AME 455 Final Project Due: 5/2/08 (Experimental)

The objective of this project is to implement a closed-loop control of a real system. As mentioned in class, modern control systems are computer-based, i.e. take measurements with some frequency, calculate the error between desired input and the measured output of the system, and send a command signal to the actuator controlling the system. The control system selected for this project consists of a pendulum with a small DC motor operating at 0-5 V. Attached to the shaft of the motor is a 2" airscrew (propeller), which provides tangential thrust force, *T*. A MATLAB script file is used to command the motor and measure the position (angle) of the pendulum. The software sends a signed value between -100 and +100 which represents a % of 5V, i.e. when you send 30, the effective voltage on the motor is approximately 1.5V (30% of 5V). **The design objective is to design and implement a controller that will command the pendulum to move to 50 degrees with a 5% settling time not exceeding 2 seconds and a steady-state error not exceeding 10% (5 degrees).**

This design project requires you to complete the following design tasks:

1. Develop a mathematical model (differential equation of motion) of the pendulum describing the relation between the input force *T*, and the output angle θ, assuming that the rod is weightless, i.e. the total mass *m* is that of the motor and is concentrated at the tip of the rod. The parameters of the system are the mass of the motor, *m*; the length of the rod, *l*, viscous friction in the pivot point producing a frictional torque $c\dot{\theta}$, and the gravitational acceleration *g*. **Submit: second order differential equation for** θ.

2. The differential equation from Step 1 should be non-linear, i.e. does not have a rational transfer function in the s-domain. Show that if you choose the thrust force

$$T = mg\sin\theta + u,$$

the resulting system between the new input *u* and angle θ becomes linear and you can find its transfer function. **Submit transfer function between *u* and** θ (containing the parameters *l*, *m*, and *c*)**.**

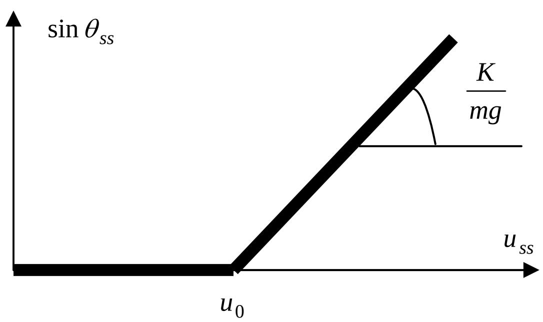
3. Note that you cannot apply directly the desired thrust $T = mg\sin\theta + u$ to the pendulum. Instead, you do so by commanding the motor to do it for you. The thrust of the propeller *T* is a function of the input voltage to the motor, $\tilde{u}$. Unfortunately, at low voltages (below some u_0) the motor doesn't turn, i.e. the voltage is not sufficient to overcome the internal friction of the motor shaft. Therefore, you could expect that the thrust is proportional to the difference between the applied voltage and this threshold voltage, i.e.

$$T = K(\tilde{u} - u_0), \quad \tilde{u} > u_0. \qquad (1)$$

Therefore, the propeller-motor also contains non-linearity represented by (1). Similar to Step 2, you are asked to eliminate the non-linear part. Using the thrust model (1), show that by selecting the input to the motor in the form $\tilde{u} = u_0 + S\sin\theta + u(t)$, you can find a suitable value

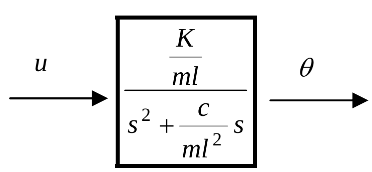
for the coefficient S such that the resulting differential equation between $u(t)$ and θ is linear and has almost identical transfer function to the one found in Step 2 (with the addition of a constant K). (Hint: select S such that the sine term cancels out).
Submit: (1) expression for the parameter S in terms of the physical parameters of the system including the motor proportionality constant K. (2) The transfer function between $u(t)$ and θ containing the parameters of the system.

4. Before proceeding with the design and implementation of a controller, you need to find a way to determine the values of the system parameters. Using the differential equation found in Step 1, find a relationship between a steady-state thrust T_{ss} and the resulting steady-state angle θ_{ss}. Notice from Step 3, that a steady-steady state thrust can be produced by setting a steady-state input to the motor, $\tilde{u}_{ss}$ according to the relationship $T_{ss} = K(\tilde{u}_{ss} - u_0)$. Using these two relations, show that if one conducts a series of tests by applying different steady-state inputs $\tilde{u}_{ss}$ and measures the steady-state deflection angles, θ_{ss}, the resulting plot should look like



