

Name Binding and Name Resolution

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CS4200 | Compiler Construction | September 24, 2020

Type Constraints

Predicates are Defined by Rules

Predicate

`typeOfExp : scope * Exp → TYPE`

Rule

```
typeOfExp(s, Add(e1, e2)) = INT() :-  
  typeOfExp(s, e1) = INT(),  
  typeOfExp(s, e2) = INT().
```

Head

Premises

For all s, e1, e2

If the premises are true, the head is true

Representing Name Binding with Scope Graphs

rules

```
typeOfExp(s, e@Add(e1, e2)) = INT() :-  
  typeOfExp(s, e1) = INT() | error $[integer expected]@e1,  
  typeOfExp(s, e2) = INT() | error $[integer expected]@e2,  
  @e.type := INT().
```

error message on constraint failure

set type attribute

\$ 9 * 10 + 3

Type: INT

x19 \$ 1 * true

integer expected

> INT() == BOOL()

> statics/base!typeOfExp(Scope("", "s_1-1"), True(), INT())

> statics/base!typeOfExp(Scope("", "s_1-1"), Mul(Int("1"), True()), INT())

> statics/base!declOk(Scope("", "s_1-1"), Exp(Mul(Int("1"), True())))

> statics/base!declsOk(Scope("", "s_1-1"), [Exp(Mul(Int(...), True()))])

> ... trace truncated ...

Type: BOOL

Constraints with Error Messages

```
constraint | error $[message [term]]@origin
```

Programs with Names

Programs with Names

```
module Names {  
  
  module Even {  
    import Odd  
    def even = fun(x) {  
      if x == 0 then true else odd(x - 1)  
    }  
  }  
  
  module Odd {  
    import Even  
    def odd = fun(x) {  
      if x == 0 then false else even(x - 1)  
    }  
  }  
  
  module Compute {  
    type Result = { input : Int, output : Bool }  
    def compute = fun(x) {  
      Result{ input = x, output = Odd@odd x }  
    }  
  }  
}
```

Name binding key in programming languages

Many name binding patterns

Deal with erroneous programs

Name resolution complicates type checkers, compilers

Ad hoc non-declarative treatment

A systematic, uniform approach to name resolution?

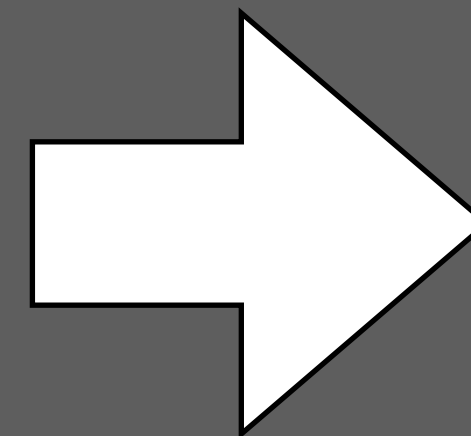
Formalizing Name Binding in Type Systems

$$\frac{O(id) = T, \text{ where } T \text{ is not a function type.}}{O, M, C, R \vdash id : T} \quad [\text{VAR-READ}]$$

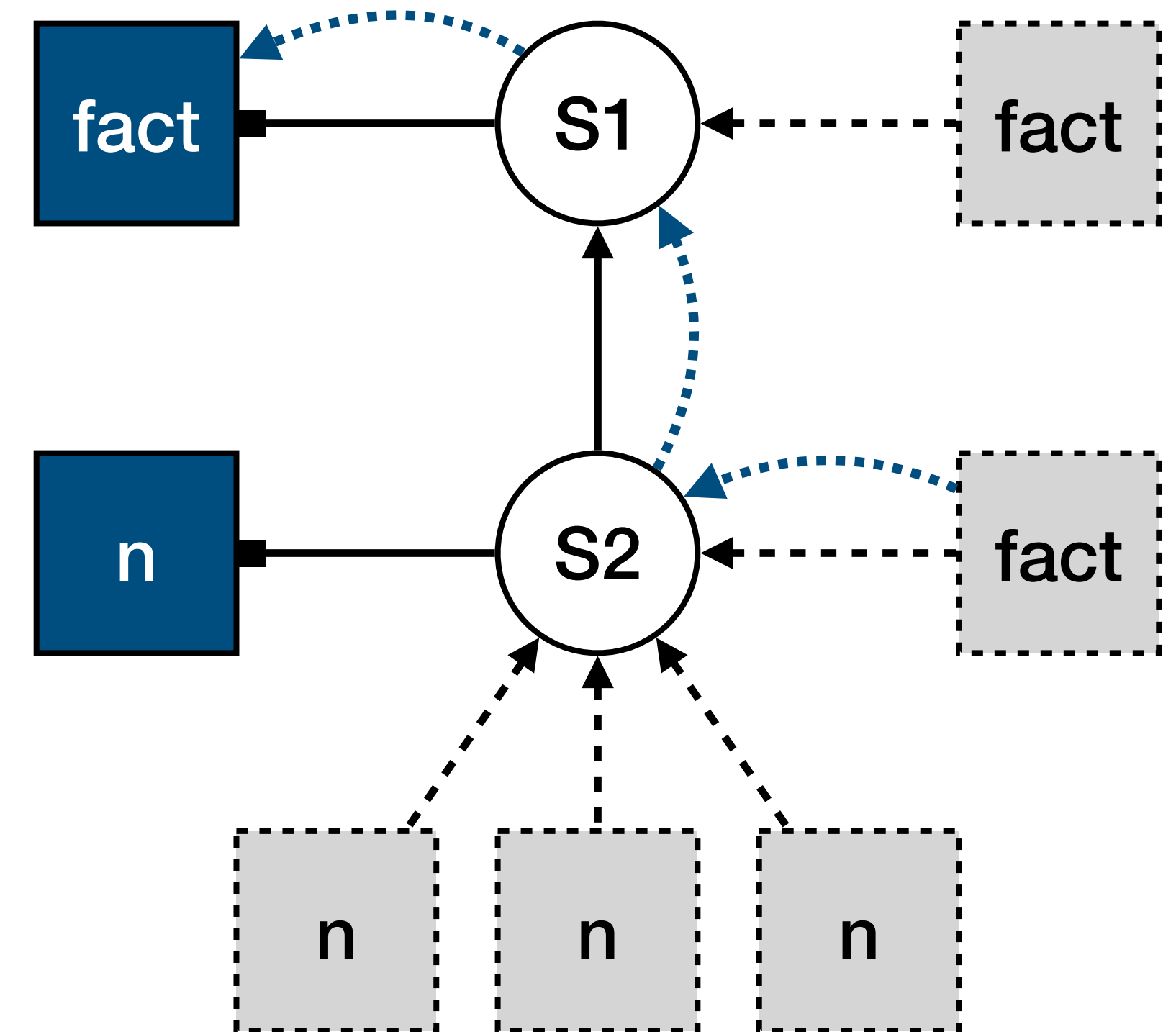
Name Resolution with Scope Graphs

Program

```
let function fact(n : int) : int =  
  if n < 1 then  
    1  
  else  
    n * fact(n - 1)  
  in  
    fact(10)  
end
```



