Modified Integral Sliding Mode Control of Motion System

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***Abstract*—The paper is oriented to the problem of the accurate trajectory tracking in motion control systems suffering from intensive parametric and signal disturbances. To assure both the robustness and the accuracy of the control process, theory of variable structure control, specifically the integral sliding mode control combined with the time suboptimal one, has been utilized. The modification of this original control method, suitable for the implementation in real motion systems and guaranteeing the undesirable chattering reduction, comprises the continuous approximation of the nonlinear control element and the corresponding switching function parameter synthesis. Numeric simulation on the motion system model demonstrates the feasibility and reliability of the proposed control algorithm along with the quality of the resultant tracking performance despite the driving torque boundary required by a technology.**

***Keywords*—motion control system, robust trajectory tracking, variable structure control, integral sliding mode, continuous approximation**

I. Introduction

Desired trajectory tracking (or command following) represents one of the most frequent tasks in motion control systems. During the motion, particularly the MIMO systems like robots or other complex mechatronic systems overcome an enormous parametric uncertainty and a wide spectrum of external forces. Thus, to keep the quality of trajectory tracking, the robust control method is a reliable answer to this problem. Moreover, with the robust control algorithms, the complex, nonlinear and mutually interacting dynamics of MIMO motion system can be decoupled to the set of linear and independent SISO dynamics, treating the external forces as disturbances [1], [2]. Additionally, to satisfy the requirements for the safety and integrity of the real motion system, the necessary limitation of the driving torques / forces can also be taken into account.

In this contribution, the attention has been focused to the robust control methods based on the theory of variable structure control (VSC) [3], [4]. It is a well-known fact that, particularly for the command tracking purposes, the quality of a feedback control can be improved by an integral element in the open loop transfer function. Theory of VSC offers an interesting method of such an element implementation – the *integral sliding mode control* (ISMC) [5], [6], [7], [8]. The main idea of ISMC is explained in Section II of this contribution.

As a basis for the ISMC application, we chose the up-to-date *time suboptimal control* [9], [10] that belongs to the robust VSC methods and assures a suitable dynamics of tracking. In agreement with the VSC theory, the motion of a controlled plant in sliding mode ensures the system's robustness with respect to both parametric and signal disturbances. Originally, variable structure control with sliding mode represents a discontinuous nonlinear control that causes an unwanted chattering in the real motion system due to neglected parasitic dynamics and nonlinearities. Furthermore, it is impossible to accomplish the sliding mode with the high frequency oscillation of driving torque in a real motion system. Therefore, in practice, the continuous modification of the control algorithm [11] should be implemented. The results of the upper mentioned techniques applied in the SISO robust motion control are summarized in Section III.

The outcome of numeric simulation of the presented robust command following control algorithm for SISO motion system with the presence of parametric and signal disturbances is given and discussed in Section IV.

II. Integral Sliding Mode Control

The main idea of the integral sliding mode control is the addition of an integral element to the variable structure control algorithm via the specific component of the *switching function* [5]. Let the affine nonlinear MIMO dynamic system (e.g. motion system) be described by the state model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} + \mathbf{d}(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = \mathbf{x}(t), \dot{\mathbf{x}} = \dot{\mathbf{x}}(t) \in R^n$ stand for the system's state vector and it's first time derivative, $\mathbf{f}(\cdot) \in R^n$ represents a nonlinear vector function, $\mathbf{B}(\cdot) \in R^{n \times m}$ is the input matrix, $\mathbf{u} = \mathbf{u}(t) \in R^m$ corresponds to the control effort vector and $\mathbf{d}(\cdot) \in R^m$ denotes the system's disturbance vector that is upper bounded and satisfies the invariance condition [5]

$$\mathbf{d}(\mathbf{x}) = \mathbf{B}(\mathbf{x})\mathbf{u}_d(\mathbf{x}) \tag{2}$$

with elements $|d_i(\mathbf{x})| \le d_i^+(\mathbf{x}), (i = 1, ..., m)$, where $d_i^+(\mathbf{x})$ is the positive scalar function.

