

Hypothesis testing

Version A has click rate ($p_A = .5$)

Experiment: Show link to $n=50$ people and see if they click on link. Version B -

Find: $S=30$ people clicked on link, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{30}{50} = .6 = \hat{p}_B$ estimate Bernoulli(.5)

Remember, the hypothesis testing framework is setup where you use your experiment to reject the hypothesis that new the new design does not increase click rate.

Therefore you want test the null hypothesis $p_B \leq p_A = .5$ and reject it if $P(\bar{X} > \hat{p}_B)$ for under this hypothesis.

α is the reject level, $\alpha = .05$.

To compute $P(\bar{X} > \hat{p}_B)$ under the null hypothesis you will use the normal approx. given by the CLT.

Part 1

(A) Derive expressions for $E[\bar{X}]$ and $\text{var}[\bar{X}]$ under the null hypothesis in terms of p_A

$$\begin{aligned} E[\bar{X}] &= E\left[\frac{1}{n} \sum_{i=1}^n x_i\right] \\ &= \frac{1}{n} \sum_{i=1}^n E[x_i] \\ &= \frac{1}{n} (np_A) = p_A \end{aligned}$$

$$\begin{aligned} \text{var}[\bar{X}] &= \text{var}\left[\frac{1}{n} \sum_{i=1}^n x_i\right] \\ &= \frac{1}{n^2} \sum_{i=1}^n \text{var}(x_i) \\ &= \frac{1}{n^2} (np_A(1-p_A)) \\ &= \frac{p_A(1-p_A)}{n} \end{aligned}$$