**Adjacency Lists:**

$$A_S = \{A, C, G\}$$

$$A_A = \{B, S\}$$

$$A_B = \{A\}$$

$$A_C = \{D, E, F, S\}$$

$$A_D = \{C\}$$

$$A_E = \{C, H\}$$

$$A_F = \{C, G\}$$

$$A_G = \{F, H, S\}$$

$$A_H = \{E, G\}$$

Take S to be the source vertex.

Given $G(V, E)$ represented in terms of adjacency lists A_v and a distinguished source vertex $s \in V$, find $d(v) \equiv d(s, v)$ for all $v \in V$.

① *Set things up:*

$d(v) \leftarrow \infty$ for all $v \neq s \in V$

$d(s) \leftarrow 0$ and $Q \leftarrow \{s\}$

② *Main loop: continues until the queue is empty*

While ($Q \neq \emptyset$) {

 Pop a vertex v off the left end of Q .

Examine each of v 's neighbours

 For each $w \in A_v$ {

 If($d(w) = \infty$) then {

Set $d(w)$ and get ready to process w 's neighbours

$d(w) \leftarrow d(v) + 1$

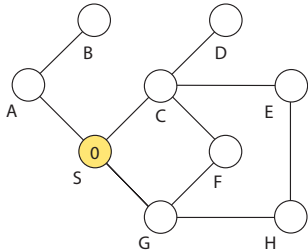
 Push w on to the right end of Q .

 }

 }

}

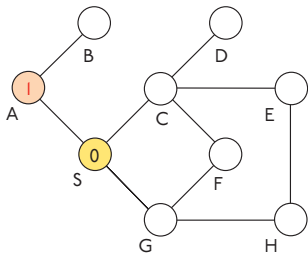
Breadth First Search: in action



v	w	Action	Queue
-	-	<i>Start</i>	$\{S\}$

Yellow vertex is the source vertex, S .

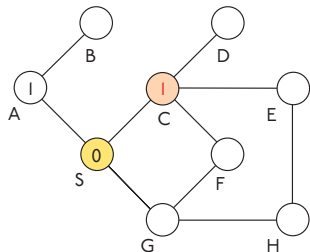
Breadth First Search: in action



v	w	Action	Queue
-	-	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$

Yellow vertex is v , red is w .

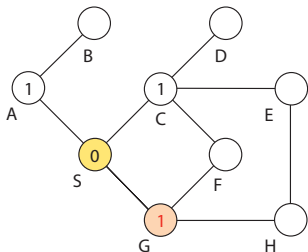
$$A_v = A_S = \{A, C, G\}$$



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$

Yellow vertex is v , red is w .

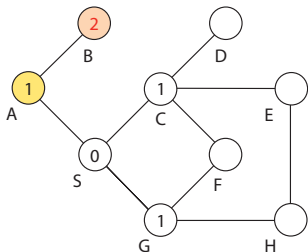
$$A_v = A_S = \{A, C, G\}$$



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$

Yellow vertex is v , red is w .

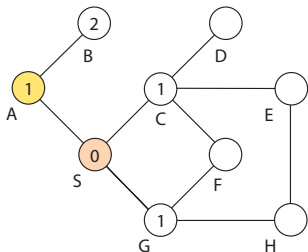
$$A_v = A_S = \{A, C, G\}$$



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$

Yellow vertex is v , red is w .

$$A_v = A_A = \{B, S\}$$

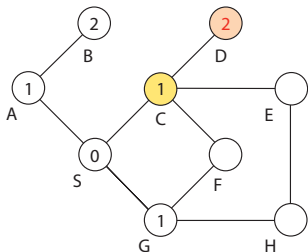


v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$

Yellow vertex is v , red is w .

$$A_v = A_A = \{B, S\}$$

Breadth First Search: in action

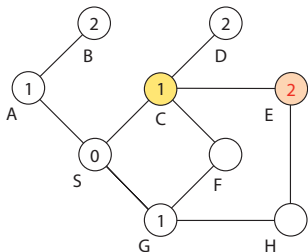


v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$

Yellow vertex is v , red is w .

$$A_v = A_C = \{D, E, F, S\}$$

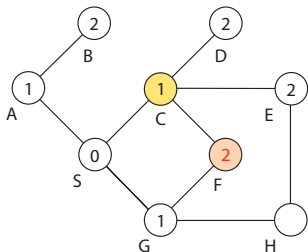
Breadth First Search: in action



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$

Yellow vertex is v , red is w .

$$A_v = A_C = \{D, E, F, S\}$$

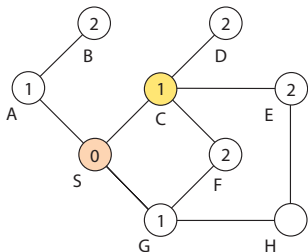


v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$

Yellow vertex is v , red is w .

$$A_v = A_C = \{D, E, F, S\}$$

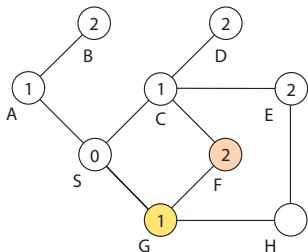
Breadth First Search: in action



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$

Yellow vertex is v , red is w .

