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A survey of welding robot intelligent path optimization

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ABSTRACT

Welding robots are widely used for most welding works in manufacturing industries due to their flexible, efficient and accurate operation. At the same time, it can improve enterprise competitiveness, productivity, product quality, and reduce producing cost. For industrial welding robot task planning, manual planning is still widely used, and it is time-consuming when there are large-scale welding joints or welding seams. To meet this challenge, automatic and efficient path optimization technologies have been studied to realize effective robot path planning. For welding robot path optimization, path optimization problem description is presented first. Then, intelligent optimization algorithms for welding robot path optimization are introduced. Furthermore, obstacle avoidance strategies are studied. Besides, welding robot multi-objective path optimization is given based on the above optimization problem description and optimization algorithm analysis. Optimization results show the effectiveness of intelligent path optimization strategies. At last, the discussions on welding robot path optimization application and future research are analyzed briefly.

1. Introduction

Automated and robotic welding is now widely used in manufacturing industry to obtain manufacturing competitiveness, high productivity, low cost, and good quality due to its flexible, efficient and accurate operation [1–3]. For an intelligent robot welding system, some functions are needed to realize desired welding effects, which are described in Fig. 1 [4]. As an important part of the intelligent welding robot, path optimization strategies are necessary to be studied.

Manual planning is still widely used for industrial welding robot task planning, and it is time-consuming when there are large-scale welding joints or welding seams. Hence, automatic and efficient path optimization technologies have been studied to improve robot path planning ability. During the welding process, the different welding path sequences will affect the welding path length, energy consumption, welding deformation and others. Hence, these factors need to be considered in welding robot path optimization, and it is always considered as a multi-objective optimization problem. The path length, welding deformation, and energy consumption are always set as optimization objectives. At the same time, obstacle avoidance is necessary to be studied as a constraint.

Structure of welding robot path optimization is presented in the Fig. 2. Based on the Path optimization description for welding robot, welding robot path optimization mainly includes following works: optimization algorithm, obstacle avoidance strategy, multi-robot cooperation, and parameters optimization.

To solve these discrete multi-objective optimization algorithms, intelligent optimization algorithms were used to obtain desired optimization effect. After numerous simulation tests, following conclusion was obtained. In the discrete optimization problem, the three-objective optimization solution is easy to fall into local convergence compared to single-objective and dual-objective optimization. As a result, it can lead to local optimization or poor diversity of solutions. In order to solve the problem that the algorithm is easy to fall into the local convergence and the non-inferior solution distribution range is small in the discrete multi-objective optimization problem, some optimization strategies needed to be integrated with the algorithms.

Following the introduction above, path optimization problem description is first given in Section 2. Then, intelligent optimization algorithms, obstacle avoidance strategies introduction, researches on welding robot collision detection, and welding robot multi-objective path optimization are addressed in Sections 3–6. Lastly, the conclusion is given in Section 7.

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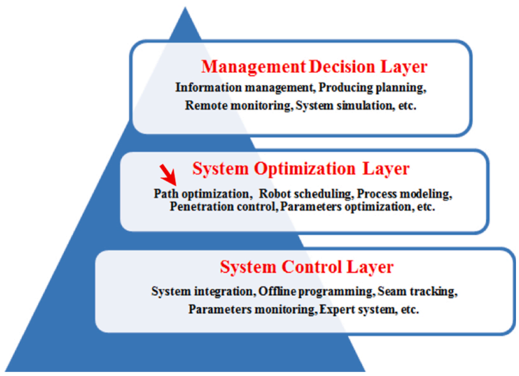

Fig. 1. Functions description of intelligent welding robot system.

2. Path optimization problem description

In this article, the optimization objectives for the welding robot path optimization include path length, welding deformation, energy consumption, welding beat, etc. Besides, the collision free path is often considered first. If two robots are used, multi-robot cooperation strategy will also be studied. Shorter path length often implies better productivity. For arc welding of a complex structure, deformation is a major issue because it has much greater heat input than spot welding and then introduces greater deformation.

2.1. Spot welding

For spot welding robot path optimization, considering a weld joints welding task $C = (c_1, c_2, \ldots, c_N)$, the distance between two weld joints can be described as $d(c_i, c_j) \geq 0$, where $c_i, c_j \in C\,(1 \leq i,j \leq N)$, c_i stands for a weld joint. When the path length is set as the optimization objective, the minimum path length of the welding robot is desired to be obtained based on the optimized welding joints sequence ($\pi = \{c_1, c_2, \ldots c_N, \}$).

$$f(\pi) = \sum_{i=1}^{N-1} d(c_i, c_{i+1}) + d(c_N, c_1) \tag{1}$$

where $f(\pi)$ denotes the distance when the welding sequence is π.

For double robot system, some other criteria have to be considered in addition to the shortest path length. Considering a welding task $C = (c_1, c_2, \ldots, c_N)$, the welding task distribution $R_1 = (c_1, c_2, \ldots, c_k)$ is assigned for welding robot 1, the rest $R_2 = (c_{k+1}, c_{k+2}, \ldots, c_N)$ for welding robot 2. And welding robot 1 starts from point A, welding robot 2 starts from point B.

First, the work space for two robots cannot overlap to avoid collision, which can be described as follows:

$$\max D(\mathrm{A},\ c_\alpha) < \min D(\mathrm{A},\ c_\beta),\ \alpha \in R_1,\ \beta \in R_2 \tag{2}$$

$$\max D(\mathrm{B},\ c_\gamma) < \min D(\mathrm{B},\ c_\delta),\ \delta \in R_1,\ \gamma \in R_2 \tag{3}$$

where $D(\cdot)$ means distance calculation.

Second, the difference between the two robots path length should be as small as possible to make sure the two robots finish the welding work at the same time nearly.

$$\text{Minimize } D(s_1,\ s_2) = |s_1 - s_2| \tag{4}$$

where $s_1,\ s_2$ indicate the welding distance of two welding robots, respectively.

At last, the paths have to be monitored in real-time to make sure welding guns will not influence each other. In order to do this, a safe distance between two welding guns should be defined, as long as they keep the distance beyond the safe distance, they will not influence each other. When the distance is smaller than the safe distance, the welding robot with a shorter welding distance will wait for a while to make sure the distance between the two welding torches is less than the safe distance.

$$D(R_1,\ R_2) - s_{safe} \geq 0 \tag{5}$$

Hence, the model of single objective two welding robot path planning can be described as follows:

$$\begin{aligned} \text{Minimize } s_1 &= \sum_{i=1}^{k-1} d(c_i, c_{i+1}) + d(c_k, c_1) \\ s_2 &= \sum_{i=k+1}^{N-1} d(c_i, c_{i+1}) + d(c_N, c_{k+1}) \end{aligned} \tag{6}$$

Subject to constrains (2)–(5).

2.2. Arc welding

Compared to the spot welding robot path optimization, welding deformation is another crucial factor to be studied for arc welding robot. For the path optimization problem description for arc welding robot path optimization, the total path length/welding time, energy consumption and welding deformation during the welding process always are studied as optimization objectives. At the same time, the obtained path should be collision-free. It is a discrete multi-objective optimization problem with some constraints.

