Assessment1-ST5222

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Problem:

We are given a Binomial distribution:

$$f(x) = \binom{n}{k} p^x (1-p)^{n-x} \qquad x \in \{0, \cdots, n\}$$

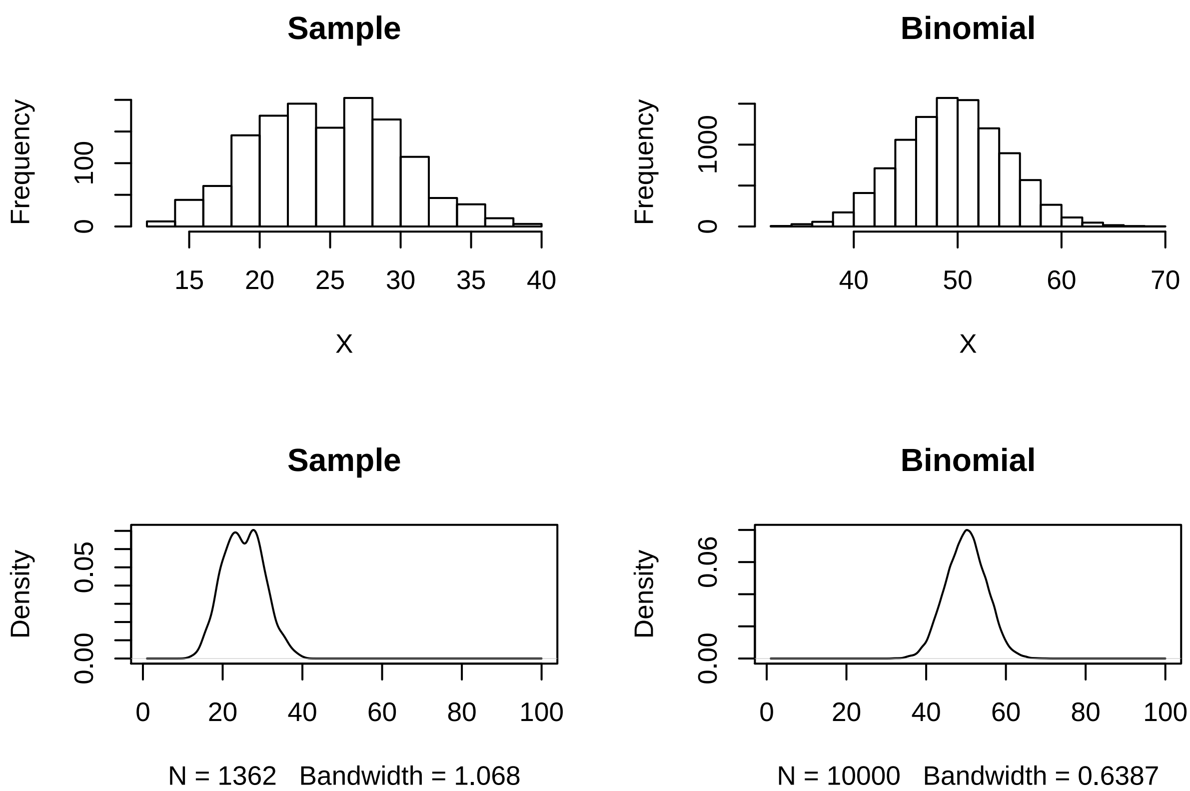
now,$n = 100$,and write a computer code to perform a Metropolis-Hastings algorithm with stationary distribution $f(x)$.Consider various p as well as proposals.
Here is my solutions.