Scope Graph



Name Resolution

Name Resolution with Scope Graphs in Statix

Declarations and References

Lexical Scope

Modules

Records

Permission to Extend

Scheduling Resolution

Declaring and Resolving Names

Declarations and References

signature

constructors

```
Var    : ID → Exp
Bind   : ID * Exp → Bind
BindT  : ID * Type * Exp → Bind
Def    : Bind → Decl
```

rules

```
declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

use before definition

Declarations and References

signature

constructors

```
Var    : ID → Exp
Bind   : ID * Exp → Bind
BindT  : ID * Type * Exp → Bind
Def    : Bind → Decl
```

rules

```
declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

rules

```
typeOfExp(s, Var(x)) = T :-
  typeOfVar(s, x) = T.
```

```
declOk(s, Def(bind)) :-
  bindOk(s, s, bind).
```

```
bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
  typeOfType(s_ctx, t) = T1,
  declareVar(s_bnd, x, T1),
  typeOfExp(s_ctx, e) = T2,
  subtype(T2, T1).
```

```
bindOk(s_bnd, s_ctx, Bind(x, e)) :- {T}
  typeOfExp(s_ctx, e) = T,
  declareVar(s_bnd, x, T).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

use before definition

Representing Name Binding with Scope Graphs

rules

declareVar : scope * ID * TYPE

typeOfVar : scope * ID \rightarrow TYPE

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

Scope Graphs: Declarations

signature

relations

var : ID \rightarrow TYPE

rules

declareVar : **scope** * ID * TYPE

typeOfVar : **scope** * ID \rightarrow TYPE

declareVar(*s*, *x*, *T*) :-
!var[*x*, *T*] **in** *s*.

declaration relation

variable *x* is declared in scope *s*
with type *T*

def a = 0
def b = a + 1
def c = a + b
> a + b + c

declaration and reference

Scope Graphs: Declarations

signature

relations

var : ID \rightarrow TYPE

rules

declareVar : scope * ID * TYPE

typeOfVar : scope * ID \rightarrow TYPE

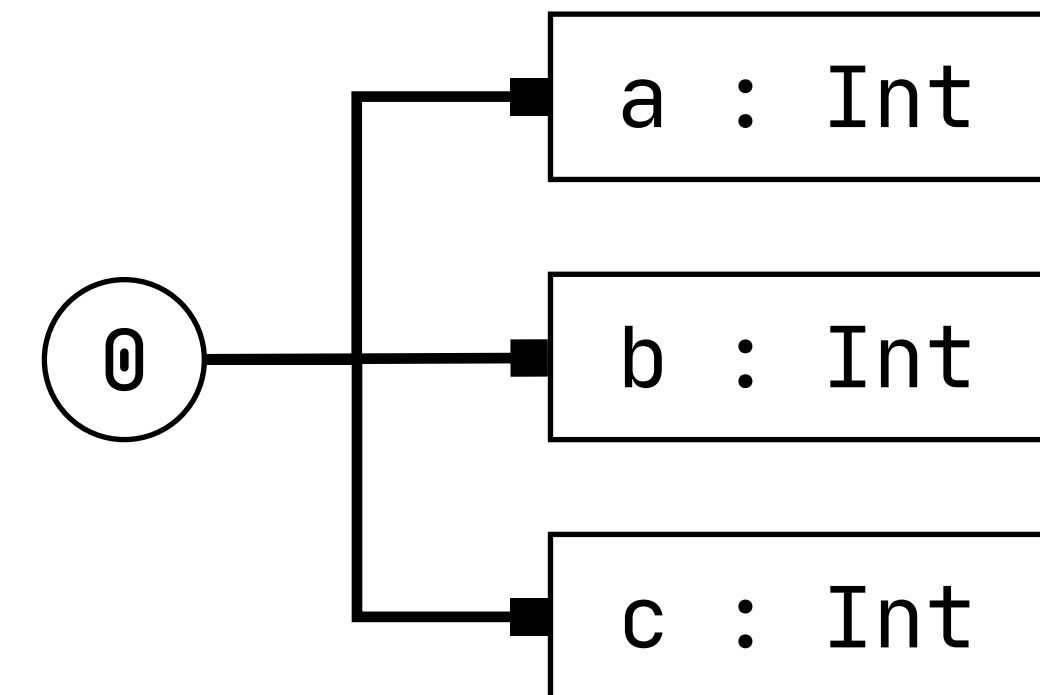
declareVar(*s*, *x*, *T*) :-
!var[*x*, *T*] in *s*.

declaration relation

variable *x* is declared in scope *s*
with type *T*

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference



Scope Graphs: Name Resolution Queries

signature
relations
var : ID \rightarrow TYPE
rules

