Fast 3D Viscous Calculation Methods

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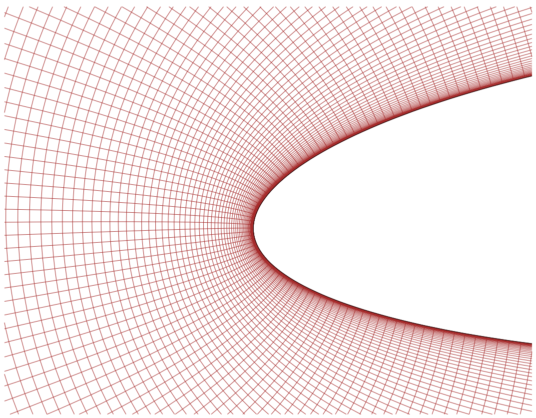
Motivation

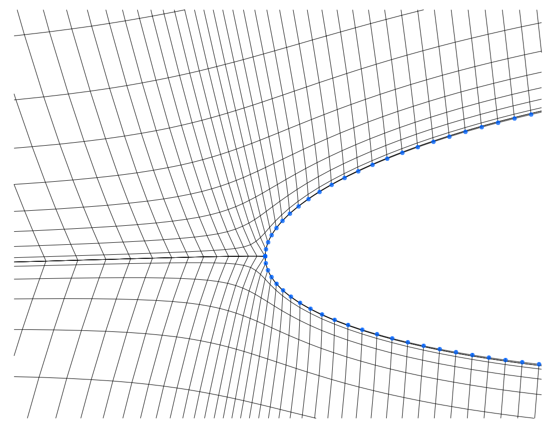
2D Inviscid+Integral-BL (IBL) methods have proven very effective

- Enormously faster than alternative Navier-Stokes for similar accuracy
- Can exploit any inviscid flow solver
- Compatible with inverse design methods
- Compatible with virtual displacements for linearized aeroelasticity



Navier Stokes

$\sim$ 500 000 variables
$\sim$ 1 hr. runtime



Potential+BL (MSIS, TRANAIR-2D)

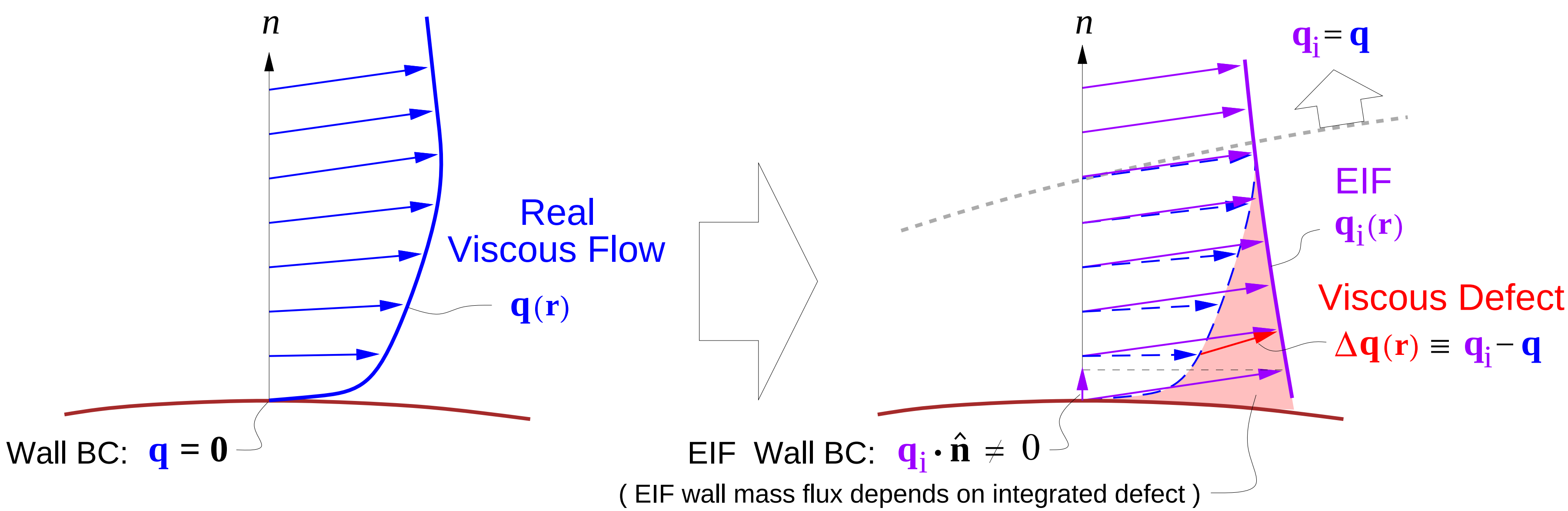
$\sim$ 5 000 variables
$\sim$ 1 sec. runtime

$\Rightarrow$ Present goal is to extend inviscid+integral-BL methods to 3D

EIF/Defect Formulation

- Real flow decomposed into irrotational Equivalent Inviscid Flow $\mathbf{q}_\mathrm{i}$ and rotational Viscous Defect $\Delta\mathbf{q}$, with . . .
  - $\mathbf{q}_\mathrm{i} = \mathbf{q}$ outside the rotational viscous layers
  - $\mathbf{q}_\mathrm{i} \cdot \hat{\mathbf{n}} \neq 0$ on solid walls, accounts for presence of viscous layer

    ($\mathbf{q}_\mathrm{i}$ is **not** the solution to the inviscid problem!)



- Originally employed by LeBalleur with boundary layer methods, but EIF/Defect formulation does **not** depend on BL approximations

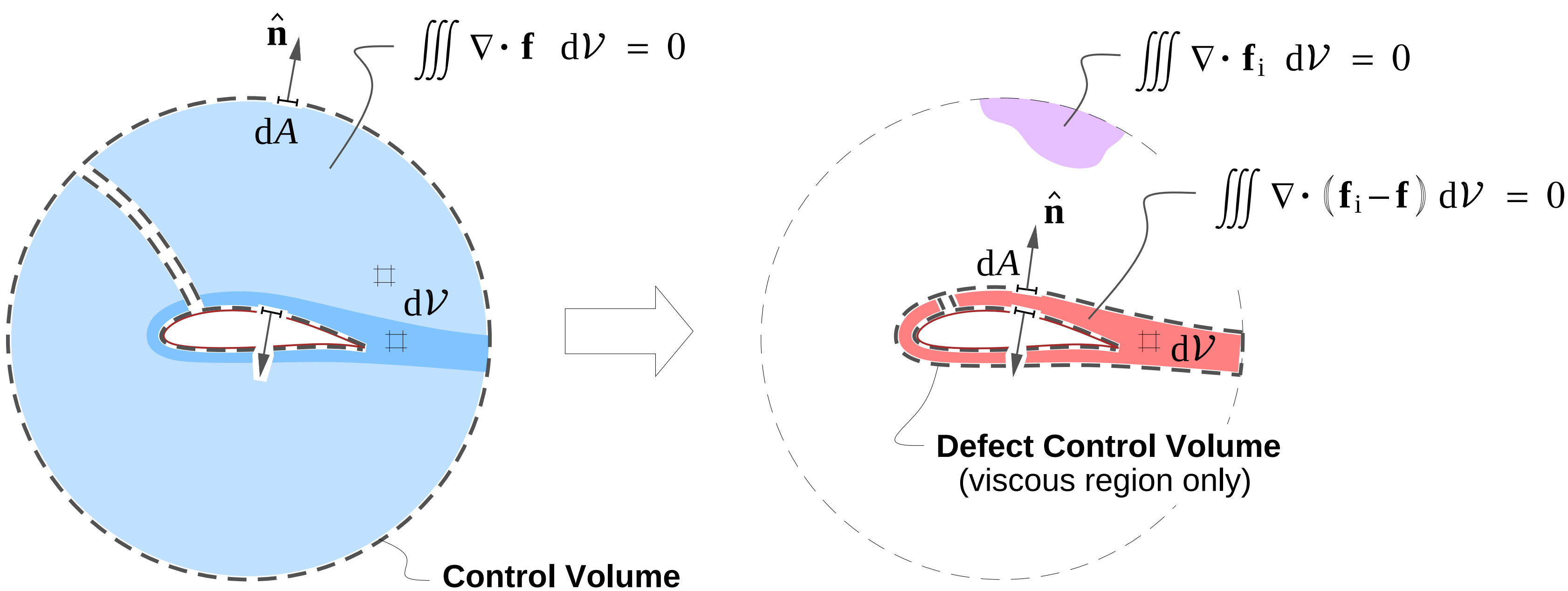
EIF/Defect Formulation

- Governing conservation laws for fluxes $\mathbf{f}$ split into EIF and Defect parts