Take a two-objective optimization problem as example, the optimization model for arc welding robot path optimization is established as follows:

$$\min F(X) = \min(F_1(X), F_2(X)) = \min\{F_1(x_1, x_2, \ldots, x_n), F_2(x_1, x_2, \ldots, x_n)\} \tag{7}$$

where $x = (x_1, \ldots, x_n) \in X$, X denotes a vector of n-dimensional decision space, $x_1, \ldots, x_n$ represent weld seams. $F(X)$ represents a two-dimensional target space vector. In this model, $F_1(X)$ and $F_2(X)$ denote the path length and total welding deformation, respectively.

Based on the above path optimization problem analysis, intelligent

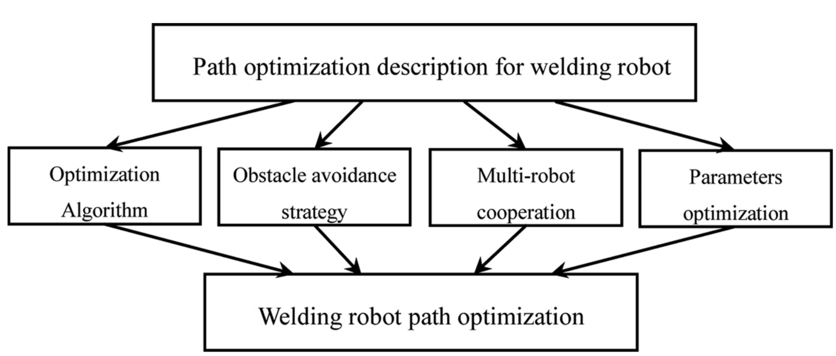

Fig. 2. Structure of welding robot path optimization.

optimization algorithms for welding robot path optimization will be introduced in Section 3, and some application will be given in Section 6.

3. Intelligent optimization algorithms

3.1. Necessity and classification

For welding robot multi-objective path optimization, the desired weld joints/seams sequence will be obtained to satisfy some objectives. There are always many weld joints/seams in a welding unit, and it is very difficult for engineering to find the desired sequence. Hence, intelligent algorithms are suitable to solve this problem.

Some intelligent algorithms are presented in Fig. 3. These algorithms can be classified three groups: evolutionary algorithms (EA), swarm intelligence algorithms, and other algorithms.

Welding robot manufacturing process is influenced by some factors which include robot path, robot trajectory, welding parameters, welding torch pose, multi-robot cooperation, etc. For welding robot path optimization, several optimization objectives need to be considered. It includes welding time, welding deformation, energy consumption, etc. Hence, it is necessary to study multi-objective intelligent optimization algorithms suitable for welding robot path optimization.

For these kinds of optimization problems, some optimization algorithms and improved strategies are studied. The concept of the multi-objective optimization problem was proposed by V. Pareto in 1896. Based on the genetic algorithms which were first established on a sound theoretical basis by Holland [5], multi-objective evolutionary algorithm (MOEA) [6], NSGA were widely studied for multi-objective optimization in these years. Besides it, particle swarm optimization (PSO) [7] was also studied for multi-objective optimization due to its effective searching ability. Furthermore, some other intelligent optimization algorithms applied for multi-objective optimization include cultural algorithm [8], simulate Anneal algorithm [9], artificial bee colony algorithm [10], etc.

As a typical combinatorial optimization problem, combinatorial explosion will be introduced when many weld joints/seams exist in an optimization unit. In this situation, how to rapidly find stable and global solution becomes difficult. To solve this problem, the designed algorithms should have good global and local searching ability. As two effective optimization algorithms, PSO and MOEA will be introduced in the following sections.

3.2. PSO

In 1995, Kennedy and Eberhart [7] presented the particle swarm optimization (PSO) algorithm based on the regularity of hunting birds. Due to its effective searching ability, PSO is useful to find multi-objective optimum. At the same time, it is easy to be integrated with other optimization methods to improve its performance. In addition, Coello Coello presented MOPSO [11], and it is a classical algorithm for multi-objective optimization problems. Hence, PSO is suitable for solve multi-objective optimization problems [12].

In the particle iteration, each particle is updated according to the reference of personal best (*pbest*) and global best particle (*gbest*). Then the ultimate solution converges to optimal or suboptimal because particles are always updated based on *pbest* and *gbest*. But when *pbest* and *gbest* fall into local optimal solution, the particle cannot jump out of the local optimal solutions either.

Eqs. (8) and (9) are the particle update equations in PSO. The Fig. 4 presents the basic principle of the PSO algorithm

$$v_i\ (t+1) = wv_i\ (t) + c_1r_1(p_i\ (t) - x_i\ (t)\,) + c_2r_2\big(p_g\ (t) - x_i\ (t)\,\big) \tag{8}$$

$$x_i\ (t+1) = x_i\ (t) + v_i\ (t+1) \tag{9}$$

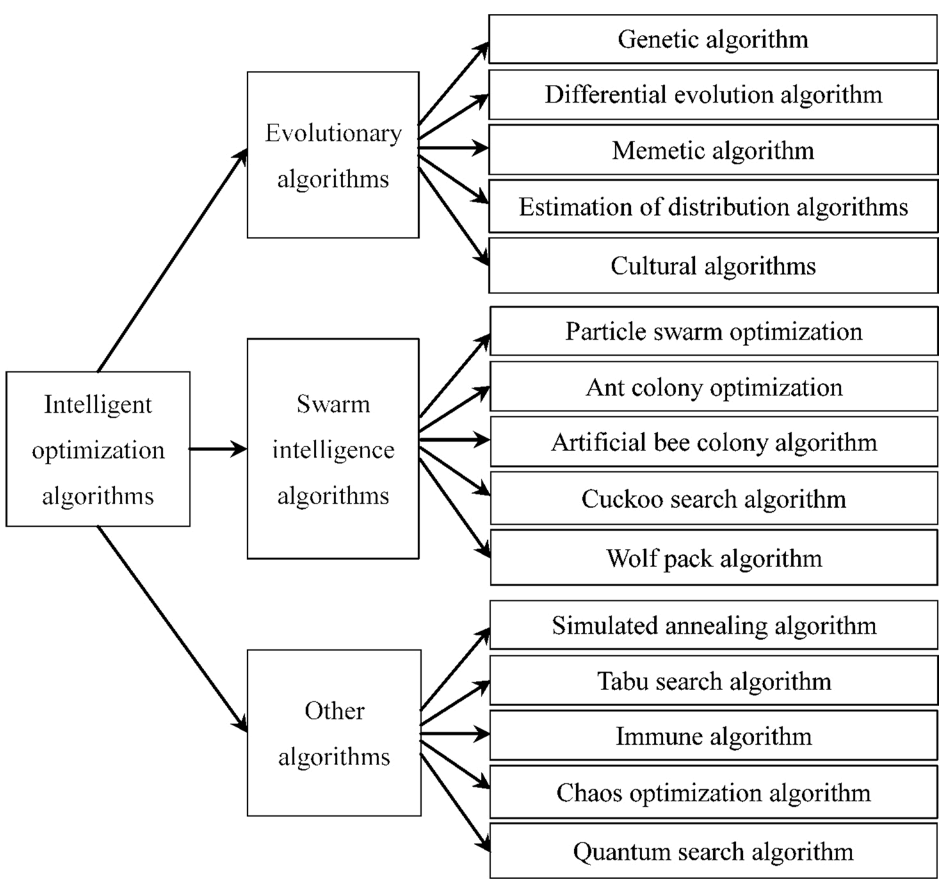

Fig. 3. Intelligent optimization algorithms.