declareVar : scope * ID * TYPE
resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))
typeOfVar : scope * ID \rightarrow TYPE

declareVar(s, x, T) :-
!var[x, T] in s.

resolveVar(s, x) = ps :-
query var
filter e and { x' :- x' = x }
min and true
in s \mapsto ps.

typeOfVar(s, x) = T :- {x'}
resolveVar(s, x) = [(_, (x', T))].

declaration relation

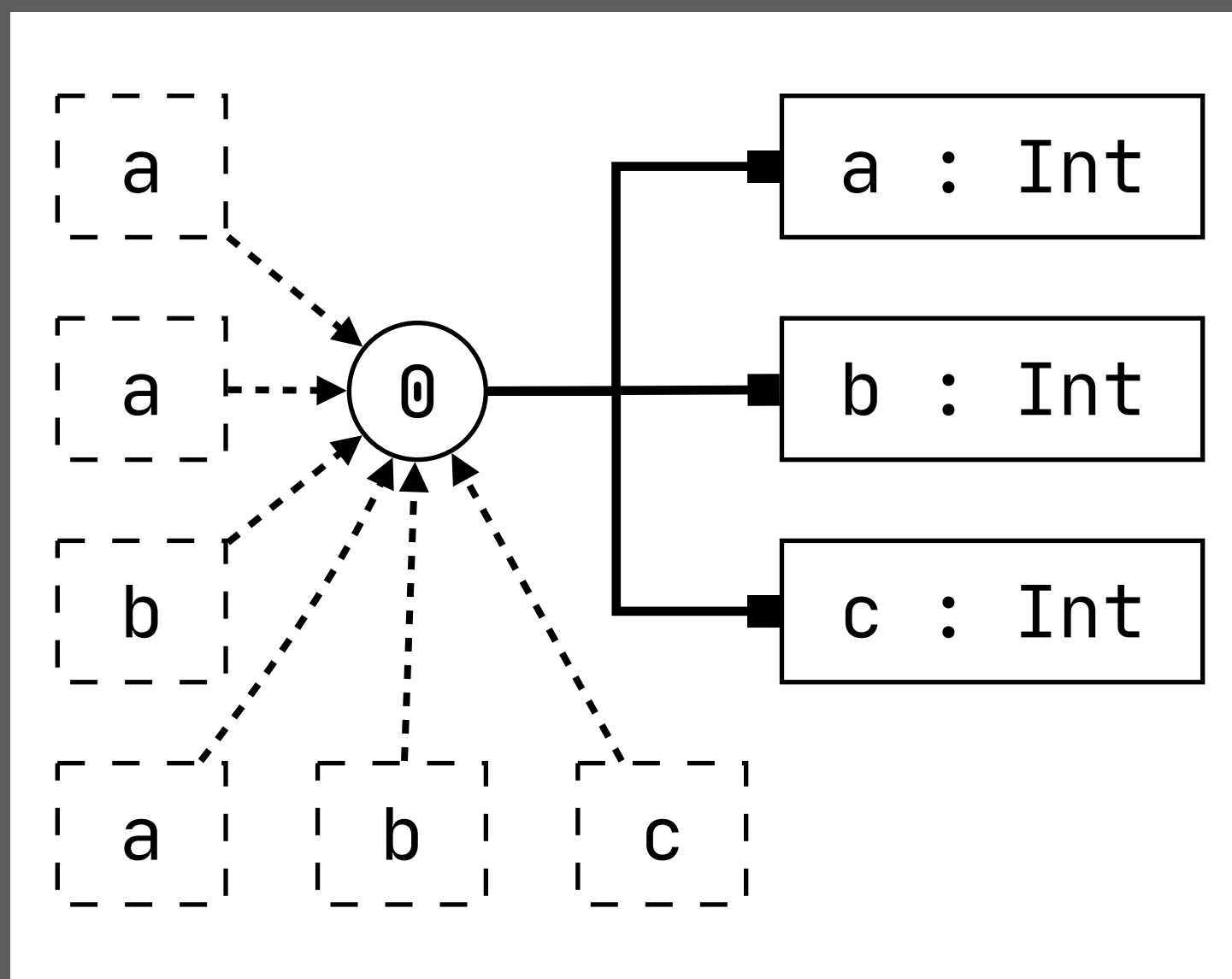
variable x is declared in scope s
with type T

variable x in scope s resolves to list
of declarations

variable x in scope s resolves to
declaration x' with type T

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference



Undefined Variable

signature
relations
var : ID → TYPE
rules

declareVar : scope * ID * TYPE
resolveVar : scope * ID → list((path * (ID * TYPE)))
typeOfVar : scope * ID → TYPE

declareVar(s, x, T) :-
!var[x, T] in s.

resolveVar(s, x) = ps :-
query var
filter e and { x' :- x' = x }
min and true
in s → ps.

typeOfVar(s, x) = T :- {x'}
resolveVar(s, x) = [(_, (x', T))].

declaration relation

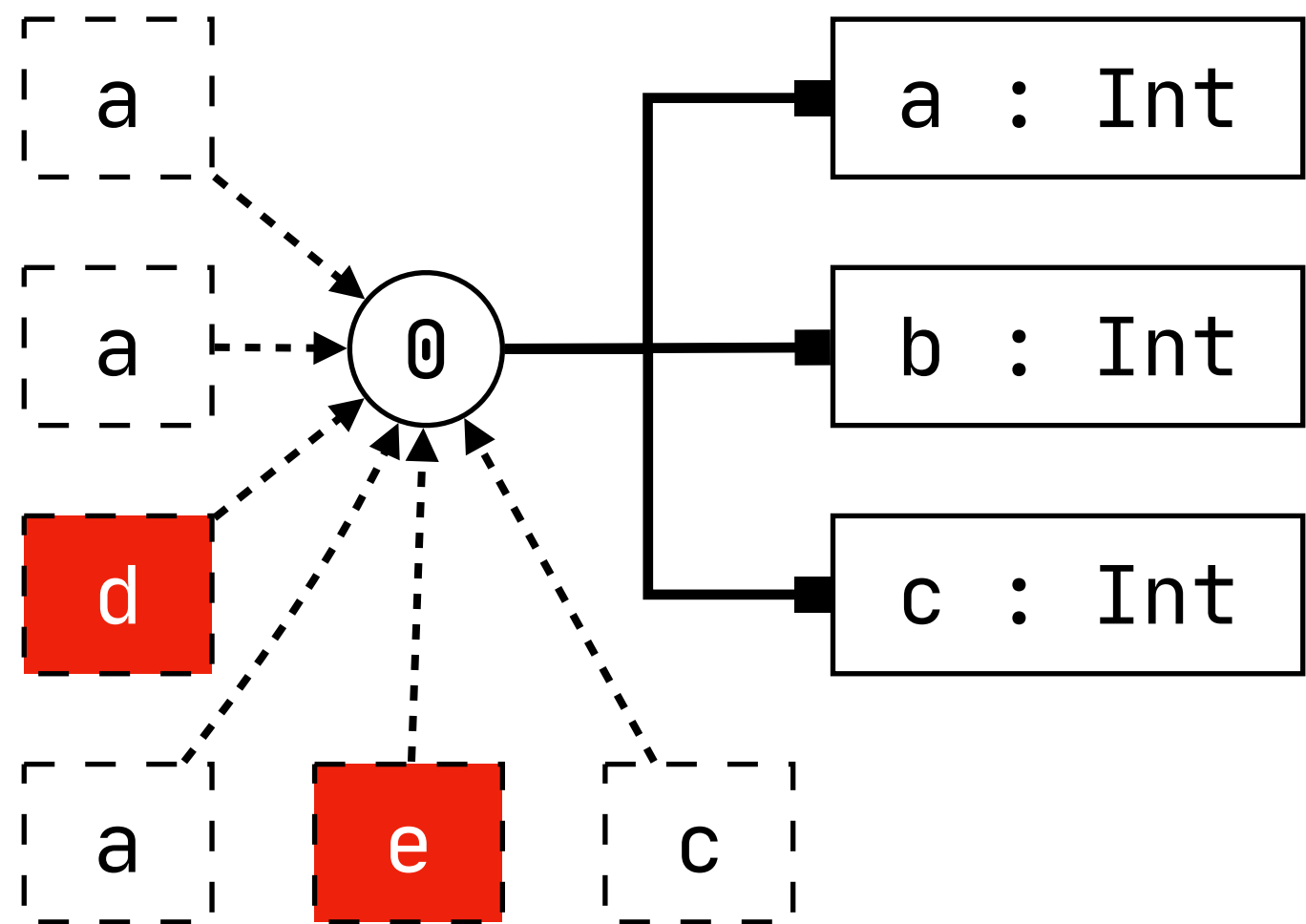
variable x is declared in scope s
with type T