Solutions:

1.Proposal–Unif , Binomial distribution(n=100, p=0.25)

```
f_0 <- function(x){
  (factorial(100)/(factorial(x)*factorial(100-x)))*0.25^(x)*(1-0.25)^(100-x)
  }
# define the Binomial distribution , n=100, p=0.25
q_0 <- function(x){1/100}
# define the proposal function q_0 (Uniform)
n <- 100
X <- numeric() # store samples
count <- 1
x_old <- 0 # initialize the Markov chain
for(i in 1:10000) # main loop to obtain samples
{
  x_new <- sample.int(101, 1)-1 # sample a integer from q_0, as a candidate
  u <- runif(1,0,1)
  acc_1 <- min((f_0(x_new)*q_0(x_old))/(f_0(x_old)*q_0(x_new)), 1)
# calculate the acceptance prob
  if(u < acc_1) # accept
  {
    X[count] <- x_new
    count <- count +1
    x_old <- x_new
  }
  else # reject the sample
  {
    x_old <- x_old
  }
}
par(mfrow = c(2, 2))
hist(X, main = "Sample", xlab = "X")
XX <- rbinom(10000, 100, 0.5)
hist(XX, main = "Binomial", xlab = "X")
P <- density(X, n = 10000, from = 1, to  = 100)
plot(P, main = "Sample")
```

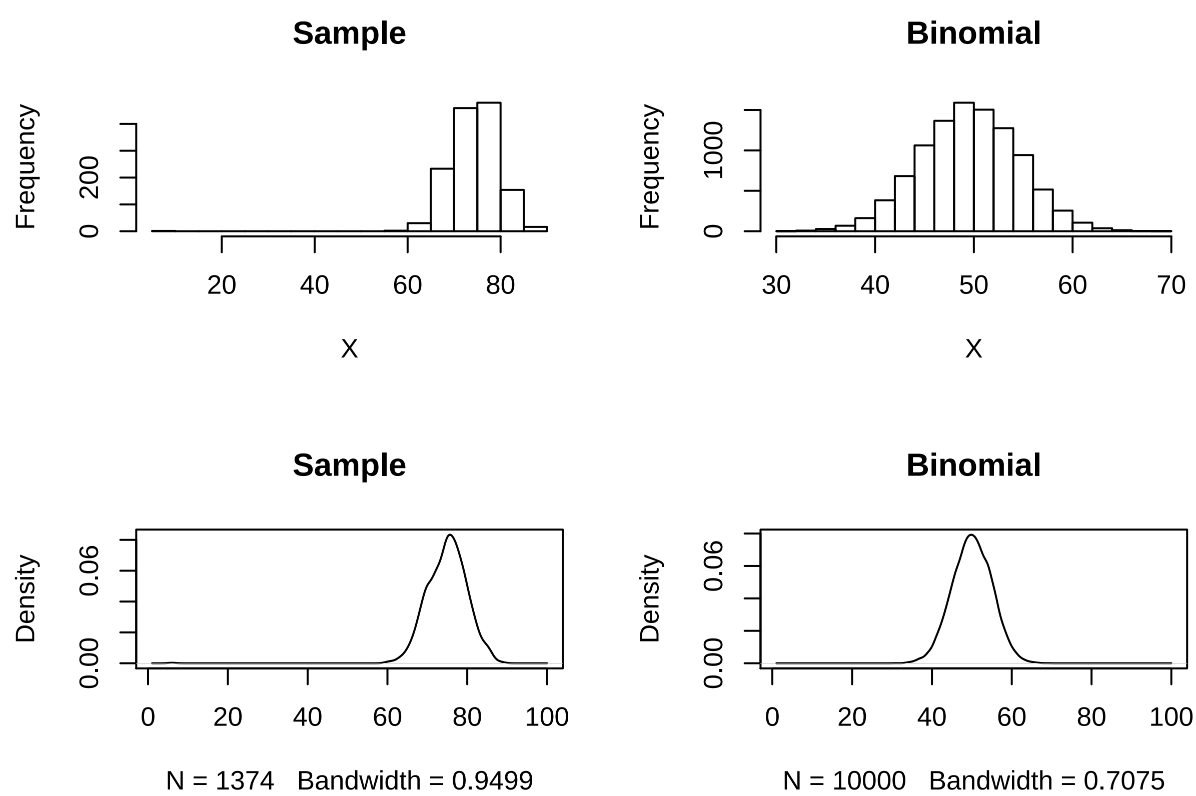
```r
B <- density(XX, n = 10000, from = 1, to  = 100)
plot(B, main = "Binomial")
```



