ST5218_Tut_3

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Q1:

```
library(tseries)
library(moments)
library(MASS)
SP500 = get.hist.quote(instrument = "^gspc",
                       start="2014-01-01", end="2016-12-31",
                       quote=c("AdjClose"), provider="yahoo",
                       compress="d")
```

```
## time series starts 2014-01-02
## time series ends   2016-12-30
```

```
daily.log.return = diff(log(SP500), lag=1)
fit.t = fitdistr(daily.log.return, "t")
```

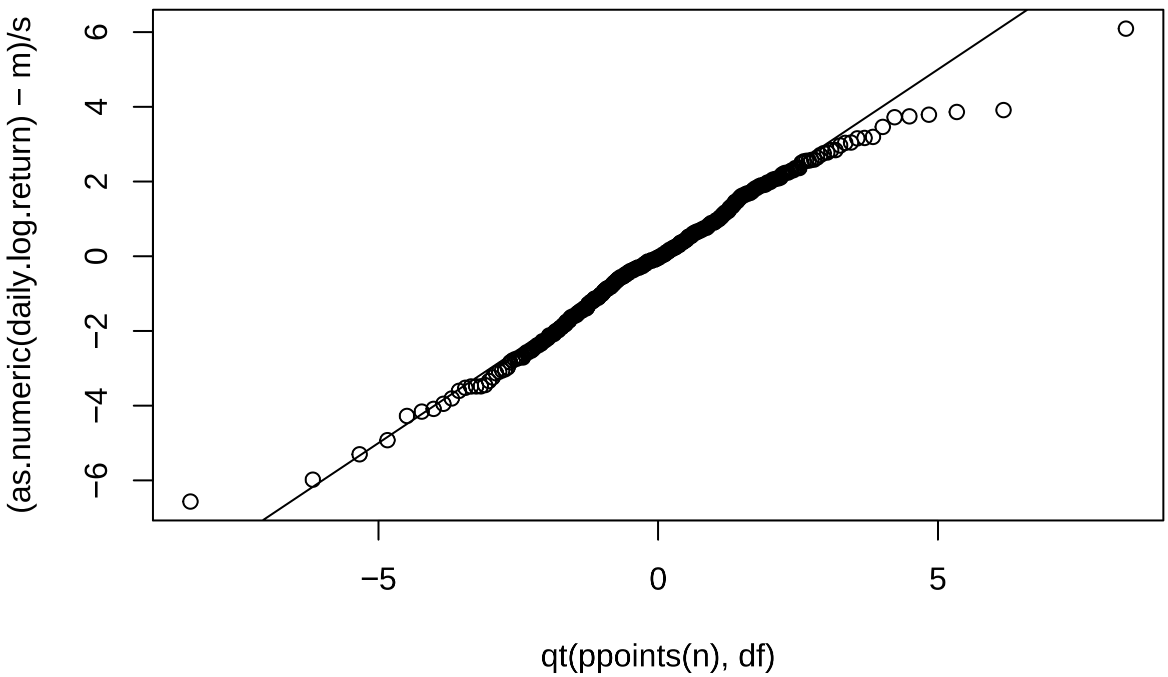
a)

```
fit.t
```

```
##          m               s               df
##    0.0005095120    0.0062012019    3.8465161900
##   (0.0002662766) (0.0003014795) (0.6412577464)
```

b)

```
m = fit.t$estimate[1]
s = fit.t$estimate[2]
df = fit.t$estimate[3]
n = length(daily.log.return)
qqplot(qt(ppoints(n), df), (as.numeric(daily.log.return)-m)/s)
lines(qt(ppoints(n), df), qt(ppoints(n), df))
```




```
# The distribution does not fit well for the data in the tails
```

c)

$Y = \frac{r-0.0005095120}{0.0062012019} \sim t(3.8465161900)$

With probability 0.001.