$$\mathbf{f} = \mathbf{f}_\mathrm{i} - (\mathbf{f}_\mathrm{i} - \mathbf{f})$$

$$\boxed{\nabla \cdot \mathbf{f} = 0} \quad \overset{\text{replaced by}}{\Longrightarrow} \quad \boxed{\nabla \cdot \mathbf{f}_\mathrm{i} = 0} \; , \quad \boxed{\nabla \cdot (\mathbf{f}_\mathrm{i} - \mathbf{f}) = 0}$$

- Inviscid-equation solutions are economical, well automated
- Defect solutions needed only on compact viscous-region domain



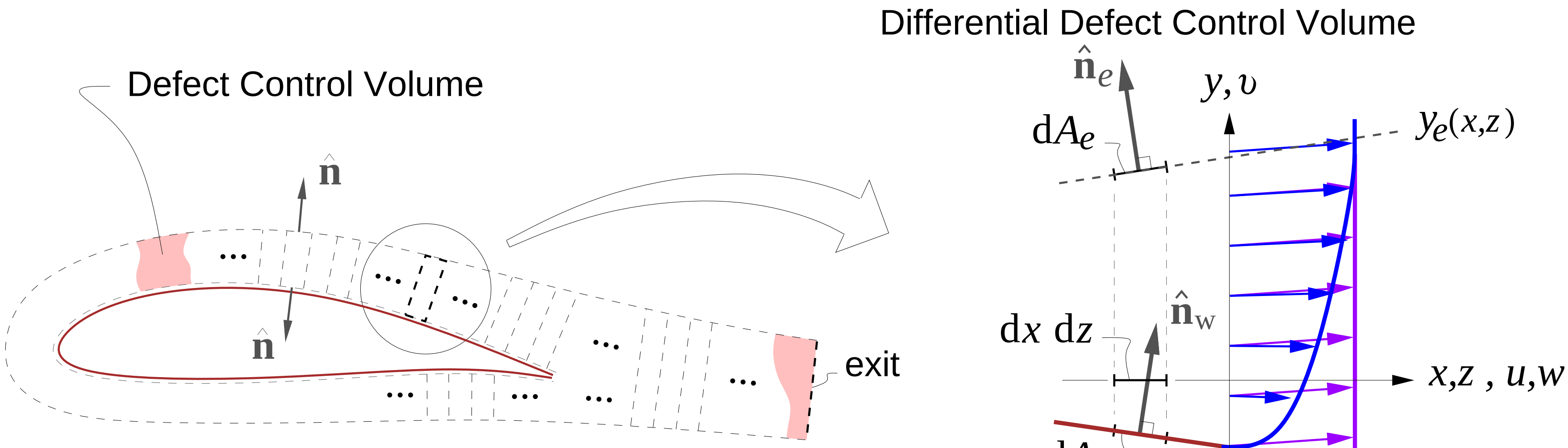
EIF/Defect Formulation

Defect conservation law on differential volume spanning viscous layer

$$\iiint \nabla\cdot(\mathbf{f}_\mathrm{i}-\mathbf{f})\,\mathrm{d}\mathcal{V} \;=\; \iint \widetilde{\nabla}\cdot\left[\int_{y_\mathrm{w}}^{y_e}\left(\tilde{\mathbf{f}}_\mathrm{i}-\tilde{\mathbf{f}}\right)\mathrm{d}y\right]\mathrm{d}x\,\mathrm{d}z$$
$$+\;\cancel{\iint(\mathbf{f}_\mathrm{i}-\mathbf{f})_e\cdot\hat{\mathbf{n}}_e\,\mathrm{d}A_e}$$
$$-\;\iint(\mathbf{f}_\mathrm{i}-\mathbf{f})_\mathrm{w}\cdot\hat{\mathbf{n}}_\mathrm{w}\,\mathrm{d}A_\mathrm{w} \;=\; 0$$

where $\quad \tilde{\mathbf{f}} \;=\; \mathrm{f}_x\,\hat{\mathbf{x}} \;+\; \mathrm{f}_z\,\hat{\mathbf{z}}$ $\qquad$ (in-plane flux)

$$\widetilde{\nabla}() \;=\; \frac{\partial()}{\partial x}\,\hat{\mathbf{x}} \;+\; \frac{\partial()}{\partial z}\,\hat{\mathbf{z}} \qquad \text{(in-plane gradient)}$$



Defect Equations

Defect equations on differential volumes spanning viscous layer

$$\int(\textsf{mass}_\mathrm{i} - \textsf{mass})\,\mathrm{d}y \;\rightarrow\; \widetilde{\nabla}\cdot\mathbf{M} - \rho_{\mathrm{iw}}\mathbf{q}_{\mathrm{iw}}\cdot\hat{\mathbf{n}}_\mathrm{w} \;=\; 0$$

$$\int(\textbf{mom}_\mathrm{i} - \textbf{mom})\,\mathrm{d}y \;\rightarrow\; \widetilde{\nabla}\cdot\bar{\bar{\mathbf{J}}} - \widetilde{\nabla}\cdot\mathbf{M}\,\mathbf{q}_{\mathrm{iw}} - \boldsymbol{\tau}_\mathrm{w} - (p_{\mathrm{iw}} - p_\mathrm{w})\hat{\mathbf{n}}_\mathrm{w} + \widetilde{\nabla}\Pi \;=\; \mathbf{0}$$

$$\int(\mathbf{q}_\mathrm{i}\cdot\textbf{mom}_\mathrm{i} - \mathbf{q}\cdot\textbf{mom})\,\mathrm{d}y \;\rightarrow\; \widetilde{\nabla}\cdot\mathbf{E} - \widetilde{\nabla}\cdot\mathbf{M}\,q_{\mathrm{iw}}^2 - \rho_\mathrm{i}\mathbf{Q}\cdot\widetilde{\nabla}q_\mathrm{i}^2 - 2\mathcal{D} \;=\; 0$$

$$\int(\mathbf{q}_\mathrm{i}\times\textbf{mom}_\mathrm{i} - \mathbf{q}\times\textbf{mom})\cdot\hat{\mathbf{y}}\,\mathrm{d}y \;\rightarrow\; \widetilde{\nabla}\cdot\mathbf{E}^\circ - \widetilde{\nabla}\cdot\mathbf{M}\,q_{\mathrm{iw}}^2\psi_{\mathrm{iw}} - \rho_\mathrm{i}\mathbf{Q}^\circ\cdot\widetilde{\nabla}q_\mathrm{i}^2 \ldots - 2\mathcal{D}^\circ \;=\; 0$$

Conserved integral defects (fluxes), source terms

$\mathbf{M} \equiv \int(\rho_\mathrm{i}\mathbf{q}_\mathrm{i} - \rho\mathbf{q})\,\mathrm{d}y$ Mass flux defect

$\bar{\bar{\mathbf{J}}} \equiv \int(\rho_\mathrm{i}\mathbf{q}_\mathrm{i}\,\mathbf{q}_\mathrm{i}^T - \rho\mathbf{q}\,\mathbf{q}^T)\,\mathrm{d}y$ Momentum flux defect

