Solutions to
Quantum Mechanics and Path Integrals
by Richard P. Feynman and Albert R. Hibbs

Arjit Seth

Chapter 1

The Quantum-Mechanical Law of Motion

1. *Action of the harmonic oscillator.*

$$
\begin{gathered}
S=\int_0^T\left(\frac{m}{2}\right)\left(\dot{x}^2-\omega^2x^2\right)\mathrm{d}x=\int_0^T L(x,\dot{x})\ \mathrm{d}t \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0 \implies \ddot{x}=-\omega^2 x \\
x(t)=Ae^{i\omega t}+Be^{-i\omega t} \\
x_a=A+B,\ \ x_b=Ae^{i\omega T}+Be^{-i\omega T} \\
\implies A=\frac{x_a}{2}+i\left(\frac{x_a\cos\omega T-x_b}{2\sin\omega T}\right),\quad B=A^* \\
S_{free}=\frac{m}{2}\dot{x}x\Big|_0^T+\int_0^T\frac{1}{2}m\omega^2x^2\,\mathrm{d}x \\
\dot{x}_a=i\omega(A-B),\ \ \dot{x}_b=i\omega\left(Ae^{i\omega T}-Be^{-i\omega T}\right) \\
S=\frac{m}{2}|\dot{x}_bx_b-\dot{x}_ax_a| \\
=\frac{m\omega i}{2}\left[\left\{\frac{x_ax_b}{2}(2i\sin\omega T)+i\left(\frac{x_ax_b\cos\omega T-x_b^2}{2\sin\omega T}\right)(2\cos\omega T)\right\}-i\left(\frac{x_a^2\cos\omega T-x_bx_a}{\sin\omega T}\right)\right] \\
=\frac{m\omega}{4\sin\omega T}\left[2\left(x_b^2+x_a^2\right)\cos\omega T-x_bx_a\left(2+2\left(\sin^2\omega T+\cos^2\omega T\right)\right)\right] \\
=\frac{m\omega}{2\sin\omega T}\left[\left(x_b^2+x_a^2\right)\cos\omega T-2x_bx_a\right]
\end{gathered}
$$