ST5201-Homework5

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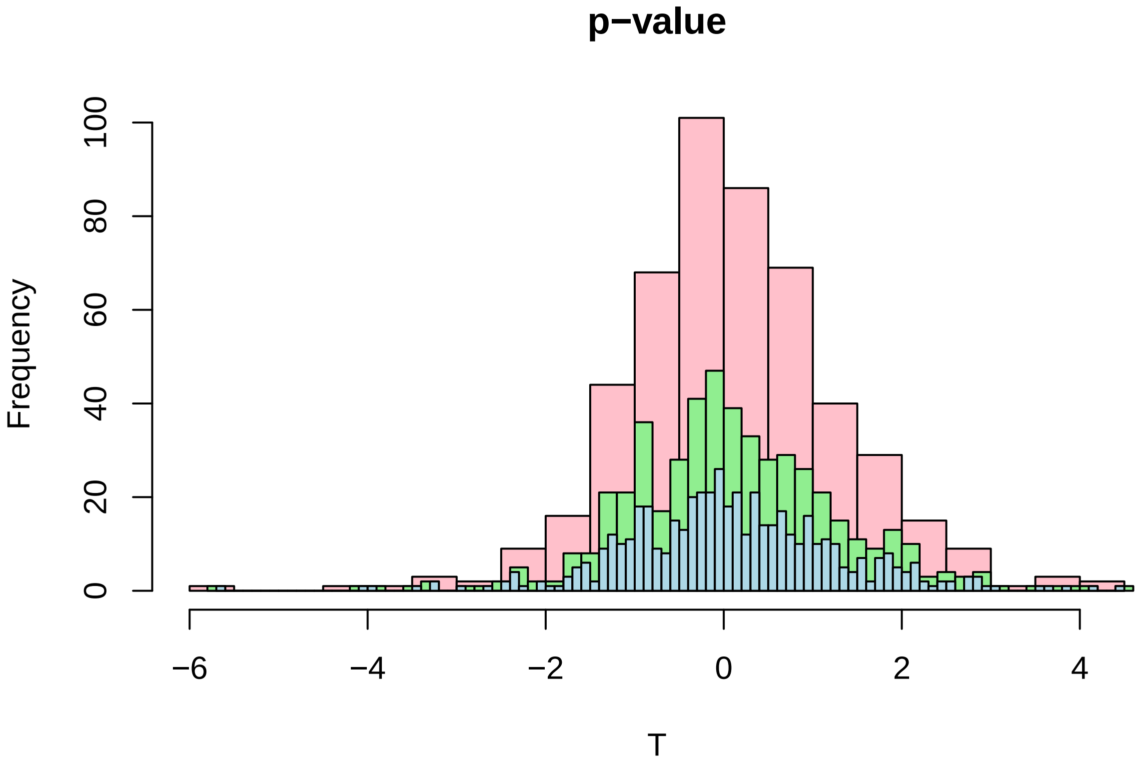
Q1

a)