$\mathbf{E} \equiv \int(\rho_\mathrm{i}\mathbf{q}_\mathrm{i}\,q_\mathrm{i}^2 - \rho\mathbf{q}\,q^2)\,\mathrm{d}y$ K.E. flux defect

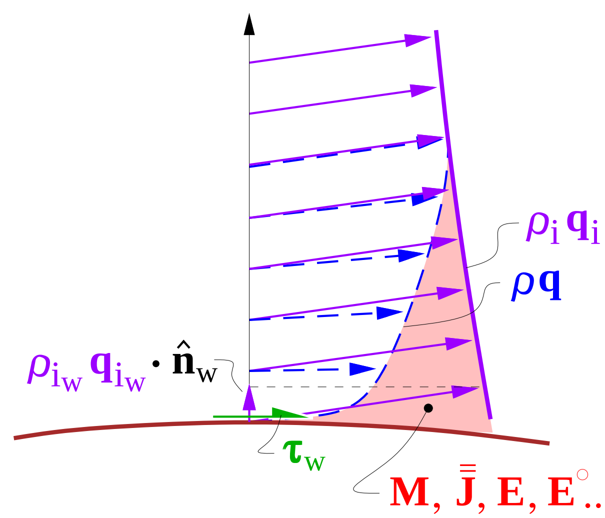
$\mathbf{E}^\circ \equiv \int(\rho_\mathrm{i}\mathbf{q}_\mathrm{i}\,q_\mathrm{i}^2\psi_\mathrm{i} - \rho\mathbf{q}\,q^2\psi)\,\mathrm{d}y$ Curvature flux defect

$\mathbf{Q} \equiv \int(\mathbf{q}_\mathrm{i} - \mathbf{q})\,\mathrm{d}y$ Volume flux defect

$\Pi \equiv \int(p_\mathrm{i} - p)\,\mathrm{d}y$ Pressure defect

$\mathcal{D} \equiv \int(\bar{\bar{\boldsymbol{\tau}}}\cdot\nabla)\cdot\mathbf{q}\,\mathrm{d}y$ Dissipation integral

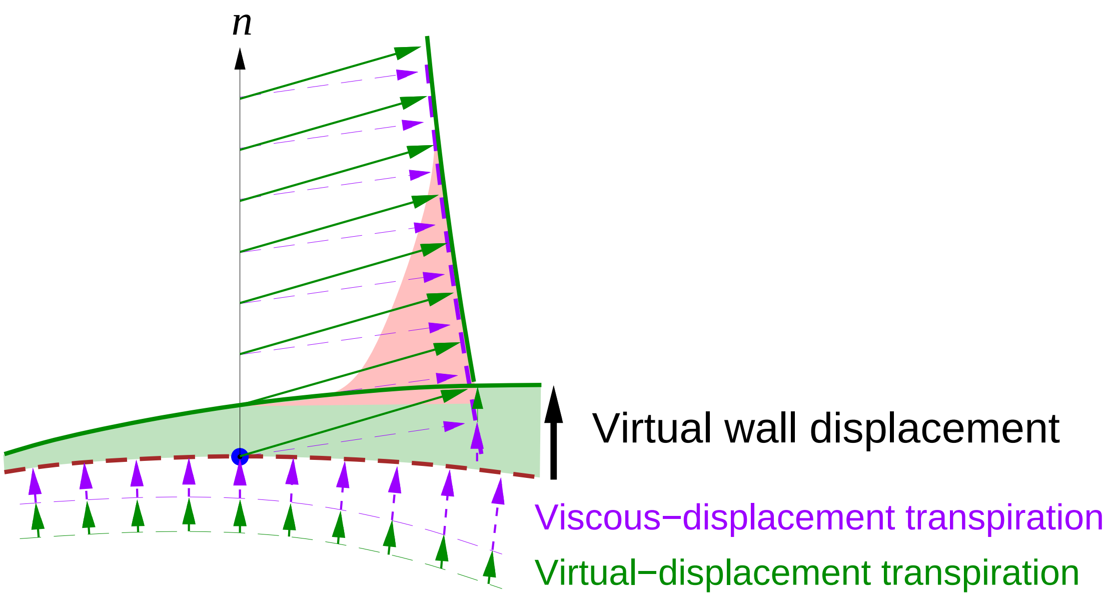
$\vdots$



In-plane flow angle variable: $\psi_{(y)} \equiv \arctan(w/u)$

Attractive Features of Defect Formulation

- Viscous defect equations are 2D (only on surfaces, wakes)
- Fully exploits any inviscid solver
- Can represent small geometry changes, steady or unsteady
  - Inverse or optimization design change without regridding
  - Linearized aeroelasticity — static, forced dynamic, flutter



Central Ideas of Present Work

- Assume EIF is represented by any fast inviscid formulation . . .
  - Vortex Lattice (AVL)
  - Panel (PANAIR, PMARC, QUADPAN, etc.)
  - Transonic Small Disturbance Potential
  - Full Potential (TRANAIR)
  - Euler (CART-3D, etc.)
- Develop complementary "surface-only" method (IBL3) to represent Viscous Defect for any EIF formulation
- Use modern sparse-matrix methods (ILU, GMRES . . .) to solve overall EIF+Defect problem

IBL3 Formulation

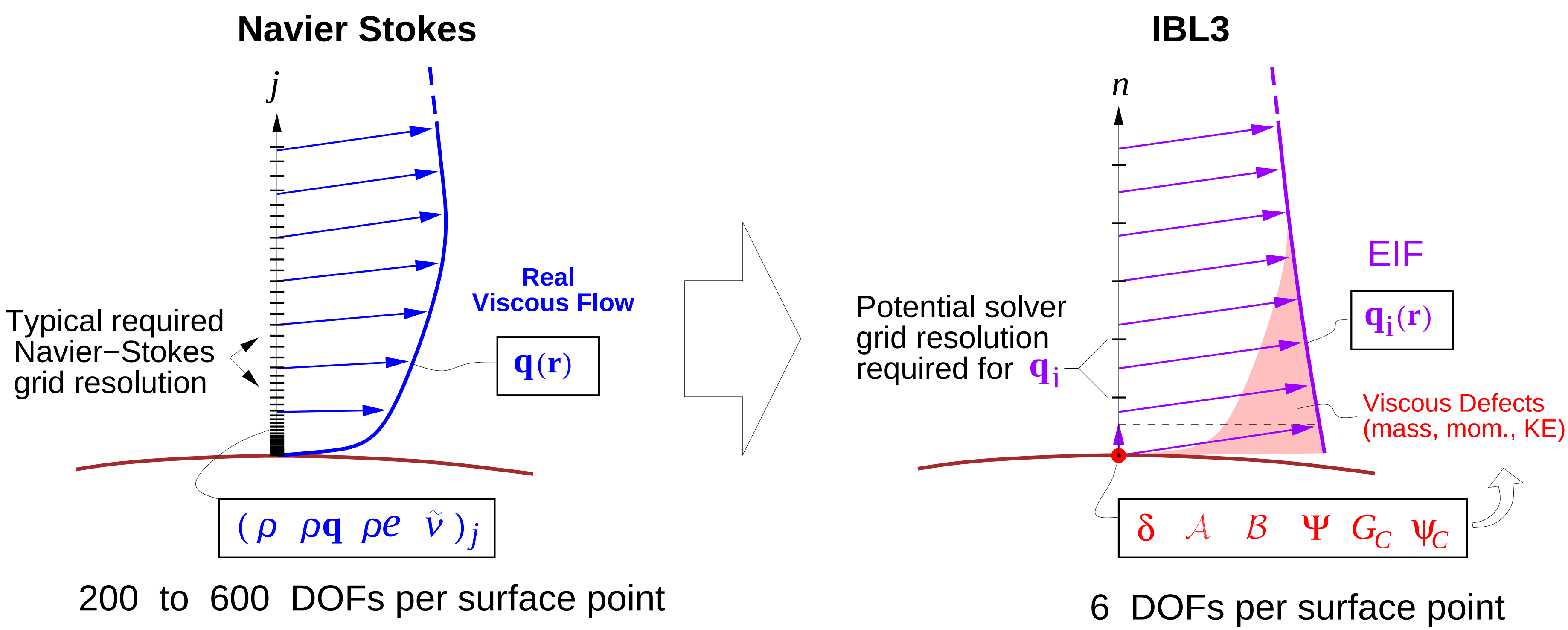
- Viscous defects represented by assumed profiles, parameterized by

$$\delta \quad \mathcal{A} \quad \mathcal{B} \quad \Psi \qquad \left(\text{roughly equivalent to} \quad \delta_1^* \quad \theta_{11} \quad \delta_2^* \quad \theta_{12}\right)$$

