Lab 4 MM5016

Contents

1 Problems

Uppgift 1.1 (System 1)**.** Use Gaussian elimination to solve the system of linear equations

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5. \end{aligned}$$

Lösning 1.1.1. First step is to check that the coefficient of x_1 of the first equation is non-zero. Next step is to eliminate all x_1 under row 1. We do this by adding an appropiate multiple of row 1 to the row we want to eliminate x_1 from. In this case it means to add the first row times 2 to second row. That yields the following system:

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ 3x_2 &= 9. \end{aligned}$$

Our system is now overtriangular, which mean we can go into the process of solving each equation by themselves by starting from the bottom and substituting into the equation above relative of the equation in question.
So we start by solving $x_2 = 3$ from the second equation. Next step is to substitute this into the above equation and get the equation

$$x_1 + 15 = 7.$$

And thus we can from this equation solve that $x_1 = -8$.

Uppgift 1.2 (System 2)**.** Use Gaussian elimination to solve the system of linear equation

$$\begin{aligned} 2x_2 + x_3 &= -8 \\ x_1 - 2x_2 - 3x_3 &= 0 \\ -x_1 + x_2 + 2x_3 &= 3. \end{aligned}$$

Lösning 1.2.1. First step is to see if the first equation has a non-zero coefficent for x_1, we see that this is not the case. So we swap this row and the closest row below which has a non-zero coefficient for x_1. In this case it's row 2. So we swap row 1 and 2. We get the following system:

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 0 \\ 2x_2 + x_3 &= -8 \\ -x_1 + x_2 + 2x_3 &= 3. \end{aligned}$$

Next step is to eliminate all x_1 from the equations below by adding an appropiate multiple of row 1 to it. We see that row 2 already is free from x_1. So we go to the next row which is row number 3. We see that we can add 1 times equation 1 to it, which yields the following system:

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 0 \\ 2x_2 + x_3 &= -8 \\ -x_2 - x_3 &= 3. \end{aligned}$$

Now we apply Gaussian elimination on this new system of 2 equations and 2 variables under row 1:

$$\begin{aligned} 2x_2 + x_3 &= -8 \\ -x_2 - x_3 &= 3. \end{aligned}$$

To solve this system we use the same algorithm as in the previous task, but the first variable is now x_2 instead. So we eliminate x_2 from the second equation by adding the first equation times $\frac{1}{2}$. Which yields the following system:

$$\begin{aligned} 2x_2 + x_3 &= -8 \\ -\frac{1}{2}x_3 &= -1. \end{aligned}$$

So we have the system:

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 0 \\ 2x_2 + x_3 &= -8 \\ -\frac{1}{2}x_3 &= -1. \end{aligned}$$

So we can start by solving the last equation and then substituting into the above one and repeat. So from the third equation we get $x_3 = 2$. Substiuting into the system we get:

$$\begin{aligned} x_1 - 2x_2 - 6 &= 0 \\ 2x_2 + 2 &= -8. \end{aligned}$$

Solving the last equation we get $x_2 = -5$. Substituting into the first equation we get:

$$x_1 + 10 - 6 = 0.$$

So we get that $x_1 = -4$
Final answer: $(x_1, x_2, x_3) = (-4, -5, 2)$.

Uppgift 1.3 (System 3)**.** Use Gaussian elimination to solve the system of linear equations

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 12 \\ 2x_1 + 4x_2 + 12x_3 &= -17 \\ x_1 - 4x_2 - 12x_3 &= 22. \end{aligned}$$

Lösning 1.3.1. x_1 is non-zero so we don't need to swap equations as for now. So we start by eliminating all x_1 from all the rows below the first row. To eliminate x_1

from the second row we add -2 times the first row to it. That yields the following equation:

$$8x_2 + 24x_3 = -41.$$

To eliminate x_1 from the third equation we add -1 times the first equation. Resulting in the equation:

$$-2x_2 - 6x_3 = 10.$$

So under row 1 we have the following system of 2 variables and equations:

$$\begin{aligned} 8x_2 + 24x_3 &= -41 \\ -2x_2 - 6x_3 &= 10. \end{aligned}$$

So we start by elimaniting x_2 from the equations under row 1. We do this by adding $\frac{1}{4}$ times the first equation to the second equation. The second equation then becomes:

$$0 = 10 - \frac{41}{4}.$$

Which is a contradiction which means there exists no solutions.