Let the control output $\mathbf{u}$ comprises two elements

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_a \tag{3}$$

where the conventional control effort $\mathbf{u}_0 = \mathbf{u}_0(\mathbf{x})$ stabilizes system (1) without the influence of disturbances (the ideal system), and $\mathbf{u}_a = \mathbf{u}_a(\mathbf{x})$ is the required auxiliary

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VSC algorithm that rejects the influence of disturbances $\mathbf{d}(\cdot)$ and eliminates the imperfections of the conventional control $\mathbf{u}_0$. Let the switching function in ISMC algorithm be

$$\mathbf{F}(\mathbf{x}) = \mathbf{F}_0(\mathbf{x}) + \mathbf{z}(\mathbf{x}) \tag{4}$$

with $\mathbf{F}(\cdot), \mathbf{F}_0(\cdot), \mathbf{z}(\cdot) \in R^m$, where $\mathbf{F}_0(\cdot)$ stands for the vector of conventional switching functions (in motion systems obviously given by the linear combination of the control error vector $\mathbf{e} = \mathbf{e}(t) \in R^n$ and it's time derivative $\dot{\mathbf{e}} = \dot{\mathbf{e}}(t) \in R^n$) and $\mathbf{z}(\cdot)$ represents the requested integral element determined below.

In VSC theory, the term *equivalent control* corresponds with the mean value of the actuating variable oscillation in sliding mode such that the system response to the equivalent control matches with the original variable structure control response in sliding mode. Thus, the equivalent control meets the condition

$$\frac{d\mathbf{F}(\mathbf{x})}{dt} = \mathbf{0} \tag{5}$$

for the initial condition $\mathbf{F}(\mathbf{x}(0)) = \mathbf{0}$.

Substituting (1), (2), (3) and (4) in (5) we obtain (saving the space by neglecting the independent variables)

$$\frac{d\mathbf{F}}{dt} = \frac{\partial \mathbf{F}_0}{d\mathbf{x}}\frac{d\mathbf{x}}{dt} + \frac{d\mathbf{z}}{dt} = \frac{\partial \mathbf{F}_0}{d\mathbf{x}}(\mathbf{f} + \mathbf{B}\mathbf{u}_0 + \mathbf{B}\mathbf{u}_a + \mathbf{B}\mathbf{u}_d) + \frac{d\mathbf{z}}{dt} = \mathbf{0} \tag{6}$$

with initial condition

$$\mathbf{z}(0) = -\mathbf{F}_0(0) \tag{7}$$

Let's generate the $\mathbf{z}$ element by the differential equation

$$\frac{d\mathbf{z}}{dt} = -\frac{\partial \mathbf{F}_0}{d\mathbf{x}}\left(\mathbf{f} + \mathbf{B}\mathbf{u}_0\right) \tag{8}$$

Substituting (8) in (6) yields

$$\frac{d\mathbf{F}}{dt} = \frac{\partial \mathbf{F}_0}{d\mathbf{x}}\left(\mathbf{B}\mathbf{u}_a + \mathbf{B}\mathbf{u}_d\right) = \mathbf{0} \tag{9}$$

Equation (9) is fulfilled under the condition $\mathbf{u}_a = -\mathbf{u}_d$ that in VSC corresponds with the control element

$$\mathbf{u}_a = -\mathbf{M}\operatorname{sgn}\left(\mathbf{F}\right) \tag{10}$$

where sgn stands for the sign function and $\mathbf{M} \in R^{m\times m}$ is a diagonal matrix with identical constant diagonal elements M (driving torque boundaries) that meet the condition

$$M \geq \left\|\mathbf{u}_d\right\| \tag{11}$$

Under the condition (11), expression (9) with control (10) satisfies the sliding mode condition in the vicinity of the switching function $\mathbf{F}$ [1], [3], consequently keeping the zero value of the $\mathbf{F}$ function. In engineering practice, the value of the right-hand side of (11) is estimated regarding the real motion system behavior.