- TS, CF-wave amplitudes and Reynolds stresses parameterized by

$$G_C \quad \psi_C \qquad \left(\text{equivalent to} \quad \ln\left[\overline{{u'_1}^2 + {u'_2}^2}\right] \;,\; \arctan\left[\|u'_2\| / \|u'_1\|\right]\right)$$

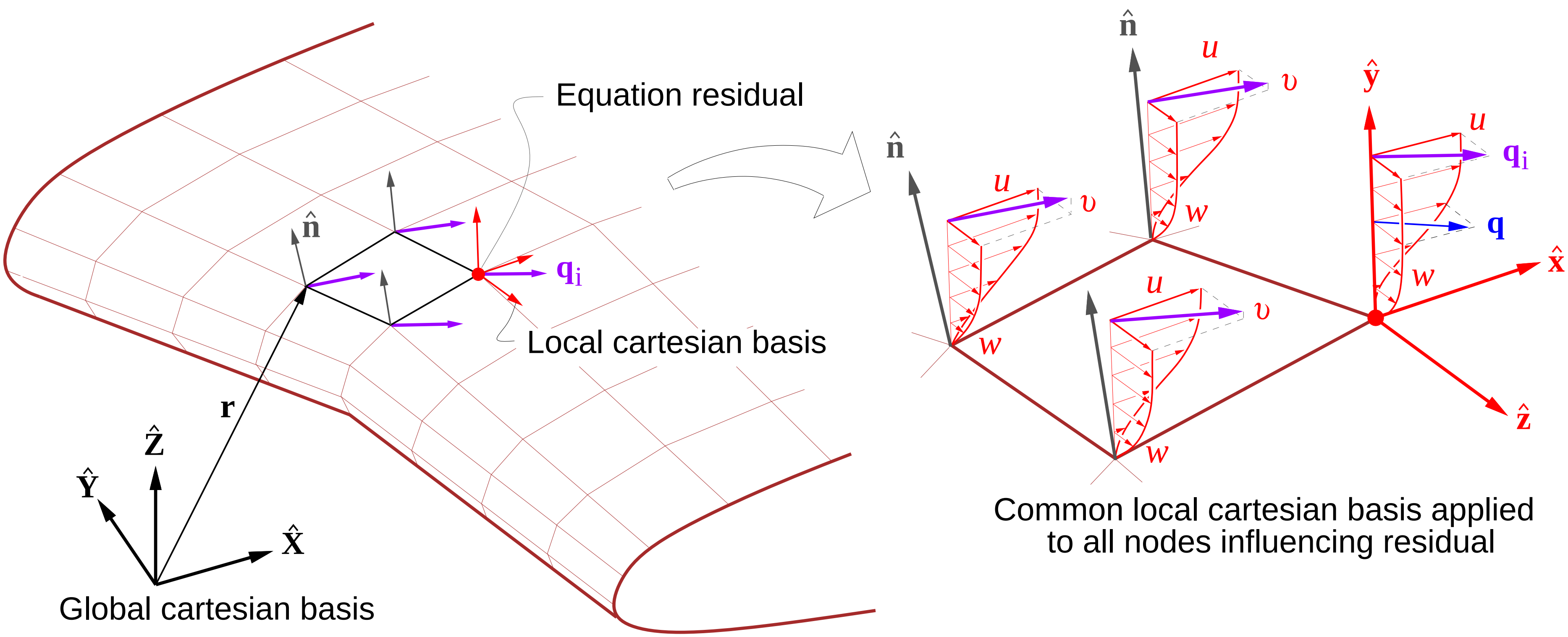
- Enormous reduction in number of unknowns from RANS



IBL3 Equation Discretization

- Surface finite element discretization allows arbitrary geometry
- Geometry $\mathbf{r}$, velocities $\mathbf{q}$, defects $\mathbf{M}, \bar{\bar{\mathbf{J}}} \ldots$ computed in global basis XYZ
- Defects put into local surface cartesian basis xyz for residual construction

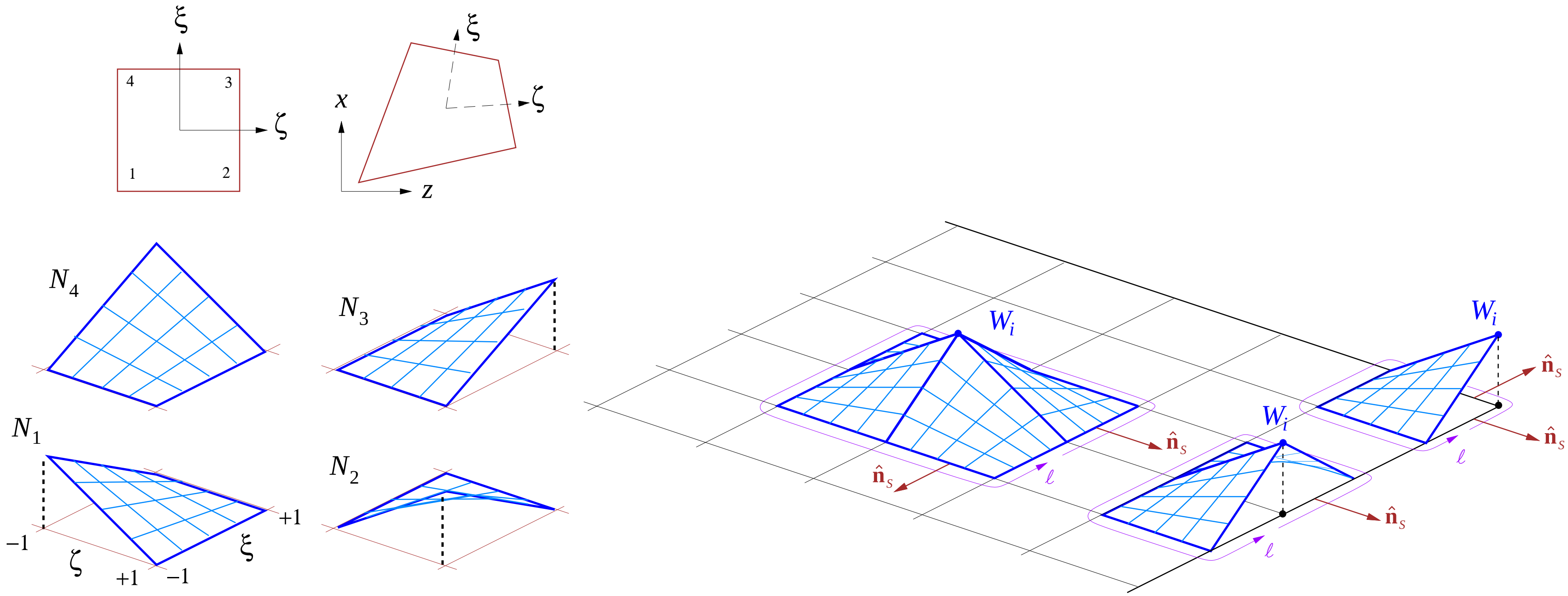
$$
\begin{aligned}
u &= \mathbf{q} \cdot \hat{\mathbf{x}} \quad , & w &= \mathbf{q} \cdot \hat{\mathbf{z}} \\
\mathrm{M}_x &= \mathbf{M} \cdot \hat{\mathbf{x}} \quad , & \mathrm{M}_z &= \mathbf{M} \cdot \hat{\mathbf{z}} \\
\mathrm{J}_x^x &= \hat{\mathbf{x}} \cdot \bar{\bar{\mathbf{J}}} \cdot \hat{\mathbf{x}} \quad , & \mathrm{J}_z^x &= \hat{\mathbf{x}} \cdot \bar{\bar{\mathbf{J}}} \cdot \hat{\mathbf{z}} \\
\mathrm{J}_x^z &= \hat{\mathbf{z}} \cdot \bar{\bar{\mathbf{J}}} \cdot \hat{\mathbf{x}} \quad , & \mathrm{J}_z^z &= \hat{\mathbf{z}} \cdot \bar{\bar{\mathbf{J}}} \cdot \hat{\mathbf{z}}
\end{aligned}
$$



