of changing variables can be many reasons. However, the key is that changing variables of my equation will not change the overall effect or product of my equation. For example, when we change the variable in integral, change from like x to u, we change the variable and make a lot of changes to the equation. However, the answer or solution to the equation does not change. !!!

Proof of DTFT time reversal property

With the understanding of the changing variables, we can start proofing the time reversal property.

$$DTFT(x[-n]) = \sum_{n=-\infty}^{\infty} \mathrm{x}[-\mathrm{n}]e^{-jwn}$$

Type equation here.

We can do a change of variable to solve this DTFT.

$$k = -n, \sum_{k} x[k]e^{-jw(-k)}$$

Since change of variables should not change the overall answer or solution of the equation, in the first equation, we basically sum from x[positive]e^jw*positive + …. + x[negative]e^jw*negative

We want to achieve the same thing while using the sum notation, so we can write as x[negative]e^jw*negative + …. + x[positive]e^jw*positive. So, the bound of the new equation can be written as:

$$\sum_{k=-\infty}^{\infty} x[k]e^{-jk(-w)} = DTFT(x[-w])$$

More remarks: notice that when we change the variable, the equation changes too but it should be the same thing.

Another way to change the index of upper bound and lower bound of the summation.

When the summation is $\sum_{k=-\infty}^{some\ real\ numbers}$ we can view the upper bound and lower bound as $-\infty < k, k \leq some\ real\ numbers$. So when we change the infinity index, $-\infty < n, n < \infty, k = -n. -\infty < -k, -k < \infty -> \infty > k, k > -\infty, so\ the\ new\ upper\ and\ lower\ bound\ is\ still\ -\infty\ to\ \infty$