MathFinance 345/Stat390
Homework 8
Due November 28

1. Complete the derivation of the arbitrage price of the BARRIER option in Section 4 of the Lecture Notes:

(a) Use the reflection principle and the Strong Markov property to justify the identity (22).
(b) Evaluate the integral in equation (22).

2. **A Perpetual Option.** Assume that the share prices of STOCK and BOND are given by equations (19) and (20), respectively. Consider an option with no date of expiration that pays the owner $\exp\{-\beta\tau\}$ (dollars) at the first time τ that the share price of STOCK reaches α (if ever). Here β and α are positive real numbers, and $S_0 < \alpha$. Calculate the arbitrage price at time 0 of this option.

3. **Knockin Options.** Assume that the prices of BOND and STOCK are governed by the differential equations

$$dB_t = rB_t\,dt \tag{1}$$

$$dS_t = rS_t\,dt + \sigma S_t\,dW_t. \tag{2}$$

for constants $r, \sigma > 0$. Consider a *knockin* put option with strike K and knockin value $H > K$. The payoff from this option at termination $t = T$ is

$$\begin{array}{rl} (K - S_T)_+ & \text{if } \max\limits_{0\le t\le T} S_t \ge H \\ 0 & \text{if } \max\limits_{0\le t\le T} S_t < H \end{array}$$

Find the arbitrage price at $t = 0$. HINT: Write the price as a discounted expectation, using indicator variables to get rid of the subscript $+$ on $(K - S_T)$. Break this expectation into two expectations, and then evaluate each by using the Cameron–Martin theorem and the reflection principle.