Name Binding and Name Resolution

Eelco Visser



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Type Constraints

```
typeOfExp : scope * Exp → TYPE
```

Predicate

typeOfExp : scope * Exp → TYPE

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Rule

```
typeOfExp(s, Add(e1, e2)) = INT() :-
  typeOfExp(s, e1) = INT(),
  typeOfExp(s, e2) = INT().
```

Predicate

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typeOfExp : scope * Exp → TYPE
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Predicate

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Premises

Predicate

typeOfExp : scope * Exp → TYPE

Rule

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Head

Premises

For all s, e1, e2

If the premises are true, the head is true

Type Attributes and Error Messages

```
$ 9 * 10 + 3
Type: INT
```

```
integer expected
> INT() == BOOL()
> statics/base!typeOfExp(Scope("","s_1-1"), True(), INT())
> statics/base!typeOfExp(Scope("","s_1-1"), Mul(Int("1"),True()), INT())
> statics/base!declOk(Scope("","s_1-1"), Exp(Mul(Int("1"),True())))
> statics/base!declsOk(Scope("","s_1-1"), [Exp(Mul(Int(...),True()))])
> ... trace truncated ...

Type: BOOL
```

Constraints with Error Messages

```
constraint error $[message [term]]@origin
```

```
module Names {
  module Even {
    import Odd
    def even = fun(x) {
         if x = 0 then true else odd(x - 1)
  module Odd {
    import Even
    def odd = fun(x) {
          if x = 0 then false else even(x - 1)
  module Compute {
    type Result = { input : Int, output : Bool }
    def compute = fun(x) {
           Result{ input = x, output = Odd@odd x }
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Name binding key in programming languages

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Many name binding patterns

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Ad hoc non-declarative treatment

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Ad hoc non-declarative treatment

A systematic, uniform approach to name resolution?

Formalizing Name Binding in Type Systems

$$O(id) = T$$
, where T is not a function type. $O, M, C, R \vdash id : T$

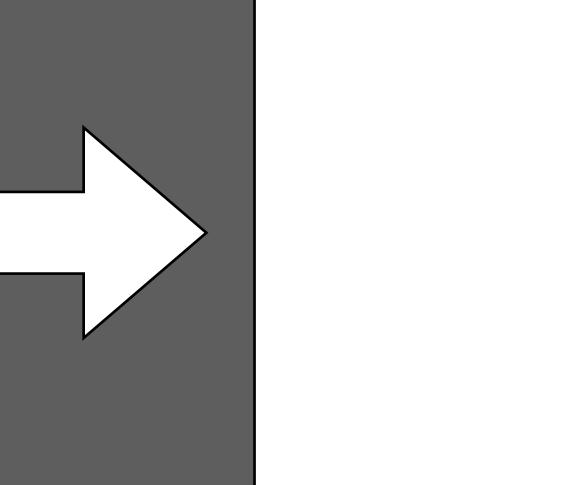
[VAR-READ]

Program

```
let function fact(n : int) : int =
      if n < 1 then
      else
        n * fact(n - 1)
 in
    fact(10)
end
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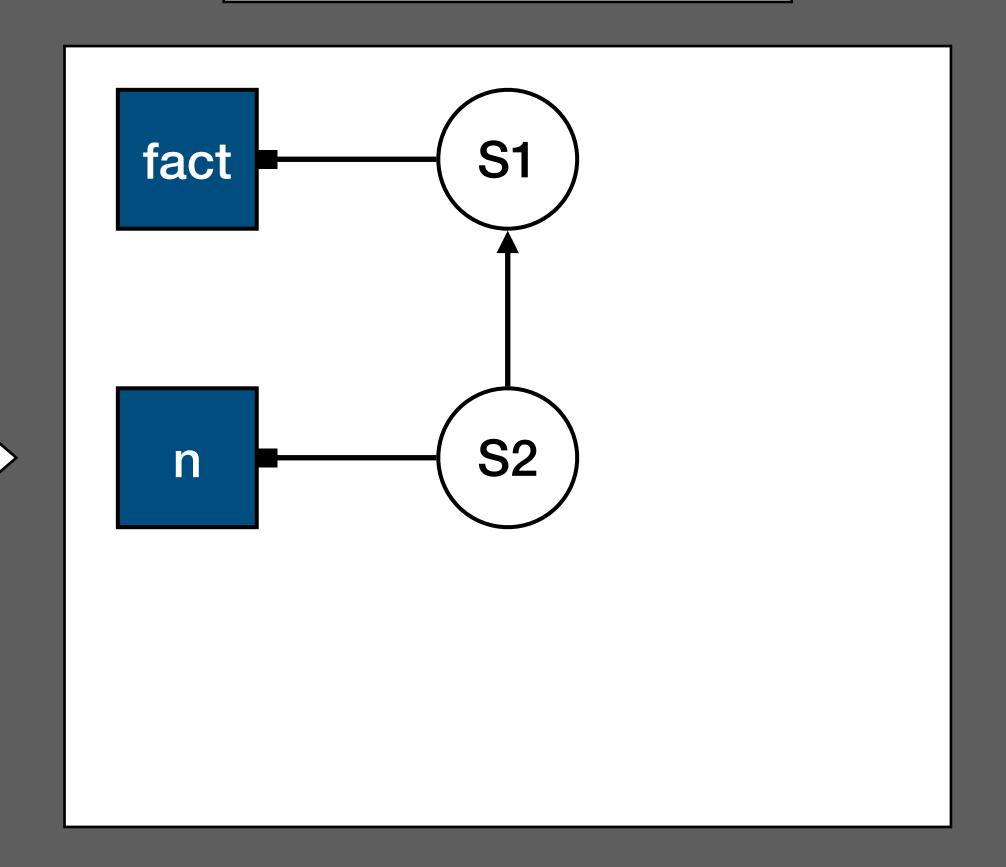
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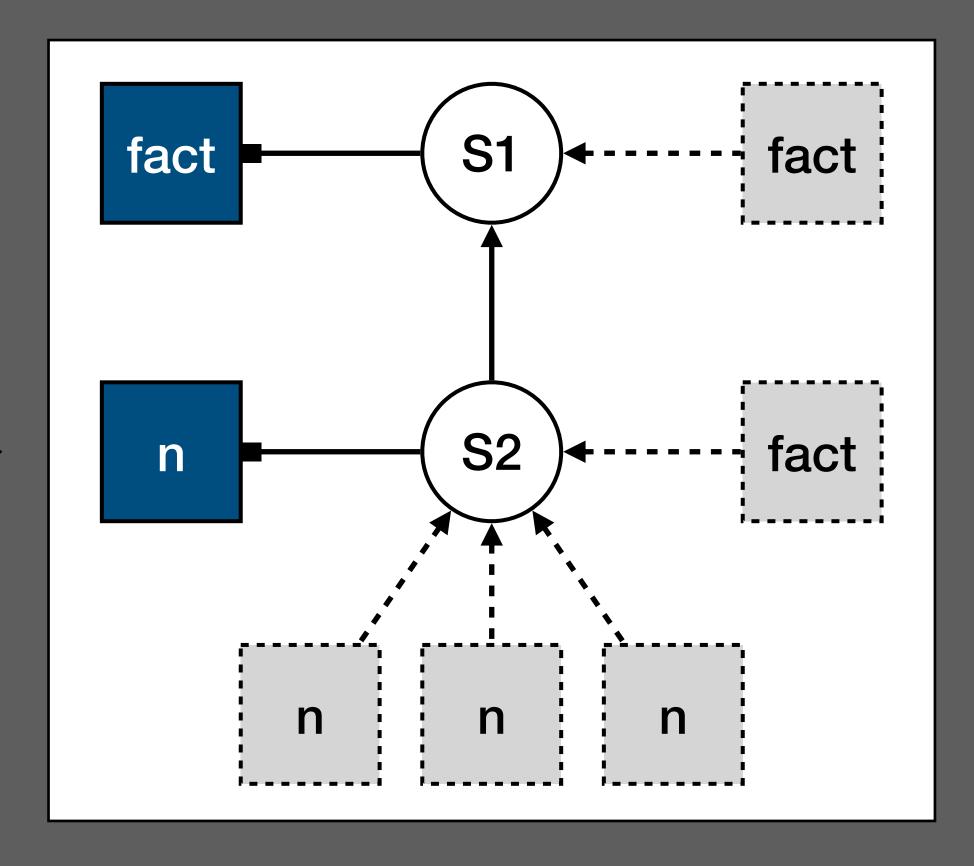
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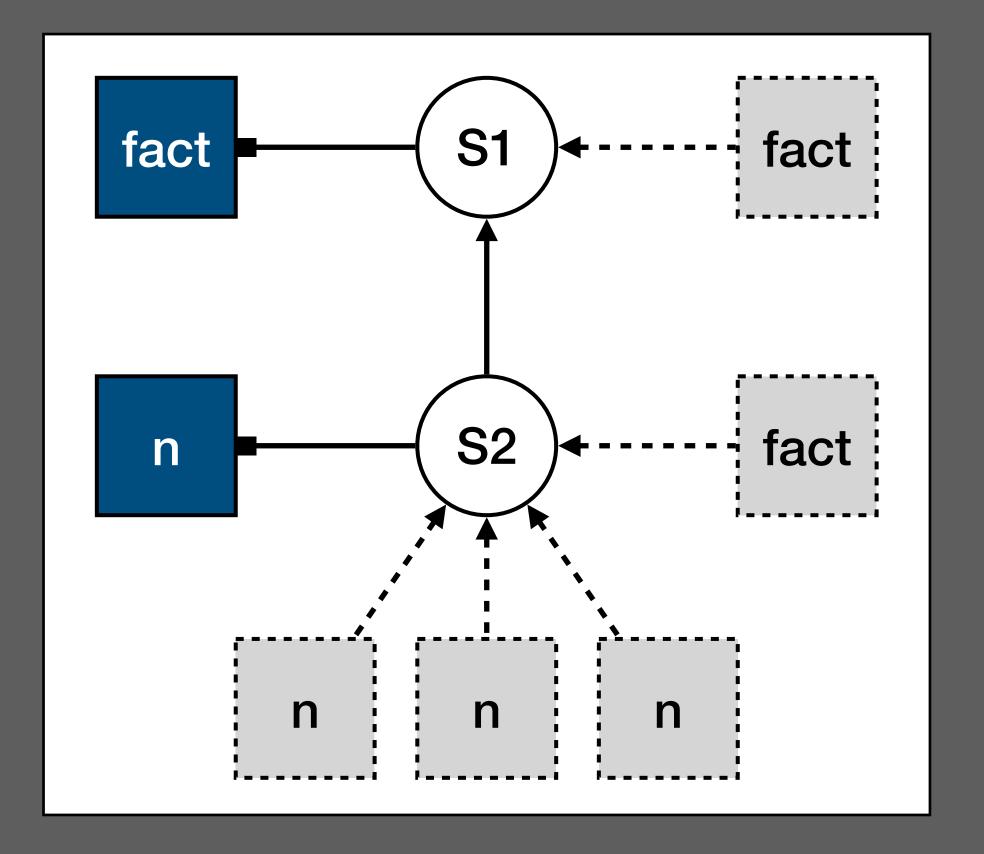
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Scope Graph

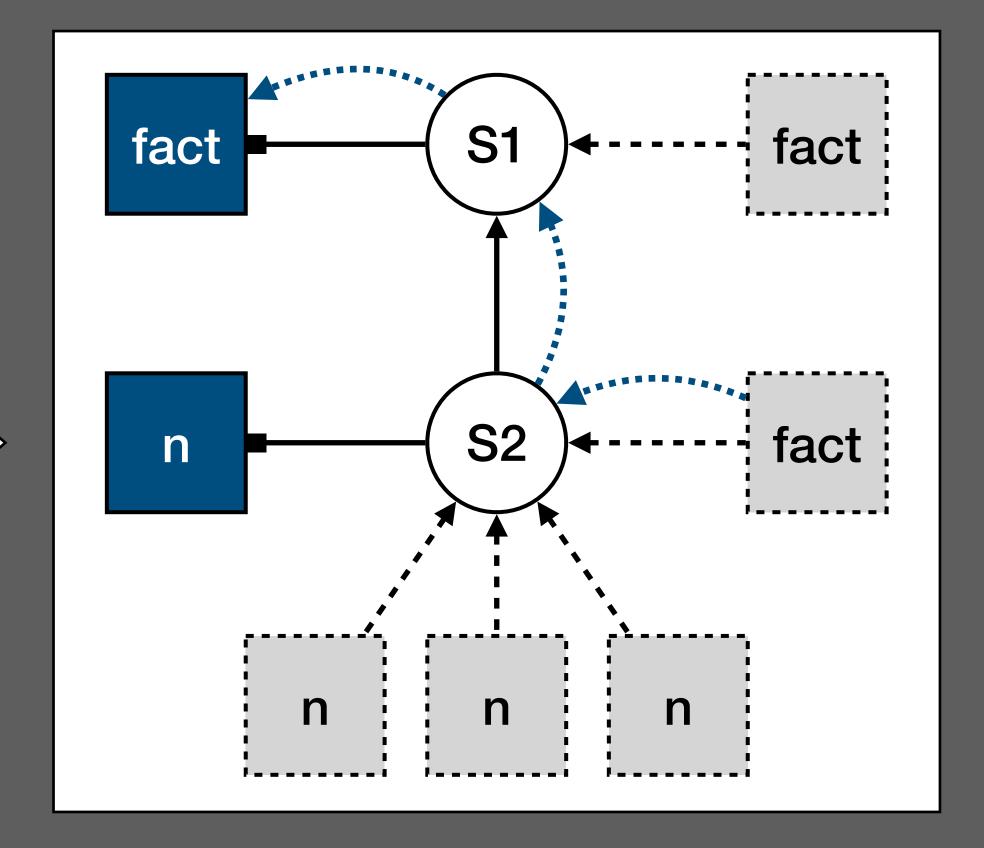


Name Resolution

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Scope Graph



Name Resolution

Name Resolution with Scope Graphs in Statix

Declarations and References

Lexical Scope

Modules

Records

Permission to Extend

Scheduling Resolution

Declaring and Resolving Names

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signature
constructors
    Var : ID → Exp
    Bind : ID * Exp → Bind
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    Def : Bind → Decl
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declsOk maps declOk(*, list(*))

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declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
   typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
   bindOk(s, s, bind).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors
    Var : ID → Exp
    Bind : ID * Exp → Bind
    BindT : ID * Type * Exp → Bind
    Def : Bind → Decl
```

```
rules

declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

declOk : scope * Decl
 declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
    typeOfExp(s_ctx, e) = T2,
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors
   Var : ID → Exp
   Bind : ID * Exp → Bind
   BindT : ID * Type * Exp → Bind
   Def : Bind → Decl
```

```
rules

declOk : scope * Decl
 declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
    typeOfExp(s_ctx, e) = T2,
    subtype(T2, T1).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors
   Var : ID → Exp
   Bind : ID * Exp → Bind
   BindT : ID * Type * Exp → Bind
   Def : Bind → Decl
```

```
rules

declOk : scope * Decl
 declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
    typeOfExp(s_ctx, e) = T2,
    subtype(T2, T1).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

decl0k : scope * Decl
decls0k maps decl0k(*, list(*))

bind0k : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules
  typeOfExp(s, Var(x)) = T:-
    typeOfVar(s, x) = T.
  declOk(s, Def(bind)) :-
    bind0k(s, s, bind).
  bindOk(s\_bnd, s\_ctx, BindT(x, t, e)) := \{T1 T2\}
    typeOfType(s_ctx, t) = T1,
    declareVar(s\_bnd, x, T1),
    type0fExp(s_ctx, e) = T2,
    subtype(T2, T1).
  bind0k(s_bnd, s_ctx, Bind(x, e)) :- {T}
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

declOk : scope * Decl
 declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
    typeOfExp(s_ctx, e) = T2,
    subtype(T2, T1).

bindOk(s_bnd, s_ctx, Bind(x, e)) :- {T}
    typeOfExp(s_ctx, e) = T,
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

```
signature
constructors

Var : ID → Exp
Bind : ID * Exp → Bind
BindT : ID * Type * Exp → Bind
Def : Bind → Decl
```

```
rules

declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
rules

typeOfExp(s, Var(x)) = T :-
    typeOfVar(s, x) = T.

declOk(s, Def(bind)) :-
    bindOk(s, s, bind).

bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
    typeOfType(s_ctx, t) = T1,
    declareVar(s_bnd, x, T1),
    typeOfExp(s_ctx, e) = T2,
    subtype(T2, T1).

bindOk(s_bnd, s_ctx, Bind(x, e)) :- {T}
    typeOfExp(s_ctx, e) = T,
    declareVar(s_bnd, x, T).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

Representing Name Binding with Scope Graphs

rules

```
declareVar : scope * ID * TYPE
```

```
typeOfVar : scope * ID \rightarrow TYPE
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

Scope Graphs: Declarations

```
signature
  relations
    var : ID \rightarrow TYPE
                                                          declaration relation
rules
  declareVar : scope * ID * TYPE
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                    variable x is declared in scope s
    !var[x, T] in s.
                                                             with type T
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

Scope Graphs: Declarations

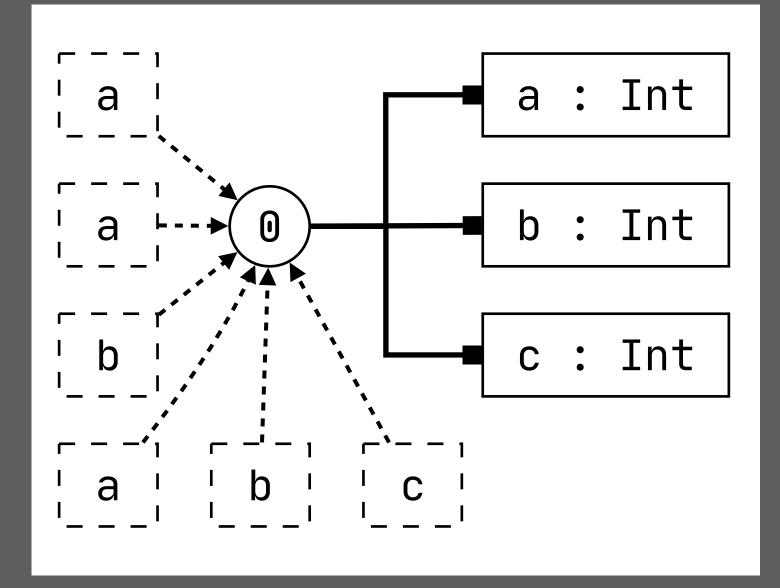
```
def a = 0
signature
  relations
                                                                                 def b = a + 1
    var : ID \rightarrow TYPE
                                                        declaration relation
                                                                                 def c = a + b
                                                                                 > a + b + c
rules
  declareVar : scope * ID * TYPE
                                                                                 declaration and reference
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                   variable x is declared in scope s
                                                                                                          a : Int
    !var[x, T] in s.
                                                           with type T
                                                                                                          b: Int
                                                                                                          c : Int
```

Scope Graphs: Name Resolution Queries

```
signature
  relations
    var : ID \rightarrow TYPE
                                                           declaration relation
rules
  declareVar : scope * ID * TYPE
  resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                      variable x is declared in scope s
    !var[x, T] in s.
                                                              with type T
  resolveVar(s, x) = ps :-
                                                    variable x in scope s resolves to list
    query var
                                                             of declarations
       filter e and \{ x' :- x' = x \}
          min and true
           in s \mapsto ps.
                                                      variable x in scope s resolves to
  typeOfVar(s, x) = T :- {x'}
                                                        declaration x' with type T
    resolveVar(s, x) = [(\_,(x', T))].
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