variable x in scope s resolves to list
of declarations

variable x in scope s resolves to
declaration x' with type T

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable



resolveVar returns empty list of declarations

Duplicate Definition

signature

relations

var : ID → TYPE

rules

declareVar : scope * ID * TYPE

resolveVar : scope * ID → list((path * (ID * TYPE)))

typeOfVar : scope * ID → TYPE

declareVar(s, x, T) :-

!var[x, T] in s.

resolveVar(s, x) = ps :-

query var

filter e and { x' :- x' = x }

min and true

in s → ps.

typeOfVar(s, x) = T :- {x'}

resolveVar(s, x) = [(_, (x', T))].

declaration relation

variable x is declared in scope s
with type T

variable x in scope s resolves to list
of declarations

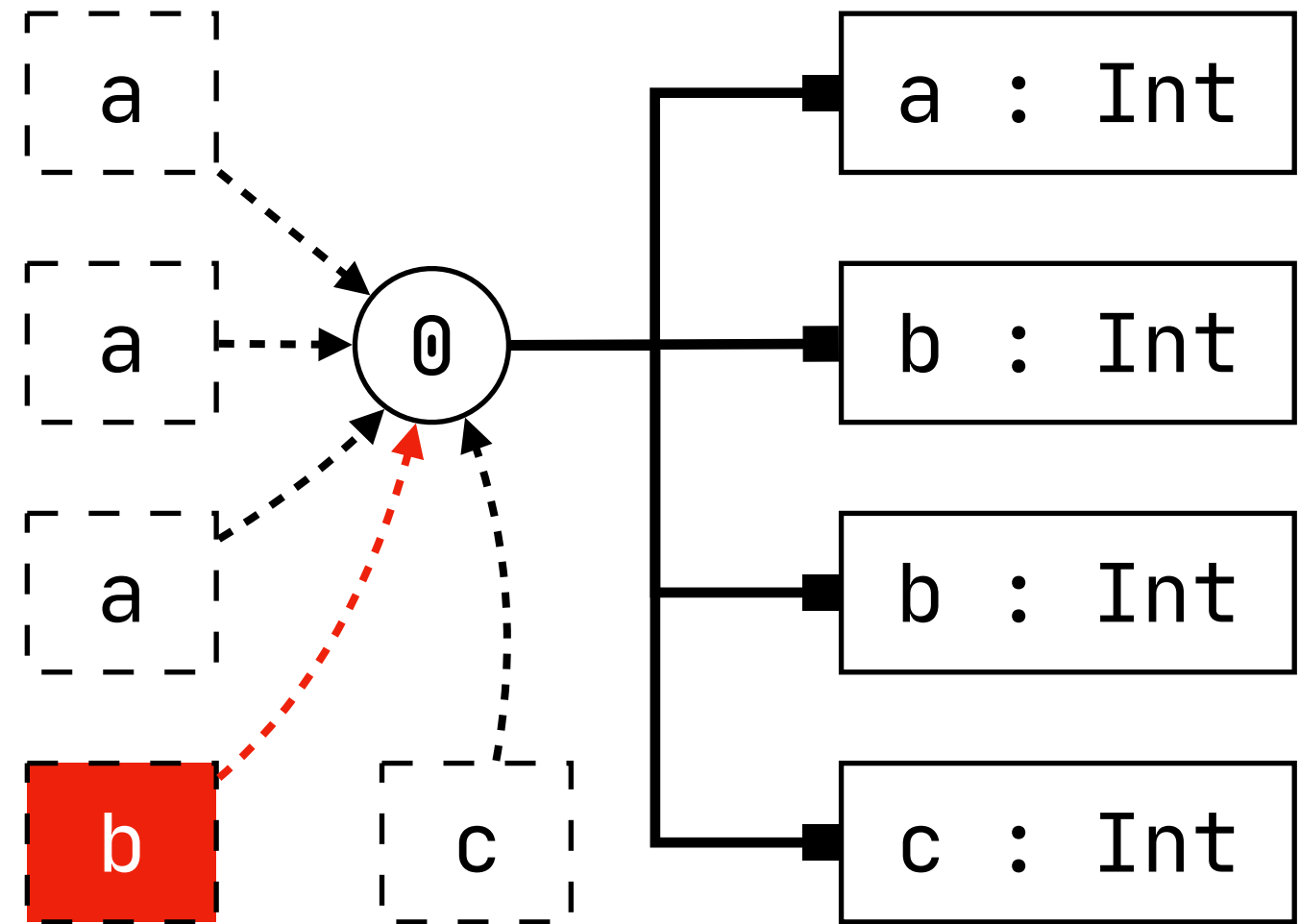
variable x in scope s resolves to
declaration x' with type T

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

what we want

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

what we get



Duplicate Definition: Permissive Resolution

signature

relations

var : ID \rightarrow TYPE

rules

declareVar : scope * ID * TYPE