2.Proposal–Unif , Binomial distribution(n=100, p=0.75)

```r
f_2 <- function(x){
  (factorial(100)/(factorial(x)*factorial(100-x)))*0.75^(x)*(1-0.75)^(100-x)
  }
# define the Binomial distribution , n=100, p=0.75
q_0 <- function(x){1/100}
# define the proposal function q_0 (Uniform)
n <- 100
X <- numeric() # store samples
count <- 1
x_old <- 0 # initialize the Markov chain
for(i in 1:10000)
{
  x_new <- sample.int(101, 1)-1 # sample a integer from q_0, as a candidate
  u <- runif(1,0,1)
  acc_1 <- min((f_2(x_new)*q_0(x_old))/(f_2(x_old)*q_0(x_new)), 1)
# calculate the acceptance prob
  if(u < acc_1) # accept
  {
    X[count] <- x_new
    count <- count +1
```

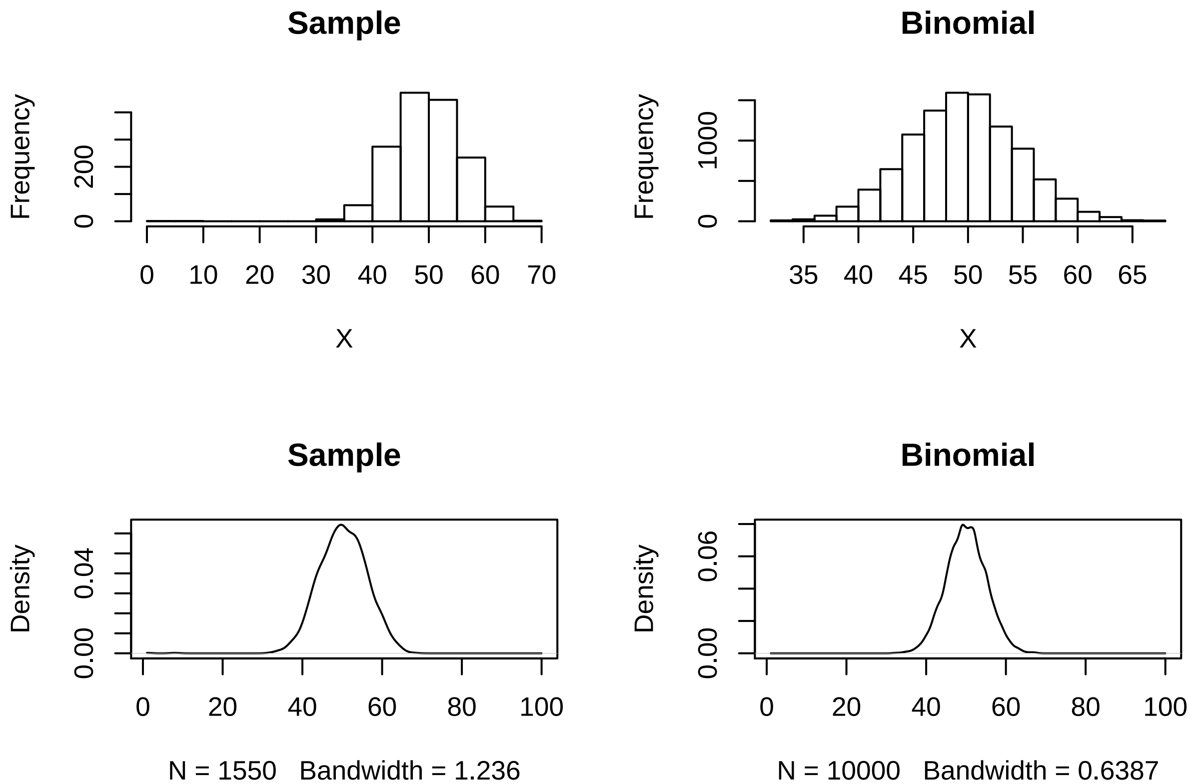
```
    x_old <- x_new
  }
  else # reject
  {
    x_old <- x_old
  }
}
par(mfrow = c(2, 2))
hist(X, main = "Sample", xlab = "X")
XX <- rbinom(10000, 100, 0.5)
hist(XX, main = "Binomial", xlab = "X")
P <- density(X, n = 10000, from = 1, to  = 100)
plot(P, main = "Sample")
B <- density(XX, n = 10000, from = 1, to  = 100)
plot(B, main = "Binomial")
```