$VaR_{0.001}(R) = 7.471026$

$VaR_{0.001}(R) = 7.471026 * 0.0062012019 - 0.0005095120 = 0.04581983$

Q2:

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

$$\sum = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

$$f_x(x,y) = \frac{\Gamma[(\nu+2/2)]}{\Gamma(\nu/2)(\nu\pi)}(\sigma_1^2\sigma_2^2(1-\rho)^2)^{-\frac{1}{2}}[1+\frac{1}{\nu(1-\rho)^2}(\frac{(x-\mu_x^2)}{\sigma_x^2}+\frac{(y-\mu_y^2)}{\sigma_y^2}-\frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y})]^{\frac{-(\nu+2)}{2}}$$

Q3:

a)

$$E(Y) = (0.2, 0.3, 0.2)^T$$

$$Cov(Y) = \begin{pmatrix} 0.8^2+1 & 0.8*0.7 & 0.8*0.9 \\ 0.7*0.8 & 0.7^2+1 & 0.7*0.9 \\ 0.9*0.8 & 0.9*0.7 & 0.9^2+1 \end{pmatrix}$$

b)

All the covariances are positive, because all that Y_i depend on common factor X with positive coefficients.

Q4:

a)

```
MMMdata = get.hist.quote(instrument="MMM",
                         start="2014-01-01", end="2014-12-31",
                         quote=("AdjClose"), provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

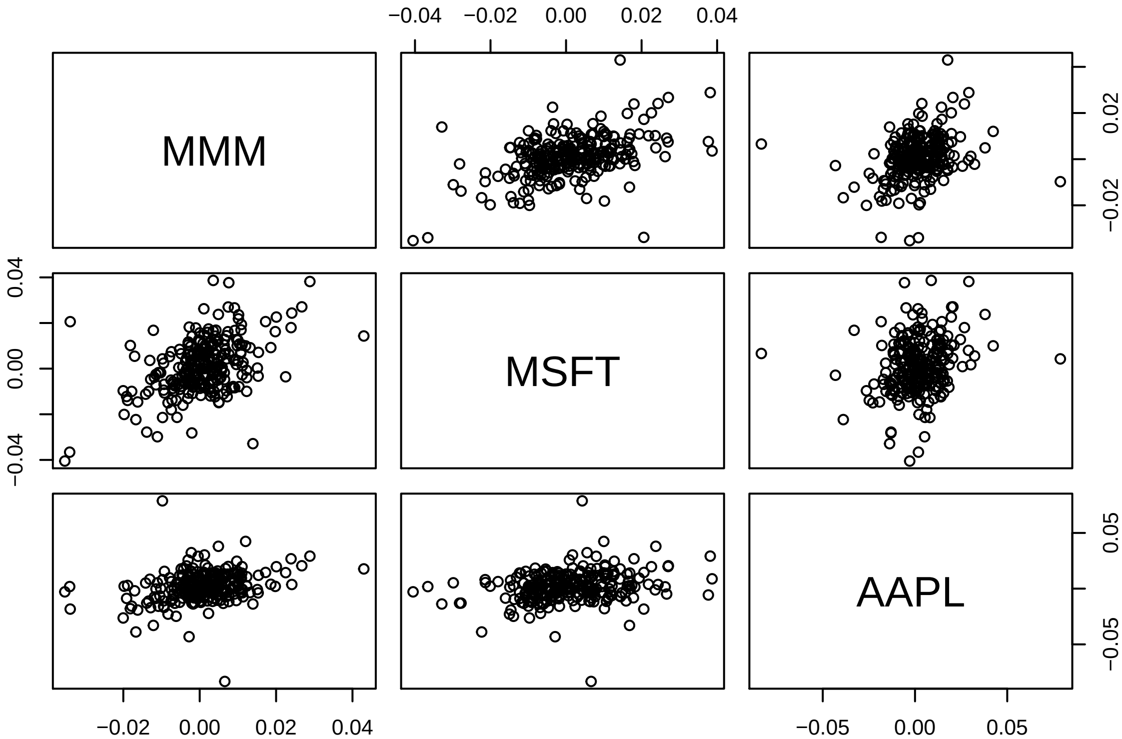
```
MSFTdata = get.hist.quote(instrument="MSFT",
                         start="2014-01-01", end="2014-12-31",
                         quote=("AdjClose"), provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

```
AAPLdata = get.hist.quote(instrument="AAPL",
                         start="2014-01-01", end="2014-12-31",
                         quote=("AdjClose"), provider="yahoo",
                         compression="d")
```

```
## time series starts 2014-01-02
## time series ends   2014-12-30
```

```r
x = merge(MMMdata, MSFTdata, AAPLdata)
R = diff(log(x))
colnames(R)[1] = "MMM"
colnames(R)[2] = "MSFT"
colnames(R)[3] = "AAPL"
pairs(R)
```




```r
cor(R)
```

```
##            MMM      MSFT      AAPL
## MMM  1.0000000 0.4742973 0.2955616
## MSFT 0.4742973 1.0000000 0.2338003
## AAPL 0.2955616 0.2338003 1.0000000
```