IBL3 Solution Approach

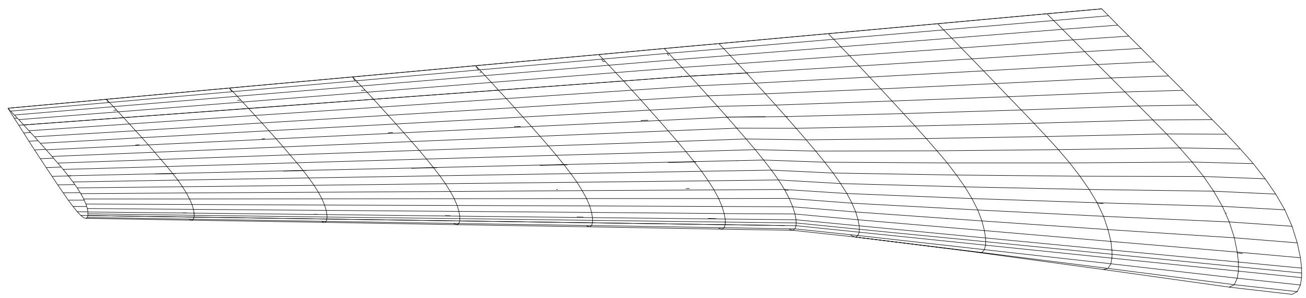
Finite-element discretization in local surface xyz basis

- Greatly simplified solution logic – no need to identify attachment lines, stagnation points
- Simple Dirichlet, Neumann PCs – no edge stencils needed
- Compatible with simultaneous solution with inviscid flow



Test Case — Low-AR Wing

- 41×12 surface paneling
- 9×12 wake paneling (not shown)

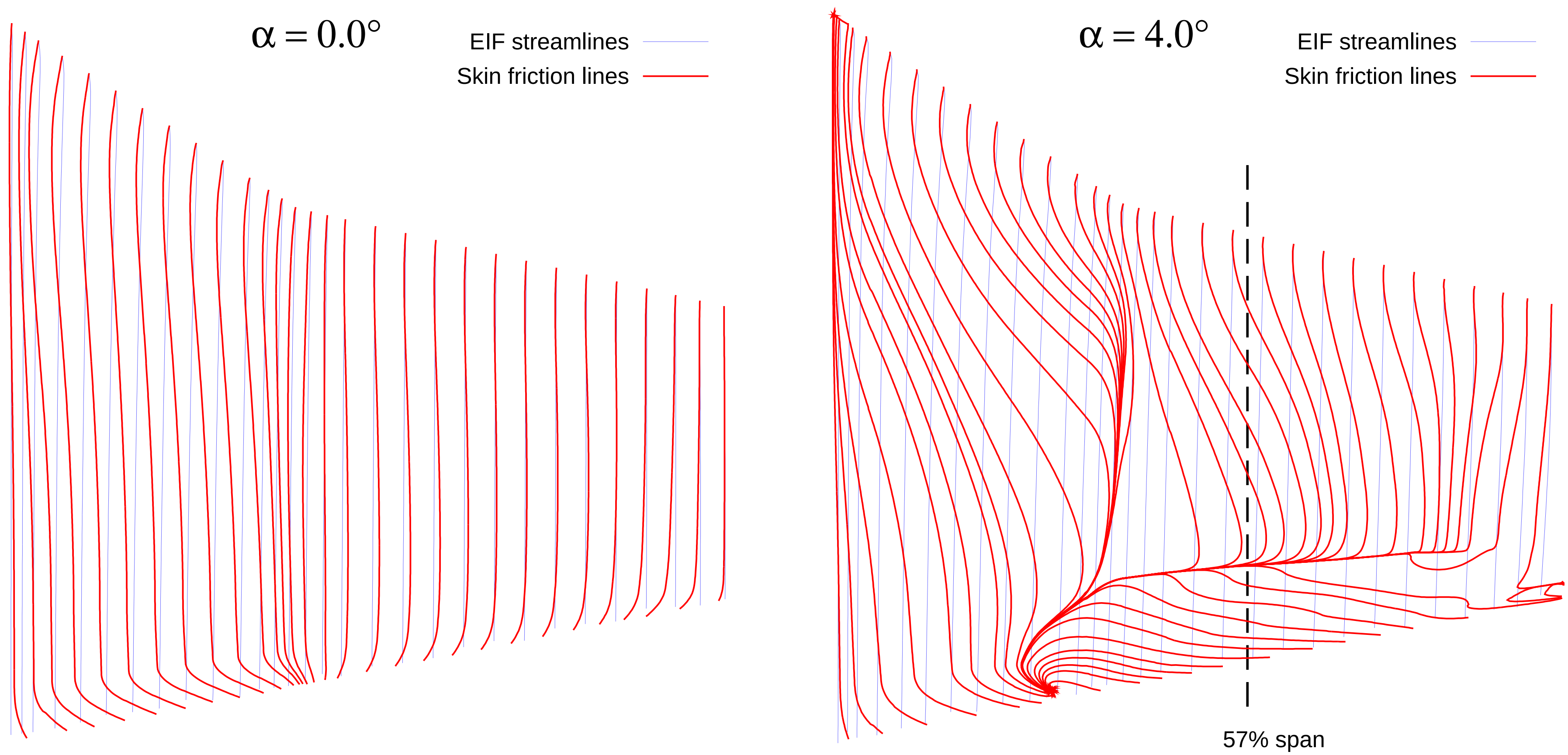


- Coupled viscous/inviscid problem with 6528 DOFs
  - 5280 viscous DOFs: δ $\mathcal{A}$ $\mathcal{B}$ Ψ G_C ψ_C at each node
  - 1000 inviscid DOFs: λ (panel source strength) on each panel

- Zero-flux viscous BCs on symmetry plane (equivalent to inviscid wall)