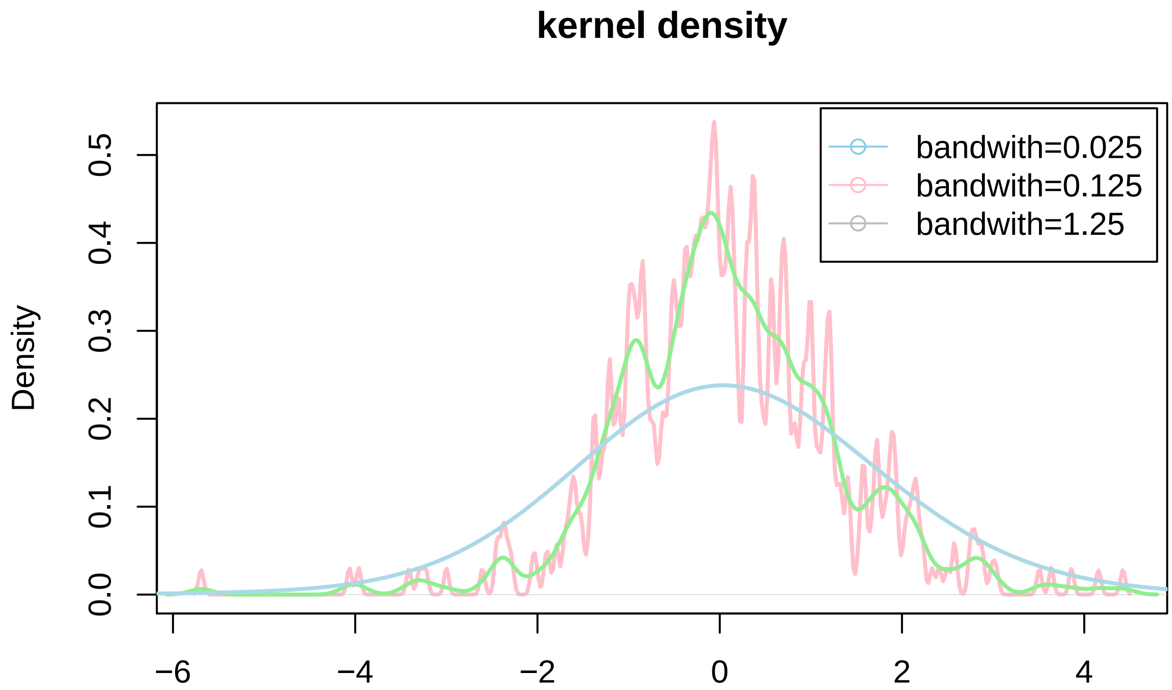
```
T=rep(0,500)
for(i in 1:500){
x=rnorm(7,5,5)
T[i]=(mean(x)-5)/sqrt(var(x)/7)
}
p1 = hist(T,breaks=20,plot = F)
p2 = hist(T,breaks=50,plot = F)
p3 = hist(T,breaks=100,plot = F)
plot(p1,col = "pink",main="p-value")
plot(p2,col = "lightgreen",add = T)
plot(p3,col = "lightblue",add = T)
```



p−value
Frequency
0
20
40
60
80
100
−6
−4
−2
0
2
4
T

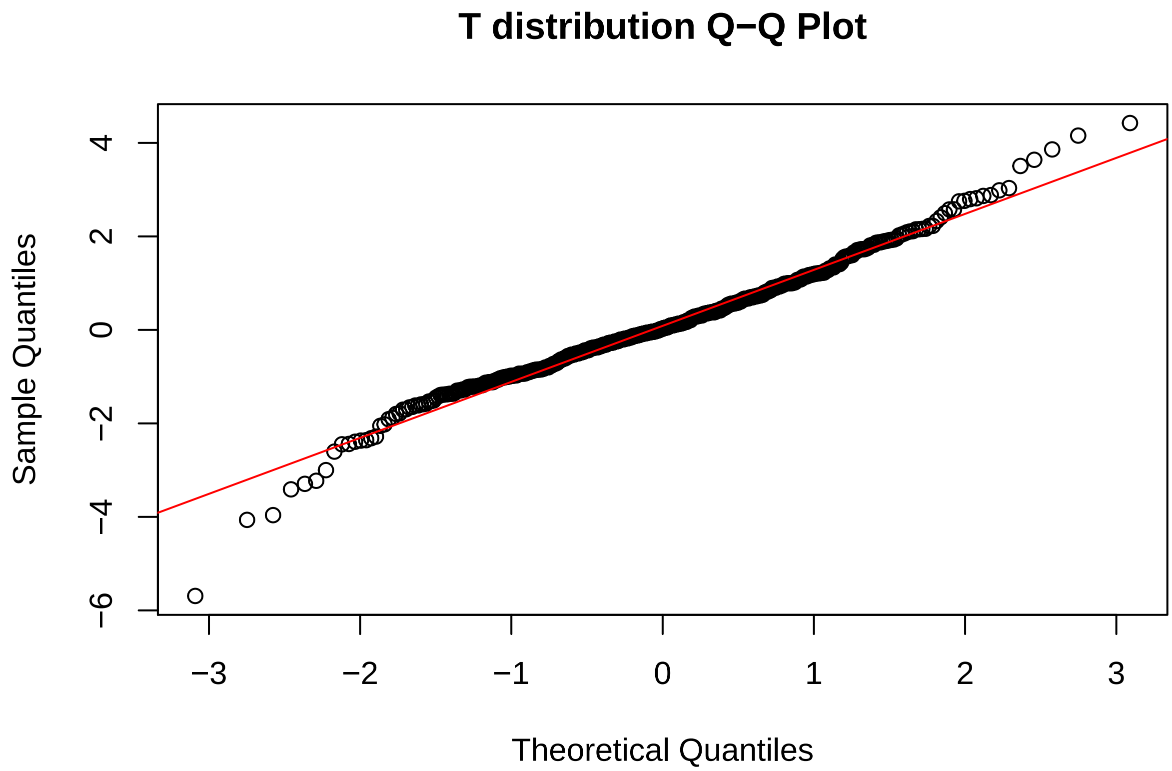
b)

```
d1=density(T,bw =0.025)
d2=density(T,bw =0.125)
d3=density(T,bw =1.25)
plot(d1, lwd = 2,col="pink",main="kernel density",xlab="")
lines(d2, lwd = 2,col = "lightgreen")
lines(d3, lwd = 2,col = "lightblue")
legend('topright',
       legend,c("bandwith=0.025","bandwith=0.125","bandwith=1.25"),
       col=c("skyblue","pink","grey"),
       pch=1,lwd = 1,inset=0.01)
```