By means of control (3) with (4), (8), (10) and (11), system (1) will be in sliding mode guaranteeing the robustness of trajectory tracking with respect to disturbances (2). Due to integral element in (4), we expect the better tracking performance than the one with the conventional variable structure control.

III. VSC Design for SISO Motion System

Let the SISO dynamics corresponding to the individual degree of freedom (DOF) of the decoupled MIMO motion system be described in the error vector plane $(e, \dot{e})$ by the state model

$$\begin{aligned} \frac{de}{dt} &= \dot{e} \\ \frac{d\dot{e}}{dt} &= -\frac{1}{T}\left(K(u - u_d) + \dot{e}\right) \end{aligned} \tag{12}$$

$$T \in \left\langle T_{\min}, T_{\max} \right\rangle \tag{13}$$

where $e = e(t), \dot{e} = \dot{e}(t) \in R$ stand for the position control error and its time derivative, K is the system's gain, T represents the system's time constant suffering from the parametric uncertainty (13), and u denotes the control effort (driving torque / force) suffering from the external disturbance u_d. The control error e is given by

$$e = x_r - x \tag{14}$$

with the reference trajectory (position) $x_r = x_r(t)$ and the output position $x = x(t)$.

Note that for the SISO motion system (12), the integral sliding mode control theory (Section II) converts to the scalar version.

Let the conventional stabilizing control effort $u_0 = K_p e$ in (3) be defined by the pole placement method with the system damping b and the proportional controller gain

$$K_p = \frac{1}{4KT_{\max}b^2} \tag{15}$$

The VSC element in (3) and (4) should have the form

$$u_a = M\operatorname{sgn}\left(F(\mathbf{e})\right),\ M > \left|u_d\right|_{\max},\ \mathbf{e} = \begin{pmatrix} e \\ \dot{e} \end{pmatrix} \tag{16}$$

$$F(\mathbf{e}) = F_0(\mathbf{e}) + z(\mathbf{e}) \tag{17}$$

with the linear switching function

$$F_0(\mathbf{e}) = \dot{e} + \alpha e \tag{18}$$

and the $z(\mathbf{e})$ element dynamics

$$\frac{dz(\mathbf{e})}{dt} = \frac{1}{T_{\max}}\left(Ku_0 + \dot{e}\right) - \alpha\dot{e} \tag{19}$$

To determine the value of the switching function parameter α and to show the positive influence of the integral sliding mode in command tracking, we chose the time suboptimal (TSO) control [9] in the role of the conventional VSC with

$$\alpha = \frac{1}{T_{\max}\left(1 - \frac{M_2}{M_1}\ln\left(1 + \frac{M_1}{M_2}\right)\right)} \tag{20}$$

where $M_1 = M + |u_d|_{\max}$ and $M_2 = M - |u_d|_{\max}$.

The advantage of TSO control is in its perfect tracking capability and the simple implementation in motion systems. Synthesis of the TSO control for robust tracking has been fully described in [9], [10]. It has been shown that the tracking performance of the VSC in motion control system can be expressed in frequency domain by means of the harmonic reference position

$$x_r = A + A\sin\left(\omega t - \pi/2\right) \tag{21}$$

Using the *tangential velocity vector method* [2], comparison of the system (12) dynamics (under the worst dynamic conditions) with the dynamics of the reference (21) yields the maximal frequency of the desired trajectory the motion system is capable of tracking

$$\omega_{\max} = \frac{-1 + \sqrt{1 + 4\frac{KM_2T_{\max}}{A}}}{2T_{\max}} \tag{22}$$