Test Case — Low-AR Wing

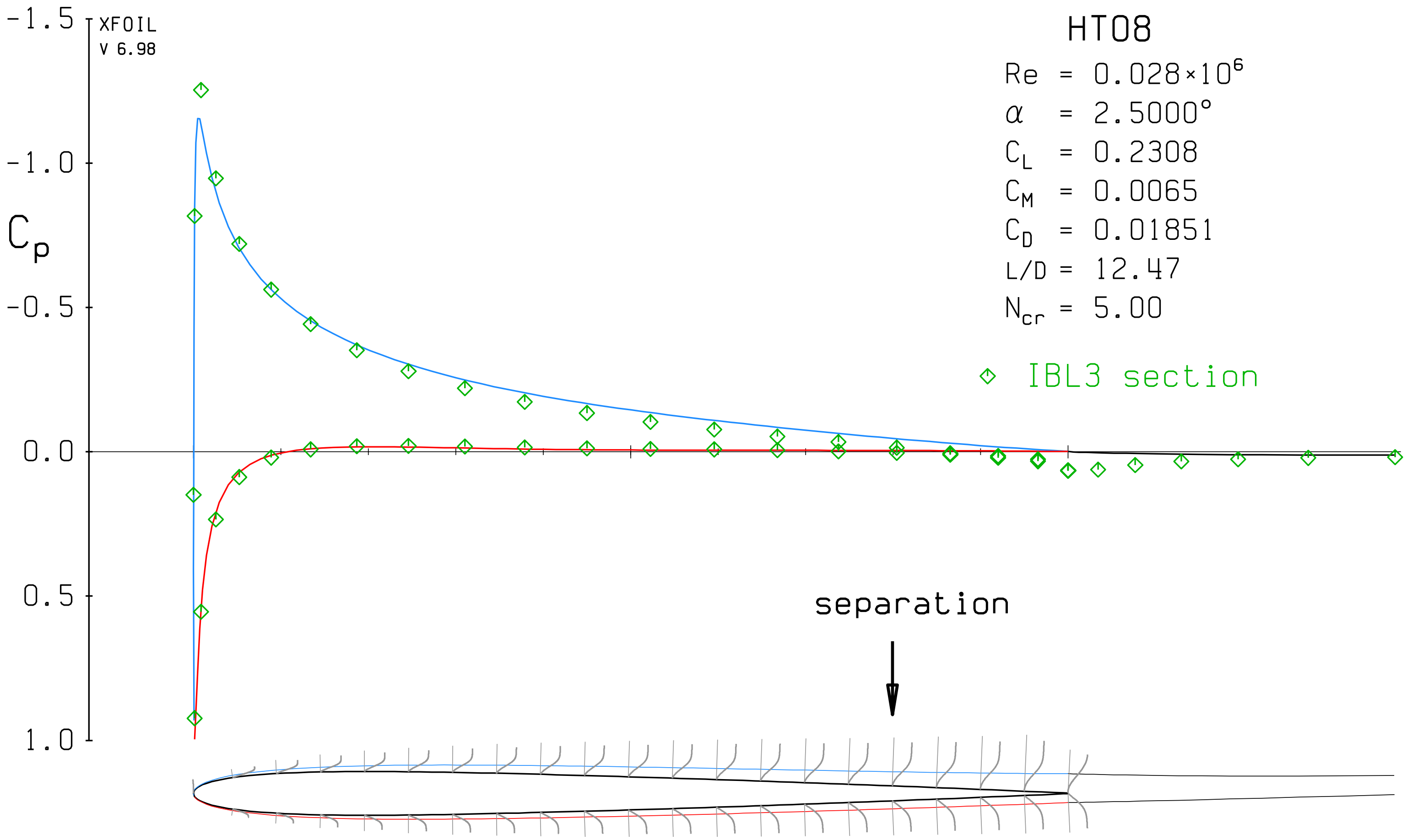
EIF and wall streamlines for laminar flow at $Re = 40\,000$



$\Rightarrow$ Separation lines are naturally captured within IBL3 surface grid

Test Case — Low-AR Wing

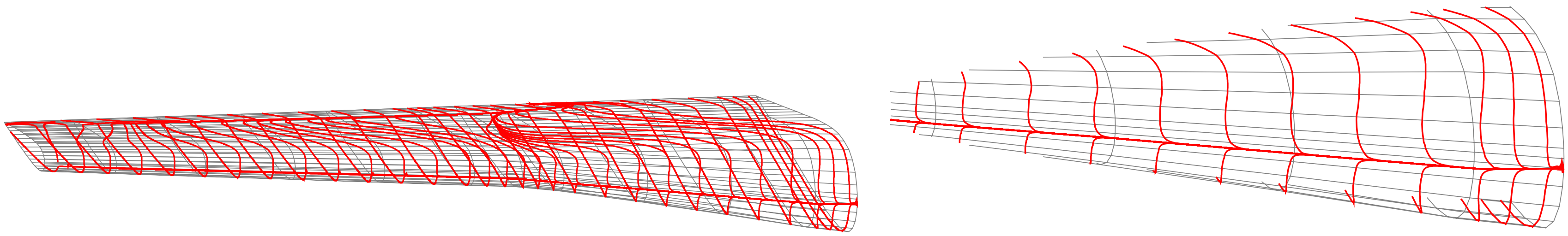
Comparison of IBL3+panel solution at 57% span with 2D (XFOIL) solution ($2D\ \alpha$ set to match local c_ℓ)



⇒ 3D and 2D separation locations match reasonably well (76% vs 79%)

Test Case — Low-AR Wing

Wall streamlines in leading edge region



$\Rightarrow$ Attachment lines are naturally captured in IBL3 surface grid

Combined transition-prediction / turbulence-lag treatment

Reynolds shear stress coefficients:

$$C_{\tau_1} \equiv \frac{-\overline{u_1'v'}}{q_{\mathrm{i}}} \quad , \qquad C_{\tau_2} \equiv \frac{-\overline{u_2'v'}}{q_{\mathrm{i}}}$$

Reynolds stress magnitude, angle variables:

$$G_C \equiv \ln C_\tau = \ln \left(C_{\tau_1}^2 + C_{\tau_2}^2\right)^{1/2}$$

$$\psi_C \equiv \arctan \frac{C_{\tau_2}}{C_{\tau_1}}$$

Governing equations:

$$\frac{\partial G_C}{\partial t} + \mathbf{q}_c \cdot \widetilde{\nabla} G_C = f_G$$

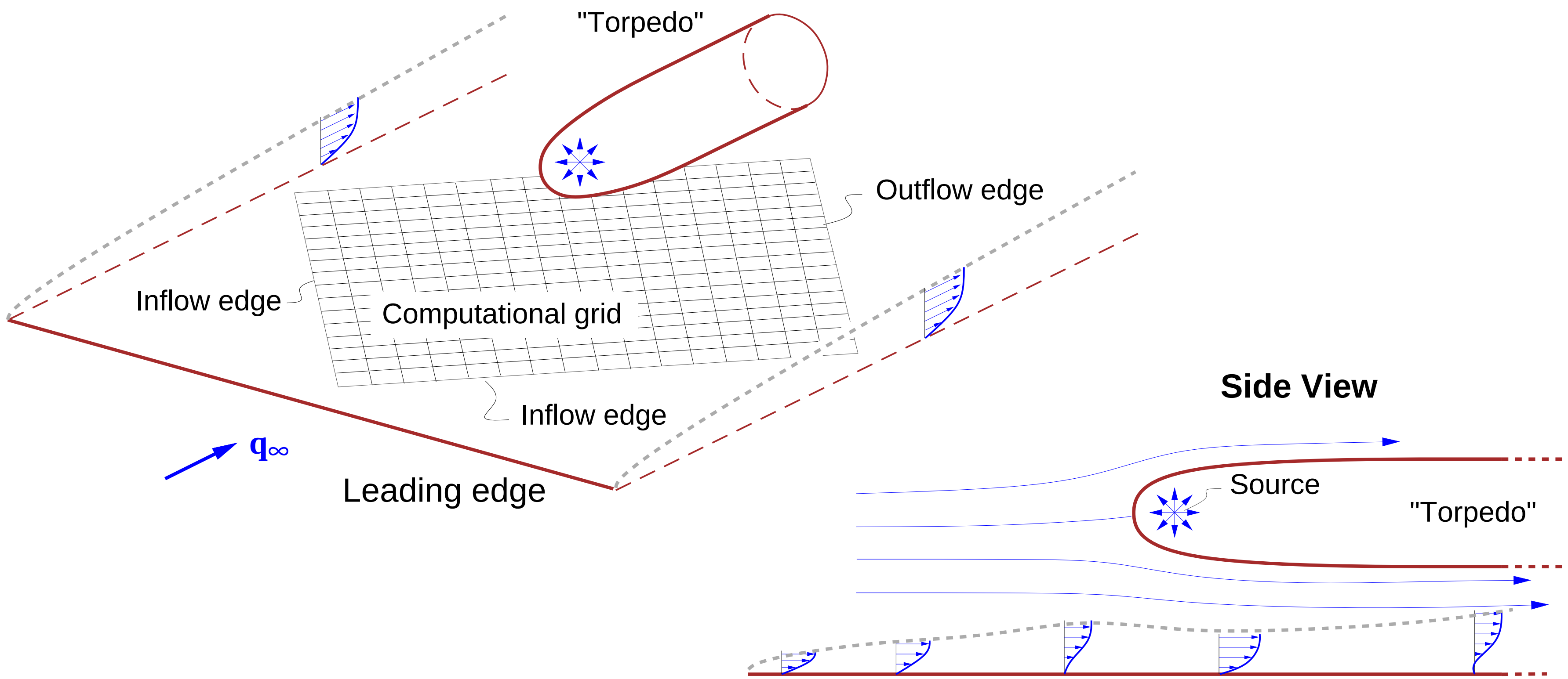
$$\frac{\partial \psi_C}{\partial t} + \mathbf{q}_c \cdot \widetilde{\nabla} \psi_C = f_\psi$$