resolveVar : scope * ID \rightarrow list((path * (ID * TYPE)))

typeOfVar : scope * ID \rightarrow TYPE

declareVar(s, x, T) :-

!var[x, T] in s,

resolveVar(s, x) = [(_, (_, _))]

| error \$[Duplicate definition of variable [x]].

resolveVar(s, x) = ps :-

query var

filter e and { x' :- x' = x }

min and true

in s \mapsto ps.

typeOfVar(s, x) = T :- {x'}

resolveVar(s, x) = [(_, (x', T))|_]

| error \$[Variable [x] not defined].

declaration relation

variable x is declared in scope s
with type T

there should only be one declaration

variable x in scope s resolves to list
of declarations

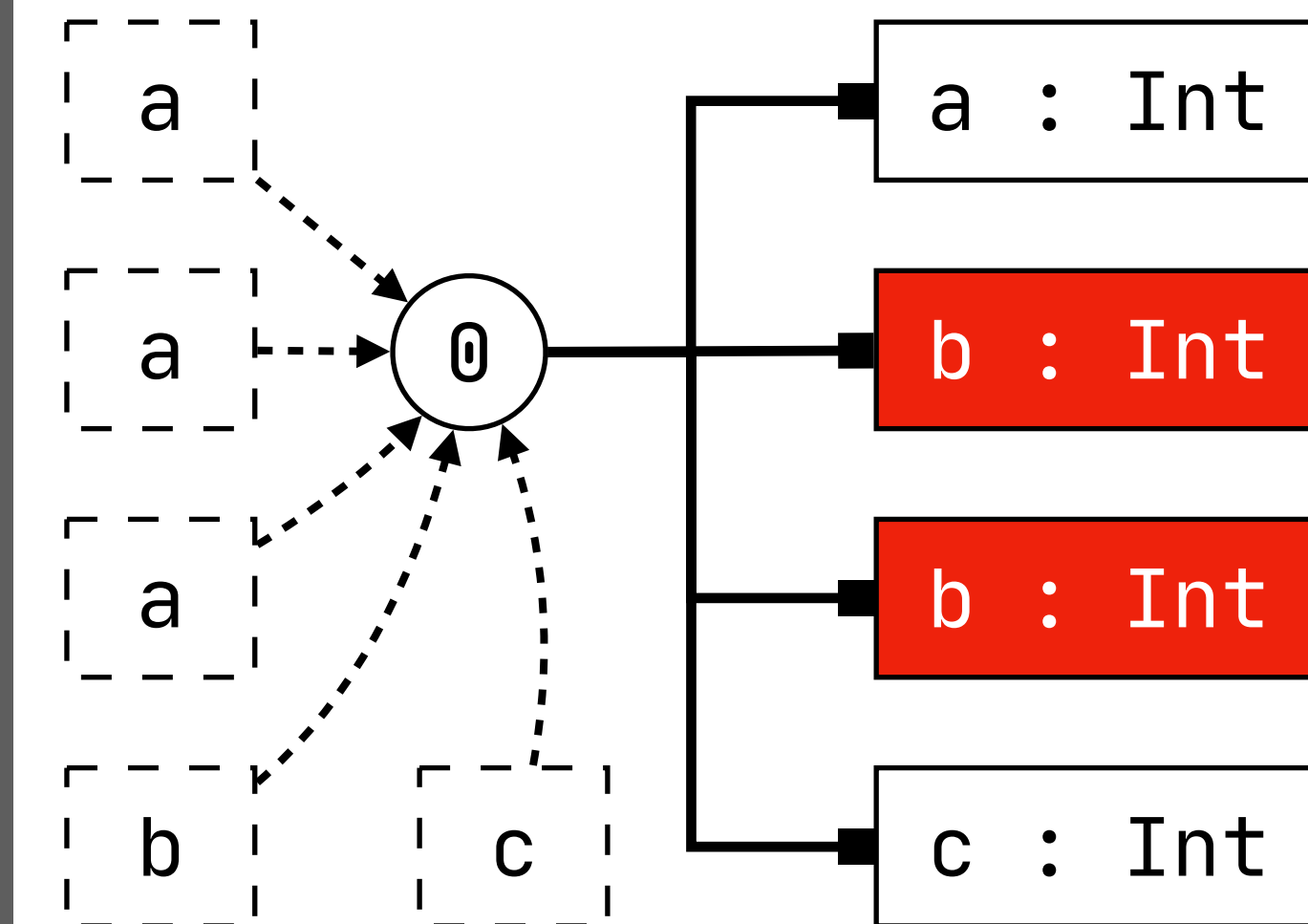
variable x in scope s resolves to
declaration x' with type T

there should be at least one
matching declaration

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```



Reference and Type Attributes

signature
relations
var : ID → TYPE
rules

declareVar : scope * ID * TYPE
resolveVar : scope * ID → list((path * (ID * TYPE)))
typeOfVar : scope * ID → TYPE

declareVar(s, x, T) :-
!var[x, T] in s,
resolveVar(s, x) = [(_, (_, _))]
