Solving ODE using Simulink—Homework 2

Haifei Wang

October 27, 2021

1 Problem

It is required to solve the differential equation

$$y'''' + 3y''' + 3y'' + 4y' + 5y = e^{-3t} + e^{-5t}\sin(4t + \pi/3), \tag{1}$$

using Simulink. The initial conditions are $y(0) = 1, y'(0) = y''(0) = 0.5$, and $y'''(0) = 0.2$.

Then, encapsulate the model and provide interfaces which can be used to set the parameters for future convenience.

2 Solving the ODE

To consider the equation from the perspective of system inputs and outputs, rewrite the equation as:

$$y'''' = e^{-3t} + e^{-5t}\sin(4t + \pi/3) - 3y''' - 3y'' - 4y' - 5y, \tag{2}$$

which is to be considered as the input of the system. Then the output of the system is:

$$y = \iiiint y'''' = \iiiint (e^{-3t} + e^{-5t}\sin(4t + \pi/3) - 3y''' - 3y'' - 4y' - 5y) \tag{3}$$

This idea can be shown in the form of a block diagram in Simulink(see Fig. 1).

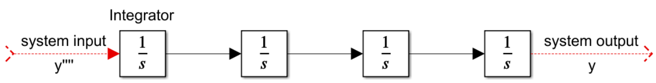


Figure 1

With this in mind, we can set up the Simulink model which can be used to solve the ODE. See Fig. 2. The version of MATLAB is R2021a.

In Fig. 2, the following blocks are used:

1. Clock. Generates the simulation time, namely, t.
2. Gain. Multiplies the signal by a constant.
3. Sine wave. Generates a sine wave with a certain amplitude, frequency and phase.
4. Math function.

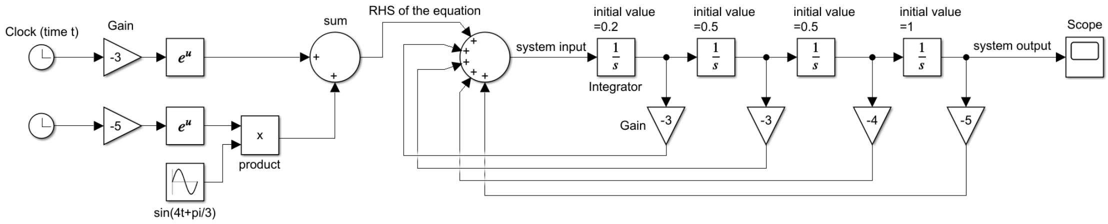
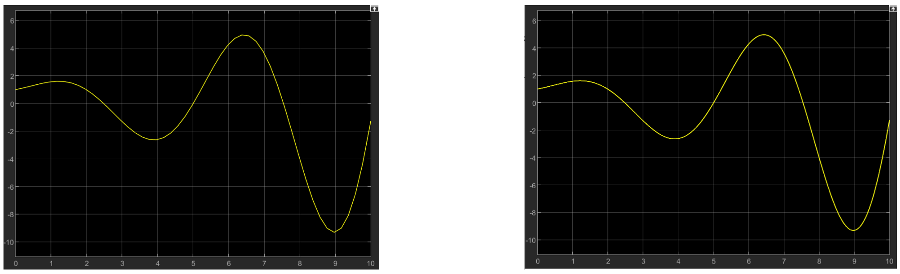


Figure 2: System used to solve the ODE.

5. Product. Multiply or divide inputs.
6. Sum. Add or subtract inputs.
7. Integrator. Integrates the input signal. The initial value can be set by users.
8. Scope. Displays the result.

The simulation result is shown in Fig. 3a.



(a) (b)

Figure 3

The result in Fig. 3a has some jagged edges. To improve the result, turn the "Max step size" in "Model Settings" from "auto" to 0.01. The result after this improvement is shown in Fig. 3b.

