连续介质力学发展史概述

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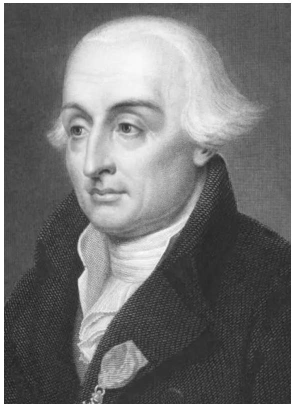
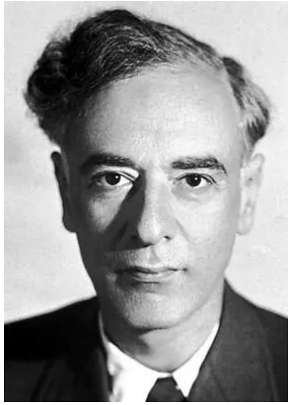
经典力学：从矢量代数到标量分析

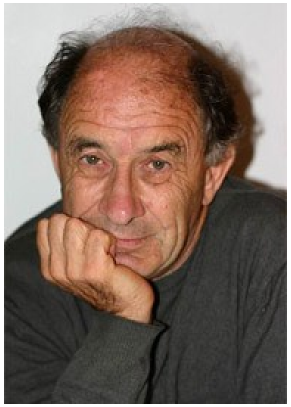
牛顿(1643-1727)



拉格朗日(1736-1813)



朗道(1908-1968)



阿诺德(1937-2010)

$$\boxed{m\ddot{\mathbf{u}} = \mathbf{f}}$$

牛顿第二定律

$$\boxed{\delta S(\mathbf{u}, \dot{\mathbf{u}}, t) = \mathbf{0}}$$

最小作用量原理(哈密顿原理、
欧拉-拉格朗日方程)

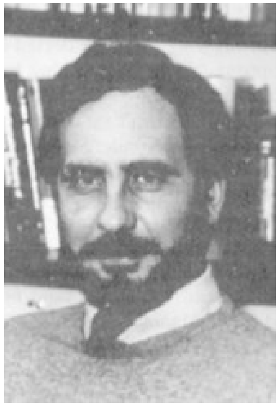
一切物理都归于几何，一切几何都归于代数与分析，数学和力学不分家。

理性力学：一群现代数学家在经典力学范畴下的唯象宇宙观

里夫林(1915-2005)

特鲁斯德尔(1919-2000)

郭仲衡(1933-1993)



思摩(1952-1994)

20世纪最顶尖的理论力学家，一部分去研究了**量子**力学，另一部分去研究了**理性**力学。前者重在探索微观粒子的客观运动规律，而后者重在挑战人作为研究主体的主观推理能力----（归纳推理能力和演绎推理能力）

理性力学基于经典力学范畴下普遍承认的客观实验事实，用数学的基本概念和严格的逻辑推理研究物质在时空中的运动规律，力求准确。

例-1 应用数学分析解决流变学问题：

1940年代，针对油漆材料，**瑞纳**用数学建模的方式研究其准确的材料行为。Wikipedia对其评价：

Research [edit]

Reiner was not only a major figure in rheology, he along with Eugene C. Bingham coined the term[2] and founded a society for its study. As well as the term rheology, and his publications, he is known for the Buckingham-Reiner Equation, the Reiner-Riwlin Equation, and Reiner-Rivlin fluids, the Deborah number and the Teapot effect – an explanation of why tea runs down the outside of the spout of a teapot instead of into the cup.

例-2 应用数学分析解决有限变形弹性力学问题：

1940年代，针对橡胶材料，**里夫林**用数学建模的方式研究其准确的材料行为。Wikipedia对其评价：

Work [edit]

His work began with his 1944 observation that "although very little force is required to detach Scotch tape from an adherend, the work expended in doing so is very large".[6] This is from the elastic effects of the adhesive, on which he commented even if "one idealized the adhesive as a perfectly elastic material there appeared to be no body of mathematical theory which would provide a basis for calculations".[6] Existing theories were only on very small deformations, so from 1945 to 1951, Rivlin was one of the creators of the modern theory of **large elastic deformations**, including theory of Neo-Hookean solids and Mooney-Rivlin solids.[7] He also made major contributions to the theory of non-Newtonian fluid flow, including in the Rivlin-Ericksen expansion.[6][8]

特鲁斯德尔 1958年发表于《科学》上的文章给出了他对理性力学的定义。

Recent Advances in Rational Mechanics

The search for underlying concepts and strict mathematical proof deepens our understanding of mechanics.