c)

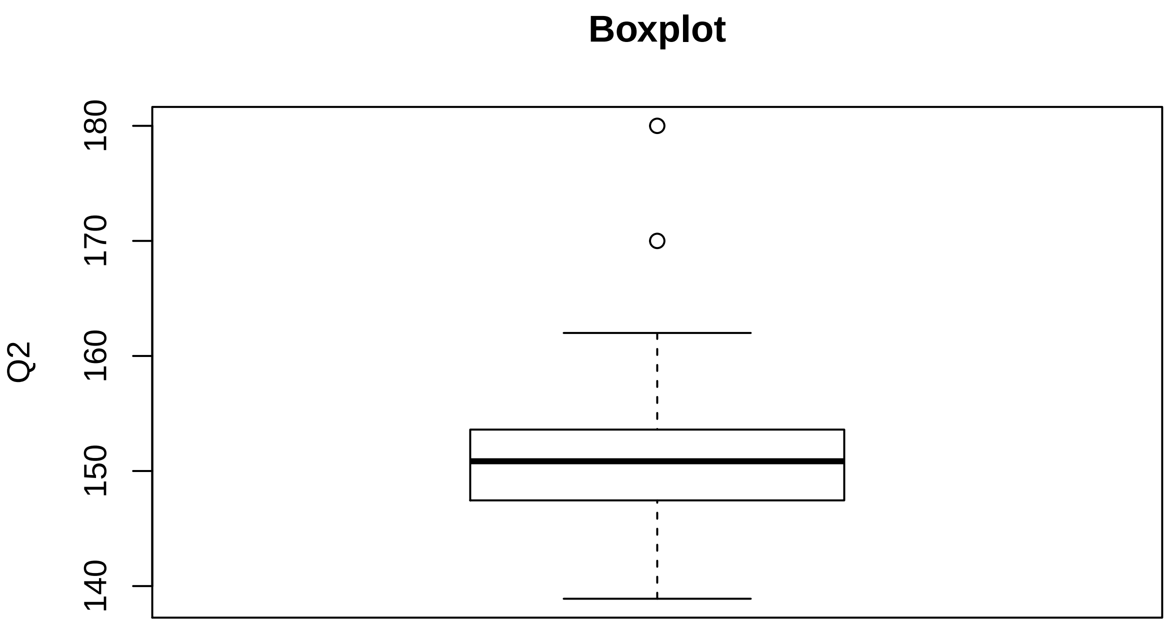
```
qqnorm(T,main = "T distribution Q-Q Plot",xlab = "Theoretical Quantiles", ylab = "Sample Quantiles",plo
qqline(rt(500, df = 6), col = 2, pch=21)
```



Q2:

a)

```
Q2=read.table(
  "/Users/xuzhu/Desktop/Notes/ST5201/Homework/hw5/Q2.txt")
x=Q2[,2]
boxplot(x,main="Boxplot",ylab="Q2")
```



b)

```
stem(x)
```

```
##
##   The decimal point is 1 digit(s) to the right of the |
##
##   13 | 9
##   14 | 012334444
##   14 | 5566677777777788888899999999
##   15 | 00000000000111122222222222223333333344444444
##   15 | 55566666777888
##   16 | 012
##   16 |
##   17 | 0
##   17 |
##   18 | 0
```

c)

```r
mean(x)
```

```
## [1] 151.029
```

```r
median(x)
```

```
## [1] 150.85
```

