Tut8_ST5218

Name:Zhu Xu
User ID:E0337988
Matriculation ID:A0191344H

9/April/2019

Question 1:

a)

$$\gamma(0) = (1 + 1.2^2 + 0.3^2)$$

$$\gamma(1) = 1 \times 1.2 + 1.2 \times (-0.3)$$

$$\gamma(2) = 1 \times (-0.3)$$

$$\gamma(3) = \gamma(4) = \gamma(5) = 0$$

Thus

$$\rho(1) = 0.332, \rho(2) = -0.119, \rho(3) = \rho(4) = \rho(5) = 0$$

b)

We have

$$\gamma(0) = 0.5\gamma(1) + 1, \quad \gamma(1) = 0.5\gamma(0), \quad \gamma(h) = 0.5\gamma(h-1) \quad for \quad h > 1$$

Thus

$$\rho(h) = 0.5^h \quad for \quad h \geq 1$$

c)

Similarly

$$\gamma(0) = \frac{1}{1 - 0.098^2}; \qquad \gamma(1) = \frac{0.98}{1 - 0.98^2}; \qquad \rho(h) = 0.98^h, \quad h \geq 1$$

d)

Fram $Z_t - 1.3Z_{t-1} + 0.4Z_{t-2} = a_t$, we have

$$\gamma(0) - 1.3\gamma(1) + 0.4\gamma(2) = 4$$
$$\gamma(1) - 1.3\gamma(0) + 0.4\gamma(1) = 0$$
$$\gamma(2) - 1.3\gamma(1) + 0.4\gamma(0) = 0$$
$$\gamma(h) - 1.3\gamma(h-1) + 0.4\gamma(h-2) = 0$$

Then

$$\gamma(0) = 34.5679, \quad \gamma(1) = 32.0988, \quad \gamma(2) = 27.9012, \quad \gamma(h) = 1.3\gamma(h-1) - 0.4\gamma(h-2)$$

Thus we have

$$\gamma(3) = 1.3 \times 27.9012 - 0.4 \times 32.0988 = 23.4320$$
$$\gamma(4) = 1.3 \times 23.4320 - 0.4 \times 27.9012 = 19.3011$$
$$\gamma(5) = 1.3 \times 19.3011 - 0.4 \times 23.4320 = 15.7186$$

Question 2:

Since

$$y_{t+1|t} = \mu + \phi_1(y_t - \mu)$$
$$y_{t+2|t} = \mu + \phi_1(y_{t+1|t} - \mu) = \mu + \phi_1^2(y_t - \mu)$$
$$y_{t+3|t} = \mu + \phi_1(y_{t+2|t} - \mu) = \mu + \phi_1^3(y_t - \mu)$$
$$y_{t+h|t} = \mu + \phi_1(y_{t+h-1|t} - \mu) = \mu + \phi_1^h(y_t - \mu) \to \mu \text{ as } h \to \infty$$

Thus write the model as

$$Y_t = \mu + \phi_1(Y_{t-1} - \mu) + a_t$$

Question 3:

After $q + 1$ steps.

Question 4:

$$\begin{aligned}
y_t &= a_t + \theta_1 a_{t-1} \\
&= a_t + \theta_1(\theta_1 a_{t-2} + a_{t-1}) \\
&= a_t + \theta_1 a_{t-1} + \theta_1^2 a_{t-2} \\
&= \cdots \\
&= a_t + \theta_1 a_{t-1} + \cdots + \theta_1^{h-1} a_{t-h+1} + \theta_1^h a_{t-h} \\
&\to AR(\infty)
\end{aligned}$$

Question 5:

a)

```r
library(tseries)
library(timeSeries)
```

```
## Loading required package: timeDate
```