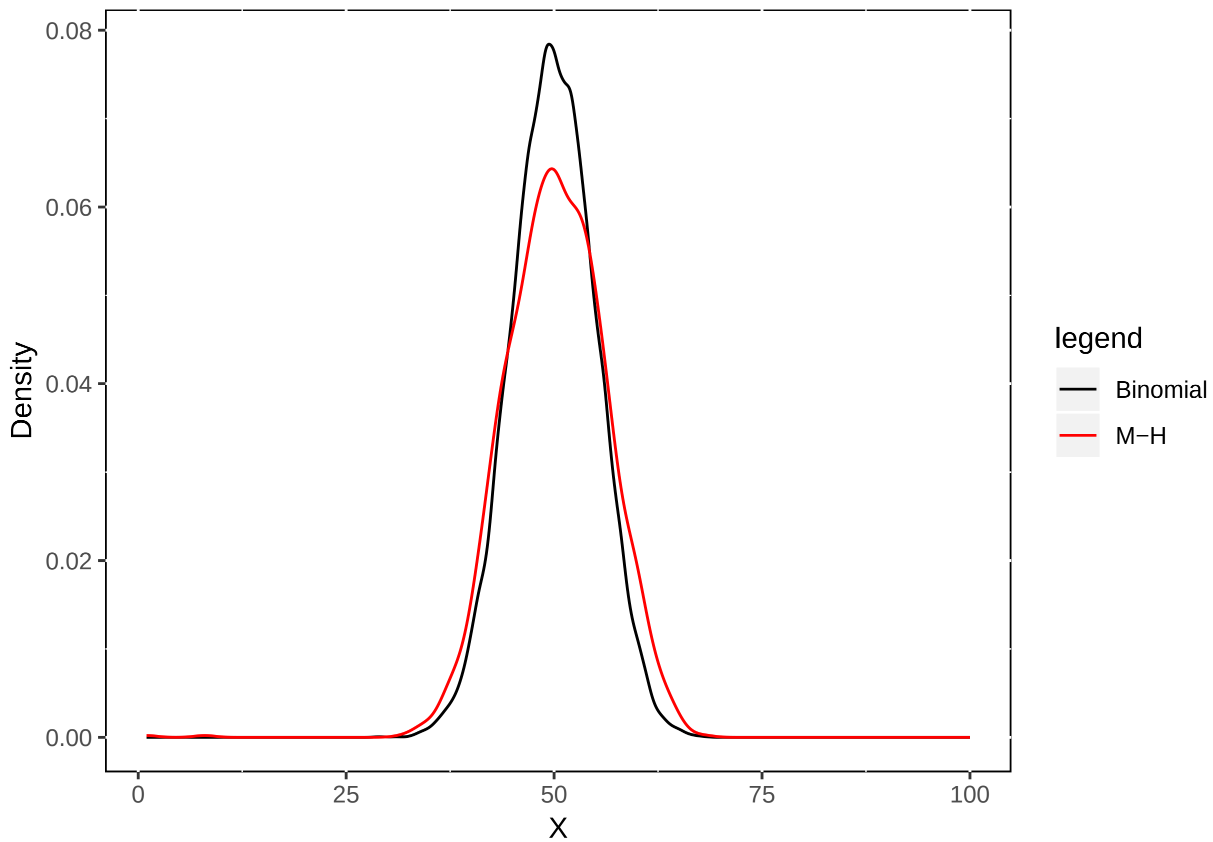
3.Proposal — unif 3, p=0.5

```
f_1 <- function(x){
  (factorial(100)/(factorial(x)*factorial(100-x)))*0.5^(x)*(1-0.5)^(100-x)
  }
# define the Binomial distribution , n=100, p=0.5
q_0 <- function(x){1/100}
# define the proposal function q_0 (Uniform)
n <- 100
```

```r
X <- numeric() # store samples
count <- 1
x_old <- 0 # initialize the Markov chain
for(i in 1:10000)
{
  x_new <- sample.int(101, 1)-1 # sample a integer from q_0, as a candidate
  u <- runif(1,0,1)
  acc_1 <- min((f_1(x_new)*q_0(x_old))/(f_1(x_old)*q_0(x_new)), 1)
# calculate the acceptance prob
  if(u < acc_1) # accept
  {
    X[count] <- x_new
    count <- count +1
    x_old <- x_new
  }
  else # reject
  {
    x_old <- x_old
  }
}
par(mfrow = c(2, 2))
hist(X, main = "Sample", xlab = "X")
XX <- rbinom(10000, 100, 0.5)
hist(XX, main = "Binomial", xlab = "X")
P <- density(X, n = 10000, from = 1, to  = 100)
plot(P, main = "Sample")
B <- density(XX, n = 10000, from = 1, to  = 100)
plot(B, main = "Binomial")

p <- 0.5
real <- rbinom(10000, 100, p)
d <- density(real, n = 10000, from = 1, to =  100)
s <- density(X, n = 10000, from = 1 ,to =  100)
Data_real <- data.frame(num = d$x, y = d$y, legend = rep(c("Binomial "), 10000, 1))
Data_sample <- data.frame(num = s$x, y = s$y, legend = rep(c("M-H"), 10000, 1))
Data <- rbind(Data_real, Data_sample)
library(ggplot2)
```