b)

```
n = length(MMMdata)
abs(cor(R)[1,2]) > qnorm(1-0.01/2)/sqrt(n-3)
```

```
## [1] TRUE
```

```
abs(cor(R)[1,3]) > qnorm(1-0.01/2)/sqrt(n-3)
```

```
## [1] TRUE
```

```
abs(cor(R)[2,3]) > qnorm(1-0.01/2)/sqrt(n-3)
```

```
## [1] TRUE
```

Q5:

$$Var(wR_1 + (1-w)R_2) = w^2\sigma_1^2 + 2w(1-w)\sigma_{12} + (1-w^2)\sigma_2^2$$

The first order condition is:

$$2w\sigma_1^2 + 2(1-2w)\sigma_{12} + 2(w-1)\sigma_2^2 = 0$$

The solution:

$$w = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

Q6:

Finding $w_1 \cdots w_n$ to minimize $Var(\sum_{i=1}^n w_k R_k)$ with $w_1 + \cdots + w_n = 1$

Using Lagrange multipliers:

$$\min_{w_1,\cdots,w_n,\lambda} \{Var(\sum_{i=1}^n w_k R_k) + \lambda(w_1 + \cdots + w_n - 1)\}$$

Taking derivatives respect to w_k and λ:

$$2w_k\sigma_k^2 + \lambda = 0, \qquad k = 1, \cdots, n$$

The solution is:

$$w_k = -\frac{\lambda}{2\sigma^2}, \quad \lambda = -\frac{2}{(\frac{1}{\sigma_1^2}) + \cdots + (\frac{1}{\sigma_n^2})}$$

$$\Rightarrow \quad w_k = \frac{1/\sigma_k}{1/\sigma_1^2 + \cdots + 1/\sigma_1^2}$$

Q7:

a)

Let w_1, w_2 be the weighting corresponding to A and B

$$w_1 + w_2 = 1$$

$$E(R_N) = w_1 E(R_1) + w_2 E(R_2) = w_1 r_1 + w_2 r_2$$

$$2.3\% w_1 + 4.5\% w_2 = 3\% \quad \Rightarrow w_1 = 0.682, \quad w_2 = 0.318$$

Thus $R_N = 0.682R_1 + 0.318R_2$

b)

$$\sigma_N^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12}w_1w_2\sigma_1\sigma_2$$

$$5.5\% = 6\% w_1^2 + 11\% w_2^2 + 2 * 0.17 * 6\% * 11\% w_1 w_2$$

$$w_1 + w_2 = 1$$

$$w_1 = 0.9404, \quad w_2 = 0.0596, \quad r_N = 2.43\%$$

or

$$w_1 = 0.411, \quad w_2 = 0.589, \quad r_N = 3.60\%$$

So the largest expected return is:

$$R_N = 0.411R_1 + 0.589R_2$$