C. Truesdell

I begin by answering the question that will occur to many a reader upon seeing the title: What is rational mechanics? It is difficult to define rational mechanics, but no more so than to define chemistry or physics or mathematics. However, a chemist writing a survey of his field is not expected to begin by defining chemistry. The difference is that in the United States, at least, rational mechanics is not a recognized science. Indeed, there are some who disbelieve in its existence (*1*).

loscope an instrument more precise than the brain. The experiential basis of the sciences is not limited to laboratory experiment.

That rational mechanics grew out of practical mechanics and cooperated with it, if sometimes unwittingly, to produce applied mechanics and mechanical engineering is obvious. In writing the first treatise on rational mechanics, Isaac Newton (*3*) established its standard of mathematical rigor as precisely that of geometry. Not always has this standard been maintained, but today, as in 1687, it remains the ideal. Newton's

1960年，特鲁斯德尔 和 图平 的专著《经典场论》出版，成为理性力学发展史上标志性的工作。

The Classical Field Theories.

By

C. TRUESDELL and R. TOUPIN[1].

With 47 Figures.

With an Appendix on Invariants by

J. L. ERICKSEN.

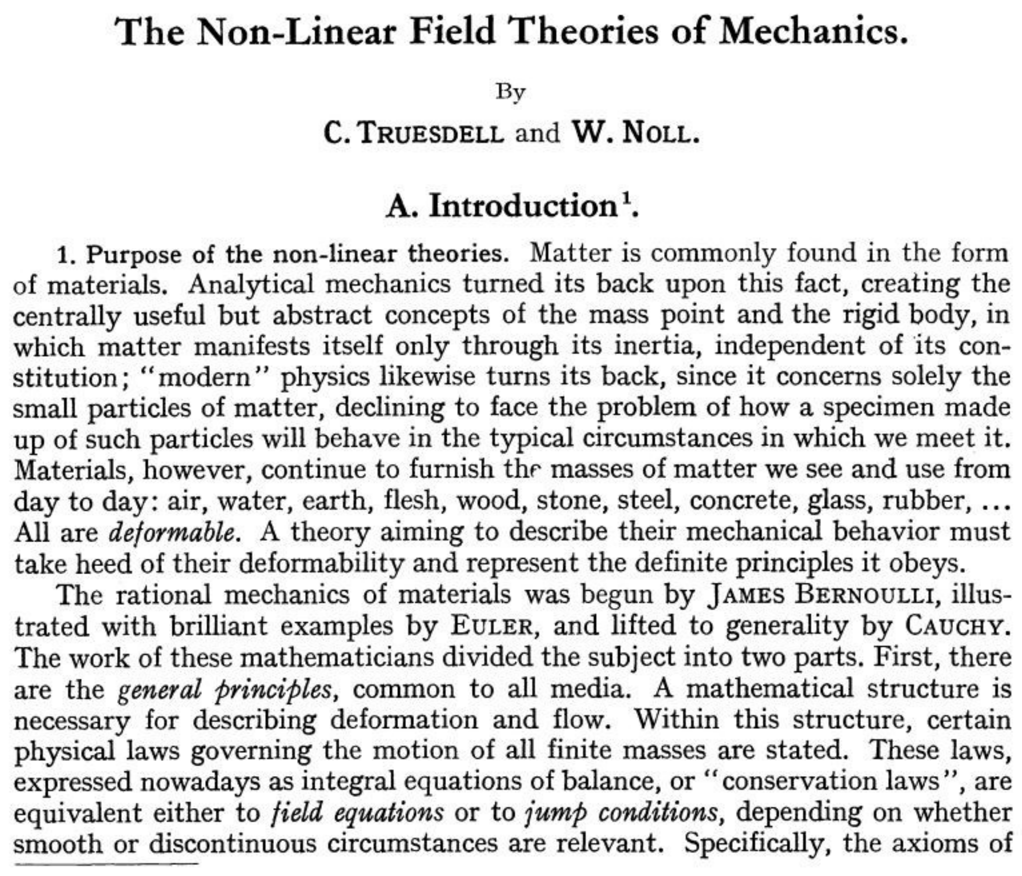
A. The field viewpoint in classical physics.

1. Corpuscles and fields. Today matter is universally regarded as composed of molecules. Though molecules cannot be discerned by human senses, they may be defined precisely as the smallest portions of a material to exhibit certain of its distinguishing properties, and much of the behavior of individual molecules is predicted satisfactorily by known physical laws. Molecules in their turn are regarded as composed of atoms; these, of nuclei and electrons; and nuclei themselves as composed of certain elementary particles. The behavior of the elementary particles has been reduced, so far, but to a partial subservience to theory. Whether these elementary particles await analysis into still smaller corpuscles remains for the future.