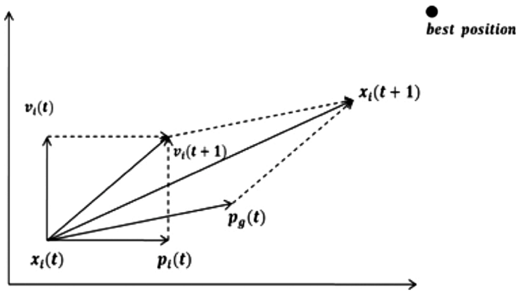

Fig. 4. PSO diagram.

As a widely used intelligent optimization algorithm, PSO [7] was used to solve path planning problems due to its simple structure, fast convergence speed and easy implementation. Meanwhile, many improvements were conducted for the PSO algorithm to solve the premature problem of PSO algorithm and accelerate the convergence rate of the algorithm. Algorithm improvements were studied on parameters [13], the position and velocity of PSO [14], local search PSO algorithm [15], fusion of different intelligent optimization algorithms [16], and updating strategy [17]. Improved PSO algorithm shows its advantages, such as fast rapid convergence and global optimization. Therefore, back-propagation neural network models based on global best adaptive mutation particle swarm optimization were established to estimate the welding penetration based on the welding characteristic parameters [18], and discrete elite PSO algorithm was studied to obtain the shortest collision-free welding path effectively [19].

3.3. MOEA

In the last two decades, a number of multi-objective evolutionary algorithms (MOEAs) have been proposed to solve various MOPs and MaOPs, lots of them have been successfully applied in many industrial areas such as logistics, machine design, robotics, communication and networks, and so forth [6]. According to the characteristics of the evolutionary mechanism, MOEAs could be roughly classified into three categories [6].

Domination-based MOEA is based on the pareto dominance. The non-dominated sorting genetic algorithm (NSGA-II) [20] and strength pareto evolutionary algorithm 2 (SPEA2) [21] are two representatives of this category.

Indicator-based MOEA assigns one indicator to compare the performance of solutions. Some remarkable MOEAs of this category are indicator-based evolutionary algorithm (IBEA) [22], S-metric selection evolutionary multi-objective optimization algorithm (SMS-EMOA) [23], and HyPE [24].

Decomposition-based MOEA aims to decompose a MOP or MaOP into a set of sub-problems, and then optimize them collaboratively using the aggregation function. Schaffer extended the simple genetic algorithm (SGA) to the vector evaluated genetic algorithm (VEGA) [25], MOEA based on decomposition (MOEA/D) [26] and reference vector guided EA (RVEA) [27]. Others partition the MOP of MaOP into several small MOPs according to the objective space, the representatives are NSGA-III [28] and SPEA based on reference direction [29].

As a widely used algorithm, NSGA-II were used for welding parameters optimization, and desired results were obtained [30,31]. To solve the discrete multi-objective optimization problem, MOEA/D algorithm was improved [32,33]. Besides it, an adaptive neighborhood discrete multi-objective optimization algorithm based on event triggering (DMOEA/D-ET) was used for the multi-objective optimization of the welding process of the balanced beam model [34].

Based on the above analysis, it can be seen that the intelligent optimization algorithms can be used for combinatorial optimization problem, and they are also suitable for welding robot path optimization.

4. Robot obstacle avoidance strategies introduction

For spot welding robot, the welding gun must move to all welding joints, and it needs to be close to or move around obstacles. In the process of moving to and away from all welding joints, some transition points are used to obtain a collision-free and effective trajectory. For arc welding robot, this problem is more difficult due to the welding gun need to move along the welding seams with the desired pose. At the same time, robot joints cannot collide with the other parts in the welding unit.

4.1. Obstacle avoidance methods

In the welding process, the path and trajectory planning need to be studied to avoid damage to the robot and workpiece. Hence, obstacle avoidance is a fundamental problem in welding robot path planning. However, most path planning with obstacle avoidance is obtained based on teaching mode programming. In this way, the planning process is time-consuming, and the optimum path is hard to achieve. Hence, robot collision-free path planning based on obstacle avoidance strategies needs to be studied to improve welding efficiency. Collision-free path planning includes two stages: environment modeling and path searching. Path searching methods include traditional methods and intelligent methods which derive from intelligent control method or optimization algorithm. Fig. 5 shows the robot obstacle avoidance methods.

4.2. Environment modeling

Environment modeling refers to the mathematical description of the environment around the welding robot, which is essential for collision-free path planning. Based on the Fig. 5, some modeling methods are presented as follows.

Grid method was proposed by Howden in 1968 [35]. The basic principle is to divide the robot working environment into numerous tiny grid units, and the grid is divided as free gird and obstacle grid. Robot can only move in the free grid and must avoid when it encounters obstacles grid. The grid map can be represented by a binary matrix, where 1 represents obstacle and 0 is free grid. Grid method and improved ant colony algorithm are used to conduct path planning in Ref. [36]. Willms [37] applied a grid method to solve path planning for real-time robots in dynamic environments.

For the grid method, grids are applied to describe the free space and obstacle space in the robot's work space. A good path can be achieved if the precision is high, but the algorithm would be time-consuming at the same time. The environment would not be described clearly if the precision are otherwise low. The grid method is often used for modeling, and it should be integrated with other algorithms to obtain a collision-free path.

C-space method presented by Lozano-Perea is a popular method to realize collision avoidance [38]. In this method, the environment information in cartesian space is transferred to the configuration space, and all the reachable state is called the free space. Then, the collision avoidance can be ensured using algorithms to find a path in the free space which can connect the initial point and target point. Yu [39] presented a C-space layered search arithmetic for the manipulator. The algorithm improved the search speed and reduced the demand for storage space.

As an effective modeling method, visibility graph, are integrated with voronoi diagram and potential fields to realize path planning in Ref. [40]. Besides complexity, visibility graph method is intuitive, and the shortest path can be found. Hierarchical bounding box [41] provides an effective method for collision detection where the complex objects are replaced with simple boxes. It includes axis-aligned bounding boxes (AABB), oriented bounding box (OBB), and Sphere, etc. Approximate voronoi boundary network was used to establish the model, and Biogeography particle swarm optimization algorithm was used to realize