To prevent the real motion system chattering excited by the control effort u_a in (16), the *continuous approximation* of the discontinuous element (relay or sign function) has been carried out. The corresponding theory has been published in [11], where two methods of the linearization have been performed. Firstly, by means of the *reaching law* [4] application, the *equivalent time suboptimal* (ETSO) control has been designed [12]. Secondly, replacing the relay control element by the saturation with the identical boundary M and the gain K_{SAT} in the linear zone, the simple continuous approximation with the switching function parameter α modification α_{SAT} has been done [11]. Comparing these two methods we can conclude, that the continuous substitute for the discontinuity in (10) or (16) should meet the following formulas [11], [12]

$$u_a = u_{aSAT} = \begin{cases} K_{SAT}F(\mathbf{e}) & for \quad |K_{SAT}F(\mathbf{e})| < M \\ M\,\mathrm{sgn}(F(\mathbf{e})) & for \quad |K_{SAT}F(\mathbf{e})| \geq M \end{cases} \tag{23}$$

$$K_{SAT} = \frac{(\alpha + k)T_{\max} - 1}{K} \tag{24}$$

$$\alpha_{SAT} = \frac{\alpha k T_{\max}}{(\alpha + k)T_{\max} - 1} \tag{25}$$

$$k = \frac{100M}{M_2\Delta}\omega_{\max}\sqrt{\frac{\omega_{\max}^2 + \frac{1}{T_{\max}^2}}{\omega_{\max}^2 + \alpha^2}} \tag{26}$$

where Δ denotes the desired tracking accuracy in % of amplitude A.

Subsequently, equations (17), (18) and (19) are replaced by

$$F(\mathbf{e}) = F_{0SAT}(\mathbf{e}) + z_{SAT}(\mathbf{e}) \tag{27}$$

$$F_{0SAT}(\mathbf{e}) = \dot{e} + \alpha_{SAT}e \tag{28}$$

$$\frac{dz_{SAT}(\mathbf{e})}{dt} = \frac{1}{T_{\max}}\left(Ku_0 + \dot{e}\right) - \alpha_{SAT}\dot{e} \tag{29}$$

Expressions (20) to (29) form the original continuous modification of ISMC applicable for the decoupled MIMO motion system guaranteeing the robustness against the parametric and signal disturbances. The block diagram of the proposed control algorithm for a single DOF can be seen in Fig. 1. The presented control structure is identical for any DOF of the MIMO motion system.

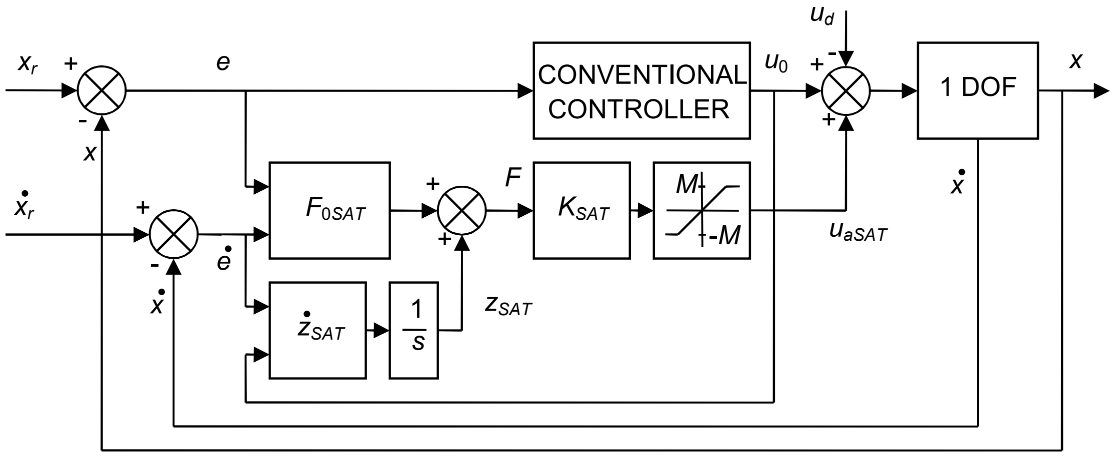

Fig. 1. Block diagram of the continuous ISMC for a single DOF.