Source functions f_G, f_ψ model:

– laminar TS,CF wave growth for $C_\tau < C_{\tau_{\mathrm{crit}}}$
– turbulence evolution, lags for $C_\tau > C_{\tau_{\mathrm{crit}}}$

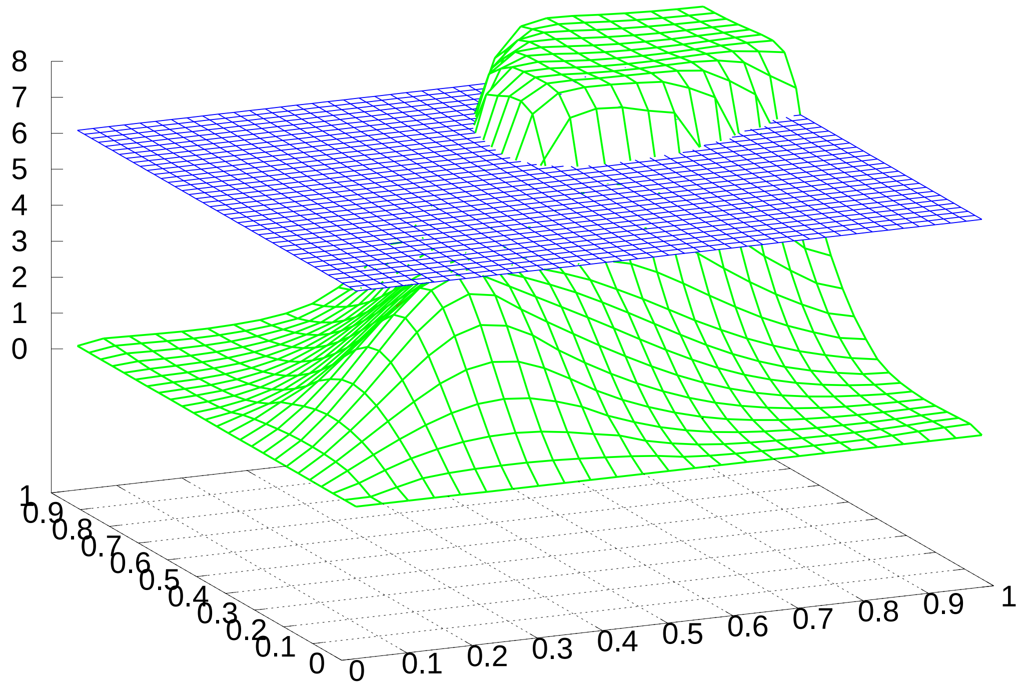
Torpedo Test Case

“Torpedo” over a wall boundary layer

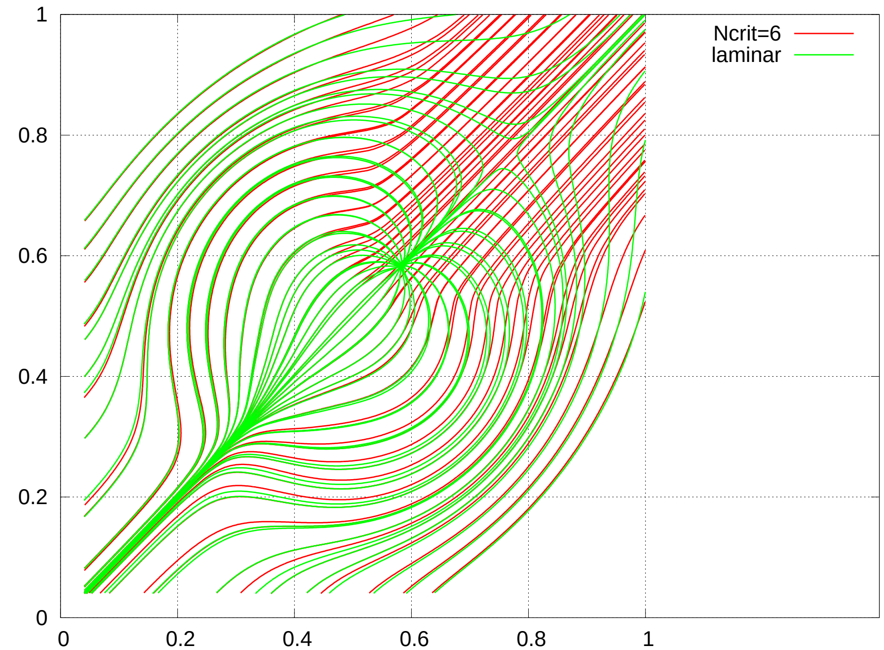


Torpedo Test Case

$$N = \frac{1}{2}\ln\left(C_\tau / C_{\tau_\infty}\right)$$



Wall friction lines



Unsteady Extension

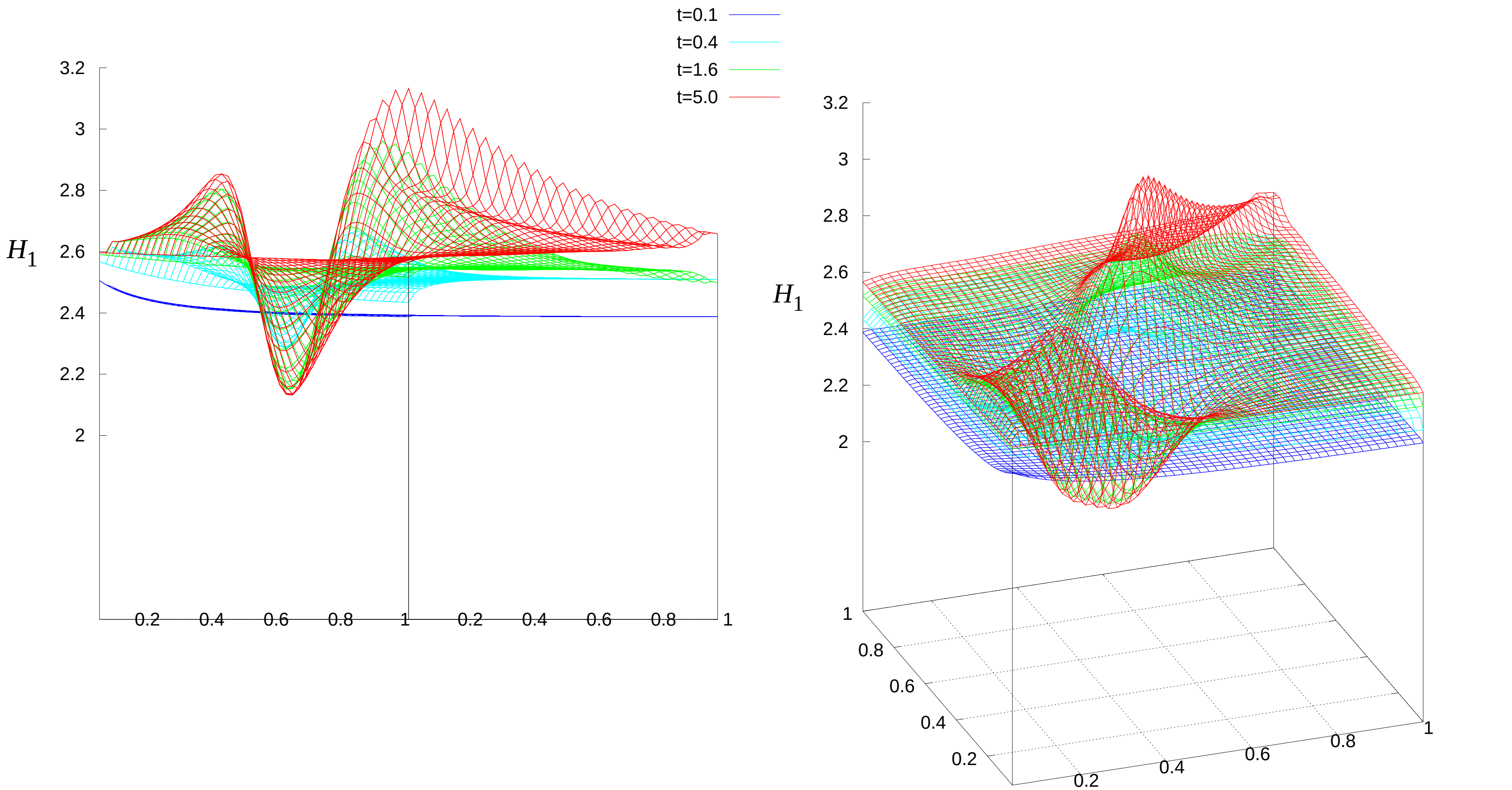
General IBL3 formulation includes . . .