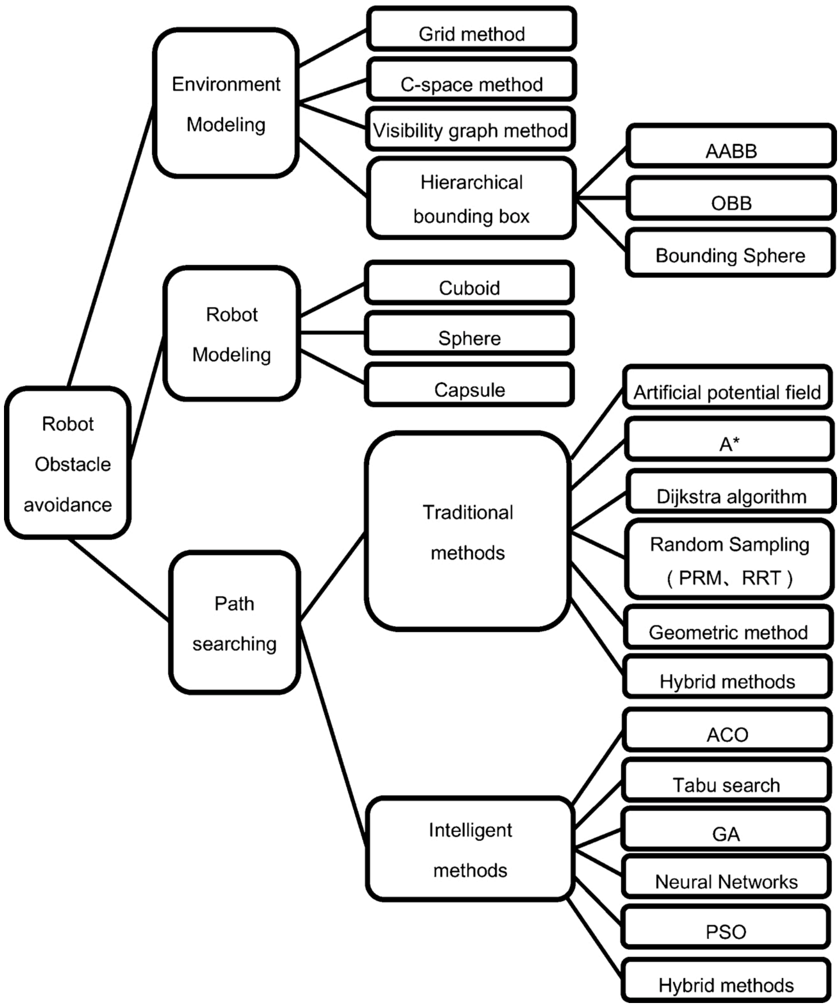

Fig. 5. Robot obstacles avoidance methods.

path planning in Ref. [42].

4.3. Robot modeling

Before computing and analyzing collision between the robot and workpiece, the robot should be modelled using some simple geometries, which include cuboid, capsule, sphere, etc. The cuboid method has good precision, but it is unsuitable for complex curve and computation-intensive. The capsule method has simple model, moderate computation, it is complicated to describe robot joint in Cartesian space at the same time. Sphere method is simple with small computation, but it is not precise enough. After analysis, the capsule method is used for robot joints modeling. The welding gun is modeled using the sphere method because of its special shape. Simple models and moderate computation can be got using these modeling methods.

4.4. Path searching methods

After environment modeling, some path searching methods are essential for finding an appropriate path. In path searching process, a reasonable path will be obtained based on the environment modeling and optimization objectives. The path searching methods can be divided as traditional and intelligent methods.

4.4.1. Traditional methods

Traditional path searching methods include artificial potential field algorithm, A*, etc. In the artificial potential field algorithm, the manipulator moves in a field of forces. The position to be reached is an attractive pole for the end effector and obstacles are repulsive surfaces for the manipulator parts [43]. Based on the attractive force and repulsive force, the manipulator will move to the object with obstacle avoidance. Li [44] presented an improved APF-based SIFORS method for autonomous mobile robot path planning in complex environments. The artificial potential field method is straightforward, but some problems may exist, such as the local optimum problem, falling into a deadlock, and moving unsteadily near the obstacle. Ref. [45] proposed a new artificial potential field algorithm for robot path planning. The effectiveness of the algorithm was verified by testing 12 different complex environments.

As effective obstacle detection algorithms, the A* and Dijkstra's algorithm have the following advantages. Unlike evolutionary algorithms, they have a deterministic nature and their performance does not depend on the initial solutions. Besides it, it is time-efficiency, specifically for 2-dimensional PP, in comparison to most of evolutionary algorithms [45]. A* algorithm was studied for the multiple-manipulator path planning, where robots were considered as a set of connected spheres and each robot was regarded as a part of the obstacle area for other robots [46].

RRT [47] algorithm is a sampling-based motion planning algorithm with probabilistic completeness. If the initial state and the target state are given, it could search the space by sampling and incrementally construct a tree structure to find the feasible path. To obtain humanoid robot's collision-free footstep planning, Ref. [48] used the swept volume approximations and the variant RRT algorithm to realize smooth collision footstep path. Ref. [49] proposed a cost-effective motion planning algorithm based on the RRT* algorithm and a stochastic optimization method to find the shortest path in the complex real environment.

4.4.2. Intelligent path searching

Intelligent path searching methods include artificial neural network, ant colony algorithm, and tabu search etc. These methods were widely used for robot path searching. An artificial neural network (ANN) trained by imperialist competitive algorithm (ICA) was proposed to obtain optimal path planning of UCAV [50]. Cao [36] proposed an improved ant colony algorithm for robot global path planning, and the pheromone evaporation rate was adjusted dynamically to enhance the global search ability and convergence speed. In addition, the artificial bee colony algorithm was used to generate an optimal path for the Unmanned Aerial Vehicles in complicate environment [51]. The genetic algorithm [52] was also used to search the optimum path for mobile robot. Obstacle avoidance and cycle time were considered to be optimized the manipulator path using the improved GA [53]. A novel methodology was presented to plan collision-free paths for a manipulator in a 3D AR environment, and a heuristic beam search algorithm was used to generate the path [54]. In Ref. [55], tabu search algorithm was used to overcome the local minimum and enhance the navigation ability of the robot in complex environments. In Ref. [56], a new PSO algorithm was used to make the mobile robot to avoid obstacles in various velocity and direction.

5. Researches on welding robot collision detection

Welding robot collision detection includes modeling and path searching. At the same time, the obstacle avoidance strategies include obstacle avoidance between welding gun and workpiece, and obstacle avoidance of robot joints. In this section, some modeling methods and welding robot obstacle avoidance strategies based on ACO combined with geometrical method are presented.

5.1. Modeling

The first step to solve the robot path planning problem is welding robot environment modeling. As the grid method is simple, the generated path is more intuitive and easier to judge the local environment, the three-dimensional grid method is selected for environmental modeling [57]. Because the traditional grid method is used in 2D space, while the working environment for the welding robot is in 3D space, the improvement of the grid method is necessary. The procedure of the 3D environment modeling with the grid method is presented as follows.

At first, the weld workpiece is simplified into some triangles. Then, the grid matrix is set up. The whole space is divided into small cubes, and the centers of the cubes are set as the welding gun's footholds. Because the diameter of the welding electrode is 4.48 mm, each side length of the cubes is set to 5 mm. Then, all centers of the cubes are mapped on the triangles. If the projected point is located in the triangle and the vertical length is less than 5 mm, the triangle is the obstacle for the center. Otherwise, the triangle is not the obstacle for the center. Furthermore, if there are no obstacle triangles for a center, the center is a free point. Otherwise, the center is an obstacle point, which means that the welding gun cannot be located in the point. At last, because the actual weld joints are located on the surface of the weldment, the weld joints cannot be located in the free points. However, in the process of path planning, the welding gun can only move among the free points. To solve this problem, a nearest free point is defined as a virtual weld joint for the weld joint. Because the distance between actual weld joints and virtual weld joints is invariable and is small in terms of the whole path length, the distance is ignored in the optimization process. The path planning mentioned below only considers the path between the virtual weld joints.

5.2. Obstacle avoidance strategies

Before welding robot path planning, collision between the robot and the environment need to be studied. Welding robot obstacle avoidance strategies include obstacle avoidance between welding gun and workpiece, and obstacle avoidance of robot joints. For obstacle avoidance between welding gun and workpiece, the grid method is selected to realize environment modeling, and the ant colony algorithm (ACO) is applied for path searching. For robot joints obstacle avoidance, based on the robot modeling, the two-level collision detection and geometrical obstacle avoidance strategy are introduced due to the complex obstacle avoidance of welding process.

