Gofinge Note for EM algorithm

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1 Introduction to expectation-maximization(EM) algorithm

1.1 An example of EM algorithm

Let's start with an example from Chuong B Do & Serafim Batzoglou's tutorial paper *What is the expectation maximization algorithm?*, and I have saved the .pdf file of this paper in the *material* folder.

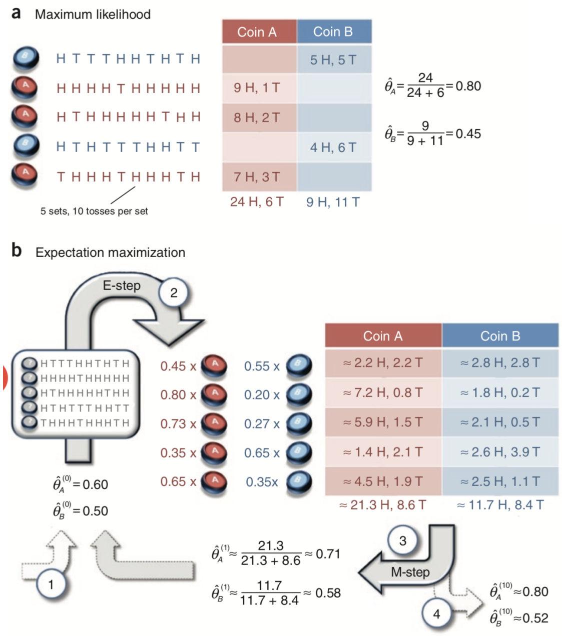

Figure 1.1: Example: As an example, consider a simple coin-flipping experiment in which we are given a pair of coins A and B of unknown biases, θ_A and θ_B, respectively. Our goal is to estimate $\theta = (\theta_A, \theta_B)$ by repeating the following procedure five times: randomly choose one of the two coins (with equal probability), and perform ten independent coin tosses with the selected coin. Thus, the entire procedure involves a total of 50 coin tosses'

If we can figure out whether the coin we flipped is A or B, we can estimate θ_A and θ_B through one of our most familiar friends, **MLE.**(Figure 1.1a) But actually, since we can not figure it out, MLE will not work in this situation. Then EM algorithm come on the stage. EM algorithm is simple because it just contain two steps in the Iteration and EM algorithm is complex is complex for its mathematical proof and further stretch in theory and application.(Figure 1.1b)

The first step of the algorithm is **Expectation step**(E step). In this step, we start by assuming $\hat{\theta}_A^{(0)} = 0.60$ and $\hat{\theta}_B^{(0)} = 0.50$, wether the assumption is true is not important. What is significant is that we can compute the expectation of Coin A and Coin B in each ten tosses with the selected coin. and compute the expectation...

1.2 Two steps in iteration of EM

Notation: Given the statistical model which generates a set X of observed data(50 samples in coin-flipping experiment), a set of unobserved latent data or missing values Z(the coin is A or B), and a vector of unknown parameters θ, along with a likelihood function $L(\theta; X, Z) = p(X, Z|\theta)$, and the marginal likelihood function of the observed data is given by:

$$L(\theta; X) = p(X|\theta) = \int p(X, Z|\theta)\, dZ$$

However, this quantity is often intractable (we don't know the coin we tossed is A or B).

Then the EM algorithm seeks to find the MLE of the marginal likelihood by iteratively applying the following two steps.
(a) Expectation Step: