Measuring the value of Planck's Constant through X-ray diffraction

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Abstract

The purpose of this experiment was to measure the value of Planck's constant *h* [1] using X-ray diffractometer with LiF crystal and copper anode, first to calibrate by comparing the accepted value of characteristic X-ray K_α and K_β of copper with the measurement from diffractometer. Second part of the experiment obtains a measurement for Planck's constant in maximum energy of emitted photon from bremsstrahlung spectrum which is $(6.6\pm0.1)\times10^{-34}$ and agrees with the accepted value within 2 standard deviations.

Introduction

Planck's constant is one of the fundamental constant in physics, introduced by German Physicist Max Planck in 1900, suggested quantised energy level of light and particle behaviour of photon in order to explain black-body radiation and ultraviolet catastrophe. Planck's constant relates frequency of photons to its energy level, E=hf, where $h=6.62607015 \times 10^{-34}$ joule second. Which lead to the development of quantum physics from wave-particle duality. [2]

Theory

When an electron beam is directed towards a metal, such as copper, electrons interact with the electromagnetic field of the metal nuclei and decelerates, loses kinetic energy, the lost energy is re-emitted as photons. The relation between energy and wavelength of emitted photon is given by

$$E = \frac{hc}{\lambda} \quad (1)$$

- c= speed of light
- λ= wavelength of emitted photon

Since electrons may decelerate differently inside copper nuclei, the emitted photon would have varied energy levels, which the diffractometer uses LiF crystal to capture and record as a function of the sine of angles using Bragg law [3], plotting a continuos bremsstrahlung radiation [4] spectrum.

$$n\lambda = 2d \sin \theta \quad (2)$$

- n = order of diffraction
- d= distance between LiF lattice layers
- θ = angle of diffraction

Characteristic X-ray [5] of copper is also produced when an electron from an inner shell, lower energy state was knocked off (in this case by electron beam), a higher energy state, outer shell electron would drop to fill the gap, the change in potential releases a photon with the corresponding energy and thus a characteristic frequency that is unique to the element.

(Accepted energies of copper Ka and Kb)

$$E_{K\alpha} = 8.038\text{keV}$$

$$E_{K\beta} = 8.905\text{keV}$$

These characteristic lines could be observed with paired peaks in orders of diffraction superimposed on top of the bremsstrahlung spectrum.

To measure Planck's constant, as a function of anode voltage, maximum energy of re-emitted photons would be the known anode voltage used to accelerate electrons if all kinetic energy is lost.

$$eV = E_K \quad (3)$$

Substitute equation 1

$$eV_{anode} = \frac{hc}{\lambda_{min}} \quad (4)$$

Substitute equation 2 and rearrange to get

$$h = \frac{2Vedsin(\theta)}{nc}$$ in joule second *(5)* and

$$E = \frac{nh}{2edsin(\theta)}$$ in eV *(6)*

Maximum photon energy can be observed as shortest wavelength on the bremsstrahlung spectrum with low cut-off angle.

Experiment

The experiment uses Phywe X-ray diffractometer [6] for measuring characteristic X-ray of and Planck's constant.

For all measurements, goniometer are set to 2:1 coupled mode, integration time of 2s along with 1mm diameter diaphragm at source to control radiation intensity entering receiver so that reading differentiates from background radiation while not spending too much time between each measurement. Crystal angle steps at 0.1° increments allowing sufficient resolution for the peaks to be resolved. Result should produce reading of X-ray intensity at each increment angle, plot intensity at y axis against crystal angle.
3 Measurements were made with different anode voltage, current and crystal angle ranges.

Set	Voltage/ kV	Anode current/mA	Scanning range/°
A	35.0	1.0	3-53
B	30.0	1.0	10-30
C	35.0	0.8	0-30

To verify the calibration of diffractometer by measuring the characteristic X-ray spectra of copper, identify the peaks on bremsstrahlung graph, use equation 6 to obtain characteristic K_α and K_β energy and compare with accepted values.

The uncertainty of energies would be

$$\Delta E = \frac{\partial E}{\partial \theta} \Delta \theta$$

since the only uncertainty would be the angle θ and the rest are constants with negligible uncertainty.

Determine Planck’s constant by identifying cut-off angle of the bremsstrahlung spectrum, substitute into equation 5 to obtain a value for Planck's constant.

The uncertainty for determining Planck's constant would be depending on uncertainty of voltage and angles

$$\Delta h=\sqrt{\left(\frac{\partial h}{\partial V}\Delta V\right)^2+\left(\frac{\partial h}{\partial \theta}\Delta\theta\right)^2}$$

Uncertainty of the anode here is assumed to be ± 0.1kV, as settings allow resolution to 0.1kV resolution.

Results

Measuring copper characteristic X-ray peaks

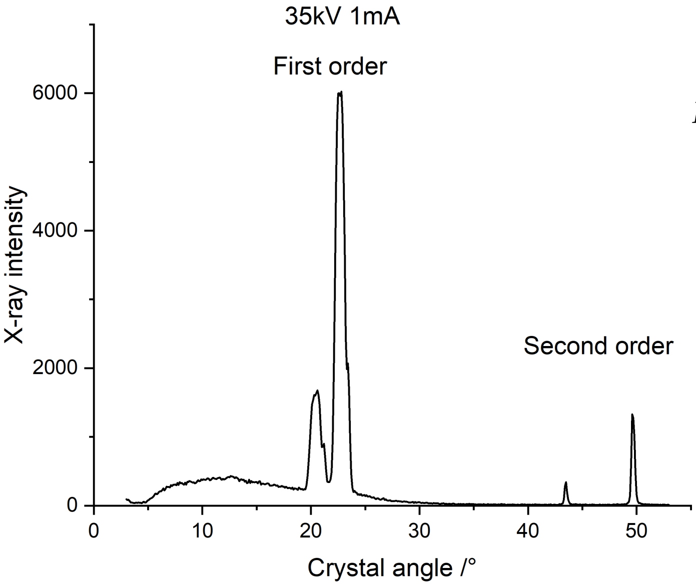

Figure 1: Set A

Peaks no.	Angle/degree	keV
1	20.3648 ± 0.0343	8.85±0.01
2	22.79754 ± 0.00824	7.944±0.003
3	43.48019 ± 0.00228	8.9469±0.0004
4	49.70816 ± --	8.071