| error \$[Duplicate definition of variable [x]],
@x.type := T.

resolveVar(s, x) = ps :-
query var
filter e and { x' :- x' = x }
min and true
in s → ps.

typeOfVar(s, x) = T :- {x'}
resolveVar(s, x) = [(_, (x', T))|_]
| error \$[Variable [x] not defined],
@x.ref := x'.

declaration relation

variable x is declared in scope s
with type T

there should only be one declaration

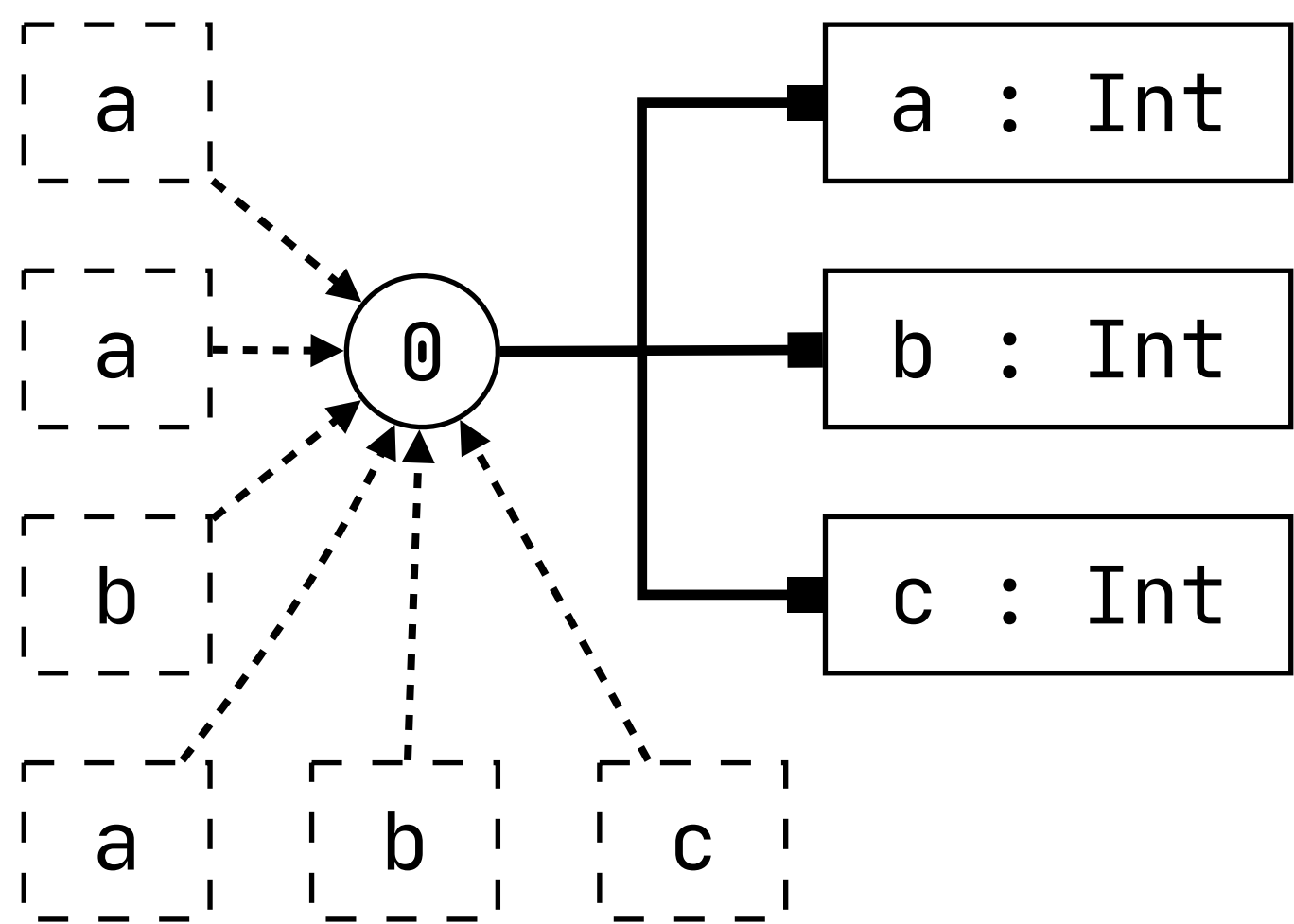
variable x in scope s resolves to list
of declarations

variable x in scope s resolves to
declaration x' with type T

there should be at least one
matching declaration

def a = 0
def b = a + 1
def c = a + b
> a + b + c

declaration and reference



Type Annotation Indirection (for Parallel Type Checking)

signature

relations

`var` : ID \rightarrow **scope**

`typeOf` : \rightarrow TYPE

rules

`declareVar(s, x, T) :-`
 `!var[x, withType(T)] in s.`

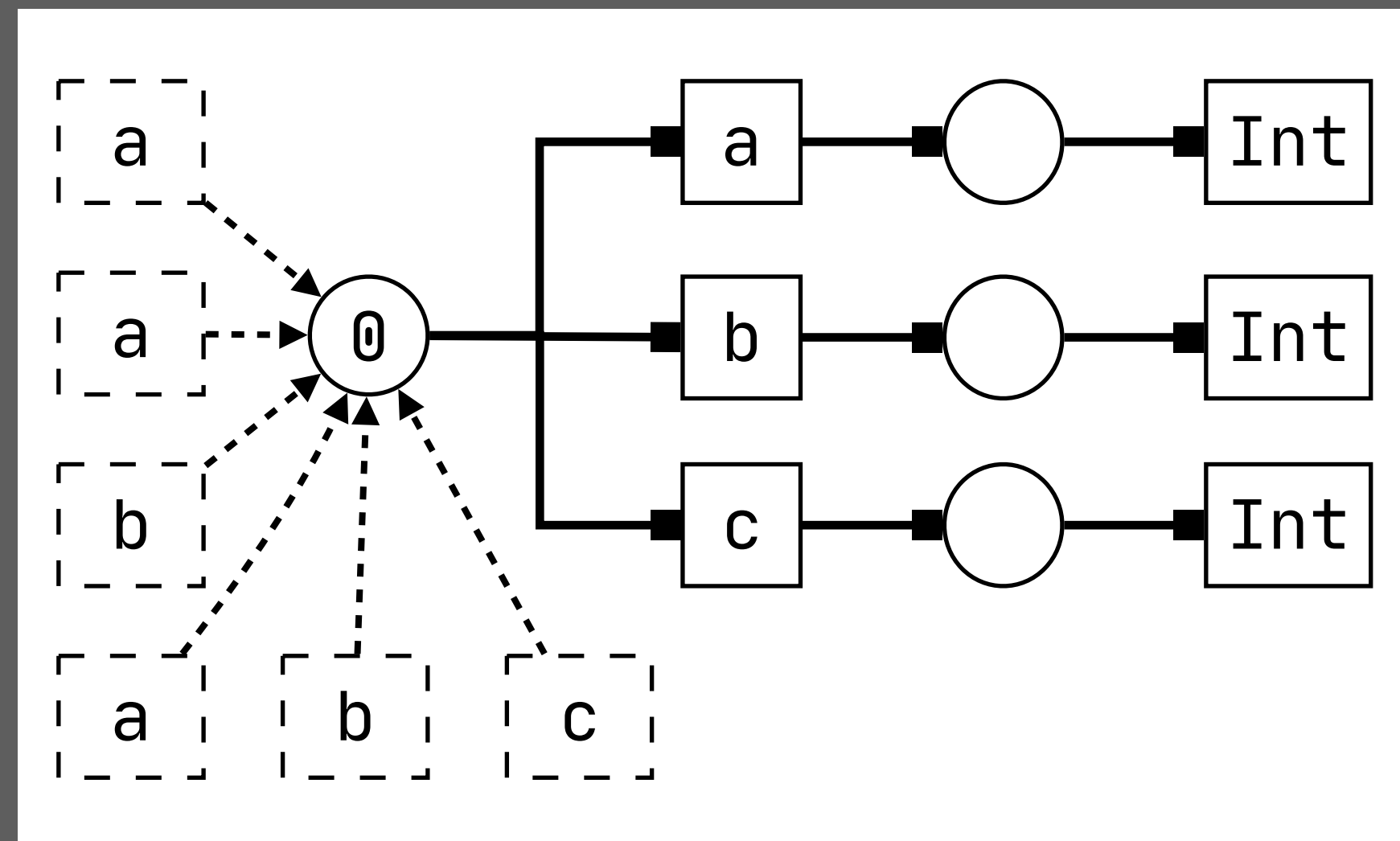
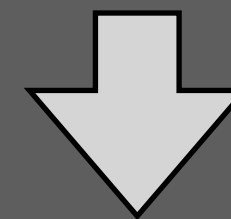
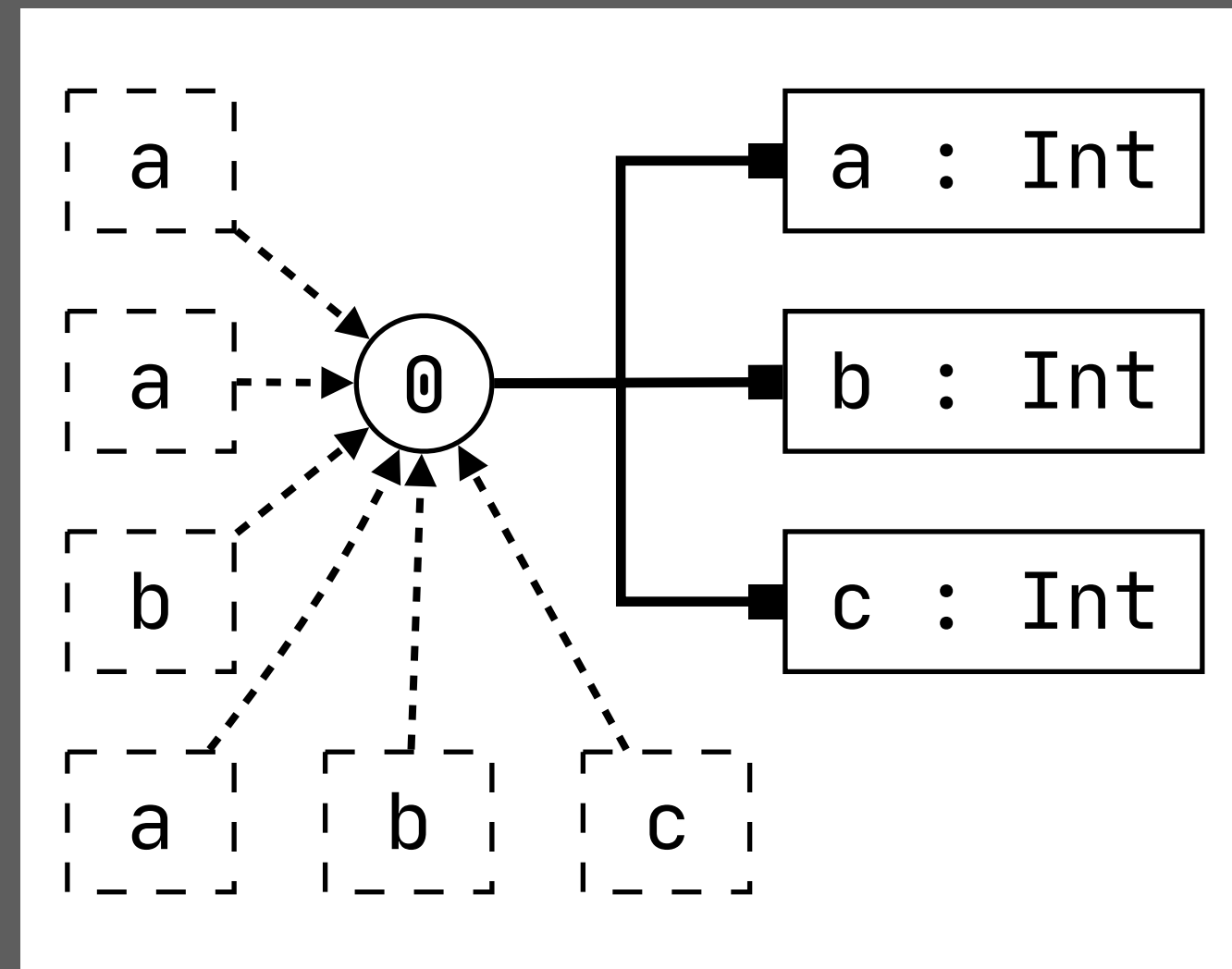
`typeOfVar(s, x) = typeOf(T) :- {x'}`
 `resolveVar(s, x) = [(_, (x', T))].`

`typeOf` : **scope** \rightarrow TYPE

