Solutions to
Gravitation: Foundations and Frontiers
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Chapter 1

Special relativity

Problem 1. Perpendicular:

$$ct' = \sqrt{c^2 - v^2}\,t$$
$$t = \frac{t'}{\sqrt{1 - v^2/c^2}} = \gamma t'$$

Parallel: Using proper time -

$$\mathrm{d}t = \gamma\,\mathrm{d}\tau$$
$$\frac{L'}{c+v} + \frac{L'}{c-v} = \gamma\frac{2L}{c}$$
$$L'\frac{2c}{c^2 - v^2} = \gamma\frac{2L}{c}$$
$$L' = \frac{1 - v^2/c^2}{\sqrt{1 - v^2/c^2}} = \frac{L}{\gamma}$$

Problem 2. Superluminal motion:

$$\Delta t' = t'_2 - t'_1$$
$$t'_1 - t_1 \approx \frac{v}{c} \Delta t \cos\theta$$
$$t'_2 - t_1 = L/c$$
$$\Delta t' = \Delta t (1 - (v/c)\cos\theta)$$
$$v_{app} = v \Delta t \sin\theta = \frac{v \sin\theta}{1 - (v/c)\cos\theta}$$

Problem 3.

Problem 4. Using a Lorentz transformation on the 'time' component of the four-vector $k^a = (\omega, \omega\mathbf{n}/c)$:

$$(a) \quad \omega_R = \gamma\left(\omega_L - \frac{\omega_L v \cos\theta}{c}\right) = \gamma\omega_L(1 - (v/c)\cos\theta)$$
$$REDO = \frac{\gamma\omega_L(\cos\theta - (v/c))}{\gamma\omega_L(1 - (v/c)\cos\theta_L)}$$

$$\int \mathrm{d}(\cos\theta)\,\mathrm{d}\phi = -\int \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = -\,\mathrm{d}\Omega$$
$$\mathrm{d}(\cos\theta_R) = \left[\frac{(v/c)[\cos\theta_L - (v/c)]}{[1 - (v/c)\cos\theta_L]^2} + \frac{1}{[1 - (v/c)\cos\theta_L]}\right] \mathrm{d}(\cos\theta_L)$$
$$\mathrm{d}\Omega' = \frac{1}{\gamma^2} \frac{\mathrm{d}\Omega}{[1 - (v/c)\cos\theta]^2}$$

Problem 5.

$$A^{i'k'} = L^{k'}_k L^{i'}_i A^{ik} = \frac{\partial x^{k'}}{\partial x^k} \frac{\partial x^{i'}}{\partial x^i} A^{ik}$$

If $A^{ik} = -A^{ki}$ then:

$$A^{k'i'} = -A^{i'k'}?$$

Problem 6.

Problem 7.

$$\eta_{ij} x^i x^j = -t^2 + x^2 + y^2 + z^2 = 0$$

Problem 8.

Problem 9.

$$\mathcal{A} = \int_{t_2}^{t_1} \mathrm{d}t\ [p\dot{q} - H(p,q)]$$

$$\delta\mathcal{A} = \int_{t_2}^{t_1} \mathrm{d}t\ \left[\dot{q}\,\delta p + p\,\delta\dot{q} - \frac{\partial H}{\partial p}\delta p - \frac{\partial H}{\partial q}\delta q\right] = 0$$

Using $\delta\dot{q} = \dot{\mathrm{d}}(\delta q)$ and integrating by parts:

$$\int_{t_2}^{t_1} \mathrm{d}t\ p\,\delta\dot{q} = \cancel{|p\,\delta q|_{t_1}^{t_2}} - \int_{t_2}^{t_1} \mathrm{d}t\ \dot{p}\,\delta q$$

$$\int_{t_2}^{t_1} \mathrm{d}t\ \left[\left(\dot{q} - \frac{\partial H}{\partial p}\right)\delta p - \left(\dot{p} + \frac{\partial H}{\partial q}\right)\delta q\right] = 0$$

Since $\delta q = 0$ at the endpoints, the second term is zero; therefore, the first term in parentheses should be zero, leading to the equations of motion:

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}$$

Problem 10.

$$\mathcal{A} = \int_{\lambda_1}^{\lambda_2} \mathrm{d}\lambda\ \left[p_a\dot{x}^a - \frac{1}{2}C\left(\frac{H}{m} + m\right)\right]$$

Problem 11.

Chapter 2

Scalar and electromagnetic fields in special relativity

Problem 1.

Problem 2.

Problem 3.

Problem 4.

Problem 5.