Receding Horizon Planning – an Introduction

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1 Introduction

To remain viable in the era of climate change, societies will require an economic system that is capable of rapid adaptation in the face of unplanned shortages and extreme weather events as well as long-term planning of investments on a societal scale for infrastructure, housing and production.

Consider a large-scale economic system consisting of m units of production. Each unit produces a collection of products, including waste by-products, and use up a collection of inputs – human produced and natural – over a given period. At any given time t – measured in units of weeks, months or possibly years – we wish to coordinate their joint production over the coming N time periods. That is, plan for future time periods

$$t+1,\ t+2,\ \dots,\ t+N.$$

Based on new information that is made available as time period t progresses into the next $t+1$, the production coordination for the subsequent N periods is revised. This is a *receding horizon planning* (RHP) approach to economic coordination.

The goal is to continuously reallocate production in a resource efficient manner to maintain long-run viability, based on continuous inputs and economic feedback (on production levels, available stocks, etc.). See Figure 1 for a conceptual illustration.

2 Planning problem

Let $n = 1, \dots, N$ index the subsequent time periods and variable $x_i(n) \geq 0$ denote the *level* of production in unit i. One choice of measure considered below is the direct labour input to unit i in, say, person-hours, but alternative measures are valid. The collection of production levels across *all* units

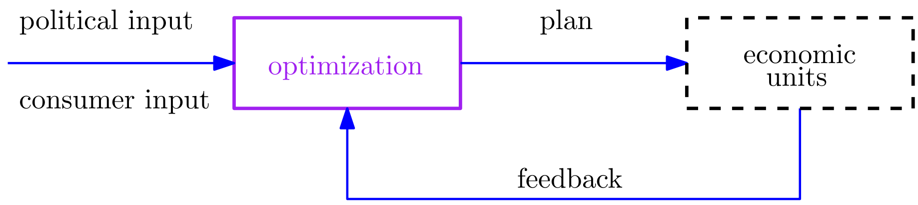

Figure 1: Conceptual overview of economic coordination via RHP. Economic targets and constraints are negotiated inputs and data is fed back through a set of protocols. An optimization procedure selects a resource-minimizing plan among all feasible plans, which is broadcast to the units of production.

of production is represented by the $m \times 1$ vector

$$x(n) = \begin{bmatrix} x_1(n) \\ x_2(n) \\ \vdots \\ x_m(n) \end{bmatrix}.$$

The planning problem consists of determining a set of target production levels over a horizon of N periods ahead. That is,

$$\{x(1), x(2), \dots, x(N)\} \tag{1}$$

is a recommended *plan* for the coming N time periods that is broadcast to all units of production. As production and consumption takes place over the subsequent period $t+1$, new economic data is made available that is *fed back* into the coordination process. Then a revised set of plans (1) are determined for the subsequent N periods.

The units of production consume and output a wide range of d distinct products. Based on investment goals, consumption demands, material constraints and available stocks, we are given a set of (net) *output* targets which we collect in a $d \times 1$ vector

$$r(n) = \begin{bmatrix} r_1(n) \\ r_2(n) \\ \vdots \\ r_d(n) \end{bmatrix}$$

for period n. The targets over the planning horizon form the set:

$$\{r(1), r(2), \dots, r(N)\} \tag{2}$$

A plan (1) is then said to be *feasible* if the joint production of all units can meet the set of targets (2).

To define the feasible plans, we introduce a $d \times m$ matrix of joint production coefficients, $J(n)$, such that

$$J(n)x(n) = \begin{bmatrix} j_{11}(n) \\ j_{21}(n) \\ \vdots \\ j_{d1}(n) \end{bmatrix} x_1(n) + \begin{bmatrix} j_{12}(n) \\ j_{22}(n) \\ \vdots \\ j_{d2}(n) \end{bmatrix} x_2(n) + \cdots + \begin{bmatrix} j_{1m}(n) \\ j_{2m}(n) \\ \vdots \\ j_{dm}(n) \end{bmatrix} x_m(n)$$

represent a first-order approximation of the d net outputs from all m units during period n. A feasible plan must satisfy the following material constraints $J(n)x(n) \geq r(n)$ for $n = 1$. Some of the *excess* outputs, $J(1)x(1) - r(1)$, can be transferred from one period to the next. The proportion transferred from k prior periods is determined by a $d \times d$ depreciation matrix $D(k)$. A feasible plan must therefore satisfy a set of material resource constraints:

$$\sum_{k=1}^{n} D(k-1)\big[J(k)x(k) - r(k)\big] \geq 0 \quad \text{for all } n = 1, 2, \ldots, N. \tag{3}$$