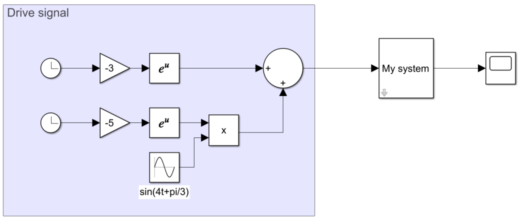
3 Encapsulation of the subsystem

Now we want to encapsulate the system for future convenience. To make the subsystem more versatile, consider the equation

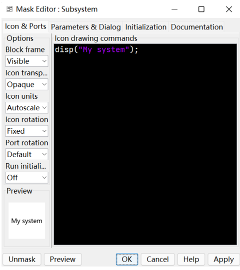
$$y'''' + a_3 y''' + a_2 y'' + a_3 y' + a_4 y = u(t), \tag{4}$$

and the initial conditions $y(0) = y_{00}, y'(0) = y_{10}, y''(0) = y_{20}$, and $y'''(0) = y_{30}$. Consequently, the parameters to be interfaced are a_1, a_2, a_3, a_4, y_{00}, y_{10}, y_{20}, and y_{30}. To modify them in a user-friendly interface, we must use "Mask" in the right-click menu of the subsystem.

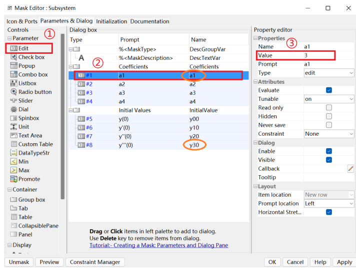
First, in "Icon & Ports", use `disp("My system")` as the icon drawing command (see Fig. 4b). Then, in "Parameters & Dialog", set the relevant parameters (see Fig. 4c, 4d, 4e). Finally, add a description for the subsystem (see Fig. 4f). The final effect is shown in Fig. 4g, which is essentially a solver of linear ODE with constant coefficients of order 4.



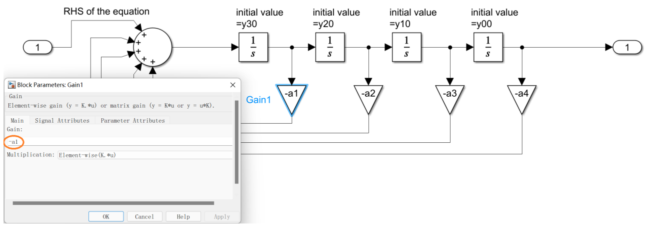
(a) Encapsulation of the system in Fig. 2.



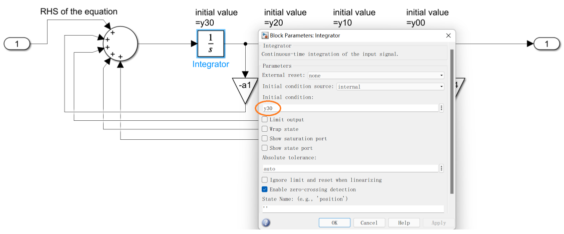
(b) Icon drawing command



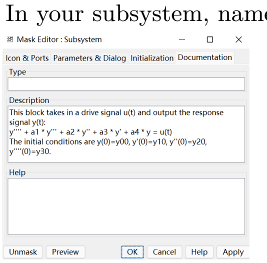
(c) In "Parameters & Dialog", create the parameters and name them respectively as `-a1`, `-a2`, `-a3`, `-a4`, `y30`, `y20`, `y10`, `y00`.



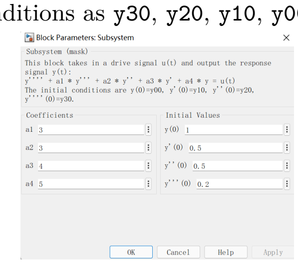
(d) In your subsystem, name the gain as `-a1`, `-a2`, `-a3`, `-a4`.



(e) In your subsystem, name the initial conditions as `y30`, `y20`, `y10`, `y00`.



(f) Add a description for your own subsystem.



(g) This dialog box will pop out whenever you double click on the subsystem. Here you can modify the parameters conveniently.

Figure 4: Interfacing the subsystem