5.2.1. Obstacle avoidance between robot and work piece

Obstacle avoidance is very important for robot safety operation. Collision-free paths planning for a manipulator in a 3D AR environment was studied in Ref. [54]. In Ref. [58], collision avoidance was merged into one single objective cost function, and a modified particle swarm optimization was proposed to realize redundant angle optimization for welding robot. In Ref. [59], a collision-free torch path for a ship welding robot was generated after collisions detection based on a motion planning kit. In Ref. [60], collision avoidance was considered to realize minimum cycle-time spot welding robot path optimization. In Ref. [57], the grid method was selected to realize environment modeling, and the ant colony algorithm (ACO) was applied for path searching. Local search starts from the initial solution, and begin to search the vicinity field. If a particle can find a better solution, then replace the initial solution.

ACO is a swarm intelligence optimization algorithm that stems from real ants. In 1991, Dorigo [61] firstly proposed the ACO and successfully applied it to solve different combinatorial optimization problems. The foraging behavior of the ant colony can be regarded as a distributed collaborative optimization mechanism. It is difficult to find the shortest path from the nest to the food source for a single ant, while the ant colony can find the shortest path because the ants can communicate with each other by releasing pheromones on the route passed. The basic ACO mode is presented as follows [52]:

$$P_{ij}^k = \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{j\in\Lambda} \tau_{ij}^\alpha \eta_{ij}^\beta} \tag{10}$$

$$\tau_{ij}(n+1) = \rho \times \tau_{ij}(n) + \sum_{k=1}^{M} \triangle\tau_{ij}^k \tag{11}$$

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{Q}{L_k}, \ \textit{if ant k goes through the path } (i,j) \\ \qquad\qquad 0 \ \textit{otherwise} \end{cases} \tag{12}$$

where P_{ij}^k is the transition probability of ant k from i to j, i is the location of the ant, and j is the location that the ant can arrive at. τ_{ij} is the pheromone intensity from i to j, and η_{ij} is heuristic information, where $\eta_{ij} = 1/d_{ij}$. α is the weight of the pheromone, β is the weight of heuristic information, Λ is the collection of the nodes that ants can reach, and ρ is the evaporation coefficient of pheromone on the route. $\Delta\tau_{ij}^k$ is the pheromone intensity ant k leaves on the route from i to j, and M is the amounts of ants. n is the number of iterations, d_{ij} is the objective function, which is the Euclidean distance between two points, Q is the quality coefficient of the pheromone, and L_k is path length of ant k goes from nest to food source.

When the ACO is applied for the welding robot path planning, the starting point of the welding gun is the nest of the ant colony and the destination is the food resource. Hence, the path planning based on the ACO is the food searching process for the ant colony. The communication between the ants is realized based on pheromones. The shortest path can be found through the mutual cooperation among the individuals.

5.2.2. Robot joints obstacle avoidance

Based on the above work, the path is collision-free for the welding gun and the work piece. However, in the actual industrial production process, it is not enough. The trajectory of the robot is based on the all robot joints movement, and it is necessary to ensure a collision free path for the robot joints and work piece.

To meet the actual obstacle avoidance requirements, the obstacle avoidance trajectory for the robot joints was studied based on geometric collision detection [62]. According to the welding path, there is no collision between the two robots. Hence, the collision detection focused on the detection between the robot joints and the work piece. Collision detection here included rough detection, accurate detection and path selection.

The spatial division was chose to realize rough collision detection after bounding box technology and spatial division method [62] were considered. Based on the workpiece division, the part that may be collided can be found.

It can be seen from the results of rough collision detection, the collision tends to be produced at the edge line of every plane. The bounding box [63], artificial field [64], and geometric methods [65] can be used for accurate collision detection. After these three methods were analyzed, geometric method was selected to realize accurate collision detection.

After the collision was detected between the robot and the work piece in the welding process, some obstacle avoidance strategies then were applied to avoid the collision and promise the following welding procedure.

Due to the complex obstacle avoidance of welding process, geometric method [65] was applied to avoid collision, and the middle point was found to realize obstacle avoidance. Geometric method is a mathematical method of calculating the distance from a robot to a work piece. Based on the simplified robot model, the distance between the joints of the robot and the work piece is calculated to avoid collision. According to the weld joints distribution in different regions, the transition points with the shortest path length were obtained based on the geometrical method.

For obstacle avoidance, the work-piece model was established first. Then, the transition point for obstacle avoidance was obtained using geometric reasoning. The geometric obstacle avoidance strategy is described as follows [65].

Because the two weld joints were not on the adjacent planes, and the middle transient point was found outside the work-piece. The torch moves from the starting point of the work-piece to the middle transient point, and then reaches the terminal point. For transition point selection, the safety distance was set first according to the length of the torch. Then, a transition line was obtained, and the distance between the line and work-piece was bigger than the length of the torch. Furthermore, the transition point with the shortest path was found through iteration after the transient line discretization. Obtained transient point is shown by the blue dots in Fig. 6 [65]. Besides, the in-point and out-point can be got using the same method described in the above paragraph. At last, the final path is presented in the Fig. 6. The path starts from the start point to the out point, through the transition point to the in-point, and to the terminal point finally.

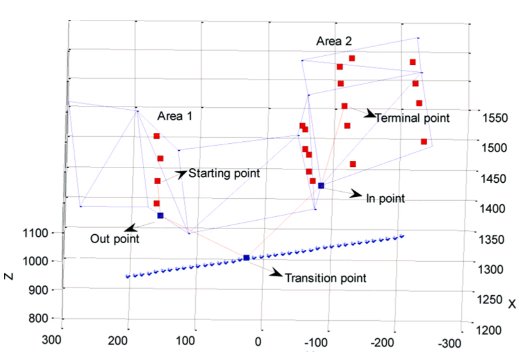

Fig. 6. Weld point on nonadjacent surfaces.

6. Welding robot multi-objective path optimization

6.1. Spot welding robot path optimization

For Spot welding robot path optimization, it is most important to finish welding work in the shortest time without collision. Hence, obstacle avoidance is always studied besides considering path length or welding time.

Some research works were done to study robot path planning and obstacle avoidance. Task sequencing and path planning for remote laser welding was studied based on TSP and meta-heuristic algorithm [66]. The virtual point was introduced to convert the multi-traveling salesman problem into a single traveling salesman problem. Artificial bee colony algorithm was used to optimize welding time [67]. Robot paths were generated to achieve minimum accumulative joint motion subject to robot reachability and path consistency [68].

Genetic algorithm was used to minimize the cycle time of the robotic spot welding operations [69]. The minimum cycle time and minimum distortion were taken as optimization objectives, and elastic net method and genetic algorithm were used to obtain the optimal welding path in Ref. [70]. Evolutionary algorithms were used to optimize a sheet-metal spot welding process with the minimum cycle-time and collision-free path [60]. In Ref. [71], welding tasks were considered as traveling salesman problems (TSP), and the point to point transfer time was used as the TSP parameter. The genetic algorithm was adopted to obtain minimum time path in spot welding tasks. In Ref. [72], collision-free and time-optimal path for manipulator was studied based on a modified GA.

Sometimes, a single robot system can't quickly complete some complex tasks such as car body welding. Hence, double-robot welding systems are developed for efficiency improvement. All the welding joints were divided into two groups by using the elite particle swarm algorithm [73], and the double-robot path planning was treated as the welding path planning of the two individual robots. In Ref. [65], the shortest path length and energy consumption were considered as optimization objectives, and obstacle avoidance was set as the constraint condition. The clustering guidance multi-objective particle swarm algorithm was applied to solve the spot welding robot path planning problem (Fig. 7).

