MATH3821 Assignment 1

Stephen Sung

Question 1

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(c)

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(g)

Question 2

Given n independent binary random variables $Y_1 \cdots Y_n$ with

$$P(Y_i = 1) = \pi_i \text{ and } P(Y_i = 0) = 1 - \pi_i$$

The probability function of Y_i is:

$$\pi_i^{Y_i}(1 - \pi_i)^{1-Y_i}$$

where $Y_i = 0$ or 1

(a) For a probability function to belong to the exponential family of distributions, it must follow the formula:

$$f(y; \theta, \phi) = K(y, \frac{p}{\phi}) \exp(\frac{p}{\phi}\{y\theta - c(\theta)\})$$

For the given probability density function:

$$\begin{aligned}
f(y; \pi) &= \pi_i^y(1 - \pi_i)^{1-y} \\
&= \exp(\log \pi_i^y(1 - \pi_i)^{1-y}) \\
&= \exp(\log \pi_i^y + \log(1 - \pi_i)^{1-y})) \\
&= \exp(y \log \pi_i + (1 - y) \log(1 - \pi_i)) \\
&= \exp(y \log(\frac{\pi}{1 - \pi}) + \log(1 - \pi))
\end{aligned}$$

With $p = 1$ and $\phi = 1$, the above equation follows the form of the exponential family of distribution where $K(y, \frac{p}{\phi}) = 1$, $\theta = \log(\frac{\pi}{1-\pi})$ and $c(\theta) = \log(1 - \pi)$

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Question 3

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Question 4

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Question 5

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(b)

(c)

(d)