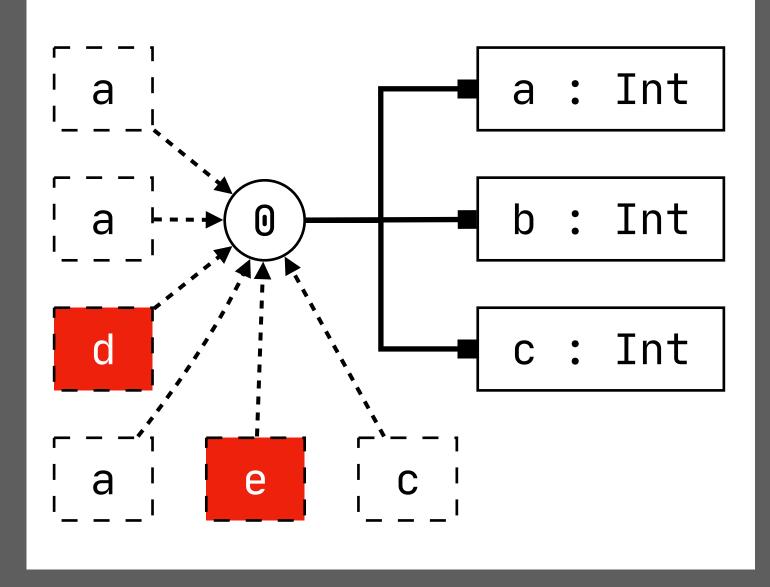


Undefined Variable

```
signature
  relations
    var : ID \rightarrow TYPE
                                                          declaration relation
rules
  declareVar : scope * ID * TYPE
  resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                     variable x is declared in scope s
    !var[x, T] in s.
                                                              with type T
  resolveVar(s, x) = ps :-
                                                    variable x in scope s resolves to list
    query var
                                                            of declarations
       filter e and \{x':-x'=x\}
          min and true
           in s \mapsto ps.
                                                     variable x in scope s resolves to
  typeOfVar(s, x) = T :- {x'}
                                                       declaration x' with type T
    resolveVar(s, x) = [(\_,(x', T))].
```

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable



resolveVar returns empty list of declarations

Duplicate Definition

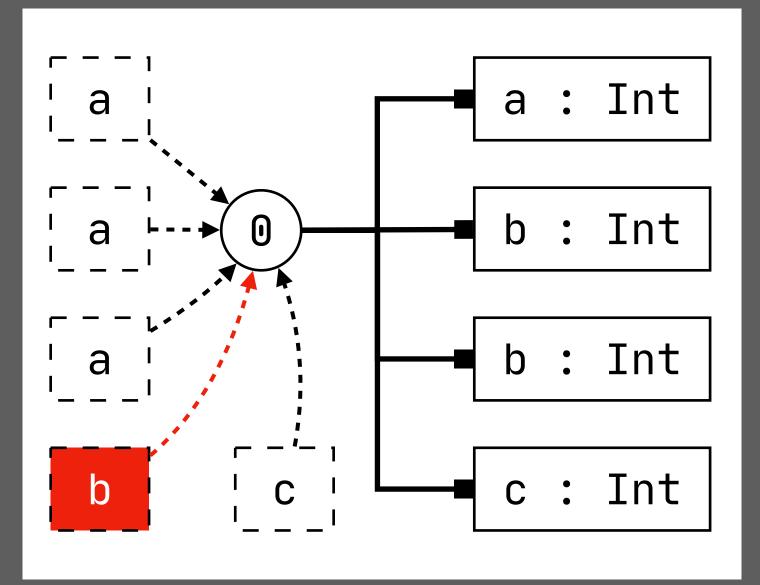
```
signature
  relations
    var : ID \rightarrow TYPE
                                                           declaration relation
rules
  declareVar : scope * ID * TYPE
  resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                      variable x is declared in scope s
    !var[x, T] in s.
                                                              with type T
  resolveVar(s, x) = ps :-
                                                    variable x in scope s resolves to list
    query var
                                                             of declarations
       filter e and \{ x' :- x' = x \}
          min and true
           in s \mapsto ps.
                                                      variable x in scope s resolves to
  typeOfVar(s, x) = T :- {x'}
                                                        declaration x' with type T
    resolveVar(s, x) = [(\_,(x', T))].
```

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c

what we want

what we get

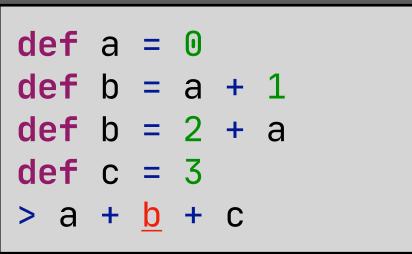


Duplicate Definition: Permissive Resolution

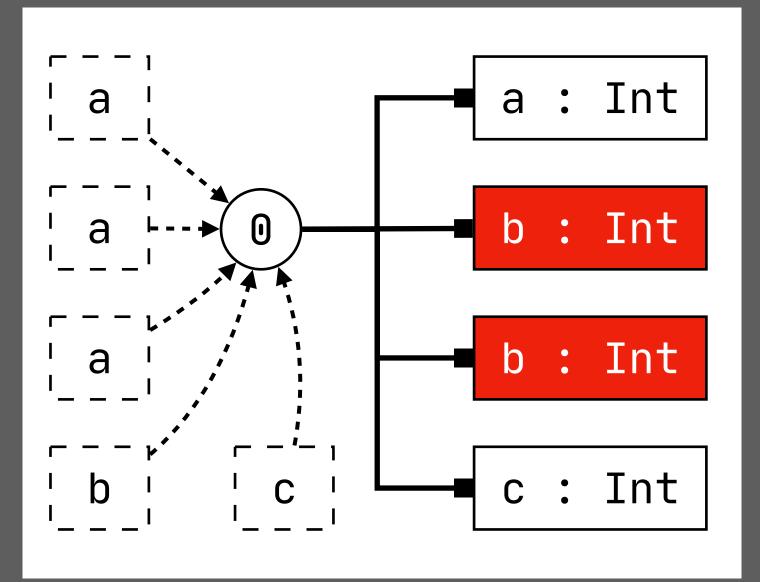
matching declaration

```
signature
  relations
    var : ID \rightarrow TYPE
                                                          declaration relation
rules
  declareVar : scope * ID * TYPE
  resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                     variable x is declared in scope s
    !var[x, T] in s,
                                                             with type T
    resolveVar(s, x) = [(_, (_, _))]
         error [Duplicate definition of variable [x]].
                                                   there should only be one declaration
  resolveVar(s, x) = ps :-
                                                   variable x in scope s resolves to list
    query var
                                                            of declarations
       filter e and \{x':-x'=x\}
          min and true
           in s \mapsto ps.
  typeOfVar(s, x) = T :- \{x'\}
                                                     variable x in scope s resolves to
                                                       declaration x' with type T
    resolveVar(s, x) = [(_,(x', T))|_]
         error $[Variable [x] not defined].
                                                      there should be at least one
```

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```



duplicate definition

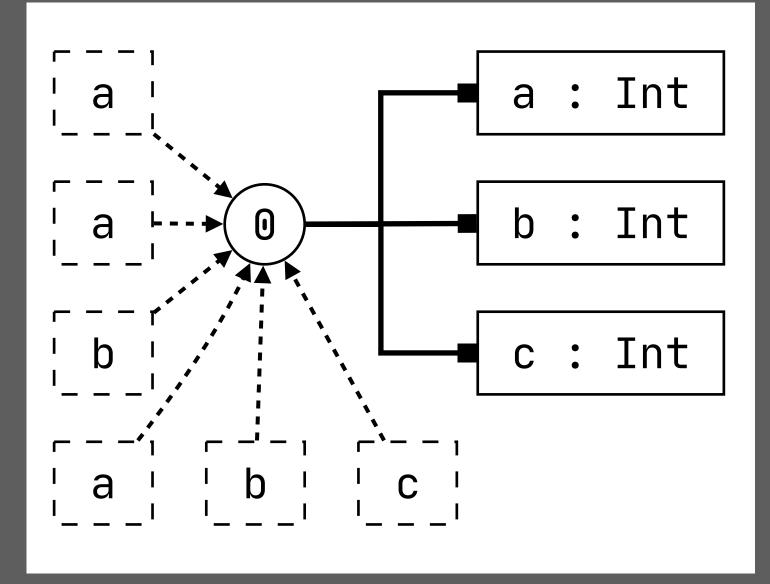


Reference and Type Attributes

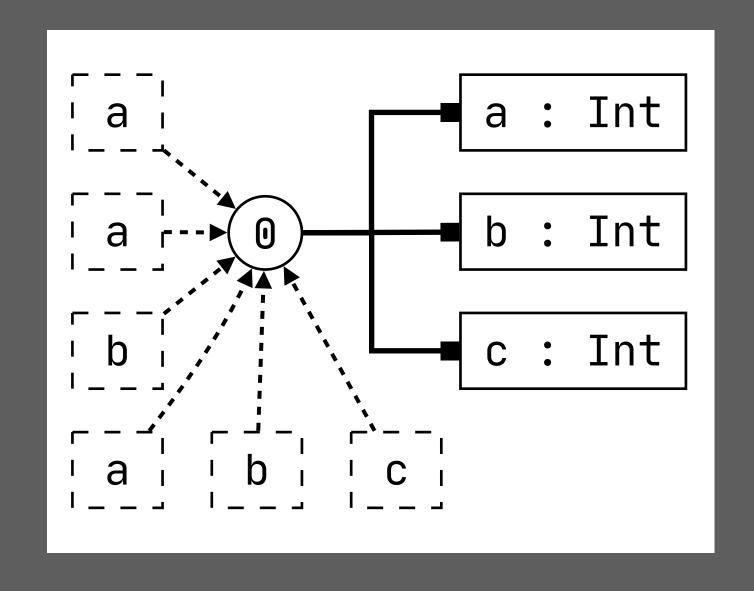
```
signature
  relations
    var : ID \rightarrow TYPE
                                                           declaration relation
rules
  declareVar : scope * ID * TYPE
  resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
  typeOfVar : scope * ID → TYPE
  declareVar(s, x, T) :-
                                                     variable x is declared in scope s
    !var[x, T] in s,
                                                              with type T
    resolveVar(s, x) = [(_{-}, (_{-}, _{-}))]
       error Duplicate definition of variable [x]],
    @x.type := T.
                                                    there should only be one declaration
  resolveVar(s, x) = ps :-
                                                    variable x in scope s resolves to list
    query var
                                                            of declarations
       filter e and \{x':-x'=x\}
          min and true
           in s \mapsto ps.
                                                     variable x in scope s resolves to
  typeOfVar(s, x) = T :- \{x'\}
                                                        declaration x' with type T
    resolveVar(s, x) = [(\_,(x', T))]_{\_}
         error $[Variable [x] not defined],
                                                       there should be at least one
    @x.ref := x'.
                                                          matching declaration
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

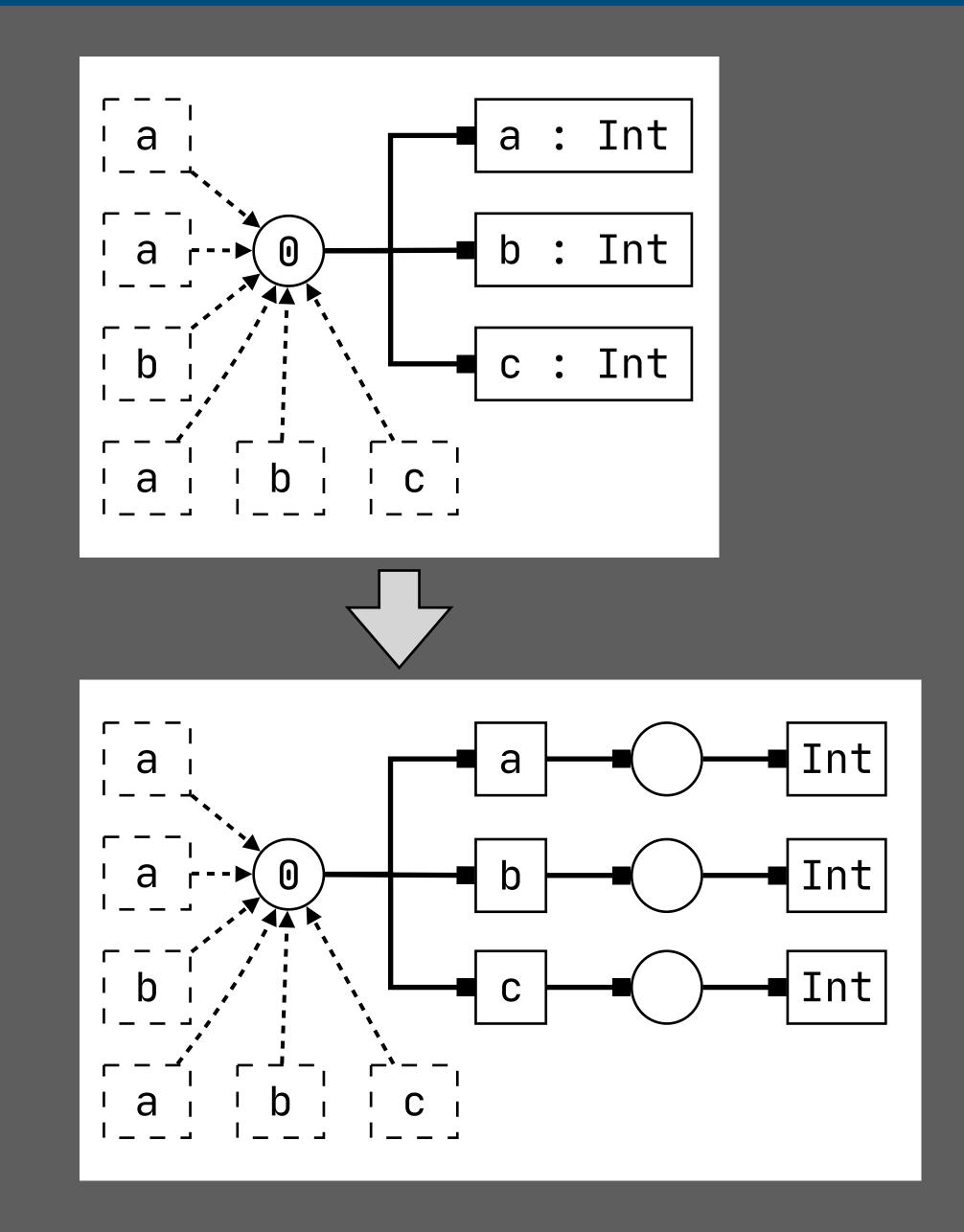
declaration and reference



Type Annotation Indirection (for Parallel Type Checking)

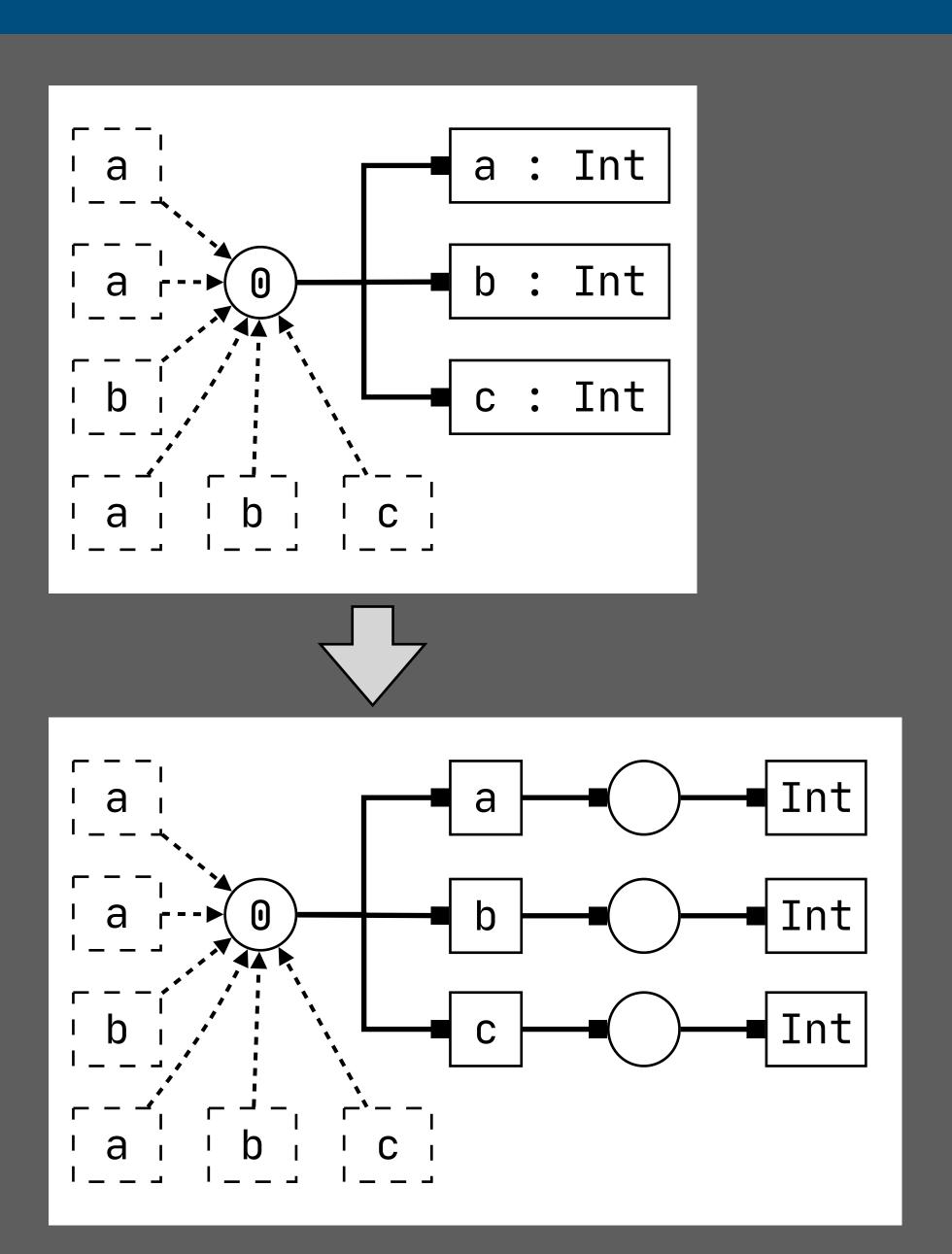


Type Annotation Indirection (for Parallel Type Checking)



Type Annotation Indirection (for Parallel Type Checking)

```
signature
  relations
    var : ID \rightarrow scope
    typeOf : → TYPE
rules
  declareVar(s, x, T) :=
    !var[x, withType(T)] in s.
  typeOfVar(s, x) = typeOf(T) :- \{x'\}
    resolveVar(s, x) = [(_,(x', T))].
  typeOf : scope → TYPE
  typeOf(s) = T : -
    query typeOf
      filter e and true
      min /* */ and true
      in s \mapsto [(\_, T)].
  withType : TYPE → scope
  withType(T) = s : -
    new s, !typeOf[T] in s.
```



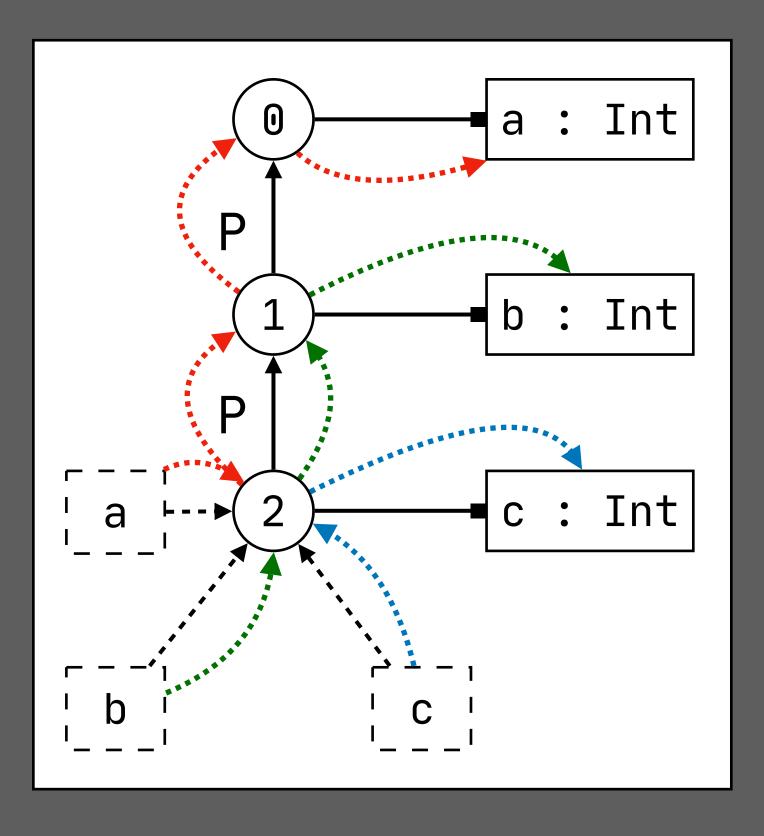
How about shadowing?

Lexical Scope

Labeled Scope Edges: What Scopes are Reachable?

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
```