Q3:

a)

$$\mu = 140$$
$$\{\bar{X} \geq \mu - t_{n-1}(\frac{a}{2})\frac{S}{n}\} \cup \{\bar{X} \leq \mu + t_{n-1}(\frac{a}{2})\frac{S}{n}\}$$

b)

```r
t.test(x,mu=140)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 19.375, df = 99, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 140
## 95 percent confidence interval:
##  149.8995 152.1585
## sample estimates:
## mean of x
##   151.029
```

c)

```r
confint<-function(x,alpha=0.10)
{
  n=length(x)
  xb=mean(x)
  sigma=sqrt(n/(n-1))*sd(x)
  tmp=sd(x)/sqrt(n)*qt(1-alpha/2,n-1)
  df= n-1
  data.frame(mean=xb,df=df,a=xb-tmp,b=xb+tmp)
}
confint(x)
```

```
##      mean df        a        b
## 1 151.029 99 150.0839 151.9741
```

d)

```r
confint2<-function(x,alpha=0.10)
{
  n=length(x)
  xb=mean(x)
  sigma2=var(x)
  a=n*sigma2/qchisq(alpha/2,n-1)
  b=n*sigma2/qchisq(1-alpha/2,n-1)
  df=n-1
  data.frame(sigma2=var(x),df=df,a,b)
}
confint2(x)
```

```
##     sigma2 df        a        b
## 1 32.40168 99 42.05479 26.29468
```

Q4:

a)

$$z_1 = X; \quad z_2 = X + Y \qquad \Rightarrow |J^{-1}| = 1, \quad X = z_1, \quad Y = z_2 - z_1$$
$$f(x,y) = \tfrac{1}{2\pi} exp\{-\tfrac{x^2+y^2}{2}\}$$

$$\begin{aligned} f_{z_1,z_2}(z_1,z_2) &= |J^{-1}| f_{X,Y}(z_1.z_2) \\ &= \frac{1}{2\pi} exp\{-\frac{1}{2}[z_1^2 + (z_2 - z_1)^2]\} \\ &= \frac{1}{2\pi} exp\{-\frac{1}{2}(2z_1^2 + z_2^2 - 2z_1 z_2)\} \end{aligned}$$

$$\begin{aligned} f(x, x+y) &= \frac{1}{2\pi} exp\{-\frac{1}{2}[2x^2 + (x+y)^2 - 2x(x+y)]\} \\ &= \frac{1}{2\pi} exp\{-\frac{x^2+y^2}{2}\} \end{aligned}$$

b)

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\ f(x+y) &= \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{4}(x+y)^2} \\ f(x)f(x+y) &= \frac{1}{2\sqrt{2\pi}} exp\{-\frac{x^2 + \frac{1}{2}(x+y)^2}{2}\} \neq f(x, x+y) \end{aligned}$$

Thus X and X+Y are not independent.

c)

$$z_3 = X - Y; \quad z_4 = X + Y \qquad \Rightarrow |J^{-1}| = \tfrac{1}{2}, \quad X = \tfrac{z_3+z_4}{2}, \quad Y = \tfrac{z_4-z_3}{2}$$
$$f_{Z_3,Z_4}(z_3,z_4) = |J^{-1}| f_{X,Y}(z_3,z_4) = \tfrac{1}{4\pi} exp\{-\tfrac{z_3^2+z_4^2}{4}\}$$
$$f(x-y, x+y) = \tfrac{1}{4\pi} exp\{-\tfrac{1}{4}[(x-y)^2 + (x+y)^2]\}$$

d)

$$f(x-y) = \tfrac{1}{2\sqrt{\pi}} exp\{-\tfrac{(x-y)^2}{4}\}$$
$$f(x-y)f(x+y) = \tfrac{1}{4\pi} exp\{-\tfrac{(x-y)^2}{4}\} exp\{-\tfrac{(x+y)^2}{4}\} = f(x-y, x+y)$$

Thus $X+Y$ and $X-Y$ are independent.