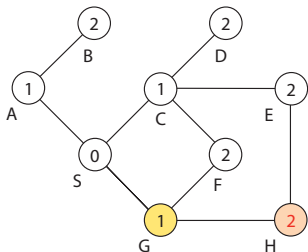
$$A_v = A_C = \{D, E, F, S\}$$



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$

Yellow vertex is v , red is w .

$$A_v = A_G = \{F, H, S\}$$

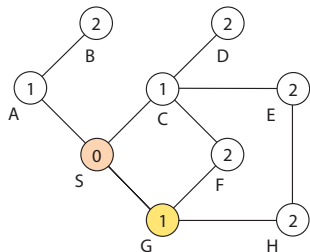


v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$

Yellow vertex is v , red is w .

$$A_v = A_G = \{F, H, S\}$$

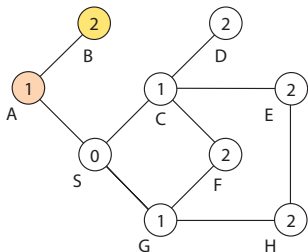
Breadth First Search: in action



Yellow vertex is v , red is w .

$$A_v = A_G = \{F, H, S\}$$

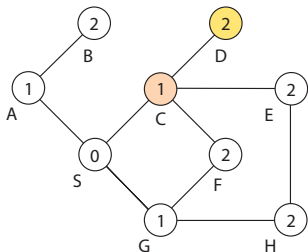
v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$



Yellow vertex is v , red is w .

$$A_v = A_B = \{A\}$$

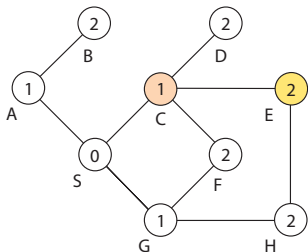
v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$



Yellow vertex is v , red is w .

$$A_v = A_D = \{C\}$$

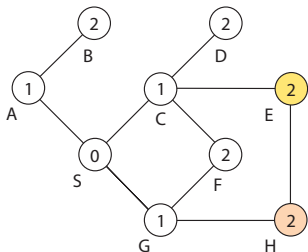
v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$



Yellow vertex is v , red is w .

$$A_v = A_E = \{C, H\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$

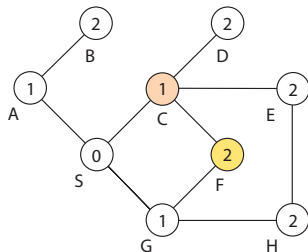


Yellow vertex is v , red is w .

$$A_v = A_E = \{C, H\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$

Breadth First Search: in action

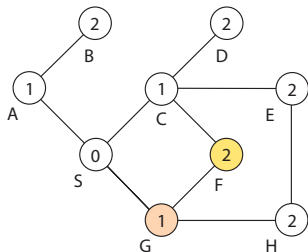


Yellow vertex is v , red is w .

$$A_v = A_F = \{C, G\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$
F	C	none	$\{H\}$

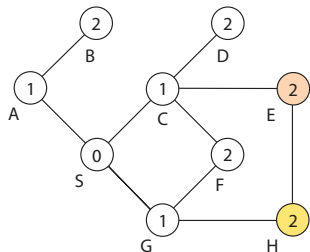
Breadth First Search: in action



Yellow vertex is v , red is w .

$$A_v = A_F = \{C, G\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$
F	C	none	$\{H\}$
F	G	none	$\{H\}$

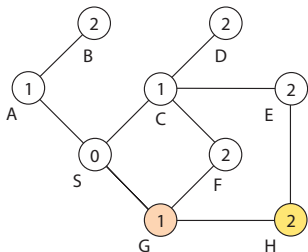


Yellow vertex is v , red is w .

$$A_v = A_H = \{E, G\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$
F	C	none	$\{H\}$
F	G	none	$\{H\}$
H	E	none	$\{\}$

Breadth First Search: in action

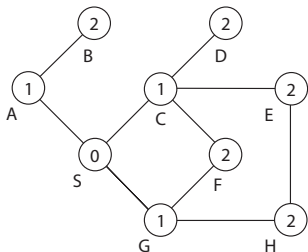


Yellow vertex is v , red is w .

$$A_v = A_H = \{E, G\}$$

v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$
F	C	none	$\{H\}$
F	G	none	$\{H\}$
H	E	none	$\{\}$
H	G	none	$\{\}$

Breadth First Search: in action



v	w	Action	Queue
–	–	<i>Start</i>	$\{S\}$
S	A	set $d(A) = 1$	$\{A\}$
S	C	set $d(C) = 1$	$\{A, C\}$
S	G	set $d(G) = 1$	$\{A, C, G\}$
A	B	set $d(B) = 2$	$\{C, G, B\}$
A	S	none, as $d(S) = 0$	$\{C, G, B\}$
C	D	set $d(D) = 2$	$\{G, B, D\}$
C	E	set $d(E) = 2$	$\{G, B, D, E\}$
C	F	set $d(F) = 2$	$\{G, B, D, E, F\}$
C	S	none	$\{G, B, D, E, F\}$
G	F	none	$\{B, D, E, F\}$
G	H	set $d(H) = 2$	$\{B, D, E, F, H\}$
G	S	none	$\{B, D, E, F, H\}$
B	A	none	$\{D, E, F, H\}$
D	C	none	$\{E, F, H\}$
E	C	none	$\{F, H\}$
E	H	none	$\{F, H\}$
F	C	none	$\{H\}$
F	G	none	$\{H\}$
H	E	none	$\{\}$
H	G	none	$\{\}$