In Ref. [57], grid method was used for modeling, and ant colony algorithm was applied as search strategy to realize obstacle avoidance between welding gun and workpiece. For obstacle avoidance of robot joints, the robot was modeled using the sphere and the capsule. Besides, two-level collision detection and geometrical collision avoidance were used to obtain a collision-free path. At last, an improved particle swarm optimization algorithm was used to realize global path planning.

Above studies realized robot path optimization, and several factors

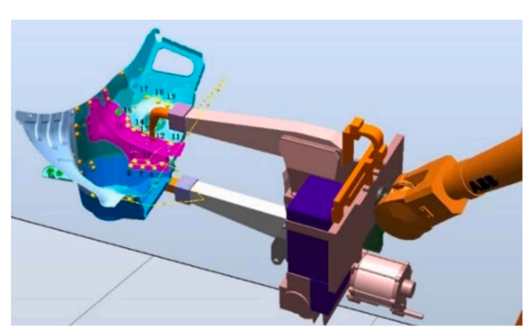

Fig. 7. Simulation test.

were considered for path optimization. It can be seen that the intelligent algorithms were used to obtain optimum welding path sequences as effective methods. Besides it, one key problem is the effective obstacle detection strategy.

6.2. Arc welding robot path optimization

For arc welding robot path planning, welding deformation is the most important factor to be considered. Moreover, obstacle avoidance, path length and energy consumption also should be taken into account for welding robot optimization. Some research works were also conducted in these aspects.

Genetic algorithms were applied to solve welding task sequencing problems, and robot operations for weld sequences were simulated graphically based on the robot simulation software IGRIP [74]. In Ref. [75], the robot path planning was performed by solving a traveling salesman problem (TSP) over the seam positions in the Cartesian space, and a simple heuristic approach was developed. An optimal robot path with minimum joint movement was obtained using a beam search algorithm in Ref. [76]. It can be seen that only single objective is considered to conduct path planning, and it is not enough for arc welding robot path planning. Hence, some research works were conducted for multi-objective arc welding robot path planning.

In Ref. [77], optimization of the path length and total welding deformation for arc welding robot is studied. Convergence and diversity performance of the presented DDN-MOPSO algorithm was verified based on the TSP problems of 80 cities. To shorten the calculation time, the surrogate model is designed to calculate the value of welding deformation according to the DOE. After the optimization problem description, the two-objective Pareto front was obtained using the DDN-MOPSO algorithm. Simulation results showed the effectiveness of the improved algorithm.

Besides it, the shortest path length, energy consumption and welding deformation are considered as optimization objectives, and welding deformation is studied based on the proxy model to improve optimization efficiency. Then, an adaptive neighborhood multi-objective discrete optimization algorithm based on event triggering (DAMOEA/D-ET) was proposed and validated. The grid method and the decomposition multi-objective algorithm (MOEA/D) were used to coordinate. The adaptive mesh coefficient and the adaptive neighborhood strategy were used to improve the quality and distribution of the non-inferior solution. Finally, the proposed intelligent algorithm was applied to solve the problem of welding robot path planning. The simulation results show that the proposed optimization strategy can obtain desired optimization effect. Balance beam model and algorithms comparison results are presented in Figs. 8 and 9.

It can be found that the welding deformation is crucial for welding quality, and it is essential to study welding deformation for arc welding. As a result, some works were done for deformation, energy consumption due to its importance. To solve the three-objective optimization problem, Shao et al. [78] studied the relationship between welding parameters and welding sequence, and established the proxy model. Then, the multi-objective particle swarm optimization algorithm was used to optimize the welding parameters. The robot speed was optimized to minimize welding distortion while keeping complete joint penetration [79]. For energy consumption, the major factors and devices determining consumed energy were found by developing an estimation model [80]. And some research work on energy consumption were conducted for industrial robot in Refs. [81–83].

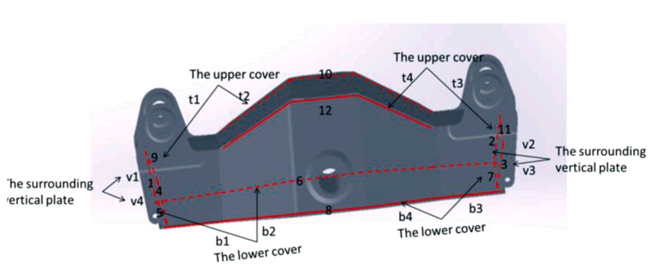

Fig. 8. Balance beam model.

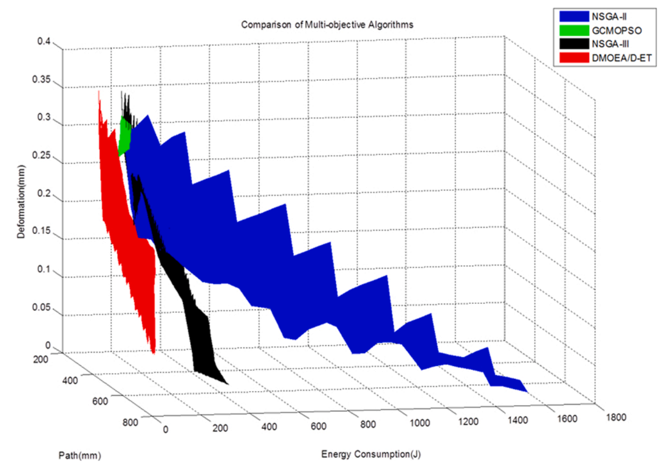

Fig. 9. Algorithms comparison results.

Based on the above analysis, it can be found the welding deformation calculation is important for path optimization. Welding deformation is always calculated using some software and it is time-consuming. Hence, the surrogate model was applied to enhance computation efficiency. Efficiency and precise of the deformation calculation is desired. At the same time, effective and efficient obstacle avoidance strategies are also crucial for arc welding robot path optimization.

7. Conclusions

For welding robot path intelligent optimization, researches on path optimization problem description, environment modeling, obstacle avoidance strategies, and welding robot multi-objective path optimization are presented in this article. It can be seen that intelligent optimization algorithms are effective for welding robot path planning. At the same time, obstacle avoidance strategy and deformation calculation are two crucial problems for welding robot path planning.

The future work will focus on the real welding robot system optimization to test the effectiveness of the proposed strategies in practical industrial application, improve the optimization effects, and improve welding automation and intelligence.