Thus in the physics of today, corpuscles are supreme. It might seem mandatory, when we are to deal with extended matter and electricity, that we begin with the laws governing the elementary particles and derive from them, as mere corollaries, the laws governing apparently continuous bodies. Such a program is triply impractical:

A. The laws of the elementary particles are not yet fully established. Even such senior disciplines as quantum mechanics and general relativity remain open to possible basic revision and not yet satisfactorily interconnected.

1965年，特鲁斯德尔 和 诺尔 的专著《力学的非线性场论》出版，进一步囊括了当时的研究成果，其中张量统一采用了抽象符号记法。



The Non-Linear Field Theories of Mechanics.

By

C. TRUESDELL and W. NOLL.

A. Introduction[1].

1. Purpose of the non-linear theories. Matter is commonly found in the form of materials. Analytical mechanics turned its back upon this fact, creating the centrally useful but abstract concepts of the mass point and the rigid body, in which matter manifests itself only through its inertia, independent of its constitution; "modern" physics likewise turns its back, since it concerns solely the small particles of matter, declining to face the problem of how a specimen made up of such particles will behave in the typical circumstances in which we meet it. Materials, however, continue to furnish the masses of matter we see and use from day to day: air, water, earth, flesh, wood, stone, steel, concrete, glass, rubber, ... All are *deformable*. A theory aiming to describe their mechanical behavior must take heed of their deformability and represent the definite principles it obeys.

The rational mechanics of materials was begun by JAMES BERNOULLI, illustrated with brilliant examples by EULER, and lifted to generality by CAUCHY. The work of these mathematicians divided the subject into two parts. First, there are the *general principles*, common to all media. A mathematical structure is necessary for describing deformation and flow. Within this structure, certain physical laws governing the motion of all finite masses are stated. These laws, expressed nowadays as integral equations of balance, or "conservation laws", are equivalent either to *field equations* or to *jump conditions*, depending on whether smooth or discontinuous circumstances are relevant. Specifically, the axioms of

郭仲衡院士给出了他对理性力学的定义。

理 性 力 学 简 介

北京大学数学力学系　郭仲衡

“1.像几何学一样，理性力学必须建立在显然正确的公理上；

2.力学的进一步事实由数学证明给出。” 1743年达朗贝尔的这两句话点出了理性力学的核心。

牛顿的“自然哲学”（1687）是理性力学的第一部著作。在前人探索的基础上，他总结出力学运动三定律。从这些简单的公理出发，物体（质点）力学运动的全部主要性质便由演绎得出。

理性力学的另一个先驱者J·伯努利从事变形体力学的研究。他用四种方法推导出沿长度受任意载荷的弦的平衡方程（1691—1704）。通过实验他发现，伸长和张力并不满足胡克所提示的线性关系；他一直认为线性关系不能作为物性的一般原理。他首先得到杆的弯曲理论，当杆为直线时，就变成“elastica”的微分方程（1694）。

1788年拉格朗日发展了分析力学，其中许多内容是符合达朗贝尔的框架的。

在后来漫长的历程里，变形体力学的一些基本概念如应力、应变等逐渐建立。1822年柯西宣布“应力原理”。从此，这原理成为连续介质理性力学的基础。在这里，应力向量仅与截面的法向量有关只是他的假设（一百多年后才得到证明），并且还没有偶应力的概念。在小变形范围内，弹性理论是柯西完成的。1894年芬格（Finger）完成了超弹性（hyperelastic，即有弹性势的）体的有限变形理论。有向物体的思想是迪昂（Duhem，1893）提出的，其理论则在1907年由科瑟拉（Cosserat）兄弟建立。

在那些时候，“理性力学”是指按达朗贝尔提法对力学问题进行的一切研究。吉布斯（Gibbs，1902）还认为他关于统计力学的书是对理性力学的贡献。“理性力学（rational mechanics）”一词大体沿用到十九世纪末本世纪初，但法国和意大利例外，那里的大学至今还设有理性力学教研室，西尼奥里尼（Signorini）的职称是理性力学教授。阿佩尔（Appell）的专著就称为“理性力学论（Traité de Mécanique Rationelle）”。