IV. Numeric Simulation Results

The perfect trajectory tracking of the proposed control algorithm is illustrated by the numeric simulation on the dynamic model of a single DOF motion system represented by a revolute joint (tracking of the angular position x_r) with an external disturbance u_d acting against the driving torque u. Such a simulation is acceptable, because the mutual interactions among the individual DOFs of the MIMO motion system with a robust control algorithm can be treated as any ordinary external disturbance (e.g. gravitational force). Moreover, only the maximal value of the sum of the external forces or even only its rough estimation should be known in practice. Nevertheless, a harmonic type of external disturbance u_d has been chosen for the simulation purpose.

Both, the simulated plant parameters and the parameters of the control algorithm are summarized in Table I. Note that the motion system suffers from severe parametric and external disturbances.

TABLE I. System Parameters

Parameter	Value
System's gain K	0.0883 radN^{-1}m^{-1}
System's minimal time constant $T_{\min}$	16 ms
System's maximal time constant $T_{\max}$	98 ms (>400% of $T_{\min}$)
Driving torque boundary M (16)	50 Nm
Conventional control damping b	0.707
Conventional controller gain K_p (15)	83.297 Nmrad^{-1}
Harmonic reference amplitude A (21)	1 rad
Maximal frequency of tracking $\omega_{\max}$ (22)	1.0652 rads^{-1}
Harmonic reference frequency ω	1.0652 rads^{-1} (=$\omega_{\max}$)
External disturbance amplitude $\|u_d\|_{\max}$	25 Nm (50% of M)
External disturbance frequency ω_d	5.3295 rads^{-1} (5 $\omega_{\max}$)
Switching function F_0 parameter α (20)	21.1256 s^{-1}
Saturation gain K_{SAT} (24)	2.2149.10^3
Modified parameter α_{SAT} (25)	21.0785 s^{-1}
Continuous control parameter k (26)	2.8696.10^3 s^{-1}
Tracking accuracy Δ	0.1% of A
Maximal tracking error $e_{\max}$	10^{-3} rad

To underline both the suitability and the advantage of the ISMC application, a conventional linear control algorithm (proportional controller given by the pole placement method) incapable of dealing with the problem of the extreme disturbance has been chosen intentionally. The weak tracking performance of such a controller is illustrated in Fig. 2. As can be seen, system trajectory x extremely oscillates around the harmonic reference x_r (21) due to the external forces oscillation, despite the identical initial conditions of x and x_r. The situation is similar for the both boundary values of the system's time constant T interval (13): $T_{\min}$ on the left-hand side a) and $T_{\max}$ on the right-hand side b).

Absolutely different is the quality of tracking in motion system with the same conventional linear controller supplemented with the proposed ISMC. The arrangement of the next set of figures is as follows: Part a) depicts the reference angular position x_r (red line) and the corresponding system response x (blue line) versus time plot. Part b) shows the driving torque u (3) performance. Part c) represents the detail of the tracking error e (14) versus time graph with the highlighted interval of the maximal admissible tracking error (cf. Table I) $\langle -e_{\max}, e_{\max} \rangle$ at the given accuracy Δ.

Note that to demonstrate the perfect tracking capability of the presented control algorithm, the maximal possible frequency (22) of reference given by the motion system dynamics, by the driving torque boundary M as well as by the amplitude of the external disturbance $|u_d|_{\max}$ has been chosen for the simulation.