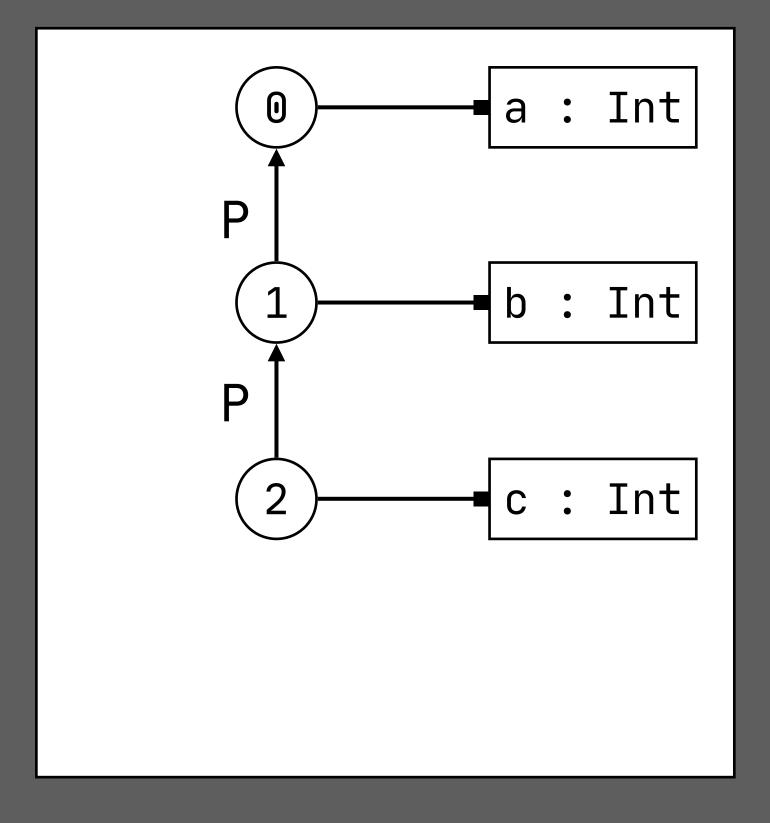
```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



New Scope and Scope Edge Constraints

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
```

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



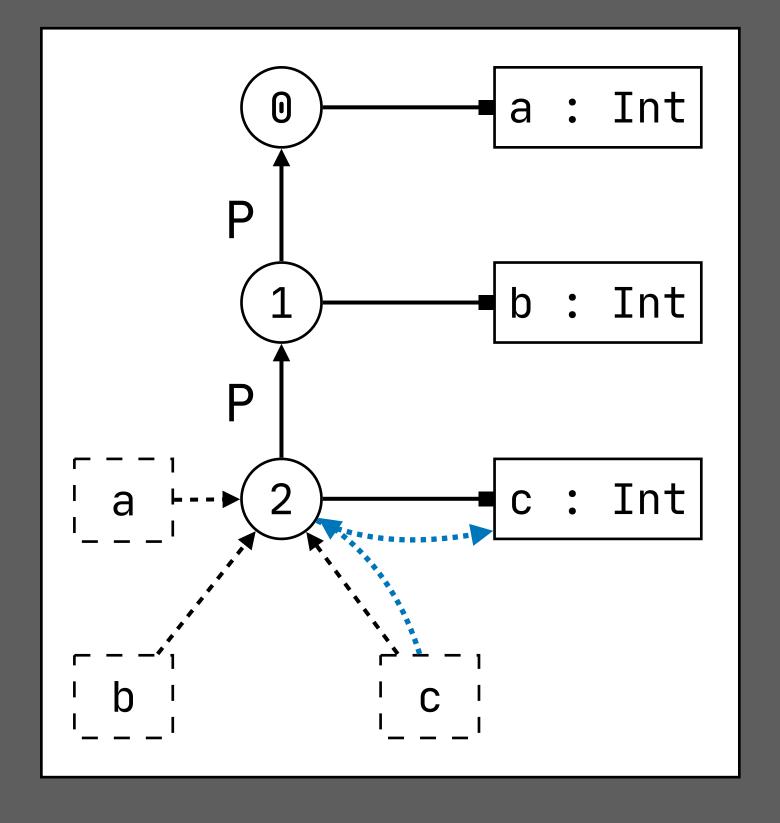
Path Wellformedness: Reachability

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
```

```
resolveVar(s, x) = ps :-
query var
filter e and \{x' :- x' = x\}
min /* */ and true
in s \mapsto ps.
```

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```

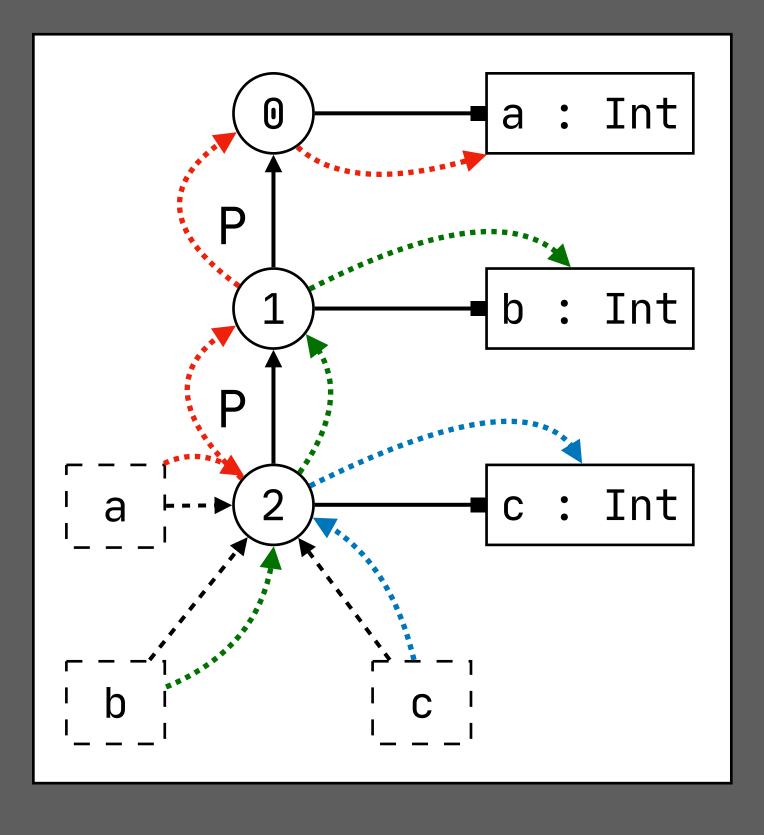


Path Wellformedness: Reachability

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
  name-resolution
  labels P
```

```
resolveVar(s, x) = ps :-
query var
filter P* and { x' :- x' = x }
min /* */ and true
in s \mapsto ps.
```

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



Duplicate Definitions Revisited

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
  name-resolution
  labels P
```

```
resolveVar(s, x) = ps :-
query var
filter P* and { x' :- x' = x }
min /* */ and true
in s \mapsto ps.
```

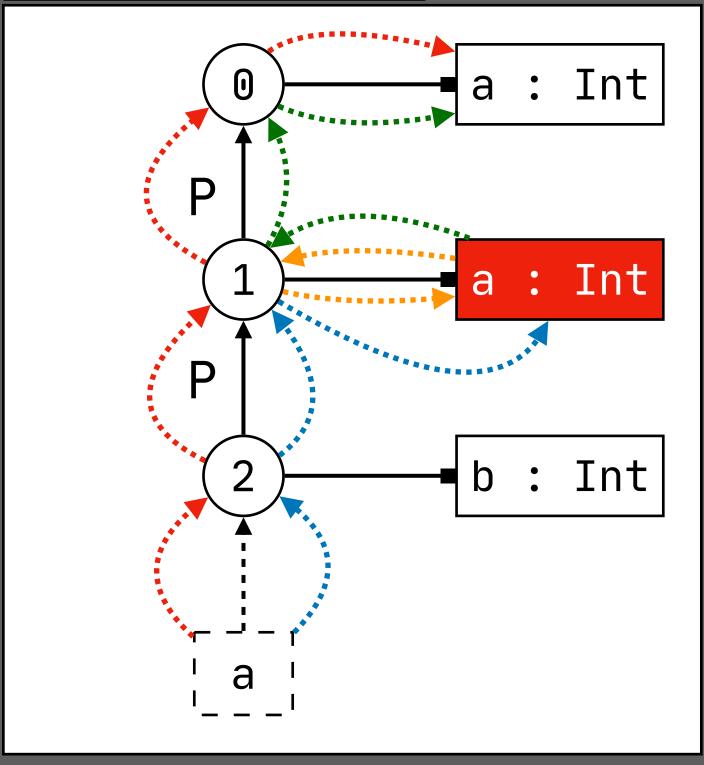
```
let a = 1 in
let \underline{a} = 2 in
let b = 3 in
   duplicate definition
                        a: Int
                        a: Int
                        b: Int
```

Duplicate Definitions Revisited

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
  name-resolution
  labels P
```

```
resolveVar(s, x) = ps :-
query var
filter P* and { x' :- x' = x }
min /* */ and true
in s \mapsto ps.
```

```
let a = 1 in
let a = 2 in
let b = 3 in
a
duplicate definition
```



Path Specificity: Visibility (Shadowing)

```
signature
  constructors
  Let : ID * Exp * Exp → Exp
  name-resolution
  labels P
```

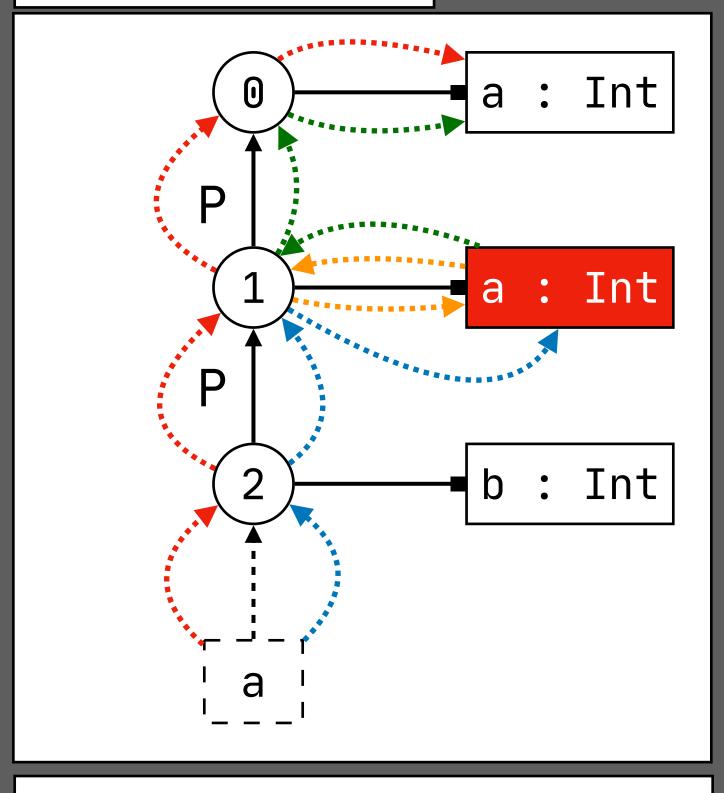
```
resolveVar(s, x) = ps :-
query var
filter P* and \{ x' :- x' = x \}
min /* */ and true
in s \longmapsto ps.
```

path P* allows resolution through zero or more P edges

prefer local scope (\$) over parent scope (P)

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



prefer blue path over red path

prefer orange path over green path

Path Specificity: Visibility (Shadowing)

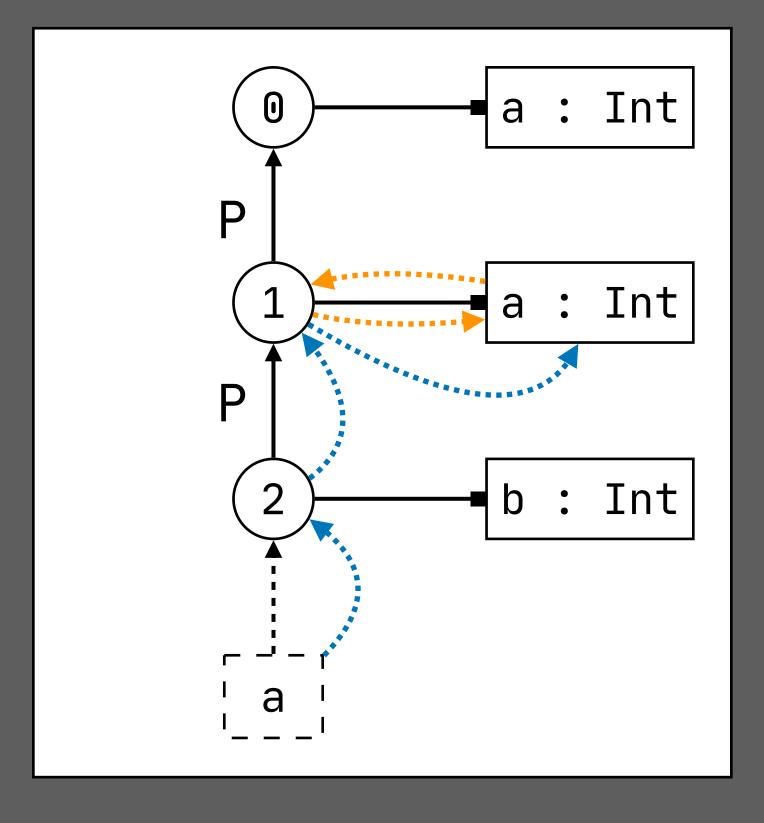
```
signature
  constructors
  Let : ID * Exp * Exp → Exp
  name-resolution
  labels P
```

```
resolveVar(s, x) = ps :-
query var
filter P* and { x' :- x' = x }
min $ < P and true
in s \mapsto ps.
```

path P* allows resolution through zero or more P edges

prefer local scope (\$) over parent scope (P)

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```



Statix Queries

query RELATION filter REGEX and MATCH min LABELORD and SHADOW in SCOPE \longmapsto RESULT

RELATION

- the relation we are querying

filter

- filter applied to individual paths to rule out 'non-wellformed' paths

min

- a filter applied to pairs of paths to select the 'minimal' paths from a set of paths

SCOPE

- the source scope of the query

RESULT

- the list of query results

Statix Queries

query RELATION filter REGEX and MATCH min LABELORD and SHADOW in SCOPE \longmapsto RESULT

RELATION

- Can be a custom relation name, decl, or () which only looks at the reachable scopes

REGEX

- Specifies path well-formedness using a regular expression (e.g. P* I*) over edge labels

MATCH

- Specifies which data in the relation to match
- To match on a name x, the match is an anonymous rule $\{x': -x' = x\}$ which is tested against all reachable declarations

LABELORD

- Determines the edge label order using inequalities (e.g. \$ < P, I < P) over edge labels

SHADOW

- Determines which declarations shadow each other
- true: all declarations shadow each other and we only get the declarations reached via the shortest path
- false: none of the declarations shadow (which could be used to find all reachable declarations)
- Anonymous rule (e.g. $\{x, x' : -x' = x\}$): only shadow between things with the same name

How about non-lexical bindings?

Scopes as Types / Modules

Modules: Scopes as Types

```
signature
                                                def c = 0
  constructors
                                                module A {
                                   scope as type
    MOD : scope \rightarrow TYPE
                                                   import B
    Module : ID * list(Decl) → Decl
                                                   def a = b + c
    Import : ID \rightarrow Decl
                                                module B {
                                                   def b = 2
rules
  declOk(s, Module(m, decls)) :- {s_mod}_
                                           lexical scope
    new s_mod, s_mod -P→ s,
    declareMod(s, m, MOD(s_mod)),
                                           scope as type
    declsOk(s_mod, decls).
                                                                  c : Int
                                                                                                        MOD(2)
                                                                                  MOD(1)
                                                                                 a : Int
                                                                                                       b: Int
```

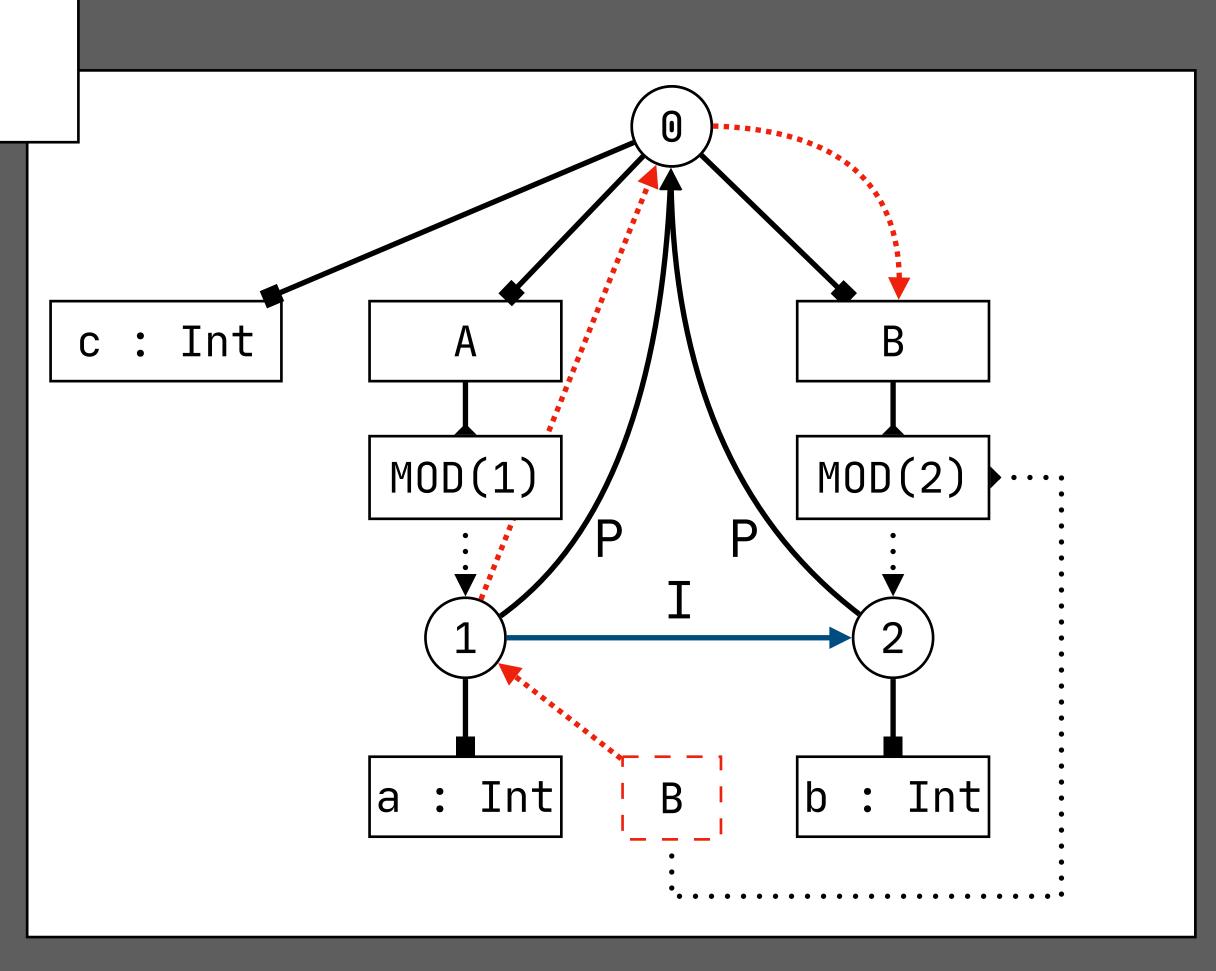
Resolving Import

```
def c = 0
signature
  constructors
                                                  module A {
                                    scope as type
    MOD : scope \rightarrow TYPE
                                                    import B
    Module : ID * list(Decl) → Decl
                                                    def a = b + c
    Import : ID → Decl
                                                  module B {
                                                    def b = 2
rules
  declOk(s, Module(m, decls)) :- {s_mod}_
                                             lexical scope
    new s_mod, s_mod -P→ s,
    declareMod(s, m, MOD(s_mod)),
                                            scope as type
    declsOk(s_mod, decls).
  declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
                                            resolve import
    s -I \rightarrow s \mod.
                                               resolve
  resolveMod(s, x) = ps :-
                                            module name
    query mod
      filter P* and \{ x' :- x' = x \}
         min $ < P and true
           in s \mapsto ps.
```

```
c : Int
                              MOD(2)
             MOD(1)
            a : Int
```