There are, however, many different plans (1) that satisfy the constraints (3). Among the feasible plans, we now seek one that where the units of production utilize the minimum amount of some primary resource, say, labour time, or output the minimal amount of some waste by-product. We let this be a 'cost' such that

$$c^{\intercal}(n)x(n) = c_1(n)x_1(n) + c_2(n)x_2(n) + \cdots + c_m(n)x_m(n)$$

equals the total utilization of the resource across all units of production operating with production level $x(n)$. We can now state a resource-minimizing RHP-problem in a formal manner:

$$\begin{aligned} \underset{\{x(n)\}}{\arg\min} \; & \sum_{n=1}^{N} c^{\intercal}(n)x(n) \\ \text{subject to} \; & \sum_{k=1}^{n} D(k-1)\big[J(k)x(k) - r(k)\big] \geq 0, \; \forall n = 1, \ldots, N \\ & x(n) \geq 0, \; \forall n = 1, \ldots, N \end{aligned} \tag{4}$$

That is, given the data $\{(r(n), c(n), J(n), D(n))\}$ compute an optimal plan (1). This is a linear optimization problem that can be solved efficiently by modern computational methods that exploit sparse matrix structures.

Solving (4) also enables evaluating alternative productive investments into existing units of production or the addition of entirely new units. It is also possible to analyze the ability to adapt to sudden economic changes arising from e.g. extreme weather events.

3 Additional constraints

The basic RHP-problem (4) can, of course, be augmented with additional economic constraints. For illustration, we consider two important constraints on the rates of change and of balance of payments. Additional logistical constraints can also be considered.

3.1 Constraints on rate of change

While we are able to reduce the resource utilization to its minimal level, there may constraints to much change on production levels is actually feasible from one period $n-1$ to the next, n. This may depend on the supply of matching labor power or physical constraints of physical processes with very slow dynamics.

For simplicity, let $\bar{\delta}_i$ and $\underline{\delta}_i$ denote the maximum relative increase and decrease in production possible for unit i, respectively. Then we may add the following constraint to (4):

$$(I-\underline{\Delta})x(n-1) \leq x(n) \leq (I+\bar{\Delta})x(n-1), \; n = 1, \ldots, N,$$

where

$$\underline{\Delta} = \operatorname{diag}(\underline{\delta}_1, \underline{\delta}_2, \ldots, \underline{\delta}_m) \qquad \bar{\Delta} = \operatorname{diag}(\bar{\delta}_1, \bar{\delta}_2, \ldots, \bar{\delta}_m)$$

and $x(0)$ denotes the current production levels.

3.2 Trade balance constraints

Virtually modern economic system on any regional scale is entirely self-sufficient but must import certain inputs.

Let $T(n)$ denote a $d \times m$ input import matrix such that

$$T(n)x(n)$$

denotes the $d \times 1$ vector of total imported inputs. For long-run balance of payments, however, the imports must be payed by exported goods and services. Let $r_{\text{exp}}(n)s(n)$ be a target vector of export products, where $s(n) \geq 0$ is a scale factor and $r_{\text{exp}}(n)$ specifies a fixed mix of products. We can then augment a trade-balance constraint to (4):

$$\sum_{n=1}^{N} p_{\text{imp}}^{\mathsf{T}} T(n)x(n) \leq \sum_{n=1}^{N} p_{\text{exp}}^{\mathsf{T}} r_{\text{exp}}(n)s(n),$$

where p_{imp} and p_{exp} are $d \times 1$ vectors of import and export prices. This adds a set of export scale factors $\{s(n)\}$ to the RHP-problem and modifies the target output vectors accordingly.

4 Existing economic data

Tackling the RHP-problem would require data from a vast number of units of production as well as a standardized list of output-types.

It is however already possible to provide some rudimentary analysis and policy recommendations using existing national accounts data, which measures many distinct outputs in units of monetary prices. E.g., dollars of steel, dollars of agricultural products, and so on. The joint production coefficient matrix, J, can be decomposed into three coefficient matrices:

$$J = B - A - T$$

with B for supply, A for the domestic inputs and T for imported inputs.

The production levels x can be quantified in units of direct labour time, which then determine the coefficient matrices above in terms of products per unit of labour time. The cost vector c can e.g. be units of labour, in which case it equals a vectors of 1s, or the amount of carbon emissions per unit of labour.

For target quantities r one can initially use the household consumption and total investment vectors, with quantities measured per unit of time.