In Fig. 3, the tracking performance of the system with the minimal time constant $T = T_{\min}$ is illustrated. Due to different initial condition $x_r(0) = 0$ of the reference trajectory and the one $x(0) = A$ of the system trajectory there exists a short transient period about 1 second (Fig. 3a) after that the system trajectory doesn't exceed the expected maximal tracking error (Fig. 3c). For comparison, in Fig. 3c there is also a dotted line (indicated by the label VSC) corresponding to the tracking error of the conventional variable structure control without an ISMC element. We can see a small improvement of the tracking for ISMC compared to conventional VSC. However, both control algorithms guarantee that the tracking error e doesn't exceed the expected maximal value despite the high amplitude of the external disturbance oscillation. As can be seen in Fig. 3b, the driving torque also reacts to the disturbance oscillation without breaking the M boundary (cf. Table I).

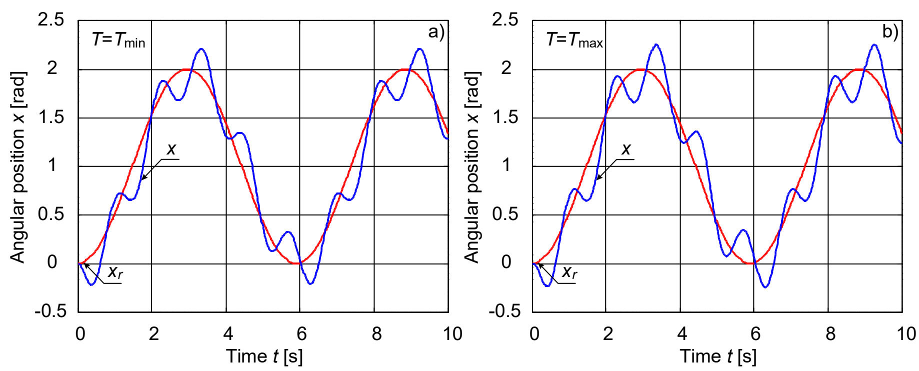

Fig. 2. Harmonic trajectory tracking in SISO motion system with conventional controller: a) $T = T_{\min}$, b) $T = T_{\max}$.

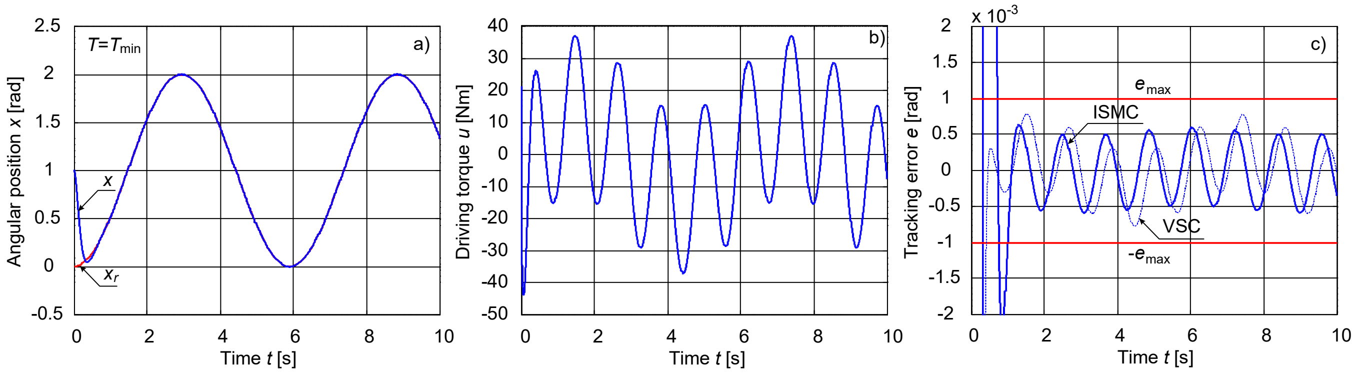

Fig. 3. Harmonic trajectory tracking in SISO motion system with ISMC, $T = T_{\min}$: a) angular position, b) driving torque, c) tracking error.