Import Edge

```
def c = 0
signature
                                                  module A {
  constructors
                                     scope as type
                                                    import B
    MOD : scope \rightarrow TYPE
    Module : ID * list(Decl) → Decl
                                                    def a = b + c
    Import : ID → Decl
                                                  module B {
                                                    def b = 2
rules
  declOk(s, Module(m, decls)) :- {s_mod}_
                                             lexical scope
    new s_mod, s_mod -P→ s,
    declareMod(s, m, MOD(s_mod)),
                                            scope as type
    declsOk(s_mod, decls).
  declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
                                            resolve import
    s -I \rightarrow s \mod.
                                             import edge
                                               resolve
  resolveMod(s, x) = ps :-
                                            module name
    query mod
      filter P* and \{ x' :- x' = x \}
         min $ < P and true
           in s \mapsto ps.
```



Resolving Variable through Import Edge

```
signature
                                                  def c = 0
  constructors
                                                  module A {
                                     scope as type
    MOD : scope \rightarrow TYPE
                                                    import B
    Module : ID * list(Decl) → Decl
                                                    def a = b + c
    Import : ID → Decl
                                                  module B {
                                                    def b = 2
rules
  declOk(s, Module(m, decls)) :- {s_mod},
                                             lexical scope
    new s_mod, s_mod -P→ s,
    declareMod(s, m, MOD(s_mod)),
                                            scope as type
    declsOk(s_mod, decls).
  declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
                                            resolve import
    s -I \rightarrow s_{mod}.
                                             import edge
  resolveVar(s, x) = ps :-
    query var
      filter P* I* and \{x':-x'=x\}
         min $ < P, $ < I, I < P and true
          in s \mapsto ps.
                                     resolve variable through
```

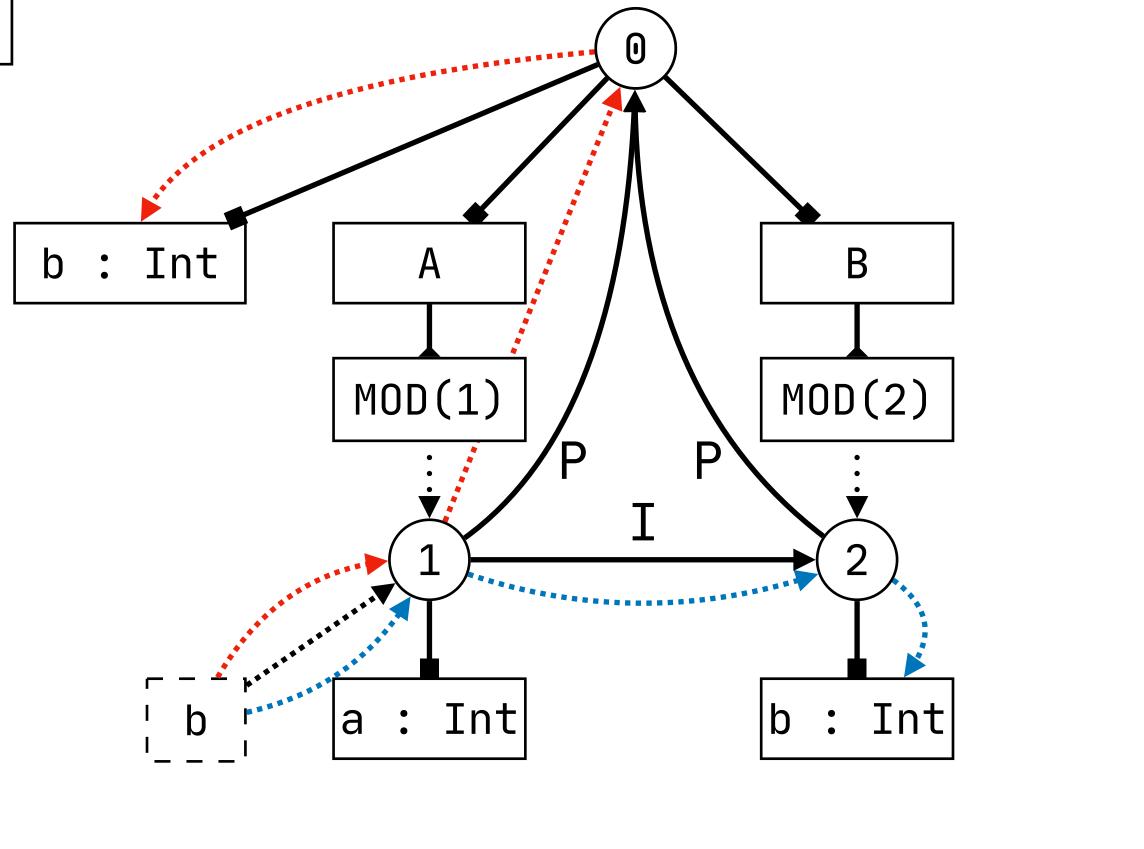
import edges

```
c : Int
                               MOD(2)
             MOD(1)
                                 : Int
```

Import Shadows Parent (Lexical Context)

```
signature
                                                   def b = 0
                                                   module A {
  constructors
                                     scope as type
                                                     import B
    MOD : scope \rightarrow TYPE
    Module : ID * list(Decl) → Decl
                                                     def a = b
    Import : ID → Decl
                                                   module B {
                                                     def b = 2
rules
  declOk(s, Module(m, decls)) :- {s_mod}_
                                             lexical scope
    new s_mod, s_mod -P→ s,
    declareMod(s, m, MOD(s_mod)),
                                             scope as type
    declsOk(s_mod, decls).
  declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
                                             resolve import
    s -I \rightarrow s_{mod}.
                                              import edge
  resolveVar(s, x) = ps :-
                                         import after parent
    query var
      filter P* I* and \{x':-x'=x\}
         min $ < P, $ < I, I < P and true prefer import
           in s \mapsto ps.
                                     resolve variable through
                                         import edges
```

prefer blue path over red path



Mutually Recursive Imports

```
signature
  constructors
   MOD : scope → TYPE
   Module : ID * list(Decl) → Decl
   Import : ID → Decl
```

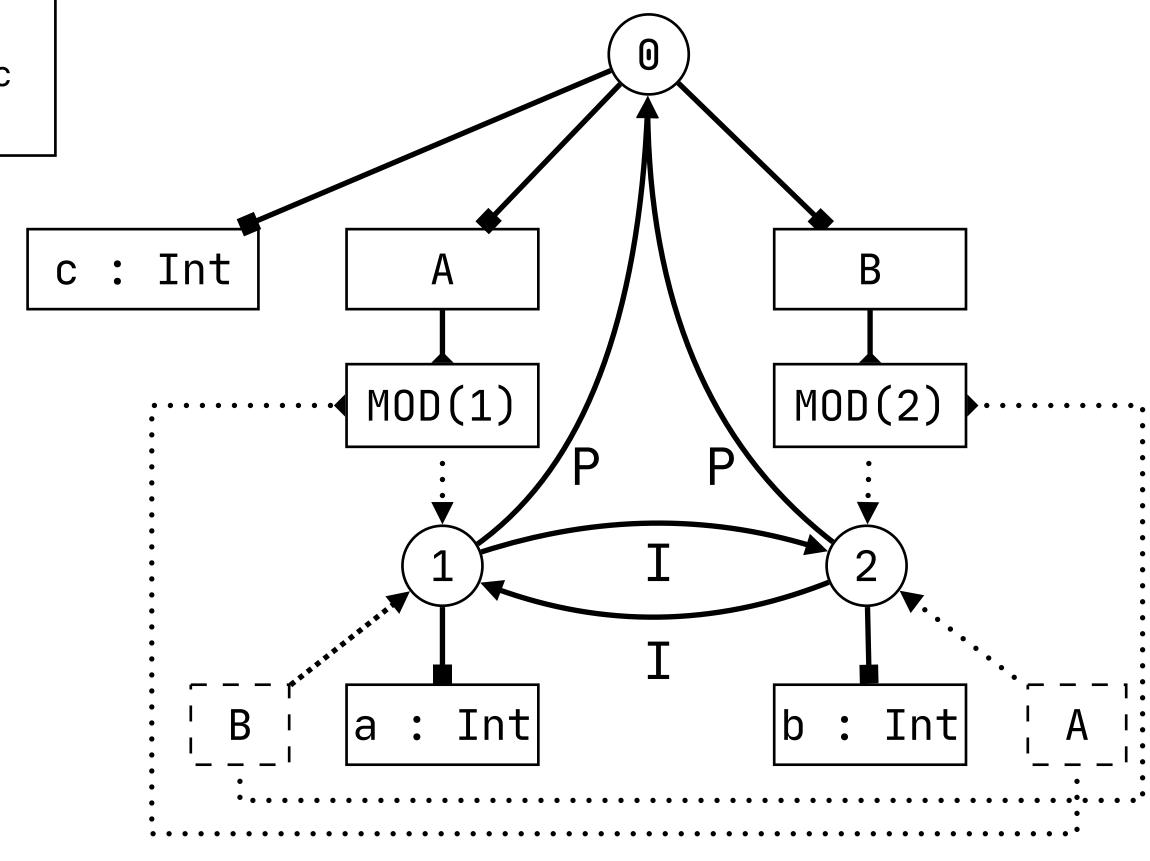
```
rules

declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
query var
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  import A
  def b = 2
  def d = a + c
}
```



Mutually Recursive Imports

```
signature
                                                    def c = 0
                                                    module A {
  constructors
                                      scope as type
    MOD : scope \rightarrow TYPE
                                                      import B
    Module : ID * list(Decl) → Decl
                                                      def a = b + c
    Import : ID → Decl
                                                   module B {
                                                      import A
rules
                                                      def b = 2
                                                      def d = a + c
  declOk(s, Module(m, decls)) :- {s_mod}
    new s_mod, s_mod -P \rightarrow s,
    declareMod(s, m, MOD(s_mod)),
                                              scope as type
    declsOk(s_mod, decls).
                                                                       c: Int
  declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
                                              resolve import
                                                                                                               MOD(2)
                                                                                        MOD(1)
    s -I \rightarrow s\_mod.
                                              import edge
  resolveVar(s, x) = ps :-
                                         import after parent
    query var
      filter P* I* and \{ x' :- x' = x \}_{\vdash}
          min $ < P, $ < I, I < P and tru prefer import
                                                                                       |a : Int|
                                                                                                              |b : Int|
           in s \mapsto ps.
                                      resolve variable through
                                          import edges
```

Transitive Import

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
query var
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```

```
module A {
  import B
  def a = b + c
module B {
  import C
  def b = c + 2
module C {
  def c = 1
                     Α
                                      MOD(2)
                  MOD(1)
```

a: Int B b: Int C c: Int

MOD(3)

Transitive Import

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
   new s_mod, s_mod -P→ s,
   declareMod(s, m, MOD(s_mod)),
   declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
   typeOfModRef(s, p) = MOD(s_mod),
   s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
query var
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```

```
module A {
  import B
  def a = b + c
module B {
  import C
  def b = c + 2
module C {
  def c = 1
                     Α
                                                           MOD(3)
                  MOD(1)
                                      MOD(2)
                                      |b : Int|
                                                          c : Int
                 |a : Int|
```

Changing Query Outcomes

(is not allowed)

Nested Modules

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
   new s_mod, s_mod -P→ s,
   declareMod(s, m, MOD(s_mod)),
   declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
   typeOfModRef(s, p) = MOD(s_mod),
   s -I→ s_mod.
```

```
module A {
   module B {
     def b = 1
   }
}
module C {
   import A
   import B
   def c = b
}
```

```
resolveMod(s, x) = ps :-
query mod
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```

```
Α
            MOD(1)
                               MOD(3)
   B
MOD(2)
                                           B
                               c : Int
```

Changing Result of Query

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
   new s_mod, s_mod -P→ s,
   declareMod(s, m, MOD(s_mod)),
   declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
   typeOfModRef(s, p) = MOD(s_mod),
   s -I→ s_mod.
```

```
module A {
    module A {
     def b = 1
    }
}
module C {
    import A
    import A
    def c = b
}
```

```
resolveMod(s, x) = ps :-
query mod
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s → ps.
```

```
Α
            MOD(1)
                               MOD(3)
   A
MOD(2)
                              c : Int
b : Int
```

Changing Result of Query

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
   new s_mod, s_mod -P→ s,
   declareMod(s, m, MOD(s_mod)),
   declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
   typeOfModRef(s, p) = MOD(s_mod),
   s -I→ s_mod.
```

```
module A {
    module A {
        def b = 1
    }
}
module C {
    import A
    import A
    def c = b
}
```

```
resolveMod(s, x) = ps :-
query mod
filter P* I* and { x' :- x' == x }
min $ < P, $ < I, I < P and true
in s → ps.
```

```
Α
            MOD(1)
                              MOD(3)
MOD(2)
                              c : Int
```

Changing Result of Query

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

```
rules

declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
    module A {
        def b = 1
    }
}
module C {
    import A
    import A
    def c = b
}
```

```
resolveMod(s, x) = ps :-
query mod
filter P* I* and { x' :- x' = x }
min $ < P, $ < P, I < P and true
in s \mapsto ps.
```

```
Α
            MOD(1)
                              MOD(3)
MOD(2)
                              c : Int
```

Alternative Encoding: Scoped Imports

```
signature
sorts DecGroups
constructors

MOD : scope → TYPE
Module : ID * DecGroups → Decl
Import : ID → Decl
ModRef : ID * ID → Exp

Decs : list(Decl) → DecGroups
Seq : list(Decl) * DecGroups
→ DecGroups
```

```
module A {
    module A {
        def b = 1
    }
}
module C {
    import A;
    import A
    def c = b
}
```

```
Α
            MOD(1)
                              MOD(3)
   A
MOD(2)
b: Int
                             c : Int
```

```
resolveMod(s, x) = ps :-
query mod
filter P+ I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```

Alternative Encoding: Scoped Imports — M Edge Label

```
signature
sorts DecGroups
constructors

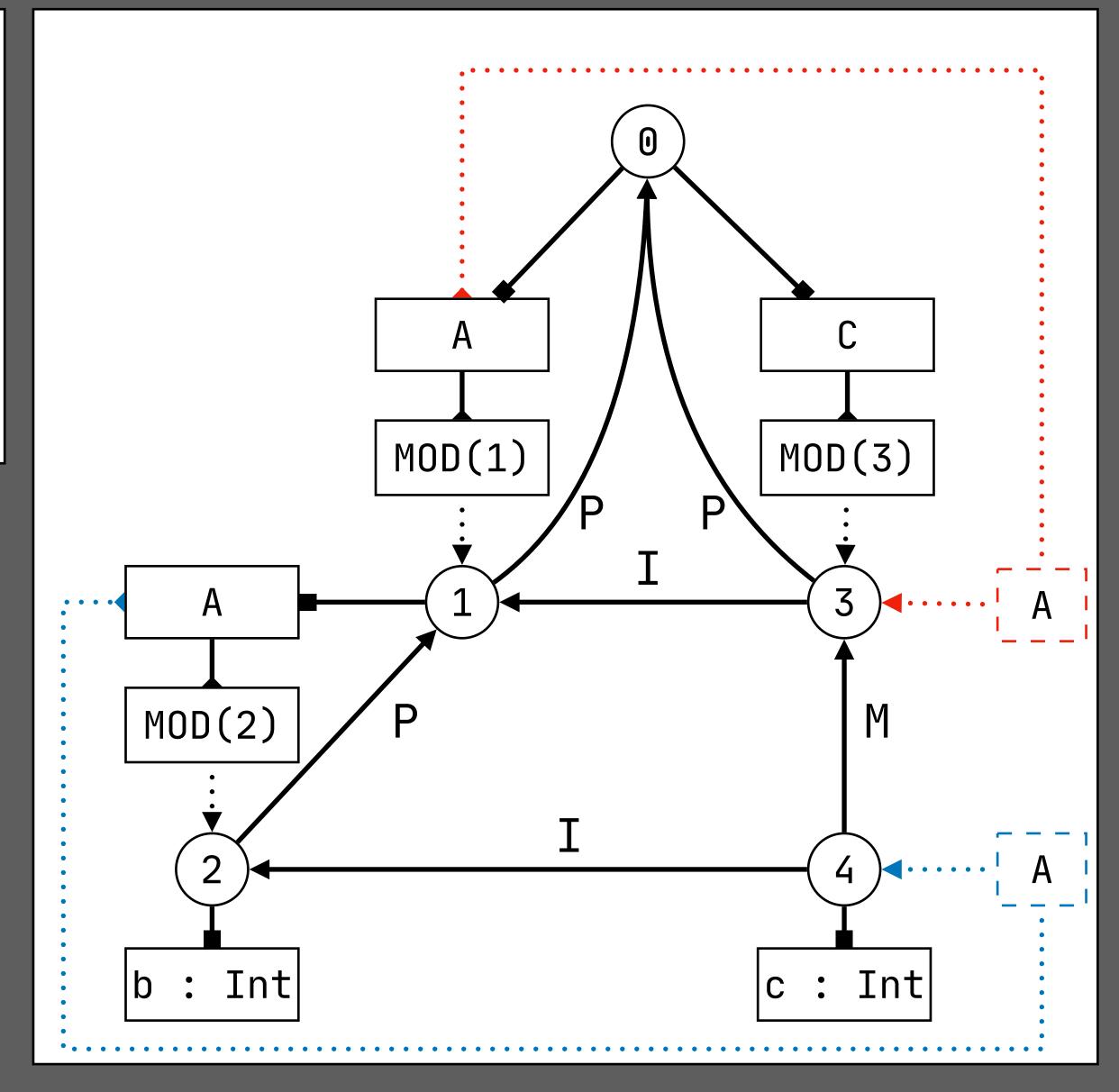
MOD : scope → TYPE
Module : ID * DecGroups → Decl
Import : ID → Decl
ModRef : ID * ID → Exp

Decs : list(Decl) → DecGroups
Seq : list(Decl) * DecGroups
→ DecGroups
```

```
module A {
    module A {
        def b = 1
    }
}
module C {
    import A;
    import A
    def c = b
}
```