- The model for the workpiece and the fixture should be more complex and accurate. At the same time, effective and efficiency obstacle avoidance strategies should be studied.
- Cooperation between two robots is necessary to be studied to obtain better optimization results.
- Welding parameters optimization deserve more attention due to its influences on welding quality.
- More works on intelligent optimization algorithm are also needed to be done to obtain better optimization effects for three optimization objectives.
- The optimization results will be tested using some offline programing software. Besides it, calibration will be studied to realize path optimization for real welding robot system.
- Practical experiments need to be conducted to improve the optimization effects.
- Welding robot scheduling, digital twins for welding robot system are essential to be studied in detailed to improve welding robot intelligence.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Liu YK, Zhang YM. Fusing machine algorithm with welder intelligence for adaptive welding robots. J Manuf Process 2017;27:18–25.
[2] Zhang LJ, Bai QL, Ning J, Wang A, Yang NJ, Yin XQ, Zhang JX. A comparative study on the microstructure and properties of copper joint between MIG welding and laser-MIG hybrid welding. Mater Des 2016;110:35–50.
[3] Xu YL, Lv N, Fang G, Du SF, Zhao WJ, Ye Z, Chen SB. Welding seam tracking in robotic gas metal arc welding. J Mater Process Technol 2017;248:18–30.
[4] Wang XW. Three dimensional vision application in GTAW process modeling and control. IJAMT 2015;80:1601–11.
[5] Holland JH. Adaptation in natural and artificial systems. Ann Arbor, MI: University of Michigan Press; 1975.
[6] Zheng JH, Zou J. Multi-objective evolutionary optimization. Beijing: Science Press; 2017. p. 2–22.
[7] Kennedy J, Eberhart RC. Particle swarm optimization. Proc of IEEE int conf on neural networks, Perth, Australia: IEEE Service Center 1995:1942–8.
[8] Guo YN, Pei Z, Cheng J, Wang C, Gong DW. Interval multi-objective quantum-inspired cultural algorithms. Neural Comput Appl 2018;30(3):709–22.
[9] Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing. Science 1983;220(4598):671–80.
[10] Karaboga D. A powerful and efficient algorithm for numerical function optimization: artificial bee colony algorithm. J Glob Optim 2007;39:459–71.
[11] Coello Coello CA, Pulido GT, Lechuga MS. Handing multiple objectives with particle swarm optimization. IEEE Trans Evol Comput 2004;8(3):256–79.
[12] Zheng XW, Liu H. A hybrid vertical mutation and self-adaptation based MOPSO. Comput Math Appl 2009;57(11):2030–8.
[13] Amoshahy MJ, Shamsi M, Sedaaghi MH. A novel flexible inertia weight particle swarm optimization algorithm. PLoS One 2016;38:281–95.
[14] Najeh B. Improved accelerated PSO algorithm for mechanical engineering optimization problems. Appl Soft Comput 2016;40:455–67.
[15] Ye Y, Yin CB, Gong Y, Zhou JJ. Position control of nonlinear hydraulic system using an improved PSO based PID controller. Mech Syst Signal Proc 2017;83:241–59.
[16] Benvidi A, Abbasi S, Gharaghani S, Dehghan MT, Masoum S. Spectrophotometric determination of synthetic colorants using PSO-GA-ANN. Food Chem 2017;220: 377–84.
[17] Wu DQ, Zheng JG. Parallel particle swarm optimization algorithm based on hybrid strategy adaptive learning. J Control Decis 2013;28:1087–93.
[18] Wang XW, Li RR. Intelligent modelling of back-side weld bead geometry using weld pool surface characteristic parameters. J Intell Manuf 2014;25(6):1301–13.
[19] Wang XW, Shi YP, Yan YX, Gu XS. Intelligent welding robot path optimization based on discrete elite PSO. Soft Comput 2017;21(20):5869–81.
[20] Deb K, Pratap A, Agarwal S, Meyarivan T. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 2002;6(2):182–97.
[21] Zitzler E, Laumanns M, Thiele L. SPEA2: improving the strength Pareto evolutionary algorithm for multiobjective optimization in Evolutionary Methods for Design, Optimisation, and Control. Barcelona, Spain: CIMNE; 2002. p. 95–100.
[22] Zitzler E, Künzli S. Indicator-based selection in multiobjective search. Heidelberg, Germany: Springer; 2004. p. 832–42.
[23] Beume N, Naujoks B, Emmerich M. SMS-EMOA: multiobjective selection based on dominated hypervolume. Eur J Oper Res 2007;181(3):1653–69.
[24] Bader J, Zitzler E. HypE: an algorithm for fast hypervolume-based many-objective optimization. Evol Comput 2011;19(1):45–76.
[25] Schaffer JD. Multiple objective optimization with vector evaluated genetic algorithms. Proc. 1st int. conf. genetic algorithms 1985:93–100.
[26] Zhang QF, Li H. MOEA/D: a multiobjective evolutionary algorithm based on decomposition. IEEE Trans Evol Comput 2007;11(6):712–31.
[27] Cheng R, Jin Y, Olhofer M, Sendhoff B. A reference vector guided evolutionary algorithm for many-objective optimization. IEEE Trans Evol Comput 2016;20(5): 773–91.
[28] Deb K, Jain H. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, Part I: Solving problems with box constraints. IEEE Trans Evol Comput 2014;18(4):577–601.
[29] Jiang S, Yang S. A strength Pareto evolutionary algorithm based on reference direction for multiobjective and many-objective optimization. IEEE Trans Evol Comput 2017;21(3):329–46.
[30] Moradpour MA, Hashemi SH, Khalili K. Multi-objective optimization of welding parameters in submerged arc welding of API X65 steel plates. J Iron Steel Res Int 2015;22(9):870–8.
[31] Gao Z, Shao X, Jiang P, Wang C, Zhou Q, Cao L, Wang. Multi-objective optimization of weld geometry in hybrid fiber laser-arc butt welding using kriging model and NSGA-II. Appl Phys A 2016;122(6):1–12.
[32] Lee J, Lee SI, Ahn J, Choi HL. Pareto front generation with knee-point based pruning for mixed discrete multi-objective optimization. Struct Multidiscip Optim 2018;58(2):823–30.
[33] Yu X, Chen WN, Gu T, Zhang HHX, Yuan HQ, Kwong S, Zhang J. Set-based discrete particle swarm optimization based on decomposition for permutation-based multiobjective combinatorial optimization problems. IEEE Trans Cybern 2018;48 (7:2139–53.
[34] Wang XW, Xia ZL, Gu XS. Multi-objective path planning of welding robot based on DMOEA/D-ET algorithm. J South China Univ Technol (Natural Sci Ed) 2019;27(4): 99–106.
[35] Howden WE. The sofa problem. Comput J 1968;11(3):299–301. https://doi.org/10.1093/comjnl/11.3.299.
[36] Cao J. Robot global path planning based on an improved ant colony algorithm. J Comput Commun 2016;4:11–9.
[37] Willms AR, Yang SX. Real-time robot path planning via a distance-propagating dynamic system with obstacle clearance. IEEE Trans Syst Man Cybern B Cybern 2008;38:884–93.
[38] Lozano-Perea T. Spatial planning: a configuration space approach. IEEE Trans Comput 1983;C-32(2):108–20.
[39] Yu NG, Wang ZK. Collision avoidance planning of manipulator based on C-space layered search arithmetic. In: Proceedings of the 2011 international conference on Electronic and Mechanical Engineering and Information Technology (EMEIT), Harbin, China, 6; 2011. p. 3258–62.
[40] Masehian E, Amin Naseri MR. A voronoi diagram-visibility graph-potential field compound algorithm for robot path planning. J Robot Syst 2004;21:275–300.
[41] Teschner M, Kimmerle S, Heidelberger B, Zachmann G, Raghupathi L, Fuhrmann A, et al. Collision detection for deformable objects. Computer graphics forum, 24. Oxford, United Kingdom: Blackwell Publishing Ltd.; 2005. p. 61–81.
[42] Mo H, Xu L. Research of biogeography particle swarm optimization for robot path planning. Neuro Comput 2015;148:91–9.
[43] Oussama K. Real-time obstacle avoidance for manipulators and mobile robots. Int J Rob Res 1986;6(5):90–8.
[44] Li G, Tong S, Cong F, Yamashita A, Asama H. Improved artificial potential field-based simultaneous forward search method for robot path planning in complex environment. In: Proceedings of the 2015 IEEE/SICE international symposium on system integration (SII), Nagoya, Japan, 11–13 December 2015; 2015. p. 760–5.
[45] Milad N, Esmaeel K, Samira D. Multi-objective multi-robot path planning in continuous environment using an enhanced genetic algorithm. Expert Syst Appl 2019;115:106–20.
[46] Tavares P, Lima J, Costa P, Moreira AP. Multiple manipulators path planning using double A*. Ind Robot 2016;43:657–64.
[47] Davoodi M, Mohades A, Sheikhi F, Khanteimouri P. Data imprecision under λ-geometry model. Inf Sci (Ny) 2015;295:126–44.
[48] Perrin N, Stasse O, Baudouin L, Lamiraux F, Yoshida E. Fast humanoid robot collision-free footstep planning using swept volume approximations. J IEEE Trans Rob 2012;28(2):427–39.
[49] Suh J, Gong J, Oh S. Fast sampling-based cost-aware path planning with nonmyopic extensions using cross entropy. J IEEE Trans Rob 2017;33:1313–26.
[50] Duan H, Huang L. Imperialist competitive algorithm optimized artificial neural networks for UCAV global path planning. Neurocomputing 2014;125:166–71.
[51] Lei L, Shiru Q. Path planning for unmanned air vehicles using an improved artificial bee colony algorithm. In: Proceedings of the 2012 31st Chinese Control Conference (CCC), Hefei, China, 25–27 July 2012; 2012. p. 2486–91.
[52] Wang Y, Mulvaney D, Sillitoe I. Genetic-based mobile robot path planning using vertex heuristics. Cybernetics and intelligent systems, 2006 IEEE conference on 2006:1–6.
[53] Gentilini I, Nagamatsu K, Shimada K. Cycle time based multi-goal path optimization for redundant robotic systems. In: International conference on Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ; 2013. p. 1786–92.
[54] Chong JWS, Ong SK, Nee AYC, Youcef-Youmi K. Robot programming using augmented reality: an interactive method for planning collision-free paths. Rob Comput Integr Manuf 2009;25:689–701.
[55] Lei XK, Liu MY, Yan MD. Tabu search based autonomous navigation algorithm for mobile robot. Proc IEEE Conf Decis Control 2011;26(9):1310–4.
[56] Min HQ, Zhu JH, Zheng XJ. Obstacle avoidance with multi-objective optimization by PSO in dynamic environment. Machine learning and cybernetics. Proceedings of 2005 international conference on 2005.
[57] Wang XW, Tang B, Yan YY, Gu XS. Double-robot obstacle avoidance path optimization for welding process. Math Biosci Eng 2019;16(5):5697–708.
[58] Doan NCN, Lin W. Optimal robot placement with consideration of redundancy problem for wrist-partitioned 6R articulated robots. Rob Comput Integr Manuf 2017;48:233–42.
[59] Ryu LH, Kim TW, Oh MJ, Ku NK, Lee KY. Workspace analysis to generate a collision-free torch path for a ship welding robot. J Mar Sci Technol 2009;14(3): 345–58.
[60] Givehchi M, Ng A, Wang LH. Evolutionary optimization of robotic assembly operation sequencing with collision-free paths. J Manuf Syst 2011;30:196–203.
[61] Colorni A, Dorigo M, Maniezzo V. Distributed optimization by ant colonies. Proceedings of the first European conference on artificial life 1991;142:134–42.
[62] Qu XY, Ma L, Yao CZ. Research of collision detection algorithm based on hybrid bounding box for complex environment. In: 2016 International Conference on Integrated Circuits and Microsystems (ICICM) IEEE; 2016. p. 248–52. https://doi.org/10.1109/ICAM.2016.7813602.