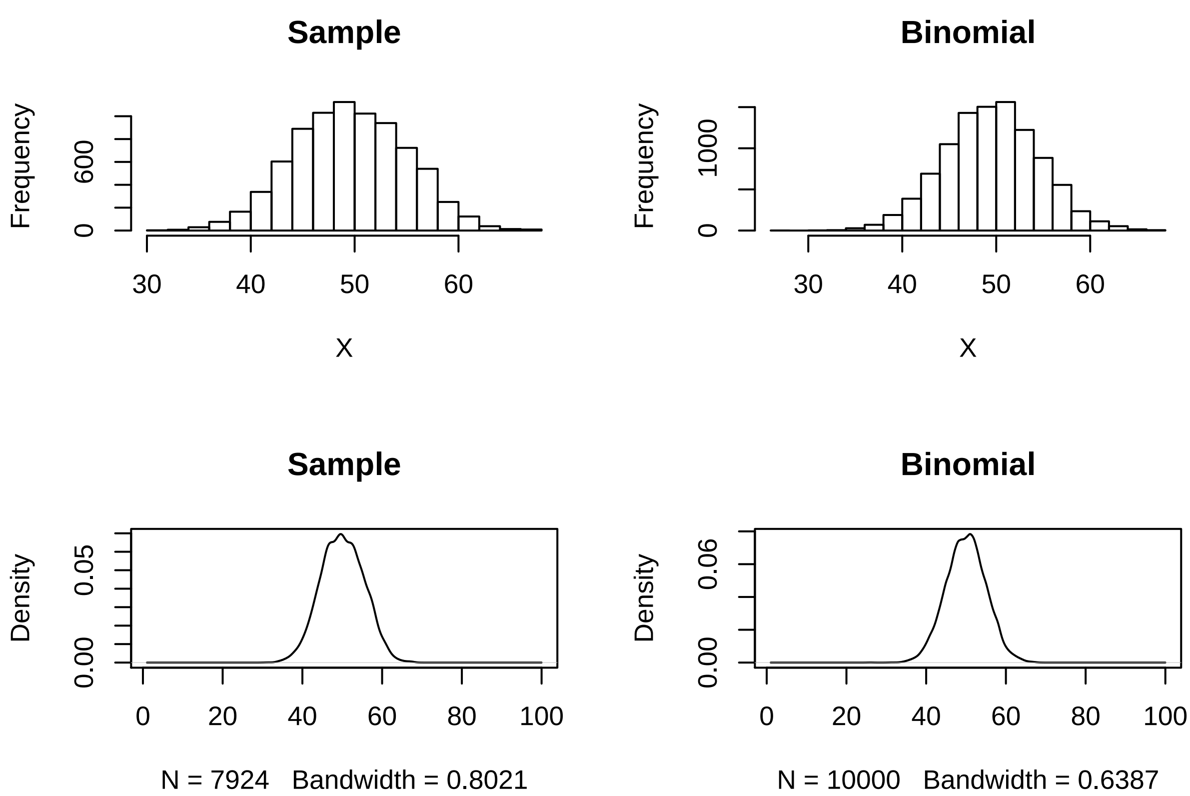

```
g <- ggplot( Data, aes(x = num, y = y, group = legend) ) +
geom_line( aes(color = legend), size = 0.5 ) +
xlab("X") + ylab("Density") +
scale_color_manual(values = c("black", "red")) +
theme(panel.background = element_rect(fill = 'white', colour = 'black'))
g
```