```
resolveVar(s, x) = ps :-
query var
filter P* (I | M)*
and { x' :- x' == x }
min $ < P, $ < I, I < P and true
in s → ps.
```

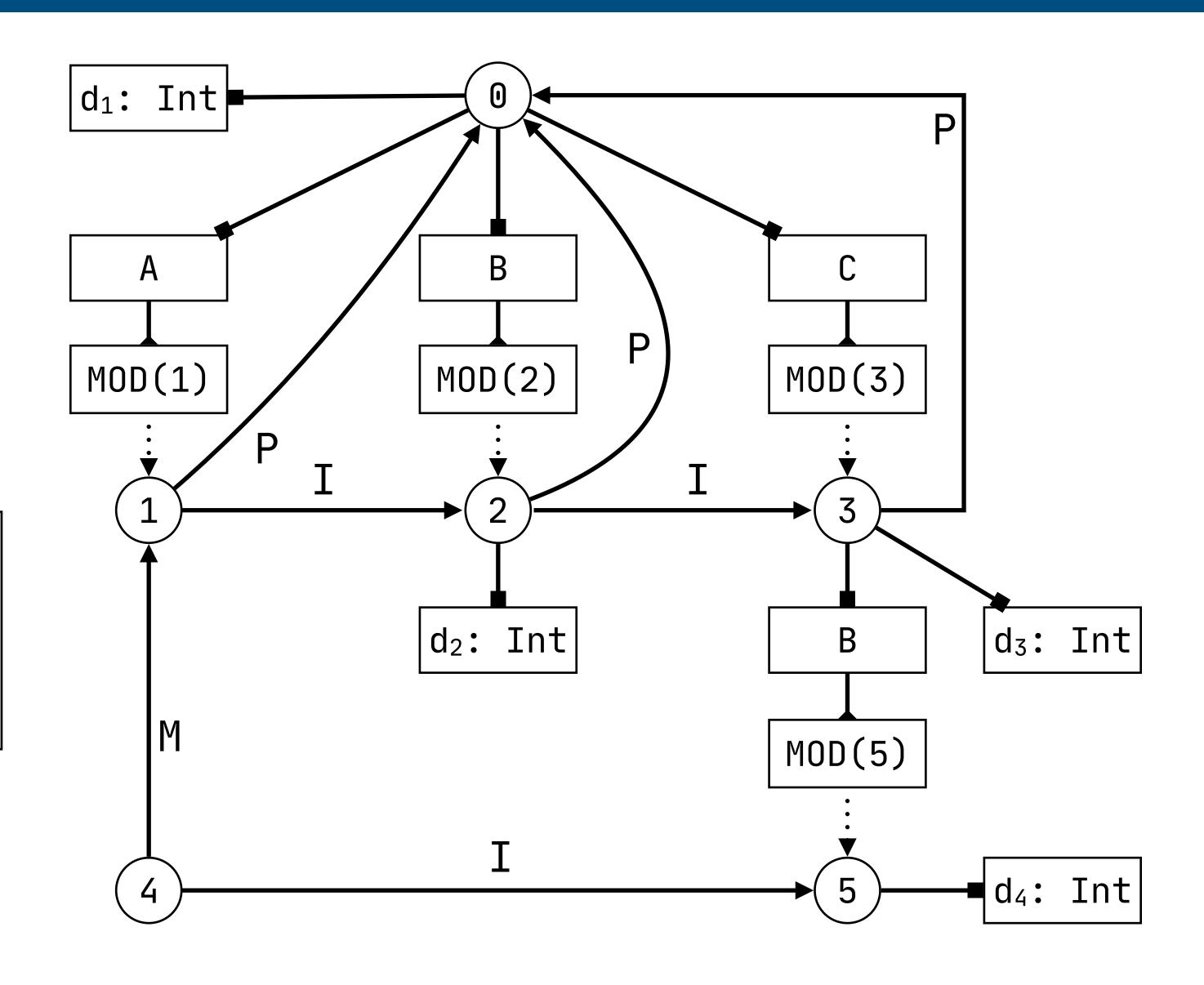
```
resolveMod(s, x) = ps :-
query mod
filter (P | M) P* (I | M)*
and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```



Exercise

```
resolveVar(s, x) = ps :-
query var
filter (P | M)* (I (I | M)*)?
and { x' :- x' == x }
min $ < P, $ < I, I < P, I < M and true
in s → ps.
```

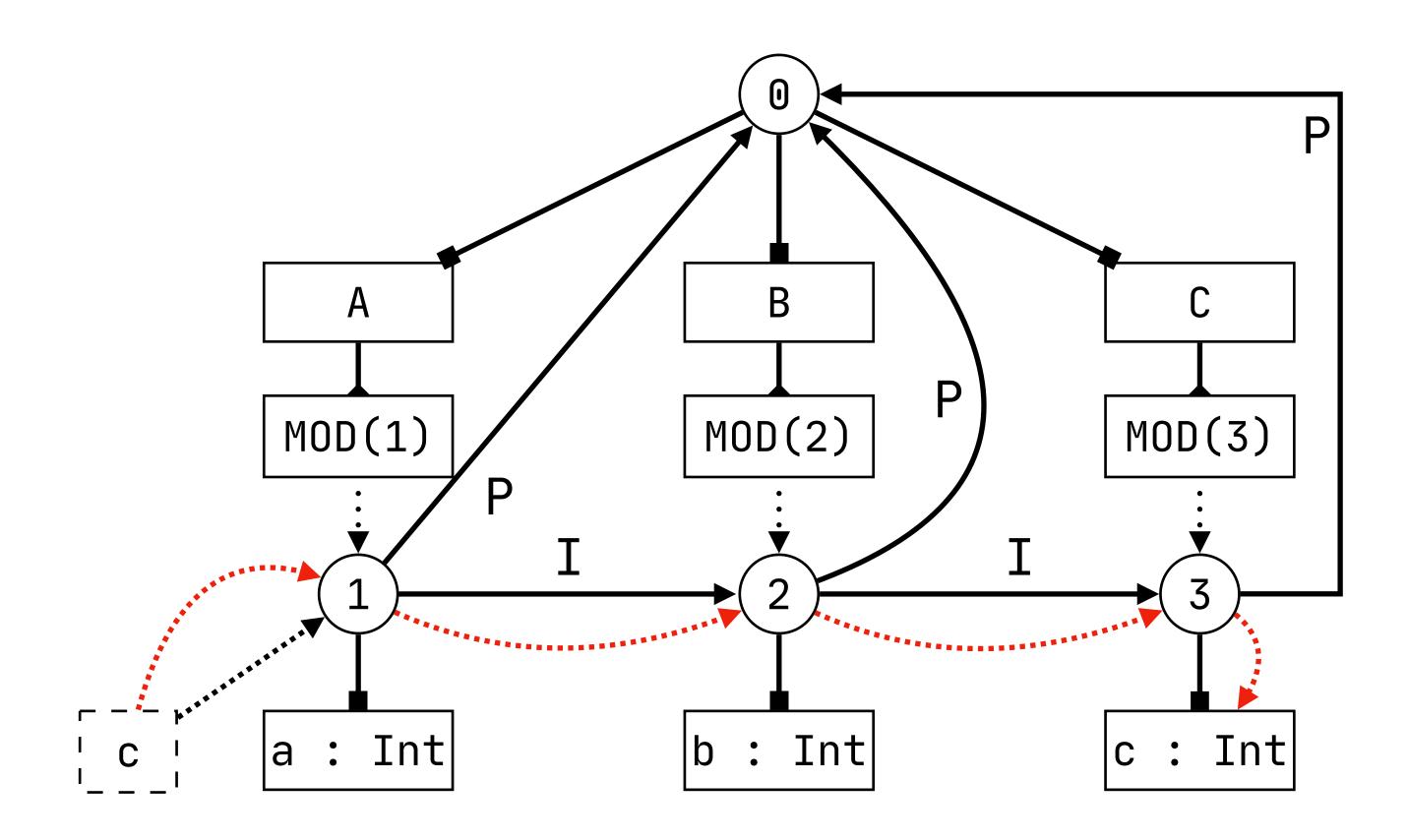
```
resolveMod(s, x) = ps :-
query mod
filter (P | M) P* (I (I | M)*)?
and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```



Exercise

```
resolveVar(s, x) = ps :-
query var
filter (P | M)* (I (I | M)*)?
and { x' :- x' == x }
min $ < P, $ < I, I < P, I < M and true
in s → ps.
```

```
resolveMod(s, x) = ps :-
query mod
filter (P | M) P* (I (I | M)*)?
and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```



Exercise

```
signature
  constructors
  MOD : scope → TYPE
  Module : ID * list(Decl) → Decl
  Import : ID → Decl
```

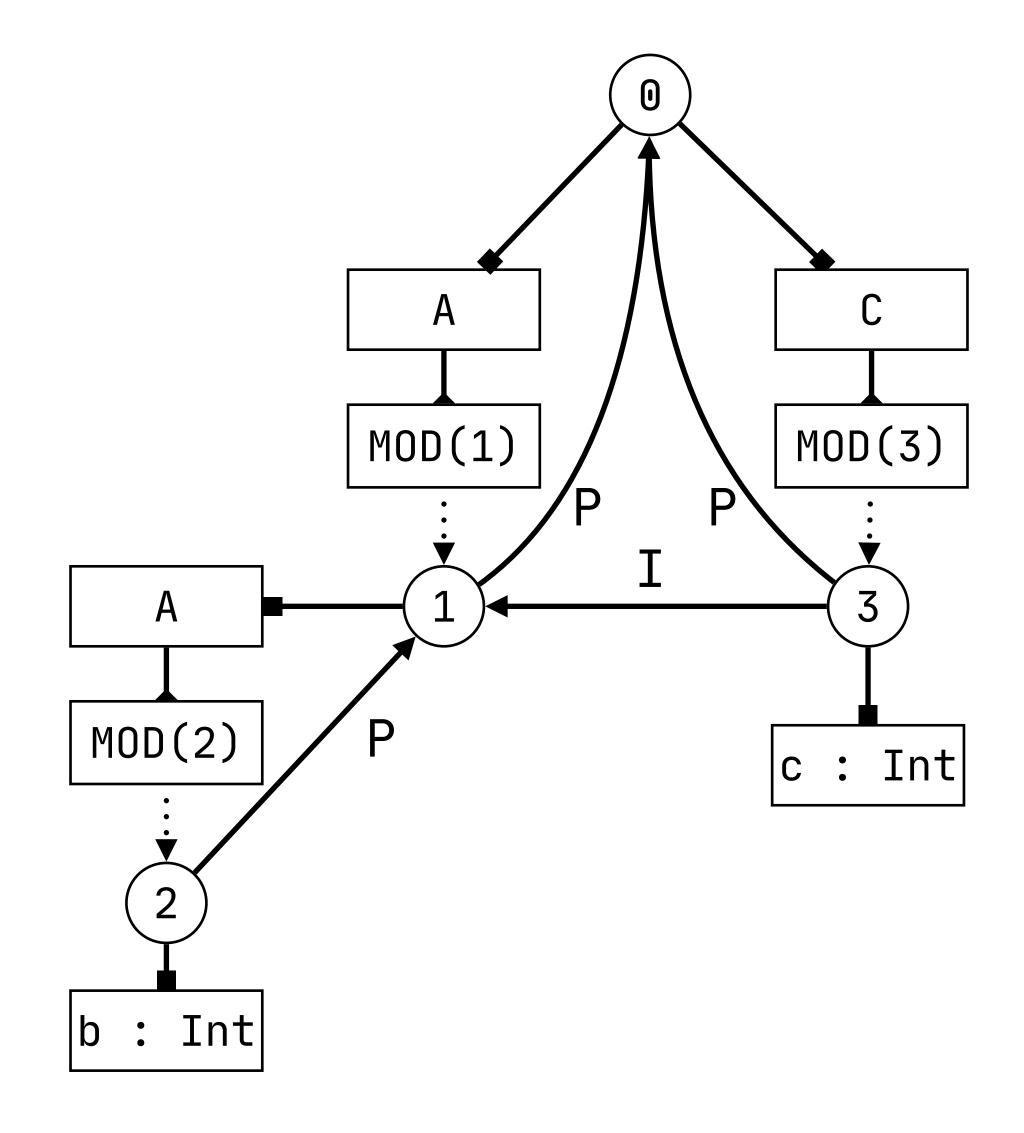
```
rules

declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).

declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
    module A {
        def b = 1
    }
}
module C {
    import A
    import A
    def c = b
}
```

```
resolveMod(s, x) = ps :-
query mod
filter P* I* and { x' :- x' = x }
min $ < P, $ < I, I < P and true
in s \mapsto ps.
```



Permission to Extend

Permission to Extend

```
signature
  constructors
   MOD : scope → TYPE
   Module : ID * list(Decl) → Decl
   Import : ID → Decl
   ExtendRemote : ID * ID * Exp → Decl
```

```
module A {
  def a = b
}
module B {
  def A.b := 2
}
```

```
rules // extend remote

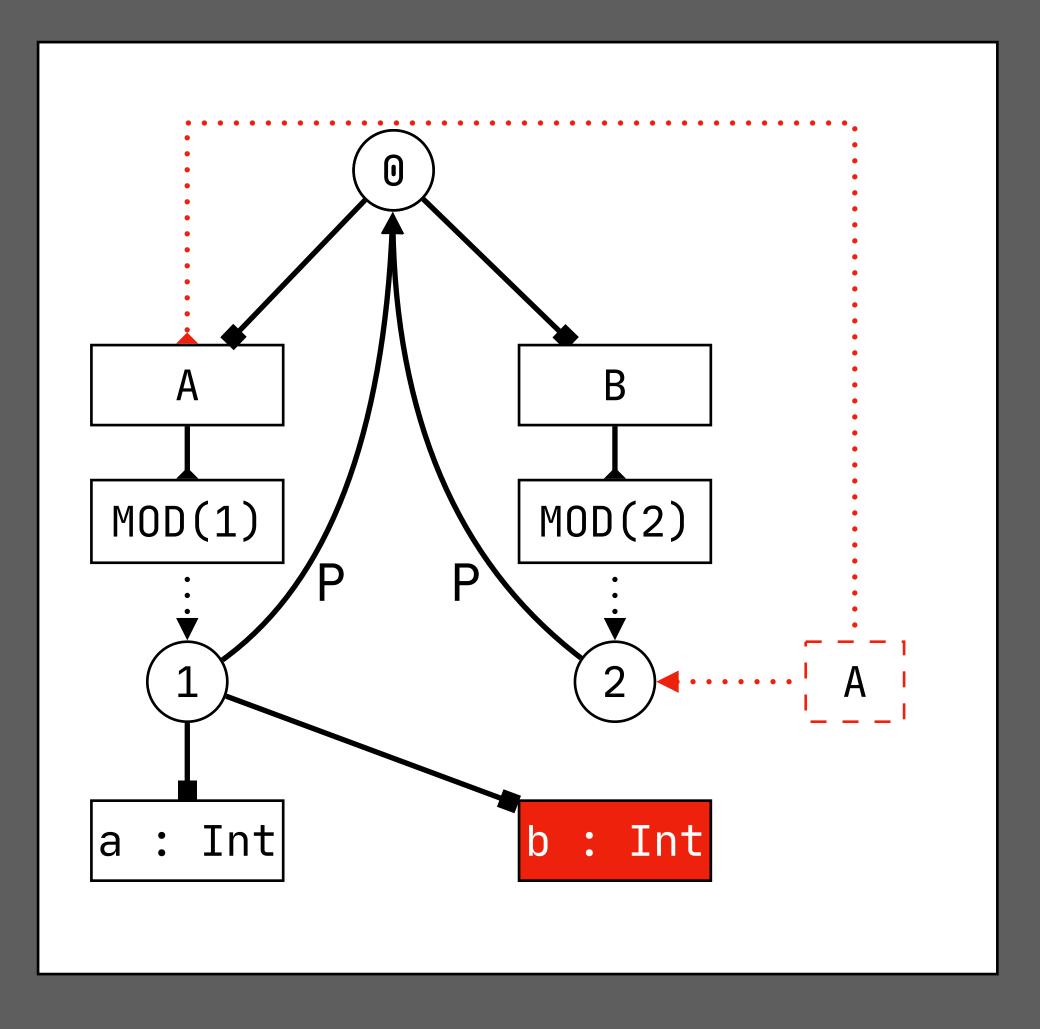
declOk(s, ExtendRemote(m, x, e)) :- {s_mod T}
  typeOfModRef(s, m) = MOD(s_mod),
  typeOfExp(s, e) = T,
  declareVar(s_mod, x, T).
  // no permission to extend

Add declaration to scope obtained through a query
```

This is not allowed in Statix

A predicate can only extend scopes over which its has ownership, i.e. that it creates or gets passed down as an argument

extend remote: def M.x := e extend module M with declaration of x



Type-Dependent Name Resolution RECOICS

Records

```
signature
  constructors

  REC : scope → TYPE
  Record : ID * list(FDecl) → Decl
  FDecl : ID * Type → FDecl
  New : ID * list(FBind) → Exp
  FBind : ID * Exp → FBind
  Proj : Exp * ID → Exp
```

```
record Point { x : Int, y : Int }
def p = Point{ x = 1, y = 2 }
> p.y
```

Record Type: Scope as Type

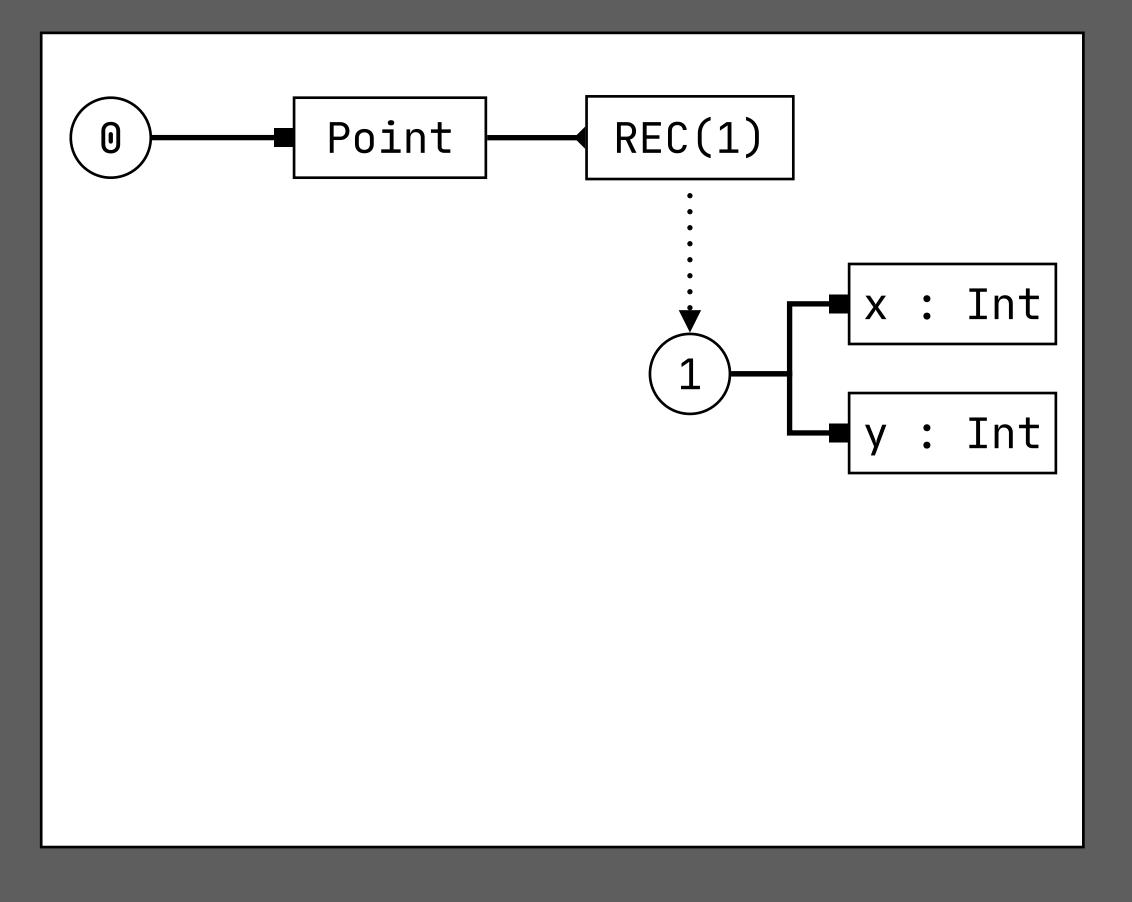
```
\begin{array}{c} \textbf{signature} \\ \textbf{constructors} \\ \textbf{REC} & : \textbf{scope} \rightarrow \textbf{TYPE} \\ \textbf{Record} & : \textbf{ID} * \textbf{list}(\textbf{FDecl}) \rightarrow \textbf{Decl} \\ \textbf{FDecl} & : \textbf{ID} * \textbf{Type} \rightarrow \textbf{FDecl} \\ \textbf{New} & : \textbf{ID} * \textbf{list}(\textbf{FBind}) \rightarrow \textbf{Exp} \\ \textbf{FBind} & : \textbf{ID} * \textbf{Exp} \rightarrow \textbf{FBind} \\ \textbf{Proj} & : \textbf{Exp} * \textbf{ID} \rightarrow \textbf{Exp} \end{array}
```

```
rules // record type

declOk(s, Record(x, fdecls)) :- {s_rec}
    new s_rec,
    fdeclsOk(s_rec, s, fdecls),
    declareType(s, x, REC(s_rec)).

fdeclOk(s_bnd, s_ctx, FDecl(x, t)) :- {T}
    typeOfType(s_ctx, t) = T,
    declareVar(s_bnd, x, T).
```

```
record Point { x : Int, y : Int }
def p = Point{ x = 1, y = 2 }
> p.y
```