[63] Zhu YF, Meng J. Efficient approach based on hybrid bounding volume hierarchy for real-time collision detection. J Syst Simul 2008;19:5099–104.
[64] Yan Y, Yan Z. Collision avoidance planning in multi-robot based on improved artificial potential field and rules. IEEE international conference on robotics and biomimetics 2009.
[65] Wang XW, Yan YX, Gu XS. Spot welding robot path planning using intelligent algorithm. J Manuf Process 2019;42:1–10.
[66] Kovács A. Integrated task sequencing and path planning for robotic remote laser welding. Int J Prod Res 2016;4:1210–24.
[67] Wang ZT, Feng ZL, Ye GY, Xu YT, Fu JZ. Path planning of double robot based on artificial bee colony algorithm. Trans China Weld Inst 2015;2:97–100.
[68] Fang HC, Ong SK, Nee AYC. Adaptive pass planning and optimization for robotic welding of complex joints. Adv Manuf 2017;5(2):93–104.
[69] Givehchi M, Ng AHC, Wang L. Spot-welding sequence planning and optimization using a hybrid rule-based approach and genetic algorithm. Rob Comput Integr Manuf 2011;4:714–22.
[70] Yang H, Shao H. Distortion-oriented welding path optimization based on elastic net method and genetic algorithm. J Mater Process Tech 2009;209(9):4407–12.
[71] Zhang Q, Zhao MY. Minimum time path planning for robotic manipulator in drilling/spot welding tasks. J Comput Des Eng 2016;3(2):132–9.
[72] Zacharia PT, Xidias EK, Aspragathos NA. Task scheduling and motion planning for an industrial manipulator. Rob Comput Integr Manuf 2013;29(6):449–62.
[73] Wang XW, Shi YP, Ding DY, Gu XS. Double global optimum genetic algorithm–particle swarm optimization based welding robot path planning. Eng Optimiz 2016;48:299–316.
[74] Kim KY, Kim DW, Nnaji BO. Robot arc welding task sequencing using genetic algorithms. IIE Trans 2002;34(10):865–80.
[75] Reinhart G, Munzert U, Vogl W. A programming system for robot-based remote-laser-welding with conventional optics. CIRP Ann Manuf Technol 2008;57(1):37–40.
[76] Fang H, Ong S, Nee A. Robot path planning optimization for welding complex joints. Int J Adv Manuf Technol 2017;90(9–12):3829–39.
[77] Wang XW, Min Y, Gu XS. Multi-objective path optimization for arc welding robot based on DDN-MOPSO. Int J Adv Robot Syst 2019. https://doi.org/10.1177/1729881419879827.
[78] Shao Q, Xu T, Yoshino T, Song N. Multi-objective optimization of gas metal arc welding parameters and sequences for low-carbon steel (Q345D) T-joints. J Iron Steel Res Int 2017;24(5):544–55.
[79] Ericsson M, Nylén P. A look at the optimization of robot welding speed based on process modeling. Weld J 2007;86(8):238–44.
[80] Um J, Stroud IA. Total energy estimation model for remote laser welding process. Procedia CIRP 2013;7(12):658–63.
[81] Mohammed A, Schmidt B, Wang LH, Gao L. Minimizing energy consumption for robot arm movement. Procedia CIRP 2014;25:400–5.
[82] Chen CY, Liao HL, Montavon G, Deng SH. Nozzle mounting method optimization based on robot kinematic analysis. J Therm Spray Technol 2016;25(6):1138–48.
[83] Bukata L, Šůcha P, Hanzálek Z. Optimizing energy consumption of robotic cells by a branch & bound algorithm. Comput Oper Res 2019;102:52–66.