Solutions to
Quantum Mechanics and Path Integrals
by Richard P. Feynman and Albert R. Hibbs

Arjit Seth

Chapter 1

The Quantum-Mechanical Law of Motion

Problem 1.

$$S = \int_0^T \left(\frac{m}{2}\right)\left(\dot{x}^2 - \omega^2 x^2\right) \mathrm{d}x = \int_0^T L(x, \dot{x})\ \mathrm{d}t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \implies \ddot{x} = -\omega^2 x$$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}$$

$$x_a = A + B,\ \ x_b = Ae^{i\omega T} + Be^{-i\omega T}$$

$$\implies A = \frac{x_a}{2} + i\left(\frac{x_a \cos\omega T - x_b}{2\sin\omega T}\right),\ B = A^*$$

$$S_{free} = \frac{m}{2}\dot{x}x\Big|_0^T + \int_0^T \frac{1}{2}m\omega^2 x^2\,\mathrm{d}x$$

$$\dot{x}_a = i\omega(A - B),\ \ \dot{x}_b = i\omega\left(Ae^{i\omega T} - Be^{-i\omega T}\right)$$

$$S = \frac{m}{2}|\dot{x}_b x_b - \dot{x}_a x_a|$$

$$= \frac{m\omega i}{2}\left[\left\{\frac{x_a x_b}{2}(2i\sin\omega T) + i\left(\frac{x_a x_b\cos\omega T - x_b^2}{2\sin\omega T}\right)(2\cos\omega T)\right\} - i\left(\frac{x_a^2\cos\omega T - x_b x_a}{\sin\omega T}\right)\right]$$

$$= \frac{m\omega}{4\sin\omega T}\left[2\left(x_b^2 + x_a^2\right)\cos\omega T - x_b x_a\left(2 + 2\left(\sin^2\omega T + \cos^2\omega T\right)\right)\right]$$

$$= \frac{m\omega}{2\sin\omega T}\left[\left(x_b^2 + x_a^2\right)\cos\omega T - 2x_b x_a\right]$$