Record Construction & Initialization

```
signature
  constructors

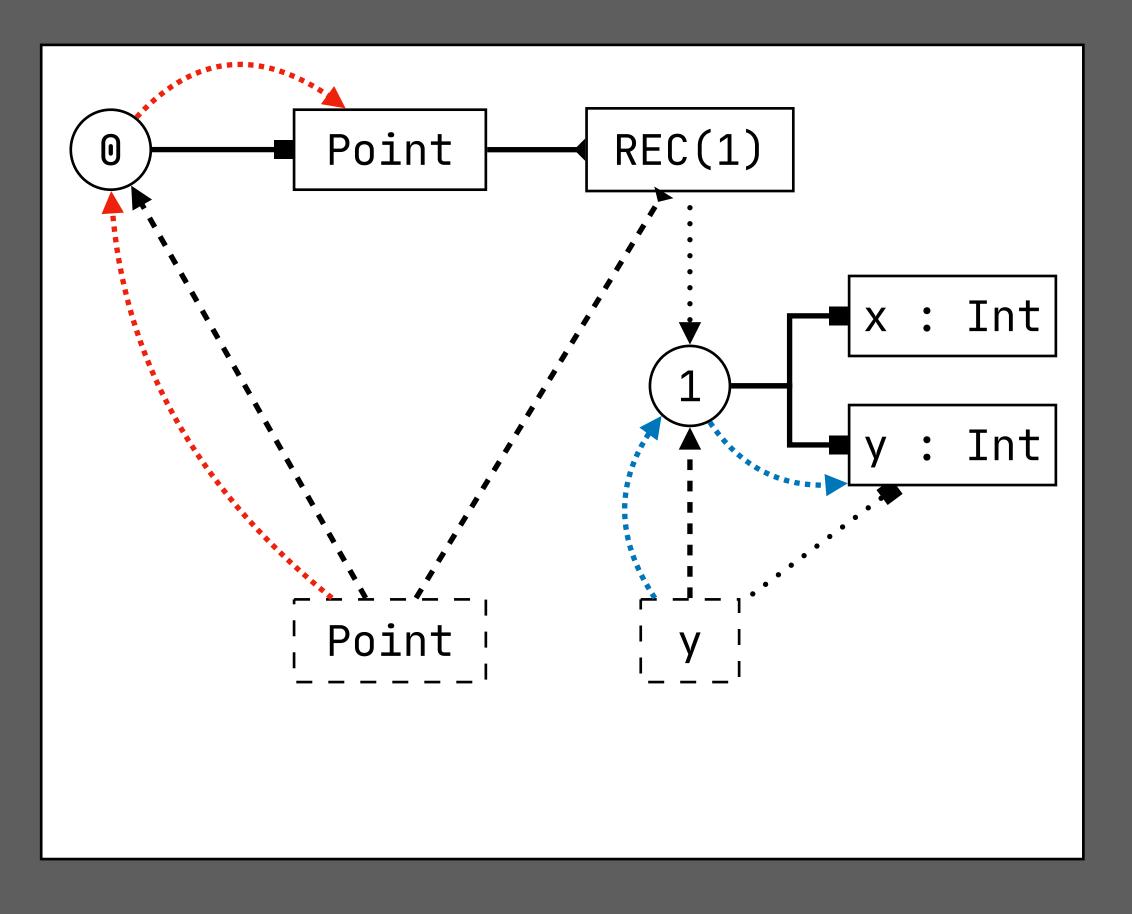
  REC : scope → TYPE
  Record : ID * list(FDecl) → Decl
  FDecl : ID * Type → FDecl
  New : ID * list(FBind) → Exp
  FBind : ID * Exp → FBind
  Proj : Exp * ID → Exp
```

```
rules // record construction

typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).

fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
def p = Point{ x = 1, y = 2 }
> p.y
```



Type-Dependent Name Resolution

```
signature
  constructors

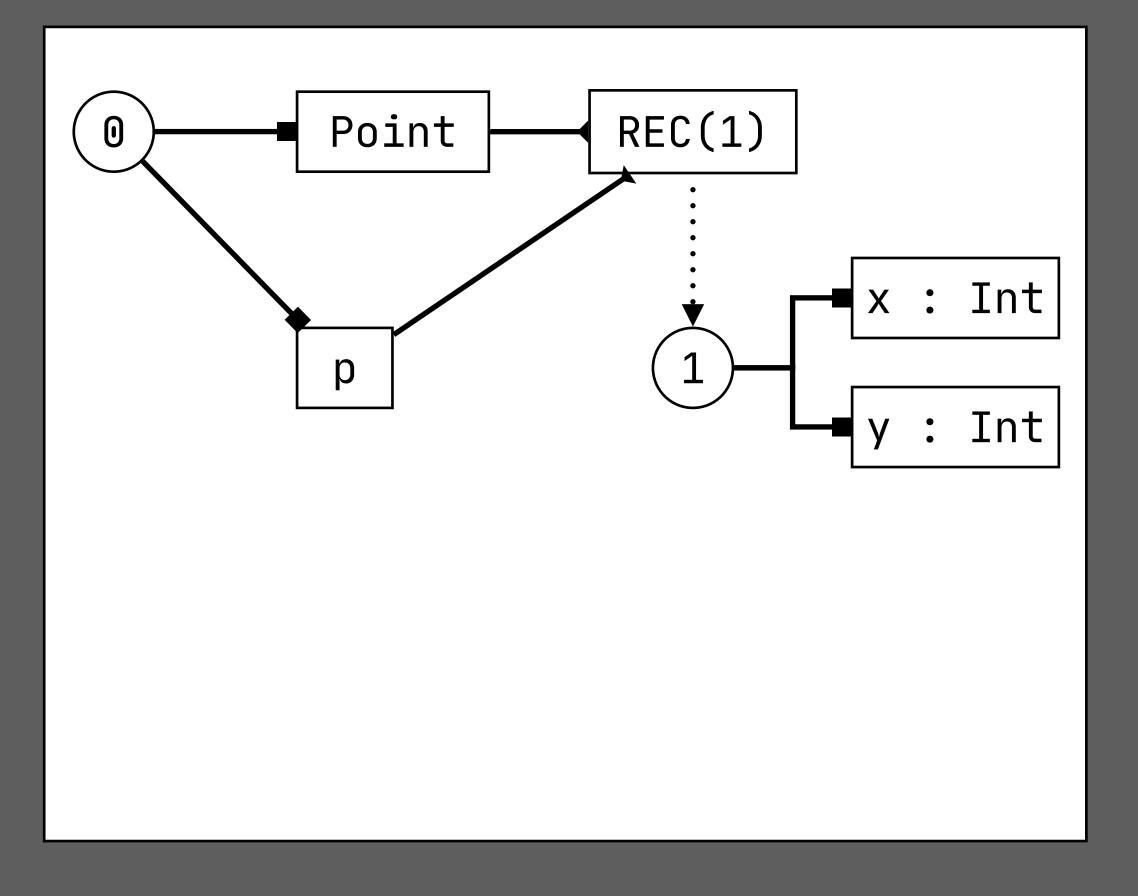
  REC : scope → TYPE
  Record : ID * list(FDecl) → Decl
  FDecl : ID * Type → FDecl
  New : ID * list(FBind) → Exp
  FBind : ID * Exp → FBind
  Proj : Exp * ID → Exp
```

```
rules // record construction

typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).

fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
def p = Point{ x = 1, y = 2 }
> p.y
```



Type-Dependent Name Resolution

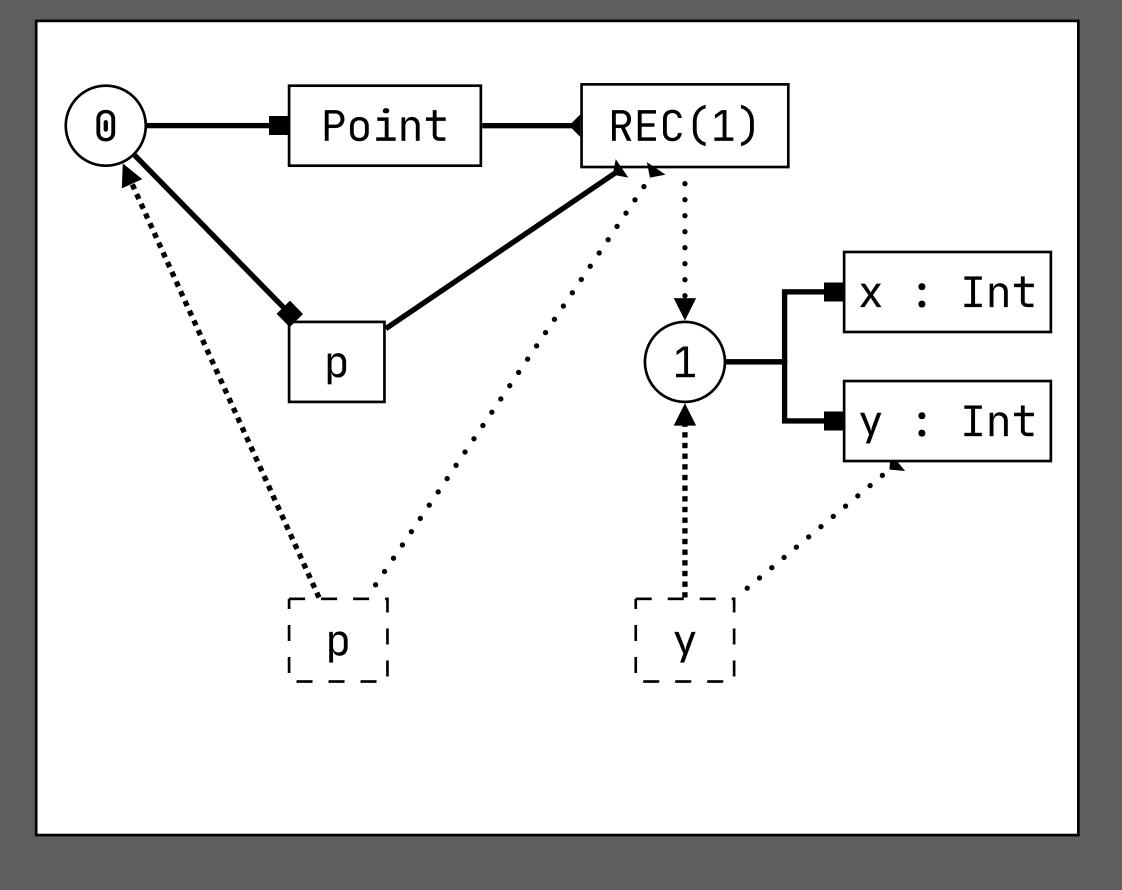
```
signature
  constructors

  REC : scope → TYPE
  Record : ID * list(FDecl) → Decl
  FDecl : ID * Type → FDecl
  New : ID * list(FBind) → Exp
  FBind : ID * Exp → FBind
  Proj : Exp * ID → Exp
```

```
rules // record projection

typeOfExp(s, Proj(e, x)) = T :- {p d s_rec S}
  typeOfExp(s, e) = REC(s_rec),
  typeOfVar(s_rec, x) = T.
```

```
record Point { x : Int, y : Int }
def p = Point{ x = 1, y = 2 }
> p.y
```



With

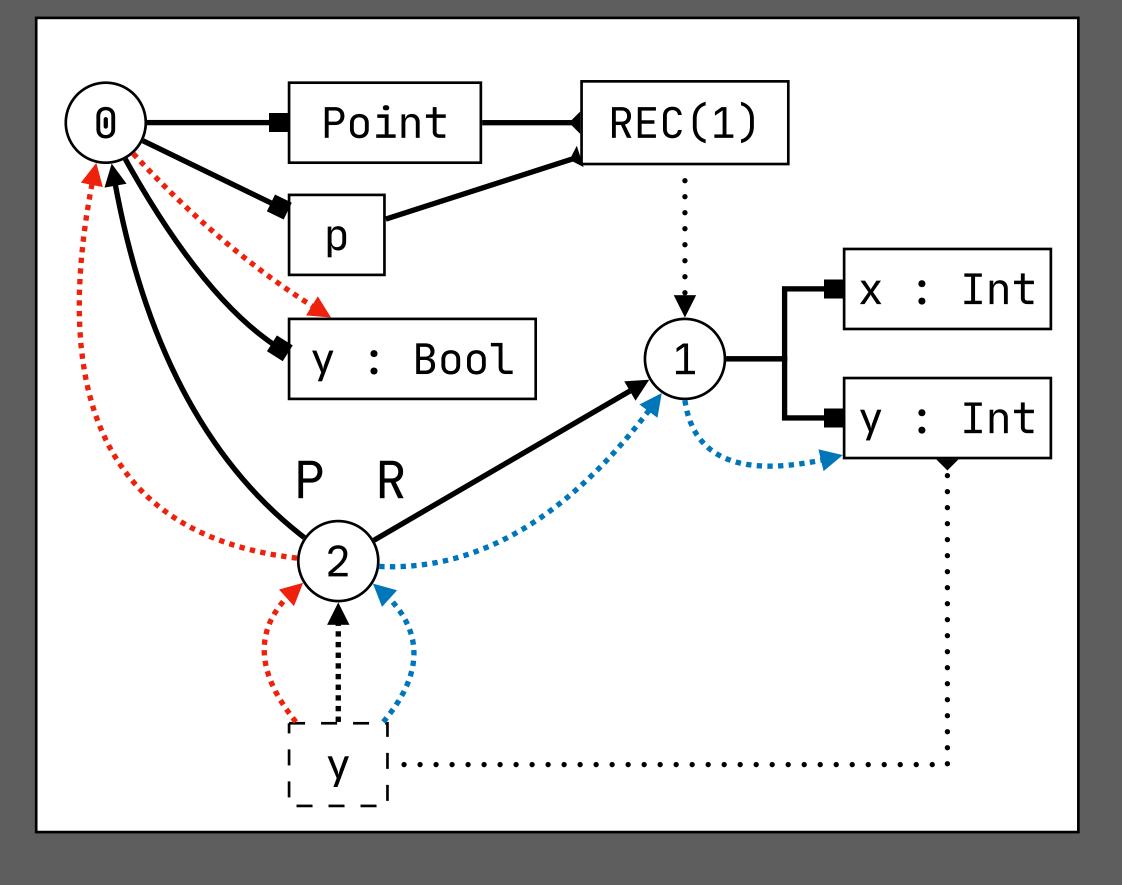
```
signature
  constructors
   REC : scope → TYPE
   Record : ID * list(FDecl) → Decl
   FDecl : ID * Type → FDecl
   New : ID * list(FBind) → Exp
   FBind : ID * Exp → FBind
   Proj : Exp * ID → Exp
```

```
rules // with record value

typeOfExp(s, With(e1, e2)) = T :- {s_with s_rec}
  typeOfExp(s, e1) = REC(s_rec),
  new s_with, s_with -P→ s, s_with -R→ s_rec,
  typeOfExp(s_with, e2) = T.
```

```
resolveVar(s, x) = ps :-
    query var
    filter P* R* and { x' :- x' == x }
    min $ < P, R < P and true
    in s → ps.
```

```
record Point { x : Int, y : Int }
def p = Point{x = 1, y = 2}
def y = true
> with p do y
```

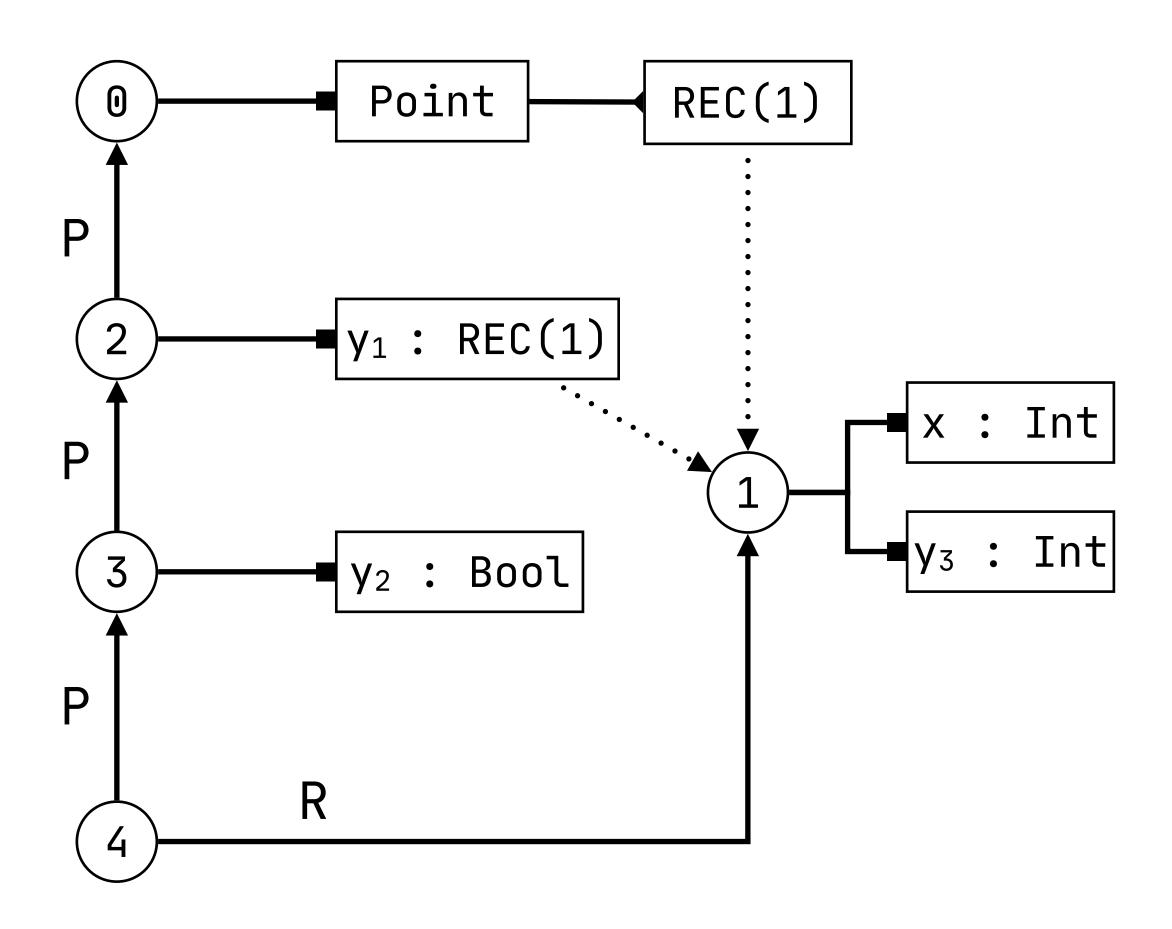


Exercise

```
record Point { x : Int, y : Int }
> let y = Point{x = 1, y = 2} in
let y = true in
with p do y
```

```
reachableVar(s, x) = ps :-
query var
filter P* R* and { x' :- x' = x }
min /* */ and true
in s \mapsto ps.
```

```
visibleVar(s, x) = ps :-
    query var
    filter P* R* and { x' :- x' == x }
    min $ < P, R < P and true
    in s → ps.</pre>
```



Scheduling Constraint Resolution

Scheduling in Type Checkers

Type checker constructs scope graph

- Module, variable declarations
- Module imports
- Scopes

Type checker queries scope graph

- Type of variable reference

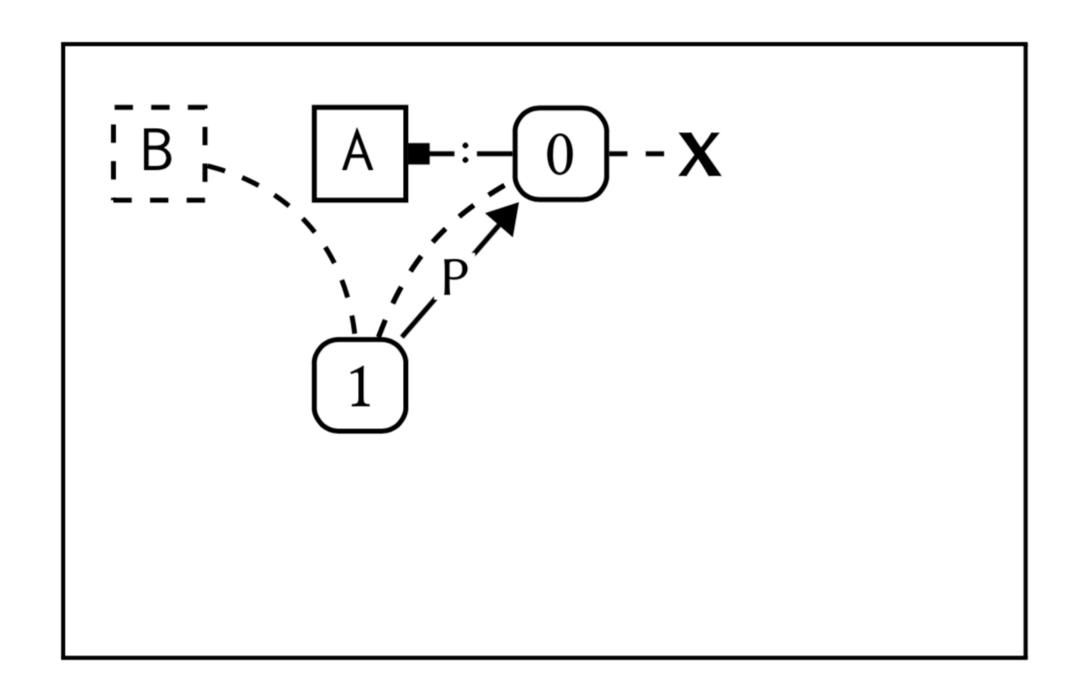
Scope graph construction depends on queries

- Imports require name resolution of module name

When is it safe to query the scope graph?

