List of Errata to A First Course in General Relativity (2nd Edition) by Bernard F. Schutz

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Chapter 1

Special Relativity

Problem 15. Suppose that the velocity v of $\hat{A}f$ \bar{O} relative to O is nearly that of light, $|\mathbf{v}| = 1 - \epsilon$, $0 < \epsilon < 1$. Show that the same formulae of Exer. 14 become:

$$\begin{array}{ll} (a) & \Delta t \approx \frac{\Delta \bar{t}}{\sqrt{2\epsilon}} \\ (b) & \Delta x \approx \frac{\Delta \bar{x}}{\sqrt{2\epsilon}} \\ (c) & w' \approx 1 - \epsilon \frac{(1-w)}{(1+w)} \end{array} \tag{1.1}$$

Correction:

(b)
$$\Delta x \approx \Delta \bar{x} \sqrt{2\epsilon}$$
 (1.2)

Problem 18. (b) Use this to solve the following problem. A star measures a second star to be moving away at speed v=0.9c. The second star measures a third to be receding in the same direction at 0.9c. Similarly, the third measures a fourth, and so on, up to some large number N of stars. What is the velocity of the Nth star relative to the first? Give an exact answer and an approximation useful for large N.

Error:

The (partial) solutions manual provides the answer as:

$$v_{N,1} = \tanh[(N \times \operatorname{arctanh}(0.9)] \tag{1.3}$$

Correction:

Since the speed of the second star with respect to the first star is $v_{2,1}=0.9$ (c=1), the velocity parameter is $u_1=\mathrm{arctanh}(0.9)$. Now, the speed of the third star with respect to the second star is $v_{3,2}=0.9$, assuming they're all moving away from the first star in the same direction. This makes the velocity parameter $u_2=\mathrm{arctanh}(0.9)$. Inductively, the velocity parameter of the N^{th} star with respect to the $(N-1)^{th}$ star will be: $v_{N,N-1}=0.9\implies u_{N-1}=\mathrm{arctan}(0.9)$

The velocity of the third star with respect to the reference frame of the first star is:

$$v_{3,1} = \tanh(u_1 + u_2) = \tanh(\operatorname{arctanh}(0.9) + \operatorname{arctanh}(0.9)) = \tanh(2\operatorname{arctanh}(0.9))$$
(1.4)

Since velocity parameters add linearly, we can show that:

$$\sum_{i=1}^{N-1} u_i = \operatorname{arctanh}(v_{N,1}) \tag{1.5}$$

Performing induction for N stars, the speed of the Nth star with respect to the first star is:

$$v_{N,1} = \tanh\left(\sum_{i=1}^{N-1} u_i\right) = \tanh[(N-1) \times \operatorname{arctanh}(0.9)]$$
 (1.6)