

# Heaps and Applications

Arjit Seth

Manipal Institute of Technology, Manipal University

## Context

These are notes created based on a talk presented by Dr. K. N. Raghavan from The Institute of Mathematical Sciences, Chennai at Computer Applications based on Modern Algebra Conference at Manipal Institute of Technology, Manipal University. The talk covers an introduction to the chromatic polynomial and its relation to acyclic orientations, Stanley's theorem and an informal introduction to heaps.

## Contents

### 1 Chromatic Polynomial

1

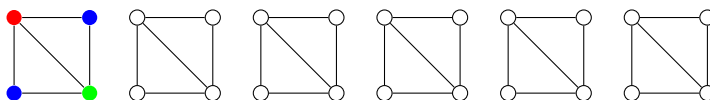
## 1 Chromatic Polynomial

George David Birkhoff attacked the four color problem by constructing a polynomial which characterises the number of vertex colourings, called the *chromatic polynomial*. The motivation is provided by example:

**Example.** Let  $G$  be the following graph:



The minimum number of colours required to colour this graph is 3 because of the adjacency of two diagonal vertices. There are 6 ways of colouring this graph with 3 colours:



**Definition 1** (Chromatic Polynomial). The *chromatic polynomial*  $\gamma_G(\lambda)$  of a graph  $G$  counts the number of its proper vertex colourings.