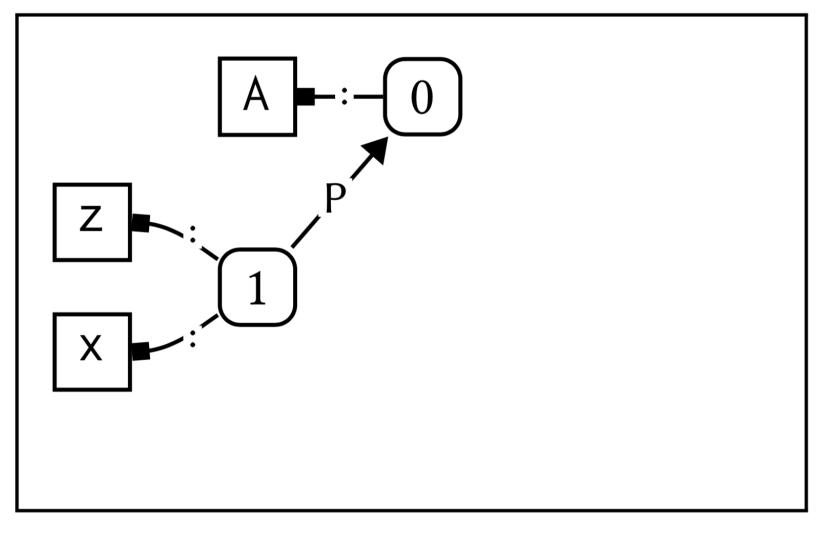
- In what order should type checker perform construction, querying?

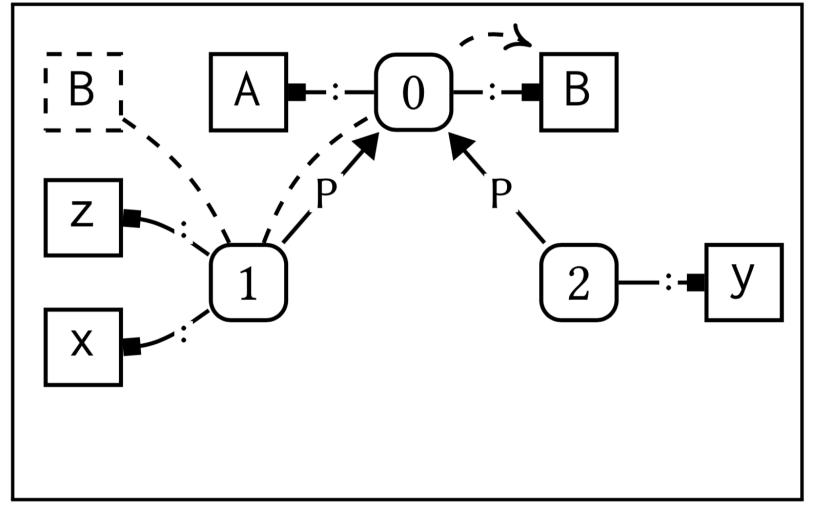
A Single Stage Type Checker (Fails)



```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

A Two Stage Type Checker: Stage 1 (Build Module Table)

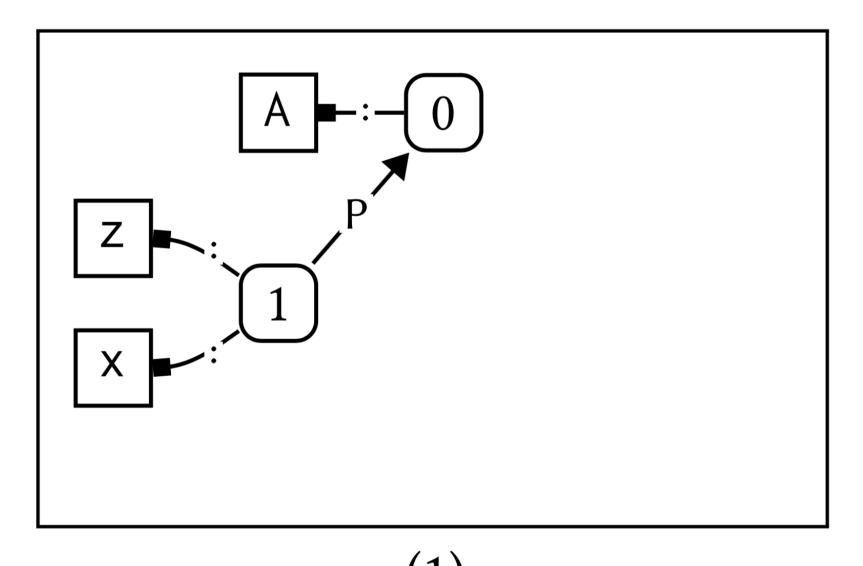


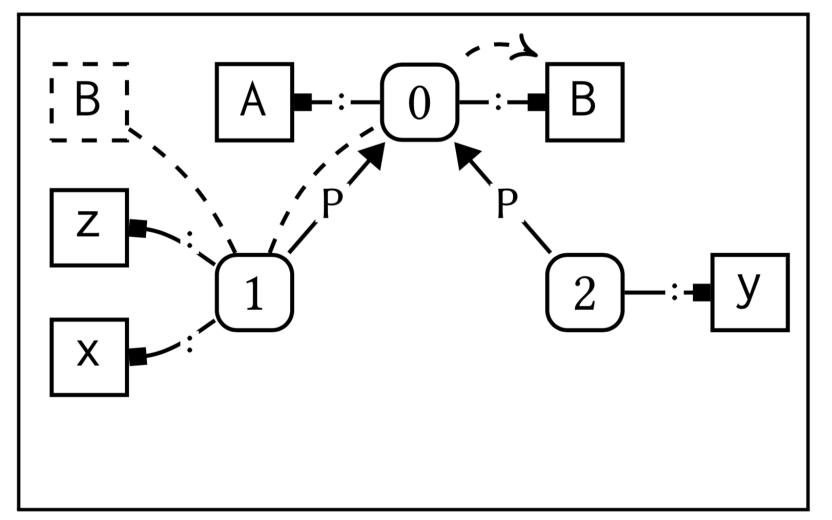


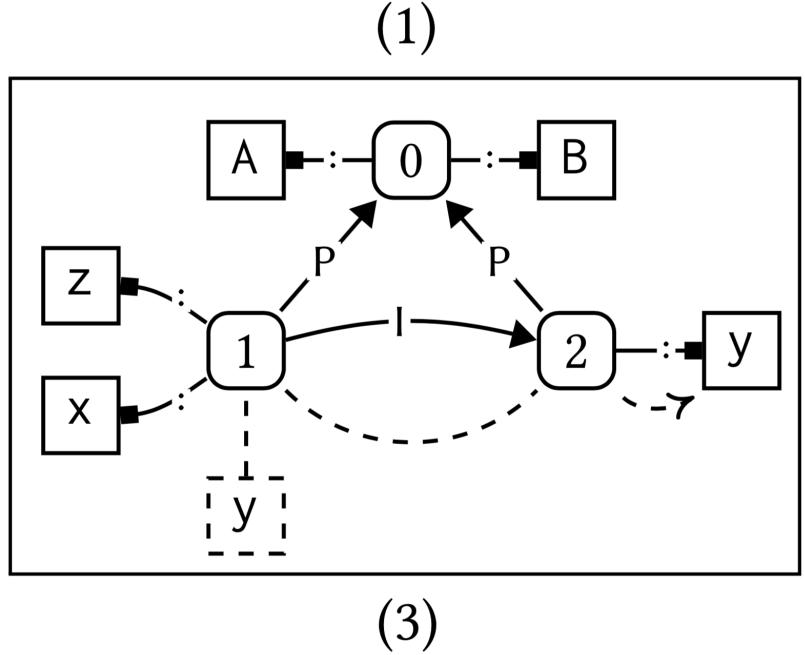
(1) (2)

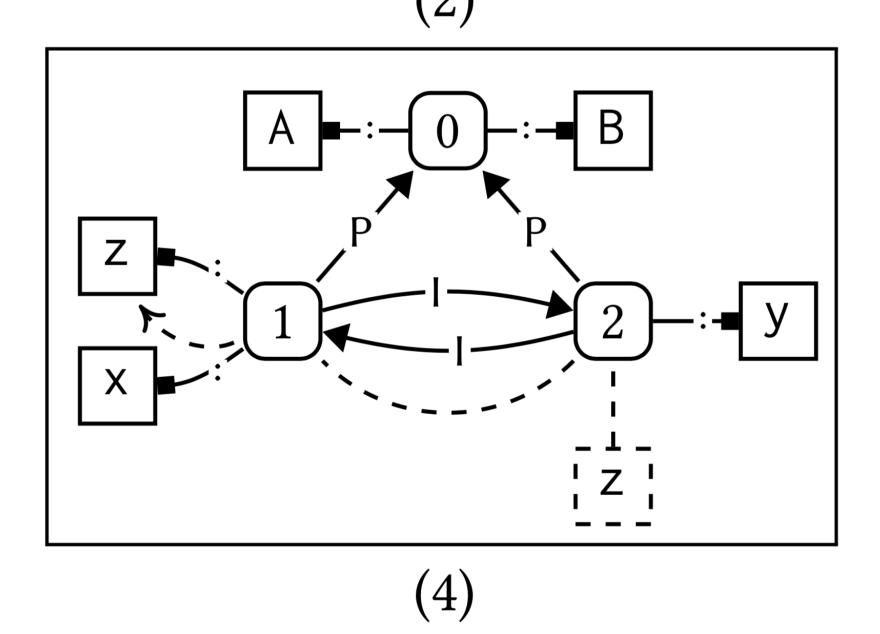
```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

A Two Stage Type Checker: Stage 2 (Check Modules)



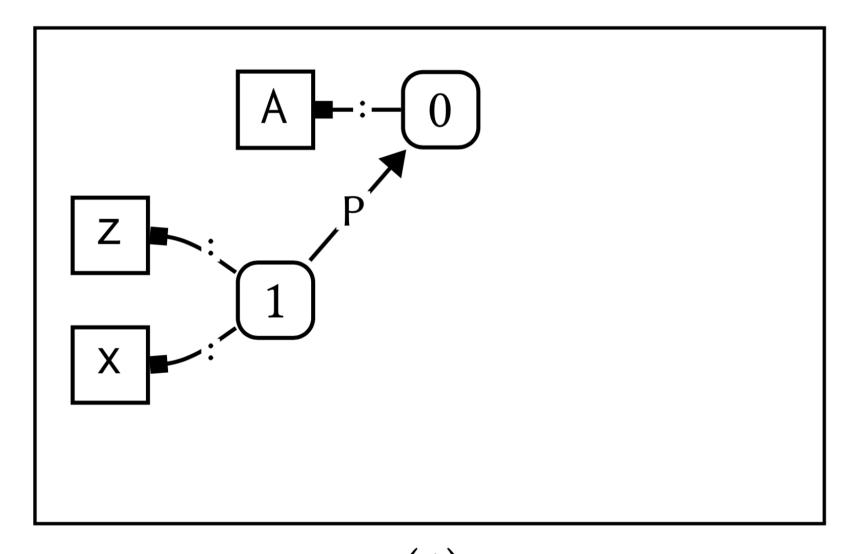


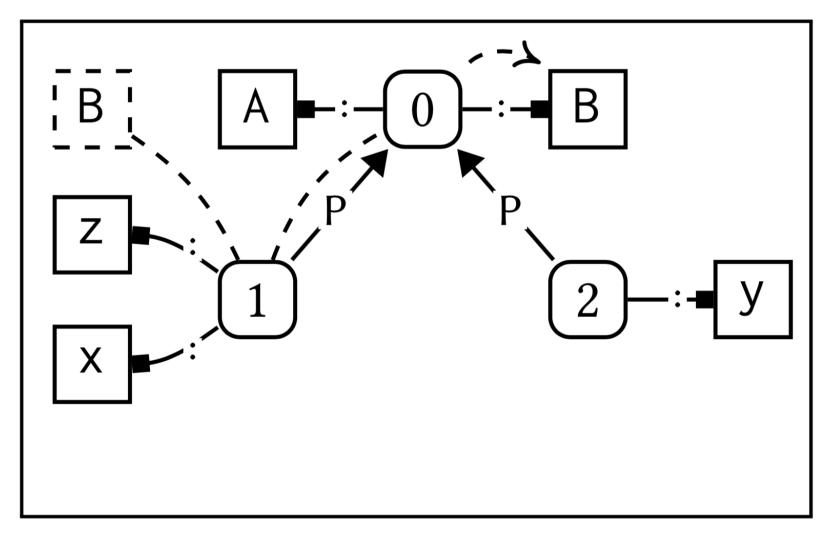




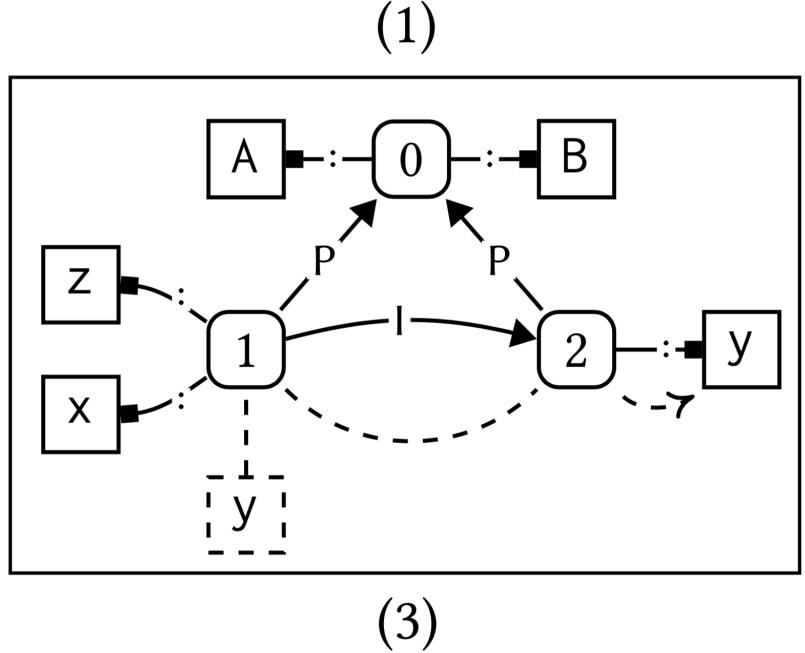
```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

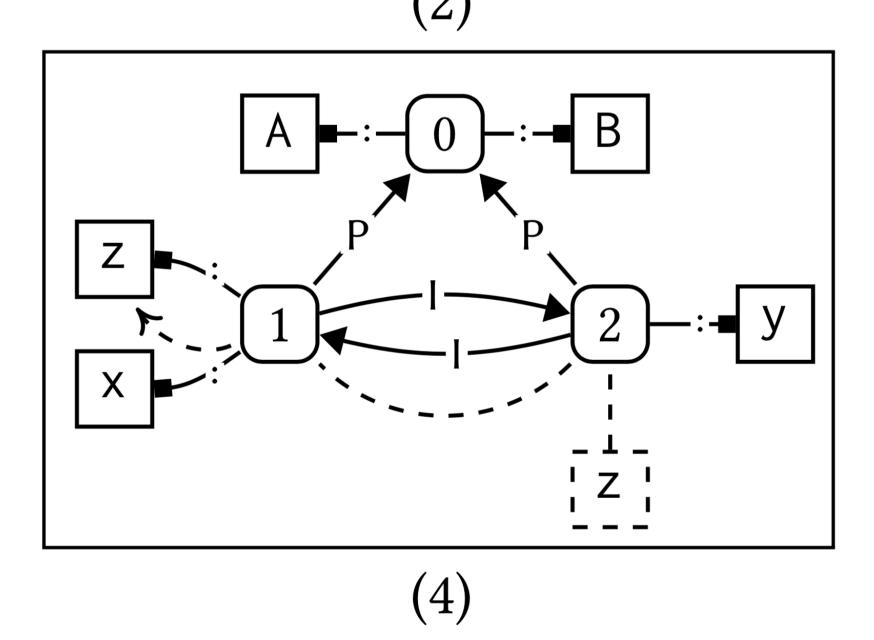
A Two Stage Type Checker: Stage 2 (Check Modules)





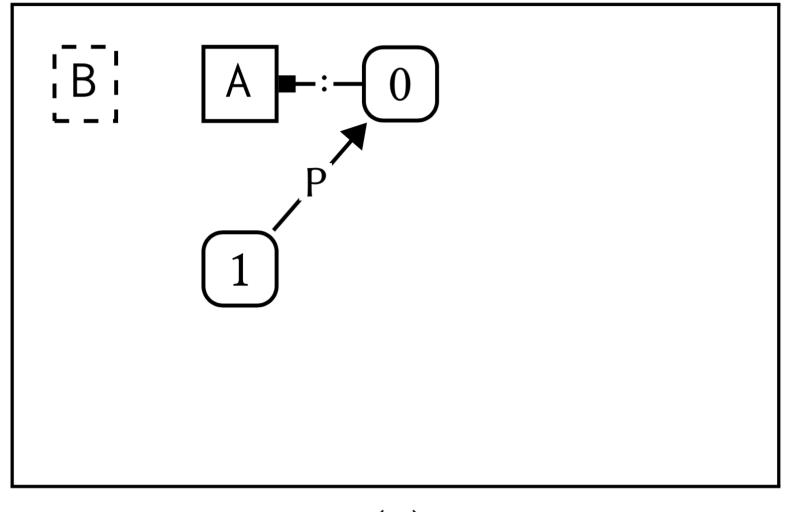
Requires that imports are resolved before variable references

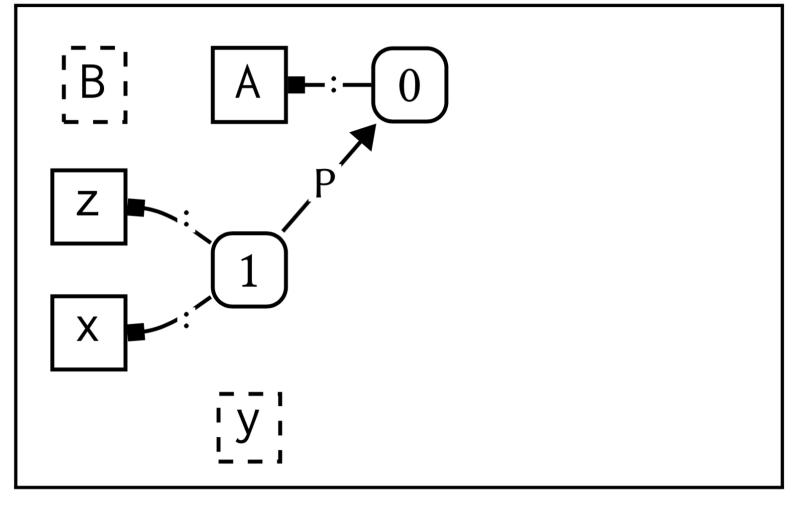




