Solutions to Gravitation: Foundations and Frontiers by T. Padmanabhan

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Chapter 1

Special relativity

Problem 1. Perpendicular:

$$ct'=\sqrt{c^2-v^2}t \ t=rac{t'}{\sqrt{1-v^2/c^2}}=\gamma t'$$

Parallel: Using proper time -

$$\begin{aligned} \mathrm{d}t &= \gamma \, \mathrm{d}\tau \\ \frac{L'}{c+v} + \frac{L'}{c-v} &= \gamma \frac{2L}{c} \\ L' \frac{2c}{c^2-v^2} &= \gamma \frac{2L}{c} \\ L' &= \frac{1-v^2/c^2}{\sqrt{1-v^2/c^2}} &= \frac{L}{\gamma} \end{aligned}$$

Problem 2. Superluminal motion:

$$\Delta t' = t_2' - t_1' \ t_1' - t_1 pprox rac{v}{c} \Delta t \cos heta \ t_2' - t_1 = L/c \ \Delta t' = \Delta t (1 - (v/c) \cos heta) \ v_{app} = v \Delta t \sin heta = rac{v \sin heta}{1 - (v/c) \cos heta}$$

Problem 3.

Problem 4. Using a Lorentz transformation on the 'time' component of the four-vector $k^a = (\omega, \omega \mathbf{n}/c)$:

$$egin{aligned} (a) & \omega_R = \gamma igg(\omega_L - rac{\omega_L v \cos heta}{c} igg) = \gamma \omega_L (1 - (v/c) \cos heta) \ & REDO = rac{\gamma \omega_L (\cos heta - (v/c))}{\gamma \omega_L (1 - (v/c) \cos heta_L)} \end{aligned}$$

$$\int \mathrm{d}(\cos heta)\,\mathrm{d}\phi = -\int \sin heta\,\mathrm{d}\theta\,\mathrm{d}\phi = -\,\mathrm{d}\Omega$$
 $\mathrm{d}(\cos heta_R) = \left[rac{(v/c)[\cos heta_L - (v/c)]}{\left[1 - (v/c)\cos heta_L
ight]^2} + rac{1}{\left[1 - (v/c)\cos heta_L
ight]}
ight]\mathrm{d}(\cos heta_L)$ $\mathrm{d}\Omega' = rac{1}{\gamma^2}rac{\mathrm{d}\Omega}{\left[1 - (v/c)\cos heta
ight]^2}$

Problem 5.

$$A^{i'k'} = L_k^{k'} L_i^{i'} A^{ik} = rac{\partial x^{k'}}{\partial x^k} rac{\partial x^{i'}}{\partial x^i} A^{ik}$$

If $A^{ik} = -A^{ki}$ then:

$$A^{k'i'} = -A^{i'k'}?$$

Problem 6.

Problem 7.

$$\eta_{ij}x^ix^j = -t^2 + x^2 + y^2 + z^2 = 0$$

Problem 8.

Problem 9.

$${\cal A} = \int_{t_2}^{t_1} {
m d}t \, \left[p \dot q - H(p,q)
ight] \ \delta {\cal A} = \int_{t_2}^{t_1} {
m d}t \, \left[\dot q \, \delta p + p \, \delta \dot q - rac{\partial H}{\partial p} \delta p - rac{\partial H}{\partial q} \delta q
ight] = 0$$

Using $\delta q = d(\delta q)$ and integrating by parts:

$$\begin{split} \int_{t_2}^{t_1} \mathrm{d}t \ p \, \delta \dot{q} &= \left| p \, \delta q \right|_{t_1}^{t_2} - \int_{t_2}^{t_1} \mathrm{d}t \ \dot{p} \, \delta q \\ \int_{t_2}^{t_1} \mathrm{d}t \, \left[\left(\dot{q} - \frac{\partial H}{\partial p} \right) \delta p - \left(\dot{p} + \frac{\partial H}{\partial q} \right) \delta q \right] &= 0 \end{split}$$

Since $\delta q = 0$ at the endpoints, the second term is zero; therefore, the first term in parentheses should be zero, leading to the equations of motion:

$$\dot{q}=rac{\partial H}{\partial p},\quad \dot{p}=-rac{\partial H}{\partial q}$$

Problem 10.

$${\cal A} = \int_{\lambda_1}^{\lambda_2} {
m d}\lambda \, \left[p_a \dot{x}^a - rac{1}{2} C igg(rac{H}{m} + m igg)
ight]$$

Problem 11.

Chapter 2

Scalar and electromagnetic fields in special relativity

Problem 1.

Problem 2.

Problem 3.

Problem 4.

Problem 5.