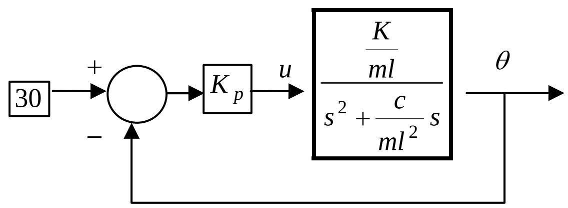
EXPERIMENT #1 Using the provided Matlab code "Open_loop.m" set various values for $\tilde{u}_{ss}$ and measure the resulting θ_{ss}. Develop a plot similar to the one shown above and extract u_0 and K/mg. What is the relationship between S and K/mg? **Submit experimental plot $\sin\theta_{ss}$ vs $\tilde{u}_{ss}$, and value for parameters u_0, K/mg, and S. NOTE: USE VALUES FOR $\tilde{u}_{ss}$ BETWEEN 0 AND 85. ABOVE 85, THE PENDULUM WILL MAKE A FULL REVOLUTION AND WILL TWIST THE WIRES. IN SUCH EVENT PRESS CTRL-C TO STOP THE PROGRAM WHILE HOLDING THE ROD STEADY AND PREVENTING FURTHER TWISTING.**

5. Recognize that $\tilde{u} = u_0 + S\sin\theta + u(t)$ with the values from Step 4, you are able to command the pendulum as if it were a linear system. Show that this later system is described by



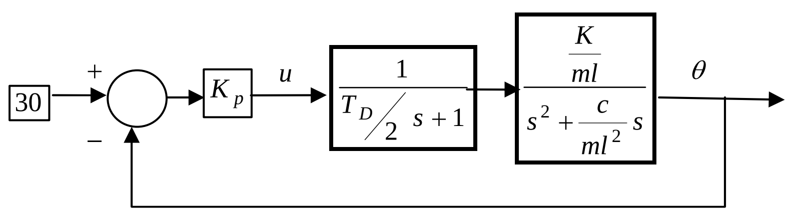
At this point you still don't know the values of the parameters, but you should be able to sketch the root-locus of the closed-loop system. Q1. Do you expect that the closed loop system will be stable for all values of the proportional gain K_p? Explain? **Submit sketched root locus and answer to Q1.**

Experiment #2: Using *Closed_Loop.m* file, run tests with various values of the proportional gain K_p starting from 0.02 and stepping up with increments of 0.03. Do you reach a critical value of the gain K_p^* for which the closed loop becomes unstable? Use a reference value for the desired angle $\theta_0 = 30$ and control law $\tilde{u} = u_0 + S \sin\theta + K_p(30 - \theta(t))$



Submit: Plots for step response using gain $K_p = 0.02;\ \ 0.05\ \ \ 0.3;\ \ 0.8$. From the plots estimate the steady-state error for each run and report it. What is the type of the above system based on its transfer function? What is its expected steady-state error due to a step input based on its transfer function? Do you observe the expected steady-state error behavior from your experiments? Do you observe the expected stability behavior from your experiments? For at least one of your simulations, induce disturbances by slightly pushing on the rod and letting it go. Does the system recover? For what gain is the recovery faster? On each plot, mark the gain used.

6. Read the attached discussion on digital control and time delay from Franklin & Powell's textbook. Using the vector *t(i)* in your matlab files (after running a pendulum experiment), estimate the sampling time $T_D = t(i+1) - t(i)$ for your computer system and model the time delay as a first-order system in series with the pendulum



For $\dfrac{K}{ml} = 33$, $\dfrac{c}{ml^2} = 0.7$, and the value of T_D found above, plot the root locus of the closed-loop system using rlocus command.. From the plot explain the observations found in step 5**. Submit: root locus plot, critical value of the gain K_p^* and explanation of how step 6 explains experimental observations from Step 5.**

7. ~~Using the transfer function developed in Step 6~~. Experimentally select a value of the gain K_p for which the system has a steady state error of less than 10% and a 5% settling time of less than 2 seconds in response to a step input. The desired final angle θ is 50^{O}. Use the selected value K_p with the matlab code used in Step 5 (*closed_loop.m*). **Submit the value of K_p satisfying the design requirements and the experimental step response. Mark on the plot the experimental steady state error and the 5% settling time, demonstrating that the design requirements have been met.**

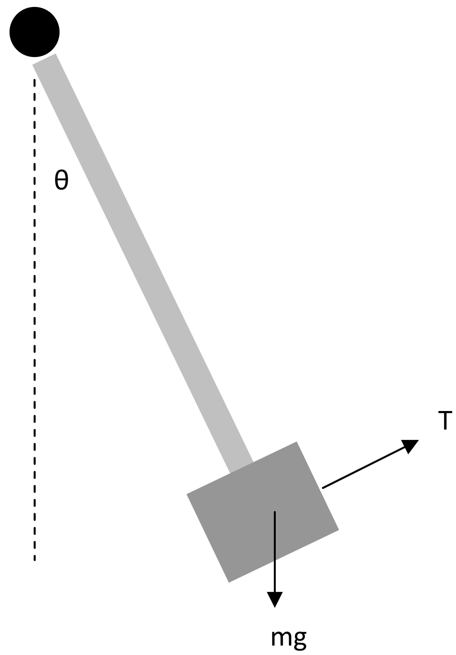


Figure 1 - Pendulum with propeller motor