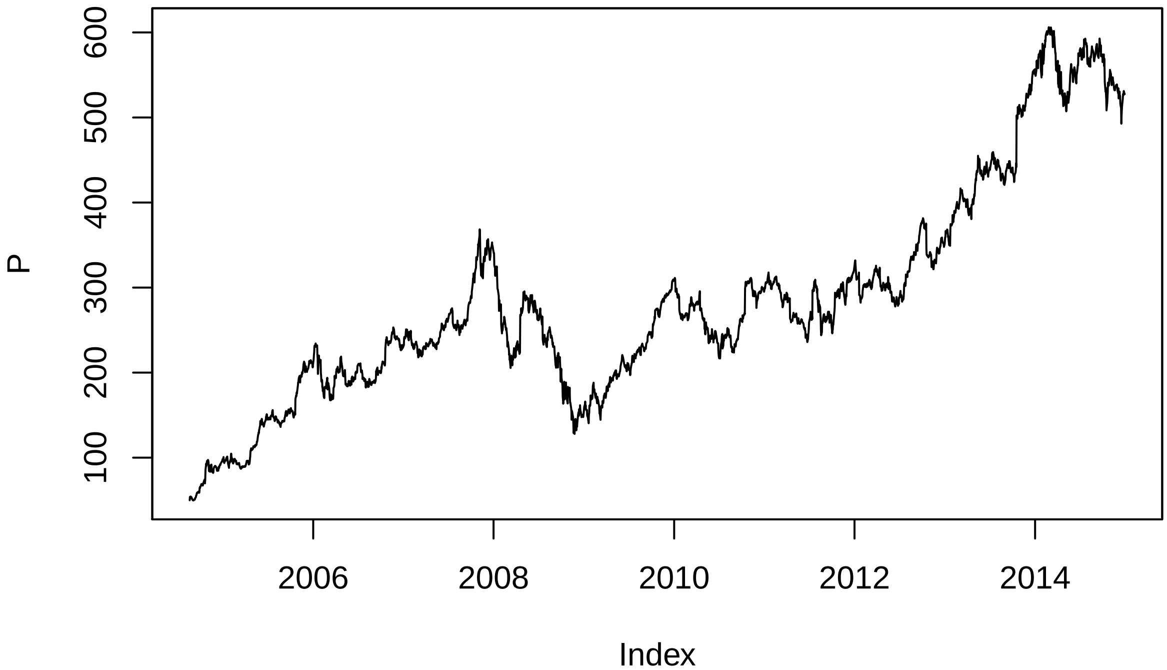
```r
P = get.hist.quote(instrument ='GOOG',start='2001-01-01',
                   end='2014-12-31', quote=c('AdjClose'),
                   provider='yahoo', compression='d')
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

```
##
## WARNING: There have been significant changes to Yahoo Finance data.
## Please see the Warning section of '?getSymbols.yahoo' for details.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.yahoo.warning"=FALSE).
```

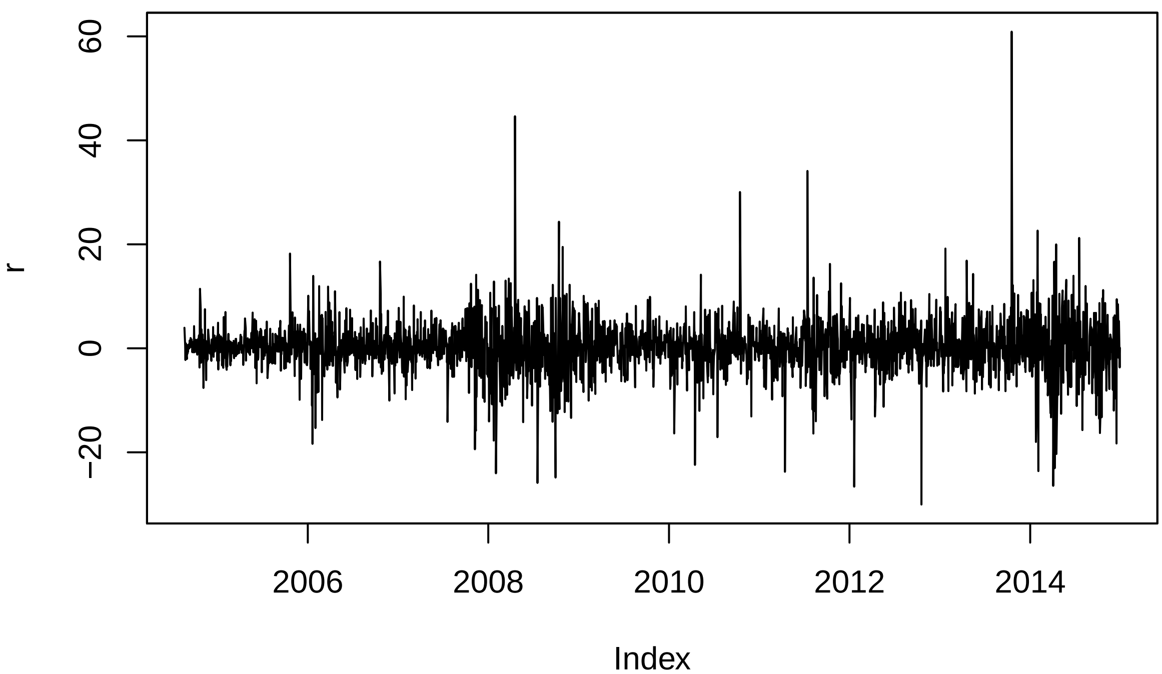
```
## time series starts 2004-08-19
## time series ends   2014-12-30
```