```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

Dynamic

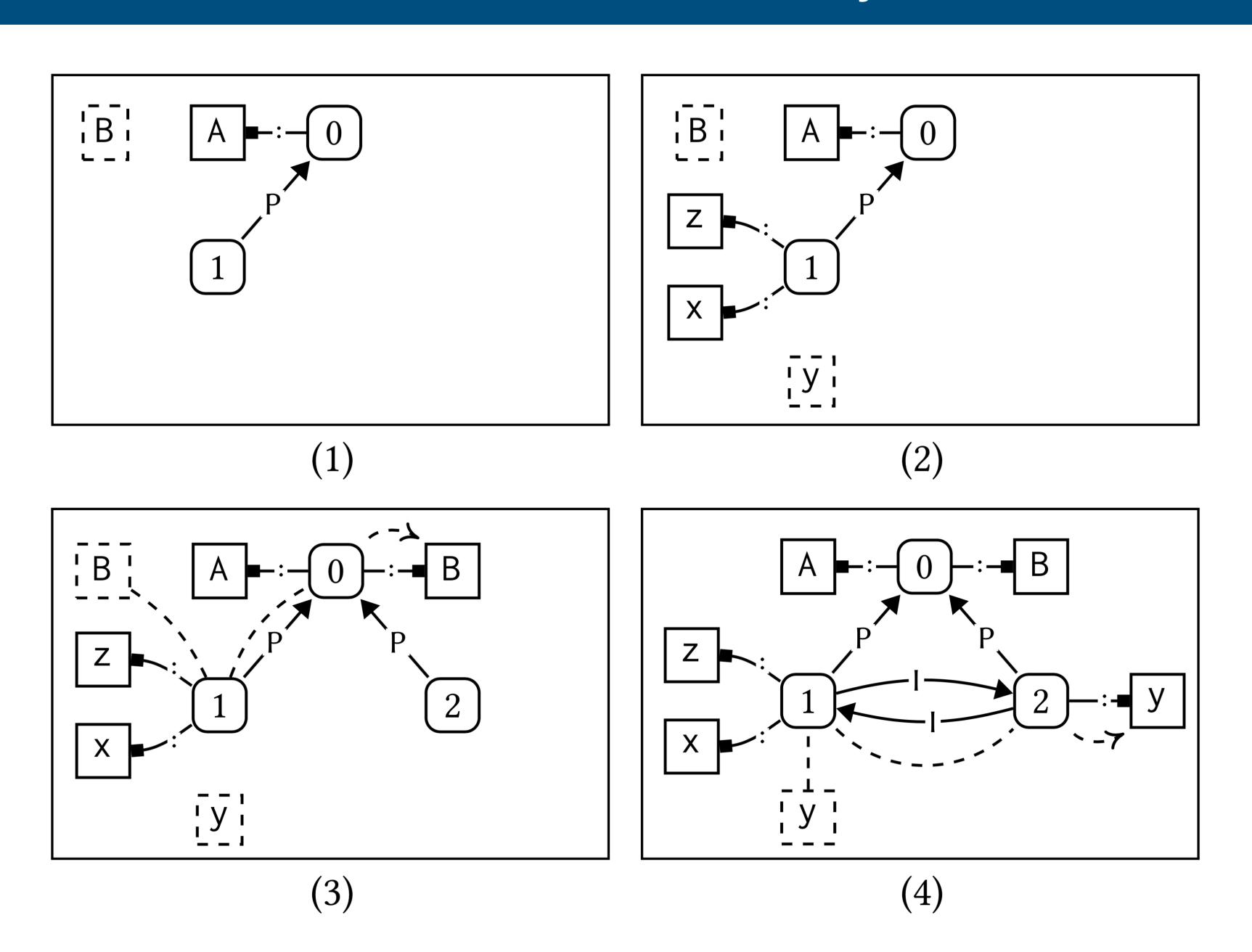




1) (2)

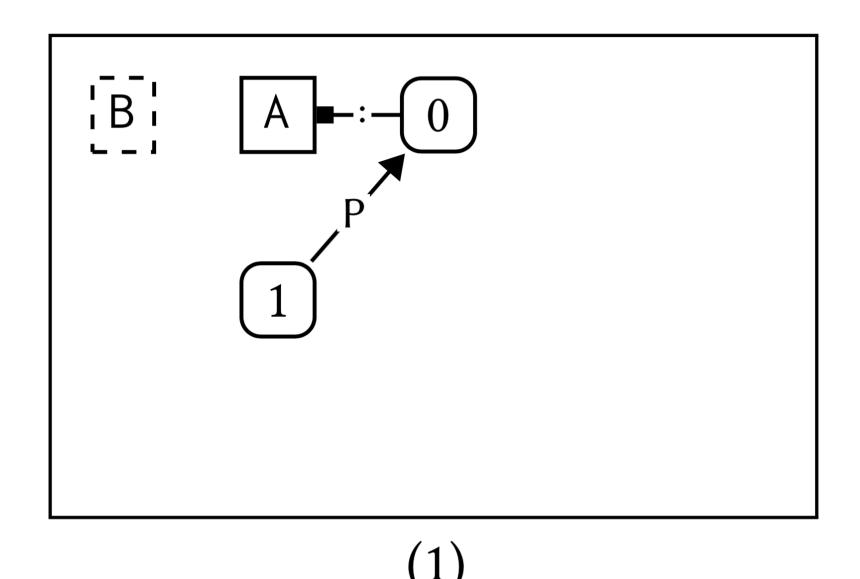
```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

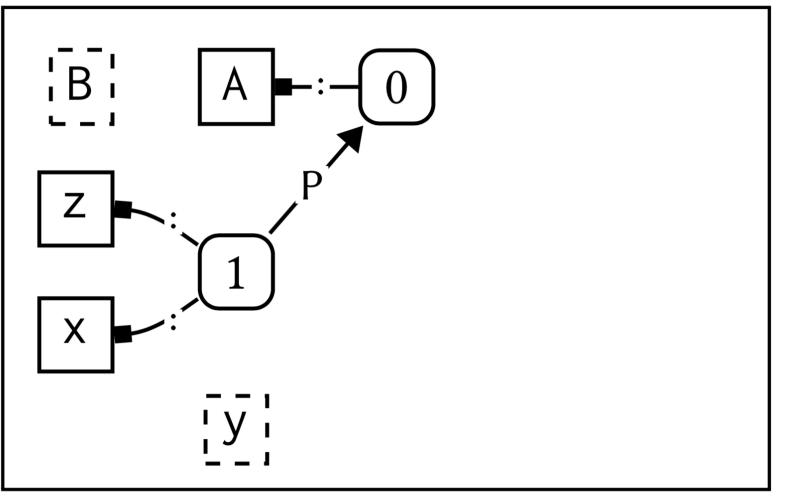
Dynamic



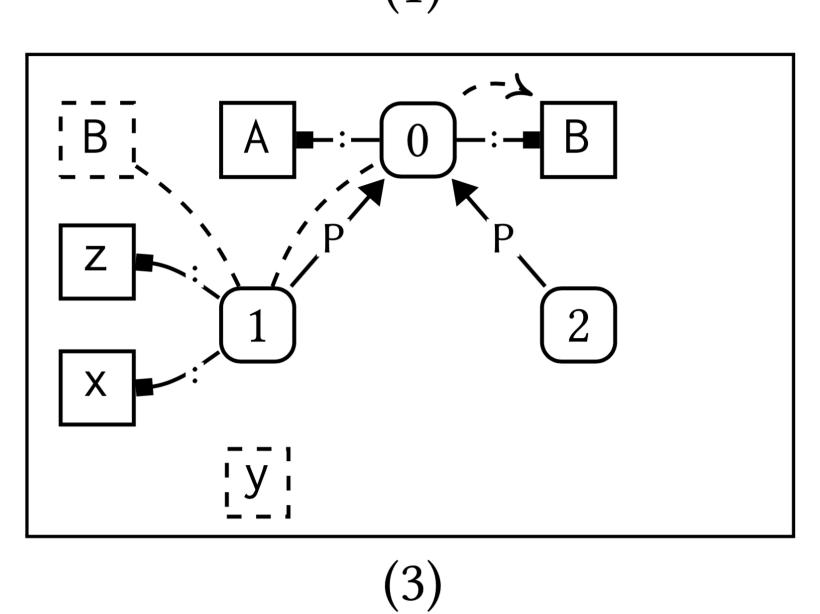
```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

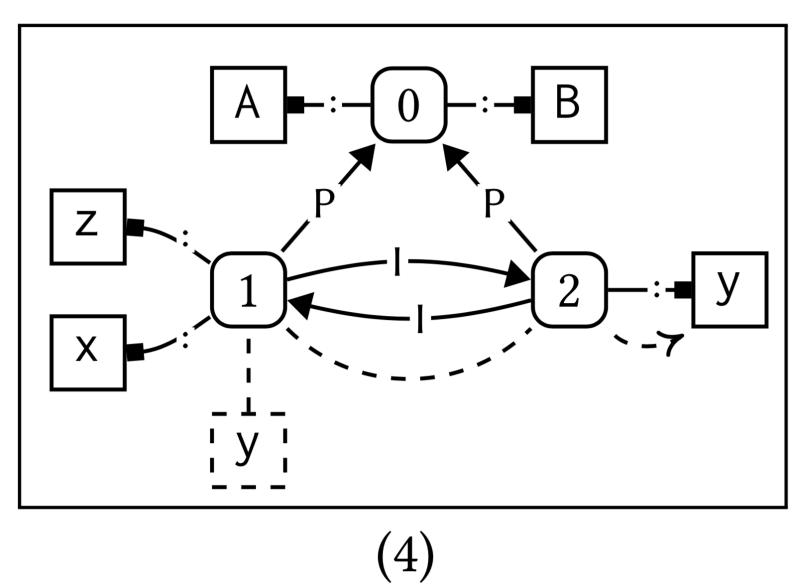
Dynamic





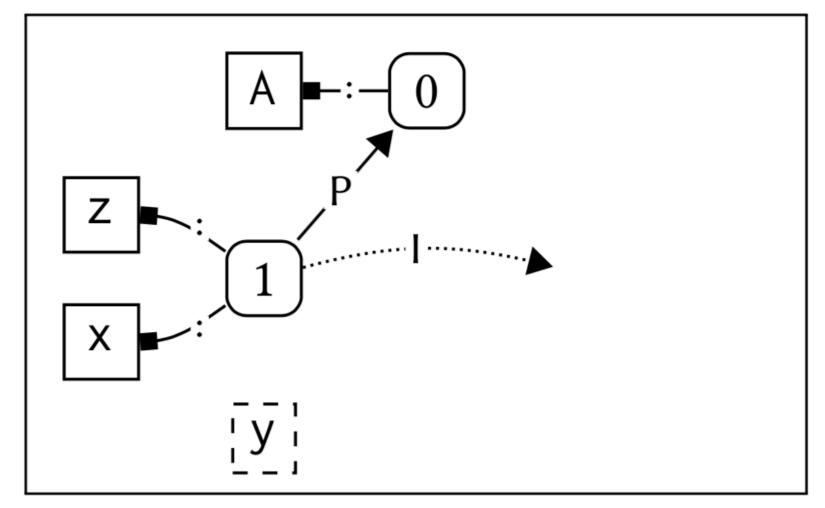
When do we have sufficient information to answer a query?



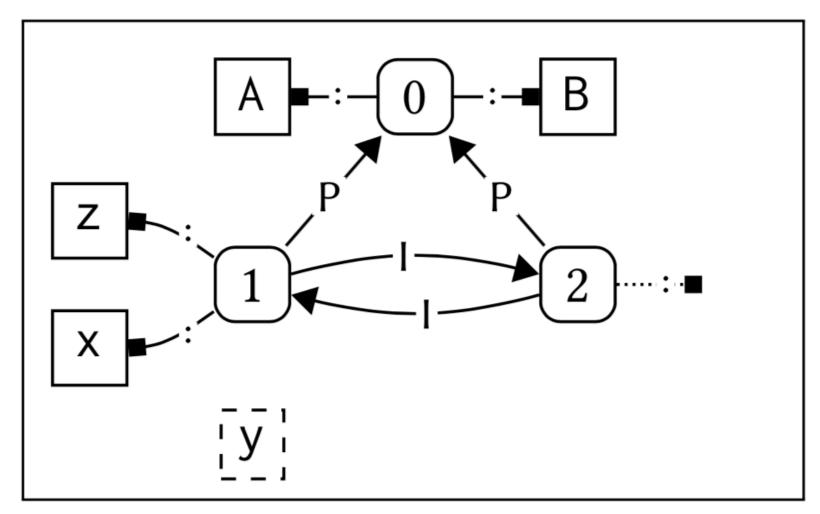


```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

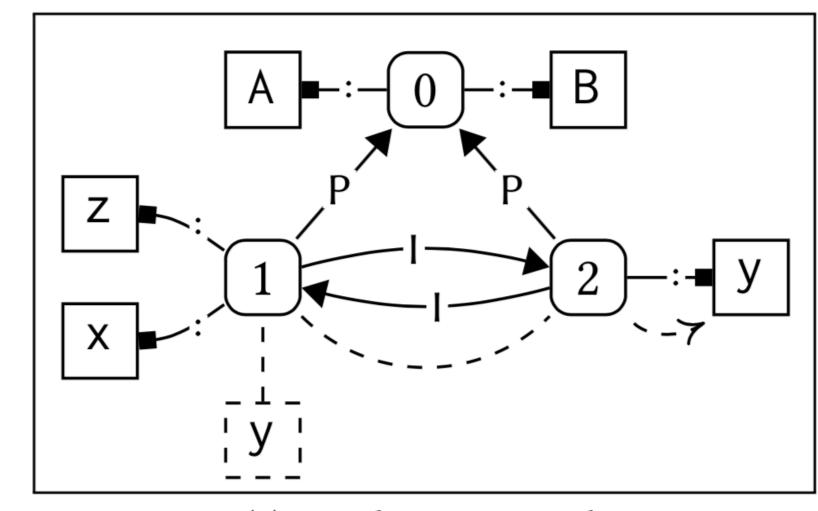
Critical Edges



(a) Intermediate scope graph



(b) Intermediate scope graph



(c) Final scope graph

```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

(Weakly) Critical Edges

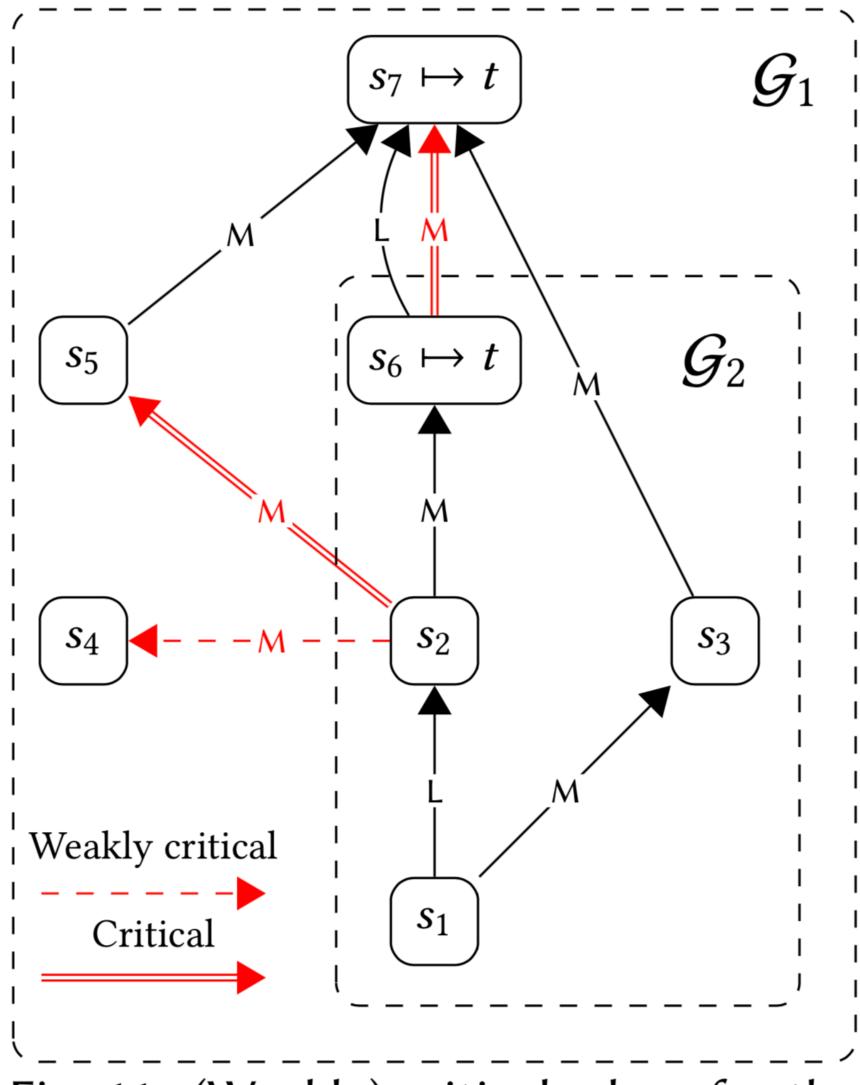


Fig. 11. (Weakly) critical edges for the query $s_1 \xrightarrow{LM*}_R D$, assuming $t \in D$

Automatically Scheduling Constraint Resolution

Scope graph represents context information

- Type checker constructs scope graph
- Type checker queries scope graph
- Scope graph construction depends on queries

When is it safe to query the scope graph?

- When there are no more critical edges for this query

Conclusion

Statix

Modeling Name Binding with Scope Graphs

- Scopes + declarations + edges (reachability)
- Queries to resolve references
- Visibility policies = path disambiguation
 - path well-formedness + path specificity
- Model wide range of name binding policies

Scheduling Constraint Resolution [OOPSLA'20]

- Declarative: no explicit scheduling / staging / stratification of traversal
- Only perform queries when outcome will not be changed (capture)
- Don't extend scopes 'remotely' (permission to extend)

Examples in this lecture: [ESOP'15] + [PEPM'16] in Statix

Generics

Scopes as Types [OOPSLA'18]

Applications

- Structural (sub)typing (records)
- Parametric polymorphism (System F)
- Nominal subtyping (FJ)
- Generic classes (FGJ)

Under investigation

- Make those encodings less clunky
- Hindley-Milner: inference supported, but how to generalize?

Ongoing Work

Incremental multi-file analysis

- Given a change, which files need to be reanalyzed?

Code completion [vision: ECOOP 2019]

- Given a hole, what can be filled in?
- Expressions, but also declarations, ...

Refactoring

- Renaming, inlining, ...

Other editor services

- Quick fixes, ...

Random term generation

- Generate program that is well-typed and well-bound