Topology

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CHAPTER 1

SET THEORY AND LOGIC

We shall assume that what is meant by a *set* of objects is intuitively clear, and we shall proceed on that basis without analyzing the concept further.

1 Fundamental Concepts

1.1 Basic Notation

Commonly we shall use capital letters $A, B, \ldots$ to denote sets, ans lowercase letters $a, b, \ldots$ to denote the ***objects*** or ***elements*** belonging to these sets.

We say that A is a ***subset*** of B is every element of A is also and element of B; and we express this fact by writing

$$A \subset B.$$

If $A \subset B$ and A is different from B, we say that A is a ***proper subset*** if B, and we write

$$A \subsetneq B.$$

The relations $\subset$ and $\subsetneq$ are called ***inclusion*** and ***proper inclusion***, respectively. If $A \subset B$, we also write $B \supset A$, which is read "B ***contains*** A."

1.2 The Union of Sets and the Meaning of "or"

Given two sets A and B, one can form a set from them that consists of all the elements of A together with all the elements of B. This set is called the ***union*** of A and B and is denoted by $A \cup B$.

1.3 The Intersection of Sets, the Empty Set, and the Meaning of "If ... Then"

Given sets A and B, another way one can form a set is to take the common part of A and B. This set is called the ***intersection*** of A and B and is denoted by $A \cap B$.

We introduce a special set that we call the ***empty set***, denoted by $\varnothing$, which we think of as "the set having no elements."

Using this convention, we express the statement that A and B have no elements in common by the equation

$$A \cap B = \varnothing.$$

We also express this fact by saying that A and B are ***disjoint***.

Mathematicians have agreed always to use "if ... then" in the first sense, so that a statement of the form "If P, then Q" means that if P is true, Q is true also, but if P is false, Q may be either true or false.

As an example, consider the following statement about real numbers:

$$If\ x > 0,\ then\ x^3 \neq 0.$$

It is a statement of the form, "If P, then Q," where P is the phrase "$x > 0$" (called the ***hypothesis*** of the statement) and Q is the phrase "$x^3 \neq 0$" (called the ***conclusion*** of the statement).

Another true statement about real numbers is the following:

$$If\ x^2 < 0,\ then\ x = 23;$$

in every case for which the hypothesis holds, the conclusion holds as well. Of course, it happens in this example that there are no cases for which the hypothesis holds. A statement of this sort is sometimes said to be ***vacuously true***.

1.4 Contrapositive and Converse

Give a statement of the form "If P, then Q," its ***contrapositive*** is defined to be the statement "If Q is not true, then P is not true."

There is another statement that can be formed from the statement $P \Rightarrow Q$. It is the statement

$$Q \Longrightarrow P,$$

which is called the ***converse*** of $P \Rightarrow Q$.

1.5 Negation

If one wishes to form the contrapositive of the statement $P \Rightarrow Q$, one has to know how to form the statement "not P", which is called the ***negation*** of P.

1.6 The Difference of Two Sets

There is one other operation on sets that is occasionally useful. It is the ***difference*** of two sets, denoted by $A - B$, and defined as the set conssiting of those elements of A that are not in B. It is sometimes called the ***complement*** of B relative to A, or the complement of B *in* A.

1.7 Collections of Sets

Given a set A, we can consider sets whose elements are subsets of A. In particular, we can consider the set of all subsets of A. This set is sometimes denoted by the symbol $\mathcal{P}(A)$ and is called the ***power set*** of A.

When we have a set whose elements are sets, we shall often refer to it as a ***collection*** of sets and denote it by a script letter.

1.8 Arbitrary Unions and Intersections

Given a collection $\mathscr{A}$ of sets, the ***union*** of the elements of $\mathscr{A}$ is defined by the equation

$$\bigcup_{A\in\mathscr{A}} A = \{x \mid x \in A \text{ for at least one } A \in \mathscr{A}\}.$$

The ***intersection*** of the element of $\mathscr{A}$ is defined by the equation

$$\bigcap_{A\in\mathscr{A}} A = \{x \mid x \in A \text{ for every } A \in \mathscr{A}\}.$$