```r
plot(P, type='l')
```



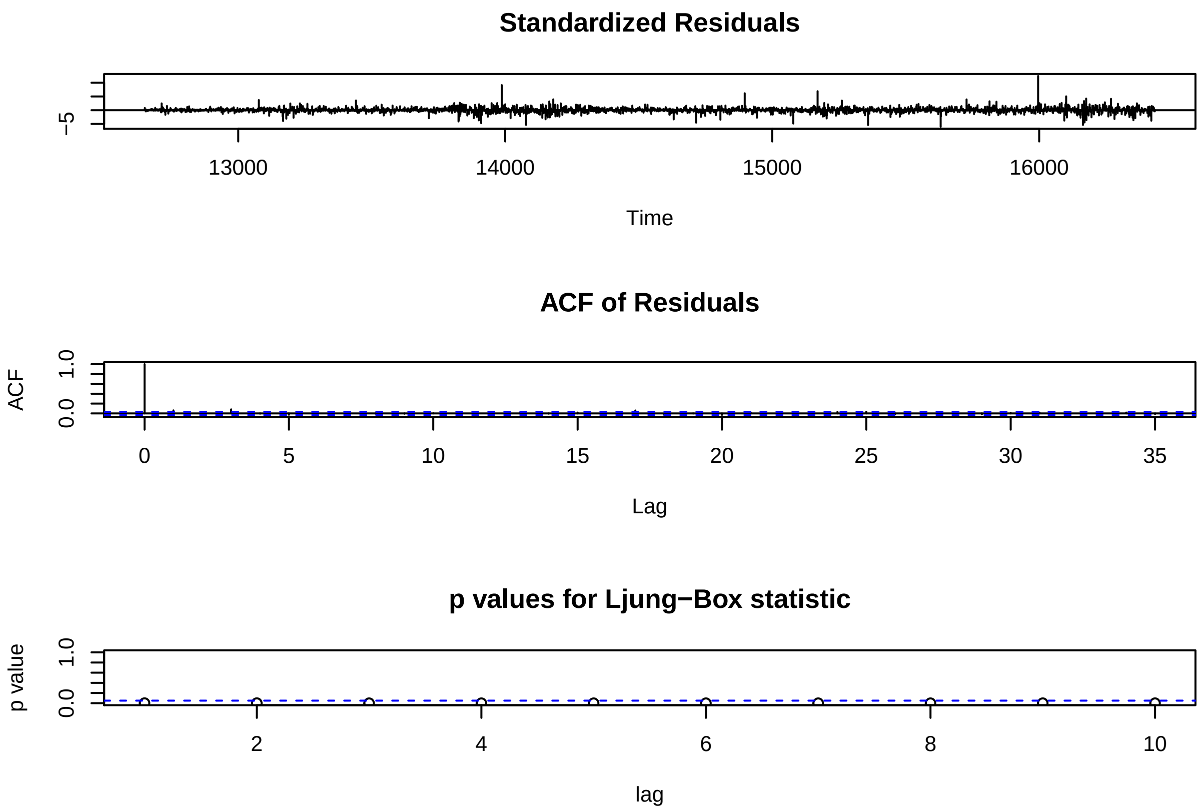
b)

