ST5218_Tut_4

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Question_1:

a)

$$w_1 = \frac{\sum^{-1} I}{I^T \sum^{-1} I}$$

b)

$$r_{target} = 0.0111$$
$$w = \tfrac{c - br_{target}}{ac-b^2} \sum^{-1} I + \tfrac{ar_{target}-b}{ac-b^2} \sum^{-1} r$$
$$\Rightarrow \quad w_2 = (0.1045060, 0.4694433, 0.1067702, 0.3192805)^T$$

c)

$$w_c = \alpha w_2 + (1-\alpha) w_1 = (0.1099927, 0.4535539, 0.1229246, 0.3135288)^T$$

d)

$$w_c = \alpha w_2 + (1-\alpha) w_1$$

e)

$$R \sim N(\mu, \sum)$$

$$w^T R \sim N(w^T \mu, w^T \sum w)$$

$$VaR = 0.01832235$$

Question_2:

$$\text{Let } L(w,\lambda) = w^T\mu - \tfrac{1}{2}w^T \sum w - \lambda(w^T I - 1) \quad \text{ where } I = (1,\cdots,1)^T$$

The first order condition is,

$$\mu - \sum w - \lambda I = 0, \qquad w^T I = 1$$
$$\Rightarrow \quad w = \sum\nolimits^{-1}(\mu - \lambda)$$

By the constraint,

$$I^T \sum\nolimits^{-1} \mu - I^T \sum\nolimits^{-1} I\lambda = 1$$
$$\lambda = \frac{I^T \sum^{-1} \mu - 1}{I^T \sum^{-1} I}$$
$$\Rightarrow \quad w = \sum\nolimits^{-1}\left(\mu - \frac{I^T \sum^{-1} \mu - 1}{I^T \sum^{-1} I}\right)$$

Question_3:

1)

$$E(R_p) = E[wR_1 + (1-w)R_2] = wr_0 + r_0 - wr_0 = r_0$$

2)

$$Cov(R_p) = w^2\sigma_0^2 + (1-w)^2\sigma_0^2 + 2w(1-w)\sigma_{12}$$
$$\frac{\partial Cov(R_p)}{\partial w} = 2w\sigma_0^2 - 2(1-w)\sigma_0^2 + 2(1-2w)\sigma_{12} = 2(2w-1)(\sigma_0^2 - \sigma_{12}) = 0$$
$$\Rightarrow \quad w = 0.5$$

3)

Since the return is always r_0, the smallest risk protfolio has the largest Sharp ratio, i.e. Tangency protfolio:

$$R_N = 0.5R_1 + 0.5R_2$$

Question_4:

a)

```
library(tseries)
library(timeSeries)
MMMdata = get.hist.quote(instrument = "MMM",
                         start="2014-01-01", end="2014-12-31",
                         quote=c("AdjClose"),provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

```
MSFTdata = get.hist.quote(instrument = "MSFT",
                         start="2014-01-01", end="2014-12-31",
                         quote=c("AdjClose"),provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

```
Tdata = get.hist.quote(instrument = "T",
                         start="2014-01-01", end="2014-12-31",
                         quote=c("AdjClose"),provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

```
x=merge(MMMdata,MSFTdata,Tdata)
R=diff(log(x))
colnames(R)[1]="MMM"
colnames(R)[2]="MSFT"
colnames(R)[3]="ATT"
mydata=data.frame(R)
mydata=as.timeSeries(mydata)
mu.vec=apply(mydata, 2, mean)
sigma.mat=cov(mydata)
mu.vec
```

```
##          MMM         MSFT          ATT
## 0.0008271646 0.0010494173 0.0001081833
```

```
sigma.mat
```

```
##               MMM         MSFT          ATT
## MMM  8.990259e-05 5.384076e-05 3.120957e-05
```

```
## MSFT 5.384076e-05 1.433333e-04 4.019968e-05
## ATT  3.120957e-05 4.019968e-05 8.096440e-05
```

b)

```r
library(fPortfolio)
```

```
## Loading required package: fBasics

## Loading required package: fAssets
```

```r
Spec = portfolioSpec(portfolio=list(targetReturn=0.0005,
                                    nFrontierPoints=500,
                                    riskFreeRate=0.000))
Constraints="Longonly"
efficientPortfolio(mydata, Spec, Constraints)
```

```
##
## Title:
##  MV Efficient Portfolio
##  Estimator:         covEstimator
##  Solver:            solveRquadprog
##  Optimize:          minRisk
##  Constraints:
##
## Portfolio Weights:
##    MMM   MSFT    ATT
## 0.3975 0.1127 0.4899
##
## Covariance Risk Budgets:
##    MMM   MSFT    ATT
## 0.3990 0.1134 0.4876
##
## Target Returns and Risks:
##   mean    Cov   CVaR    VaR
## 0.0005 0.0075 0.0184 0.0121
##
## Description:
##  Thu Feb 21 02:44:19 2019 by user:
```

c)

```r
ones=rep(1,3)
top=solve(sigma.mat)%*%ones
bot=as.numeric(t(ones)%*%solve(sigma.mat)%*%ones)
```

```
w=top/bot
w
```

```
##            [,1]
## MMM  0.3948863
## MSFT 0.1093171
## ATT  0.4957966
```

d)

```
Spec=portfolioSpec(portfolio=list(targetReturn=0.0005,
                                    nFrontierPoints=500,
                                    riskFreeRate=0.0001))
tangencyPortfolio(mydata, Spec, Constraints)
```

```
##
## Title:
##  MV Tangency Portfolio
##  Estimator:         covEstimator
##  Solver:            solveRquadprog
##  Optimize:          minRisk
##  Constraints:
##
## Portfolio Weights:
##    MMM   MSFT    ATT
## 0.5348 0.4652 0.0000
##
## Covariance Risk Budgets:
##    MMM   MSFT    ATT
## 0.4682 0.5318 0.0000
##
## Target Returns and Risks:
##   mean    Cov   CVaR    VaR
## 0.0009 0.0091 0.0202 0.0142
##
## Description:
##  Thu Feb 21 02:44:19 2019 by user:
```

e)

Line ratio unchange, do not know which portfolio is used to compare.

Question_5:

$$r_C = \alpha r_A + (1-\alpha) r_B$$

$$w = \tfrac{c-br_C}{ac-b^2} \sum^{-1} I + \tfrac{ar_C-b}{ac-b^2} \sum^{-1} \Gamma$$

$$\Rightarrow \quad \tfrac{c-b[\alpha r_A+(1-\alpha)]}{ac-b^2} \sum^{-1} I + \tfrac{a[\alpha r_A-(1-\alpha) r_B]-b}{ac-b^2} \sum^{-1} \Gamma$$

$$\Rightarrow \quad \alpha \tfrac{c-br_A}{ac-b^2} \sum^{-1} I + \alpha \tfrac{ar_A-b}{ac-b^2} \sum^{-1} \Gamma + (1-\alpha) \tfrac{c-br_B}{ac-b^2} \sum^{-1} I + (1-\alpha) \tfrac{ar_b-b}{ac-b^2} \sum^{-1} \Gamma$$

$$\Rightarrow \quad w = \alpha w_A + (1-\alpha) w_B$$