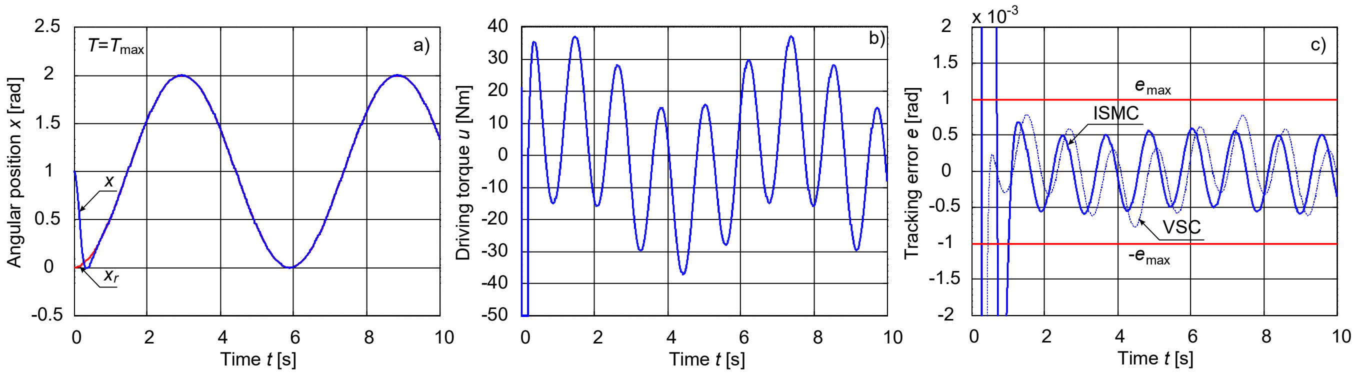

Fig. 4. Harmonic trajectory tracking in SISO motion system with ISMC, $T = T_{\max}$: a) angular position, b) driving torque, c) tracking error.

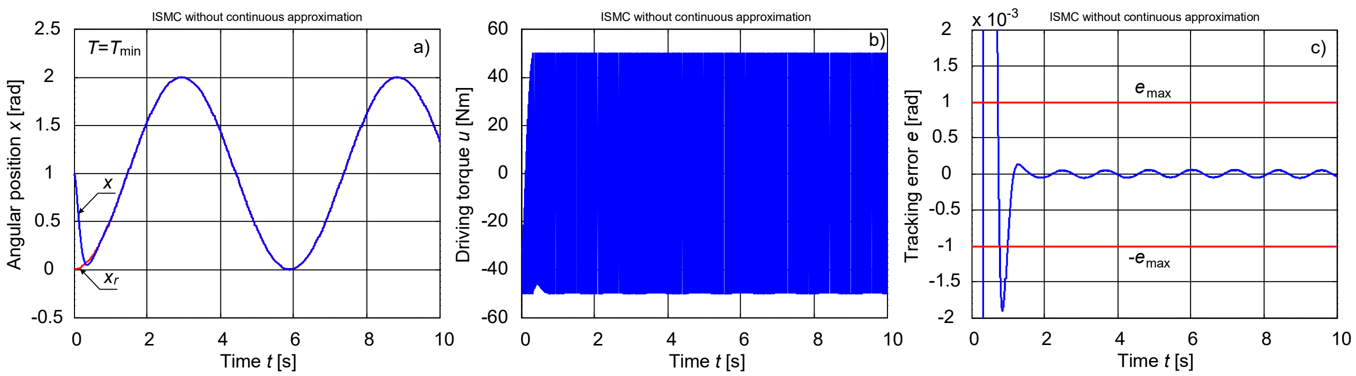

Fig. 5. Harmonic trajectory tracking in SISO motion system with ISMC without continuous approximation, $T = T_{\min}$: a) angular position, b) driving torque, c) tracking error.