```r
r = diff(P)
plot(r)
```

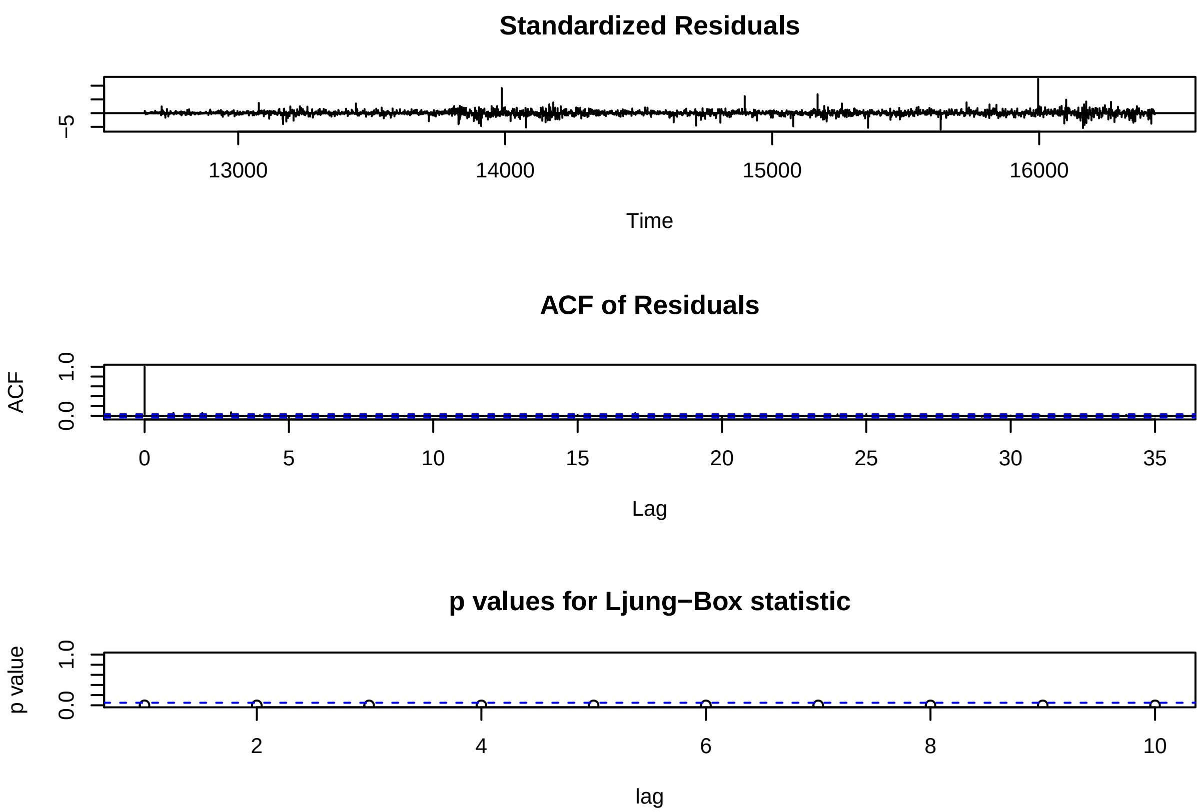


c)

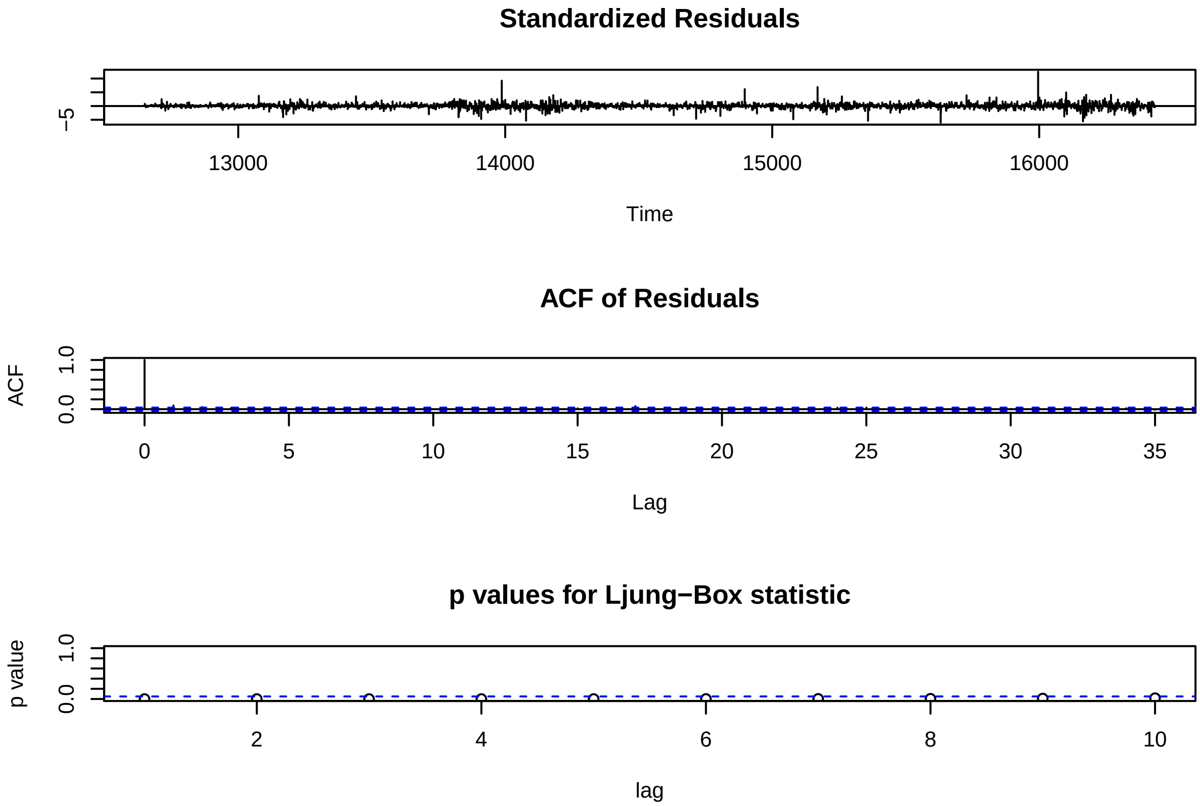
```r
model1 = arima(P, order=c(1, 1, 1))
tsdiag(model1)
```