4.Proposal — poisson lamdba=50

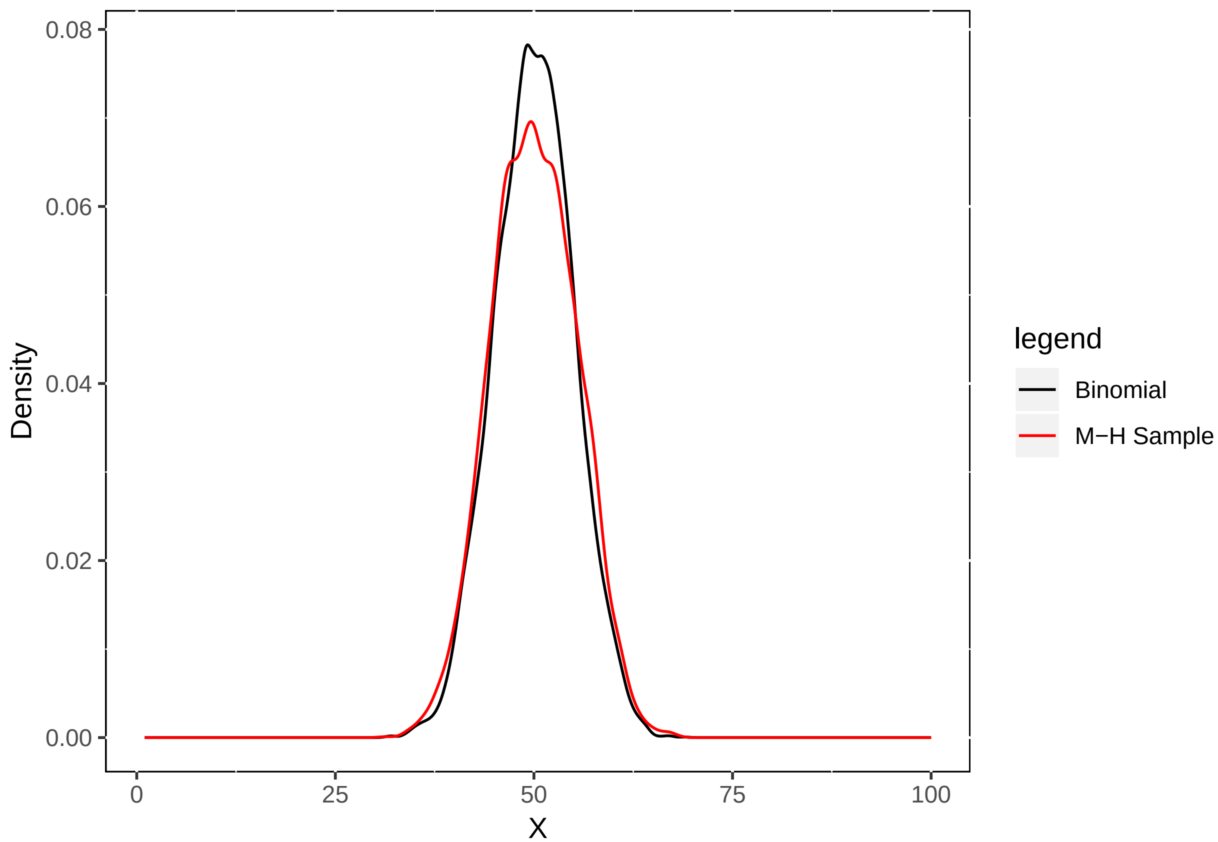
```
lambda <- 50
f_1 <- function(x){
  (factorial(100)/(factorial(x)*factorial(100-x)))*0.5^(x)*(1-0.5)^(100-x)
  }
# define the Binomial distribution , n=100, p=0.5
q_1 <- function(x){((lambda^x)*exp(-lambda))/factorial(x)}
# define the proposal function q_1 (Poisson), lambda=np=50
n <- 100
X <- numeric()  # store samples
count <- 1
x_old <- 0 # initialize the Markov chain
for(i in 1:10000)
{

  x_new <- rpois(1, lambda) # sample a candidate from q_1
  u <- runif(1,0,1)
  acc_1 <- min((f_1(x_new)*q_1(x_old))/(f_1(x_old)*q_1(x_new)), 1)
# calculate the acceptance prob
  if(u < acc_1) # accept
  {
    X[count] <- x_new
    count <- count +1
    x_old <- x_new
```

```r
  }
  else # reject
  {
    x_old <- x_old
  }
}
par(mfrow = c(2, 2))
hist(X, main = "Sample", bty = "o", xlab = "X")
XX <- rbinom(10000, 100, 0.5)
hist(XX, main = "Binomial", xlab = "X")
P <- density(X, n = 10000, from = 1, to  = 100)
plot(P, main = "Sample")
B <- density(XX, n = 10000, from = 1, to  = 100)
plot(B, main = "Binomial")
```




```r
p <- 0.5
real <- rbinom(10000, 100, p)
d <- density(real, n = 10000, from = 1, to =  100)
s <- density(X, n = 10000, from = 1 ,to =  100)
Data_real <- data.frame(num = d$x, y = d$y, legend = rep(c("Binomial "), 10000, 1))
# data of Binomial distribution
Data_sample <- data.frame(num = s$x, y = s$y, legend = rep(c("M-H Sample"), 10000, 1))
# data of sample
Data <- rbind(Data_real, Data_sample)
library(ggplot2) # use "ggplot2" package
g <- ggplot( Data, aes(x = num, y = y, group = legend) ) +
```

```
geom_line( aes(color = legend), size = 0.5 ) +
xlab("X") + ylab("Density") +
scale_color_manual(values = c("black", "red")) +
theme(panel.background = element_rect(fill = 'white', colour = 'black'))
g
```



Conclusion:

We can see that the proposal(Uniform) does not perform well, it performs better when $p = 0.5$.If we choose Poisson distribution as the proposal distribution, and let $\lambda = np$, it performs well.