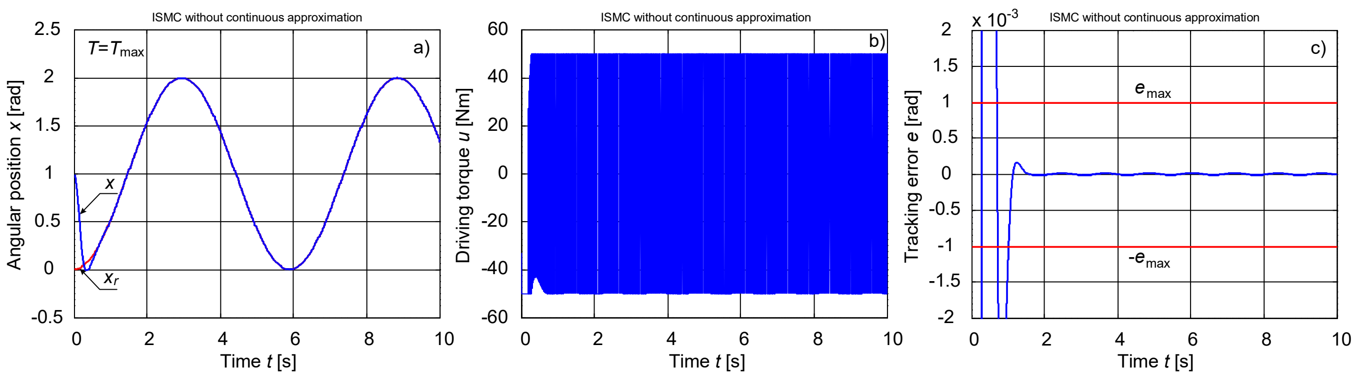

Fig. 6. Harmonic trajectory tracking in SISO motion system with ISMC without continuous approximation, $T = T_{\max}$: a) angular position, b) driving torque, c) tracking error.

Fig. 4 shows the similar tracking performance of the motion system with the maximal time constant $T = T_{max}$. The only noteworthy difference can be seen in Fig. 4b, where the driving torque u hits the $-M$ boundary at the very beginning of the transient process. However, this is the feature the VSC takes naturally into consideration without any reduction of the control process quality. We can conclude that the desired tracking accuracy has been achieved.

Figs. 5 and 6 show the difference between the presented continuous modification of the integral sliding mode control and its discontinuous (original) version. The main difference is in the driving torque u performance (Fig. 5b and Fig. 6b) that in original ISMC runs in sliding mode – the high frequency oscillation with the amplitude M – inacceptable in real motion systems. On the other hand, the tracking accuracy of the continuous ISMC is significantly worse with respect to the discontinuous one (Fig. 5c and Fig 6c). This is the price to pay for the preferred continuous behavior of the driving torque in engineering practice. Anyway, it has been shown that the presented continuous modification of the ISMC gives the possibility of setting the desired tracking accuracy Δ.

V. Conclusions

The aim of this paper is to show one of the possible and consistent solutions of the reference trajectory tracking in real MIMO motion systems, namely in robotics, where the presence of an extreme parametric uncertainty and a wide spectrum of external forces make difficult or even exclude the conventional control methods application. Theoretical background of the proposed approach is given by the theory of variable structure control yielding the robust and accurate control techniques. Particularly the integral sliding mode control, specifically suitable for the tracking purposes, has been chosen. The main contribution of this paper is in the original application of a continuous approximation of the discontinuous control element and a corresponding modification of the switching function parameter in the ISMC algorithm preserving the quality of the original VSC, which makes possible its implementation in real motion systems. Thanks to robust control that allows the decoupling of the complex and naturally nonlinear MIMO motion system to the set of independent and linear SISO systems, the scalar version of the theory, suitable for engineering practice, is presented. The control algorithm is verified via the simulation on a numeric model of a single degree of freedom of a motion system. The simulation output shows the perfect tracking performance and the reliability of the proposed control strategy.

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