```r
model2 = arima(P, order=c(2, 1, 1))
tsdiag(model2)
```



```r
model3 = arima(P, order=c(2, 1, 2))
tsdiag(model3)
```



Question 6:

a)

$$E_{y_t} = 2 + 0.7E_{y_{t-1}} + Ea_t + 0.3Ea_{a-1} + 0.4Ea_{t-2} = 2 + 0.7E_{y_{t-1}}$$

By its being stationary,$E_{y_{t-1}} = Ey_t$, thus

$$E_{y_t} = 2 + 0.7E_{y_t} = \frac{2}{1-0.7} = \frac{20}{3}$$

b)

$$a_{t-1} = y_{t-1} - y_{t-1|t-2} = 1 - \frac{20}{3} = -5.667$$

Then we predict

$$y_t = 2 + 0.7 \times 1 + 0 + 0.3 \times (-5.667) + 0.4 \times 0 = 0.999$$

c)

$y_{y-1} = 1$

$$a_{t-2} = y_{t-2} - y_{t-2|t-3} = 0.5 - \frac{20}{3} = -6.1667$$

Then

$$y_{t-1|t-2} = 2 + 0.7 \times 0.5 + 0 + 0.3 \times (-6.1667) + 0.4 \times 0 = 0.4999$$

$$a_{t-1} = y_{t-1} - y_{t-1|t-2} = 1 - 0.4999 = 0.5001$$

Thus

$$y_t = 2 + 0.7 \times 1 + 0 + 0.3 \times 0.5001 + 0.4 \times (-6.1667) = 0.3833$$