上世纪80年代末，郭仲衡院士出版了《张量理论和应用》与《非线性弹性理论》两本专著，代表了我国当时在理性力学领域的研究水平。

虽说写这本书的念头是近几年才有的，但本书体系的形成却经历了差不多30年。

作者在这领域早期受到 A. E. Green 和 W. Zerna 的《理论弹性》(1954) 和 S. Gołąb 的《张量运算》(1956) 的启蒙。Gołąb 教授是属于 Schouten 学派的。尽管有些作者还没有意识到，“芯字母”和“带撇指标”在指标记法中却早已决定性地显示出它们的优越性。

1960 年，C. Truesdell & R. A. Toupin 的《经典场论》问世。作者的导师 W. Urbanowski 教授对作者说，指标固然好，但抽象记法更佳。老师的思想感染了学生。作者在波兰发表的 22 篇论文全是本着这个精神写的。

1963 年作者回国在北大任教。当时国际学术界使用张量方法的尚属少数。但作者的信念是：张量的普及只是时间问题。作者讲授的“非线性弹性理论”固然非用张量不可。对基础课“弹性力学”，作者也尝试了用笛氏张量记法讲授。从未受过这方面训练的学生感到困难。教学效果如何作者也无把握，感到压力很大。作者采取了发补充讲义和加强辅导等措施设法将课坚持到底。学期终了，克服了重重困难的学生终于尝到了甜头，反映说：“这种方法就是好！”学生的肯定是对教师的最大支持和鼓励，增强了作者在教学上沿这路子走下去的信心。但是，“十年动乱”使作者的想法成了泡影，只留下了一本用张量书写的“非线性弹性理论”讲义。

雨过天晴。翻开杂志一看，果然不出所料，几乎每篇理论性文章无一不在不同程度上用了张量的工具。1978 年，作者参与了“全国力学规划”中的“理性力学和力学中的数学方法”部分的工

前　　言

五十年代末六十年代初，作者在波兰科学院研究有限变形理论期间作过一些札记，曾于 1963 年，1964 年在北京大学数学力学系讲授过。本书是在 1964 年讲义的基础上修改补充并添进若干结果而写成的。由于时间仓促，不免有漏误之处，请读者指正。作者感谢中国科技大学朱兆祥教授对本书修改提出的宝贵意见。

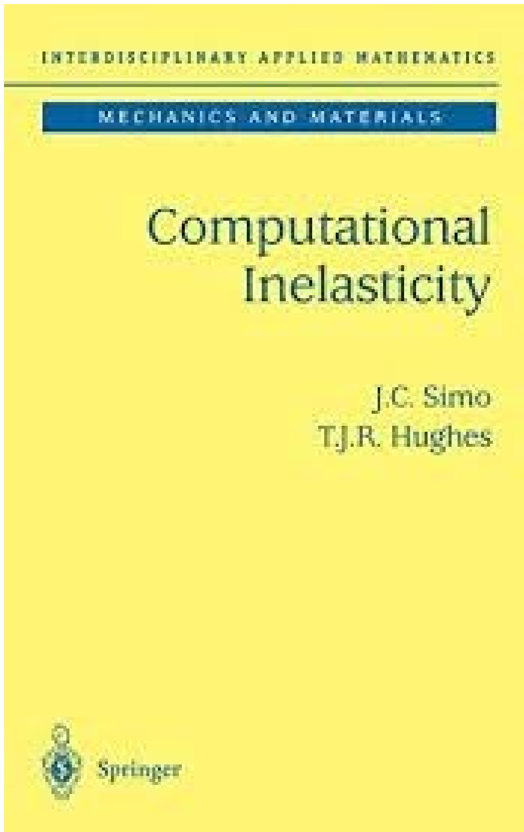
郭　仲　衡
(Guo Zhong-heng)
1978 年 12 月于北京大学

再　　言

本书脱稿付印后，作者在波兰 21 届固体力学会议 (1979 年 9 月) 作了题为“非线性弹性理论变分原理的统一理论”的报告。该文补充了本书的“变分原理”一章的不足，叙述进了一步。今列为附录，便于读者了解问题的全貌。