- Unsteady terms, allowing . . .
  - Robust time-marching startup, $\Delta t \to \infty$ recovers steady solution
  - Time-domain unsteady (nonlinear)
  - Frequency-domain unsteady (linearized)
- Artificial dissipation
  - Necessary to stabilize FEM discretization of hyperbolic IBL equations
  - Captures converging-characteristic “shocks” (separation lines)
  - Conservative — can only <u>redistribute</u> momentum defect (drag)

$$\underline{\frac{\partial \mathbf{M}}{\partial t} - \mathbf{q}_{\mathrm{i_W}} \frac{\partial m}{\partial t}} + \widetilde{\nabla} \cdot \left[\bar{\bar{\mathbf{J}}} - \underline{V_\epsilon \, \bar{\bar{\mathbf{h}}} \cdot \widetilde{\nabla} \mathbf{M}} \right] - \mathbf{q}_{\mathrm{i_W}} \widetilde{\nabla} \cdot \mathbf{M} - \boldsymbol{\tau}_{\mathrm{w}} = \mathbf{0}$$

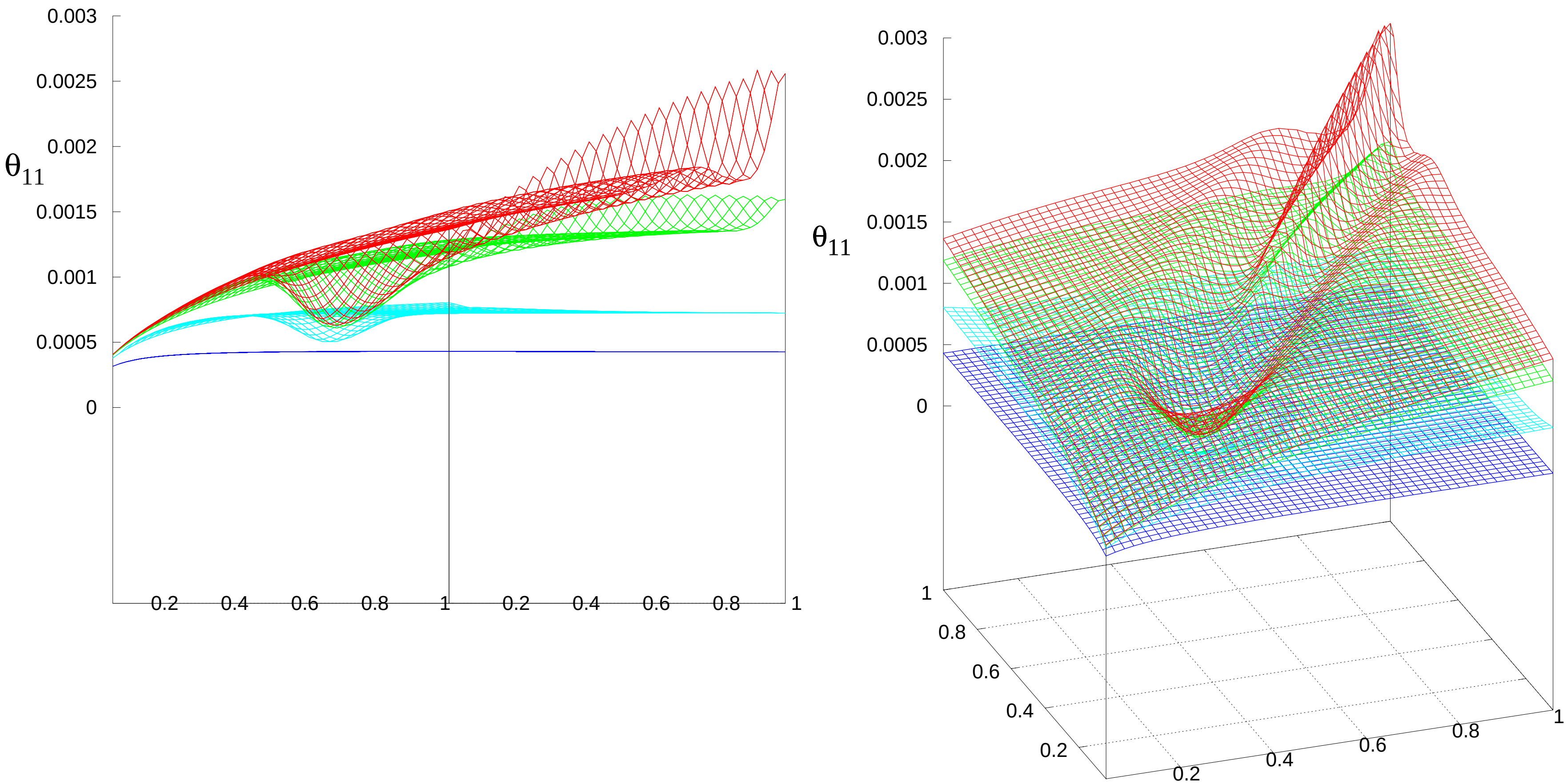
Torpedo Over Wall – Time-Ramp Test Case

Streamwise shape parameter



Time-Ramp Test Case – Torpedo Over Wall

Streamwise momentum thickness



IBL3 Work Done to Date

- Development of FE discretization on general surface grids
- Full unsteady implementation
- Laminar closure method
- Envelope-e^N 3D transition prediction method
- Development of turbulent closure method
- Combined laminar/transition/turbulent method
- IBL3 1.01 Routine Package, Documentation
- Strong coupling with simple inviscid solver (done)
- Paper: "Three-Dimensional Integral Boundary Layer Formulation for General Configurations", 21st AIAA CFD Conference, San Diego, June 2013.

IBL3 Work Underway or Planned

- Continue testing, calibration
- Better wake treatment
- Implementation into TSD code (Drela, Sato)
- Implementation into TRANAIR (by Boeing)

People

- MIT:
  - Mark Drela (IBL3, EIF coupling)
  - PhD student David Moro (VGNS)
  - Bob Haimes (geometry)
- Boeing:
  - Dave Young & TRANAIR group
  - Sho Sato (TSD code)