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1)

$$x(t) = -\frac{1}{4}cos2t$$

$$y(t) = \frac{1}{2}(t - sint * cost) = \frac{1}{2}\left(t - \frac{sin2t}{2}\right)$$

$$z(t) = f(t)$$

$$\frac{dx(t)}{dt} = \frac{sin2t}{2}$$

$$\frac{dy(t)}{dt} = \frac{1}{2}(t - cos2t)$$

$$\frac{dz(t)}{dt} = f'(t)$$

$$s(t) = \int_{t_0}^{t} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2} dt$$

$$s'(t) = |\alpha'(t)| = 1 = \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}$$

$$1 = \sqrt{\frac{\sin^2 2t}{4} + \frac{1}{4} - \frac{cos2t}{2} + \frac{\cos^2 2t}{4} + \left(f'(t)\right)^2}$$

$$= \sqrt{\frac{\sin^2 2t + \cos^2 2t}{4} + \frac{1}{4} - \frac{cos2t}{2} + \left(f'(t)\right)^2}$$

$$= \sqrt{\frac{1}{2} - \frac{2\cos^2 t - 1}{2} + \left(f'(t)\right)^2}$$

$$1^2 = \left(\sqrt{1 - \cos^2 t + \left(f'(t)\right)^2}\right)^2$$

$$1 = 1 - \cos^2 t + \left(f'(t)\right)^2$$

$$\left(f'(t)\right)^2 = \cos^2 t$$

$$f'(t) = cost$$

$$f(t) = \int cost * dt = sint + c$$

$$0 = f(0) = sin0 + c = c = 0$$

$$f(t) = \sin(t)$$

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2)

$$Let\ \alpha(s)\ be\ a\ curve\ on\ interval\ [a, b]$$

$$Let\ \beta(s)\ be\ the\ curve\ with\ changed\ orientation\ on\ interval\ [-b, -a]$$

Let s be any point in $[a, b]$

$$Then, \vec{\alpha}(s) = \vec{\beta}(-s)$$

$$\vec{\alpha}'(s) \ = -\vec{\beta}'(-s)$$
$$\vec{\alpha}''(s) \ = \vec{\beta}''(-s)$$

$$|\vec{\alpha}'(s)| = \left|-\vec{\beta}'(-s)\right| = \left|\vec{\beta}'(-s)\right|$$

$$|\vec{\alpha}'(s)\ x\ \vec{\alpha}''(s)| = \left|\left(-\vec{\beta}'(-s)\right)\ x\ \vec{\beta}''(-s)\right|$$

For a cross product, $|a\ x\ b| = |(-a)\ x\ b|$ since,

$$a\ x\ b = \det\left(\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}\right)$$

$$(-a)\ x\ b = \det\left(\begin{vmatrix} i & j & k \\ -a_x & -a_y & -a_z \\ b_x & b_y & b_z \end{vmatrix}\right)$$

Since only one row is multiplied by -1

$$a\ x\ b = -\big((-a)\ x\ b\big)$$

$$|a\ x\ b| = \left|-\big((-a)\ x\ b\big)\right| = \left|\big((-a)\ x\ b\big)\right|$$

$$|\vec{\alpha}'(s)\ x\ \vec{\alpha}''(s)| = \left|\left(-\vec{\beta}'(-s)\right)\ x\ \vec{\beta}''(-s)\right| = \left|\vec{\beta}'(-s)\ x\ \vec{\beta}''(-s)\right|$$

$$Curveture\ of\ \vec{\alpha}(s)\ at\ s = \varkappa = \frac{|\alpha'(s)\ x\ \alpha''(s)|}{|\alpha'(s)|^3} = \frac{\left|\left(-\vec{\beta}'(-s)\right)\ x\ \vec{\beta}''(-s)\right|}{\left|-\vec{\beta}'(-s)\right|^3}$$
$$= \frac{\left|\vec{\beta}'(-s)\ x\ \vec{\beta}''(-s)\right|}{\left|\vec{\beta}'(-s)\right|^3} = Curveture\ of\ \vec{\beta}(s)\ at - s$$