《张量理论和应用》前言摘录　　　　《非线性弹性理论》前言摘录

上世纪90年代末，思摩和修斯的专著《计算非弹性》的出版，标志着理性力学和计算力学的成熟结合。



This book goes back a long way. There is a tradition of research and teaching in inelasticity at Stanford that goes back at least to Wilhelm Flügge and Erastus Lee. I joined the faculty in 1980, and shortly thereafter the Chairman of the Applied Mechanics Division, George Herrmann, asked me to present a course in plasticity. I decided to develop a new two-quarter sequence entitled “Theoretical and Computational Plasticity” which combined the basic theory I had learned as a graduate student at the University of California at Berkeley from David Bogy, James Kelly, Jacob Lubliner, and Paul Naghdi with new computational techniques from the finite-element literature and my personal research. I taught the course a couple of times and developed a set of notes that I passed on to Juan Simo when he joined the faculty in 1985. I was Chairman at that time and I asked Juan to further develop the course into a full year covering inelasticity from a more comprehensive perspective. Juan embarked on this path creating what was to become his signature course. He eventually renamed it “Computational and Theoretical Inelasticity” and it covered much of the material that was the basis of his research in material modeling and simulation for which he achieved international recognition. At the outset we decided to write a book that would cover the material in the course. The first draft was written quite expeditiously, and versions of it have been circulated privately among friends, colleagues, and interested members of the research community since 1986. Thereafter progress was intermittent and slow. Some things were changed and some new chapters were added, but we both had become distracted by other activities in the early 1990s. Prior to that, we frequently discussed what would be necessary “to get it out the door,” but I do not recall the subject even coming up once in the years immediately preceding Juan’s death in 1994. Since that time I have been repeatedly urged to bring the project to completion. Through the efforts of a number of individuals, the task is now completed.

This book describes the theoretical foundations of inelasticity, its numerical formulation, and implementation. It is felt that the subject matter described herein constitutes a representative sample of state-of-the-art methodology currently used in inelastic calculations. On the other hand, no attempt has been made to present

修斯写的《计算非弹性》前言摘录

连续介质力学：理性力学的升华

在《力学的非线性场论》前言中，诺尔这样定义连续介质力学：

The period following the Second World War saw an explosive growth in the science of non-linear continuum mechanics. This growth was sparked by the appearance of (i) a beautiful series of papers by RIVLIN [1947, *9,10*] [1948, *11–16*] [1949, *15–19*] [1] showing that illuminating problems for incompressible non-linearly elastic solids and for incompressible non-Newtonian fluids could be solved for arbitrary non-linear constitutive equations, and (ii) the magisterial synthesis, clarification, and simplification by TRUESDELL [1952, *20*] [1953, *25*] of the available theories of elasticity, fluid dynamics, thermodynamics, and kinetic theory. In the period 1952–1965, these two streams coalesced to deliver a true science of continuum mechanics, equipped with fundamental principles capable of describing a rich class of material response including memory effects, with an enlarged collection of exact solutions for arbitrary families of materials, and with studies of the stability of these exact solutions on the basis of linearizations of the governing equations about them.

可以概括为：

(1) 研究描述连续介质材料响应的基本原理，即如何建立控制方程。
(2) 研究控制方程的求解。
(3) 研究解的稳定性等性质。

守恒律是建立场方程的起点。

经典场论的三大守恒律：

- 质量守恒 $\dot{\rho} + \rho \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$
- 动量守恒 $\rho \dot{\boldsymbol{v}} = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \rho \boldsymbol{g}$
- 能量守恒 $\rho \dot{e} = -p \boldsymbol{\nabla} \cdot \boldsymbol{v} + \boldsymbol{\tau} : \boldsymbol{\nabla} \boldsymbol{v} - \boldsymbol{\nabla} \cdot \boldsymbol{q}$

其中ρ密度，$\boldsymbol{v}$速度，e比内能，p压强，$\boldsymbol{\tau}$偏应力，$\boldsymbol{q}$热流密度

热力学是建立本构方程的重要支撑。

----经典热力学状态变量：T温度，s比熵，p压强，v比体积。

----经典热力学第一、二定律不同的形式：

$$de = Tds - pdv$$

$$dh = Tds + vdp$$

$$df = -sdT - pdv$$

$$dg = -dsT + vdp$$

其中h比焓，f比亥姆霍兹自由能，g比吉布斯自由能。

当代连续介质力学研究：“计算机”加持下的繁荣

- 由于过去求解能力的限制，传统对连续介质力学问题的求解往往采用先简化再解析的方式。

- 随着**计算机技术**的发展，当代对连续介质力学问题的求解基本采用计算机数值求解，速度快且准确。

- 前沿的研究转向了对各种复杂材料本构模型的研究。

- 固体
- 液体
- 气体
- 流变体
- 等离子体
- 复合材料
- 超材料
- ……………………………

科学没有对错，理性源自感性；
面对问题，大胆假设，小心求证！

谢谢！