Lab assignment 7 (Iterative method and Gradient descent method)

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Contents

1 Task 1

Uppgift 1.1 (Task1)**.** Solve the below linear equations using Gaussian elimination.

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3. \end{aligned}$$

Lösning 1.1.1. Let us first reformulate the equation as an augmented matrix:

$$\left(\begin{array}{ccc|c} 5 & -2 & 3 & -1 \\ -3 & 9 & 1 & 2 \\ 2 & -1 & -7 & 3 \end{array}\right).$$

Let us first normalise the first row, that yields:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ -3 & 9 & 1 & 2 \\ 2 & -1 & -7 & 3 \end{array}\right).$$

Let us now eliminate the second row of the first column by adding 3 times the first row:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & \frac{39}{5} & \frac{14}{5} & \frac{7}{5} \\ 2 & -1 & -7 & 3 \end{array}\right).$$

Let us now eliminate the third row of the first column by adding -2 times the first row:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & \frac{39}{5} & \frac{14}{5} & \frac{7}{5} \\ 0 & -\frac{1}{5} & -\frac{41}{5} & \frac{17}{5} \end{array}\right).$$

Let us now normalise the second row of the second column by multiplying with $\frac{5}{39}$:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & \frac{14}{39} & \frac{7}{39} \\ 0 & -\frac{1}{5} & -\frac{41}{5} & \frac{17}{5} \end{array}\right).$$

In the normal algorithm we would now add a $\frac{1}{5}$ times the second row to the third row, in order to eliminate the third row in the second column. But we will instead first scale the third row by 5, in order to make it more readable:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & \frac{14}{39} & \frac{7}{39} \\ 0 & -1 & -41 & 17 \end{array}\right).$$

Now we can simply add the second row to the third row in order to eliminate the third row of the second column:

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & \frac{14}{39} & \frac{7}{39} \\ 0 & 0 & \frac{14-41\cdot 39}{39} & \frac{7+17\cdot 39}{39} \end{array}\right).$$

From the third row we get that:

$$\begin{aligned} \frac{14-41\cdot 39}{39}x_3 &= \frac{7+17\cdot 39}{39} \\ \Longrightarrow (14-41\cdot 39)x_3 &= 7+17\cdot 39 \\ \Longrightarrow x_3 &= \frac{7+17\cdot 39}{14-41\cdot 39} \\ &= \frac{7+663}{14-1599} \\ &= \frac{670}{-1585} = -\frac{134}{317}. \end{aligned}$$

From the second row of the augmented matrix we get that:

$$\begin{aligned} x_2 + \frac{14x_3}{39} &= \frac{7}{39} \\ \Longrightarrow x_2 &= \frac{7-14x_3}{39} \\ \Longrightarrow 39x_2 &= 7-14x_3 \\ \Longrightarrow 39x_2 &= 7+\frac{14\cdot 134}{317} \\ \Longrightarrow 39x_2 &= \frac{7\cdot 317+14\cdot 134}{317} \\ \Longrightarrow 39x_2 &= \frac{2219+1876}{317} \\ \Longrightarrow 39x_2 &= \frac{4095}{317} \\ \Longrightarrow 39x_2 &= \frac{819\cdot 5}{317} && \text{we observe that 819 is divisible by 3} \\ \Longrightarrow 39x_2 &= \frac{273\cdot 3\cdot 5}{317} && \text{273 is also divisible by 3} \\ \Longrightarrow 39x_2 &= \frac{91\cdot 9\cdot 5}{317} && \text{39 is also divisible by 3} \\ \Longrightarrow 13x_2 &= \frac{91\cdot 3\cdot 5}{317} \\ \Longrightarrow x_2 &= \frac{7\cdot 3\cdot 5}{317} = \frac{105}{317}. \end{aligned}$$

From the first row we get that:

$$\begin{aligned}
& x_1 - \frac{2x_2}{5} + \frac{3x_3}{5} = -\frac{1}{5} \\
\implies & x_1 = \frac{2x_2}{5} - \frac{3x_3}{5} - \frac{1}{5} \\
\implies & 5x_1 = 2x_2 - 3x_3 - 1 \\
\implies & 5x_1 = \frac{2 \cdot 105}{317} + \frac{3 \cdot 134}{317} - 1 \\
& \quad = \frac{210}{317} + \frac{402}{317} - 1 \\
& \quad = \frac{210 + 402 - 317}{317} = \frac{295}{317} \\
\implies & x_1 = \frac{59}{317}.
\end{aligned}$$

So to summarise the solution is:

$$\begin{aligned}
x_1 &= \frac{59}{317} \\
x_2 &= \frac{105}{317} \\
x_3 &= -\frac{134}{317}.
\end{aligned}$$