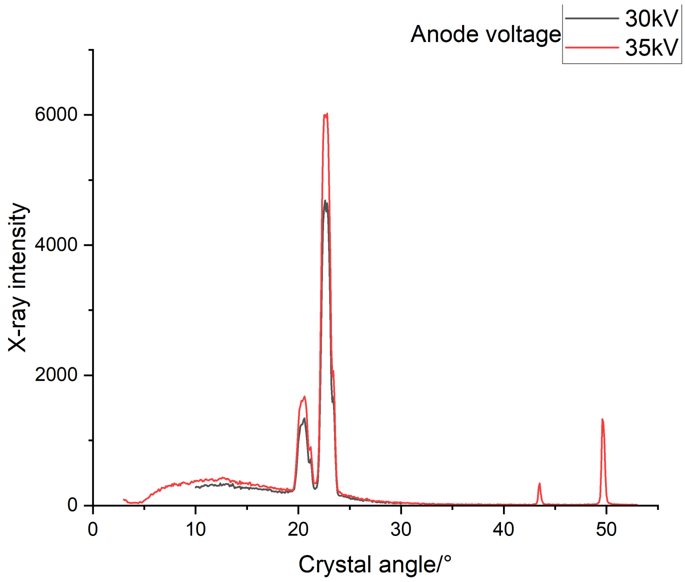

Figure 2: Compare set A and B

Peaks no.	Angle/degree	keV
1	22.69308 ± 0.00639	7.979±.0002
2	20.47273 ± 0.02755	8.800±.001

Characteristic copper K_α and K_β lines were observed for both 1st and 2nd order in set A, and only 1st order in set B, due to shorter scanning range.

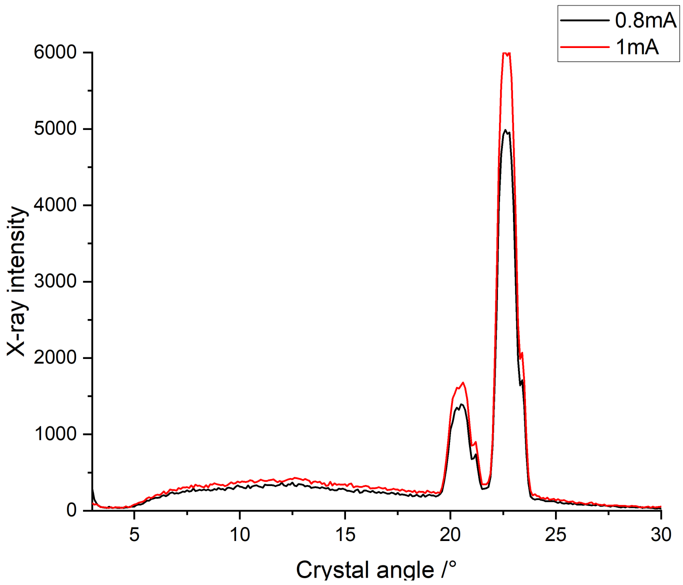

Figure 3: Compare Set A and C, both at 35kV

Characteristic X-ray around same region

Measuring Planck's constant,

- accepted value of h=6.62607015 × 10^{-34}

short-wavelength cut-off angle from Figure 3 was estimated to be around 5.0°±0.1° as it was difficult for fitting, which gives the value of experiment h = (6.6±0.1) *10^{-34} Js

Analysis

Although the obtained energies in both 1st and 2nd orders for copper characteristic X-ray are close to accepted values, confirms diffractometer was calibrated. However, the uncertainties taken from peak fitting doesn't agree within 2 standard deviations. This is most likely due to uncertainty taken from fitting being underestimated and unrealistic, resulting in a tight 2σ confidence region.

With the comparison made for varying anode voltages and currents, all 3 sets observed peaks at same diffraction angle as expected, where higher voltage and current would only result to higher magnitude at peak.

From the experiment, the measured Planck's constant gives 6.6±0.1 x10^{-34}, compared with the accepted value 6.62607015 × 10^{-34}, $|\Delta h| = 0.026 < 2\sigma$, agrees with the accepted value of h within 2 standard deviations. The results did not demonstrate the correlation between changing anode voltage and short-wavelength cut-off angle, because one of the initial scanning angle was set too high, therefore improvements can be made to reduce uncertainty by taking more measurements at the low angle, short-wavelength range.

Conclusion

Overall, this experiment successfully achieved it's objectives, first to verify the calibration of diffractometer through measuring characteristic X-ray of copper and determined a value for Planck's constant h = (6.6±0.1)×10^{-34} through cut-off angle on a bremsstrahlung spectrum, agreeing with the accepted value within uncertainty. Further improvements could be made on providing clearer evidence for relation between anode voltage and the bremsstrahlung cut-off angle.

References

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[5]Chu, Steven, Graybeal, Jack D., Hurst, George Samuel, Stoner, John Oliver. "spectroscopy". Encyclopedia Britannica, 29 Nov. 2025, https://www.britannica.com/science/spectroscopy

[6]PHYWE “X-ray diffraction” Instruction Manual **P2540105**