`typeOf(s) = T :-`
 query `typeOf`
 filter `e` and `true`
 min /* */ and `true`
 in `s` \mapsto [(_, T)].

`withType` : TYPE \rightarrow **scope**

`withType(T) = s :-`
 new `s`, `!typeOf[T]` **in** `s`.



How about shadowing?

Lexical Scope

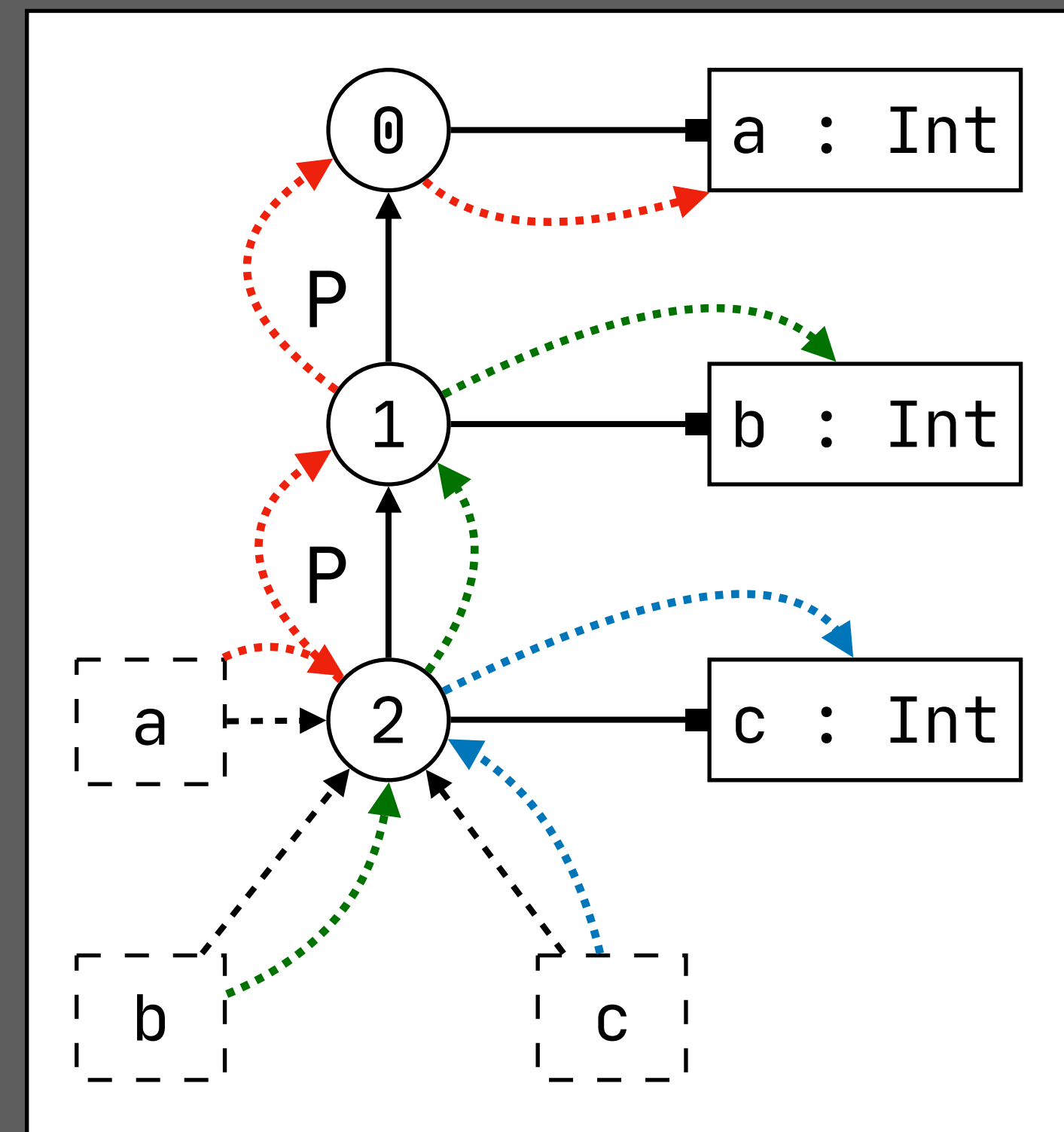
Labeled Scope Edges: What Scopes are Reachable?

signature

constructors

Let : ID * Exp * Exp \rightarrow Exp

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



New Scope and Scope Edge Constraints

signature

constructors

Let : ID * Exp * Exp \rightarrow Exp

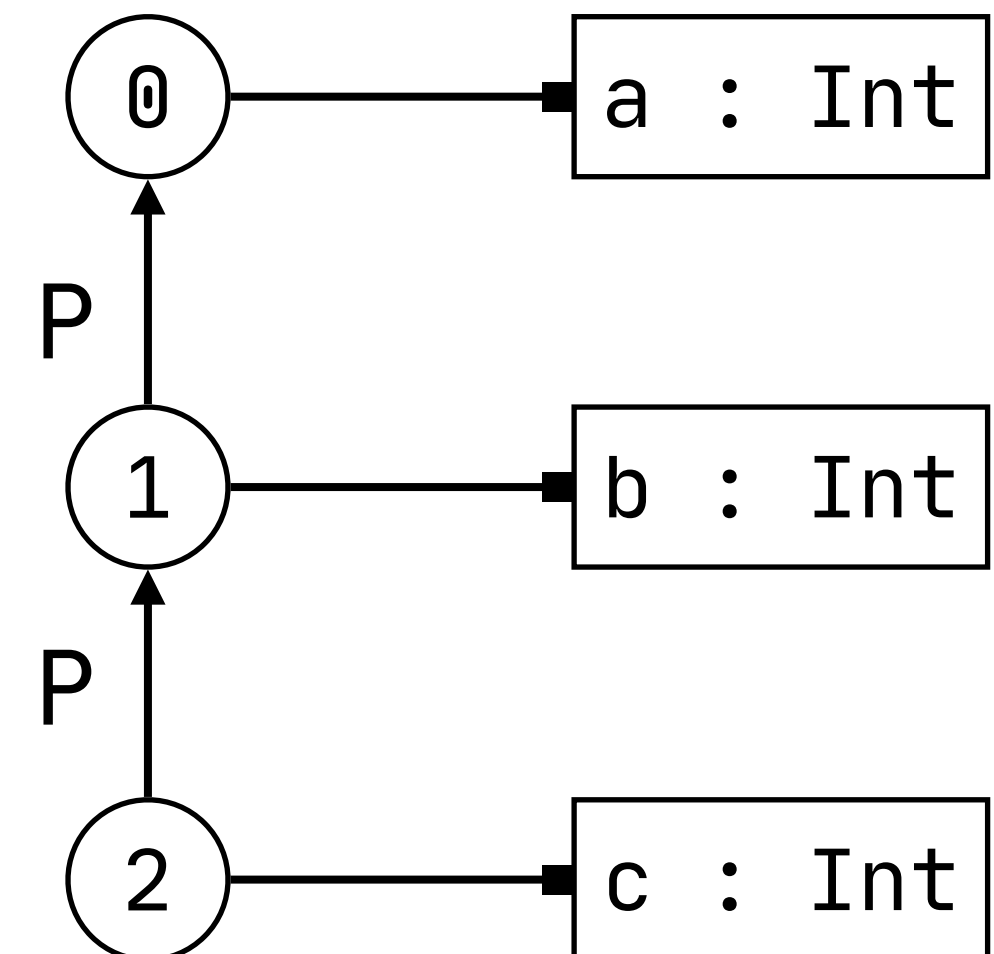
rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}  
  typeOfExp(s, e1) = S,  
  new s_let, s_let -P $\rightarrow$  s,  
  declareVar(s_let, x, S),  
  typeOfExp(s_let, e2) = T.
```

new scope

scope edge

```
let a = 1 in  
let b = 2 in  
let c = 3 in  
  a + b + c
```



Path Wellformedness: Reachability

signature

constructors

Let : ID * Exp * Exp \rightarrow Exp

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P $\rightarrow$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

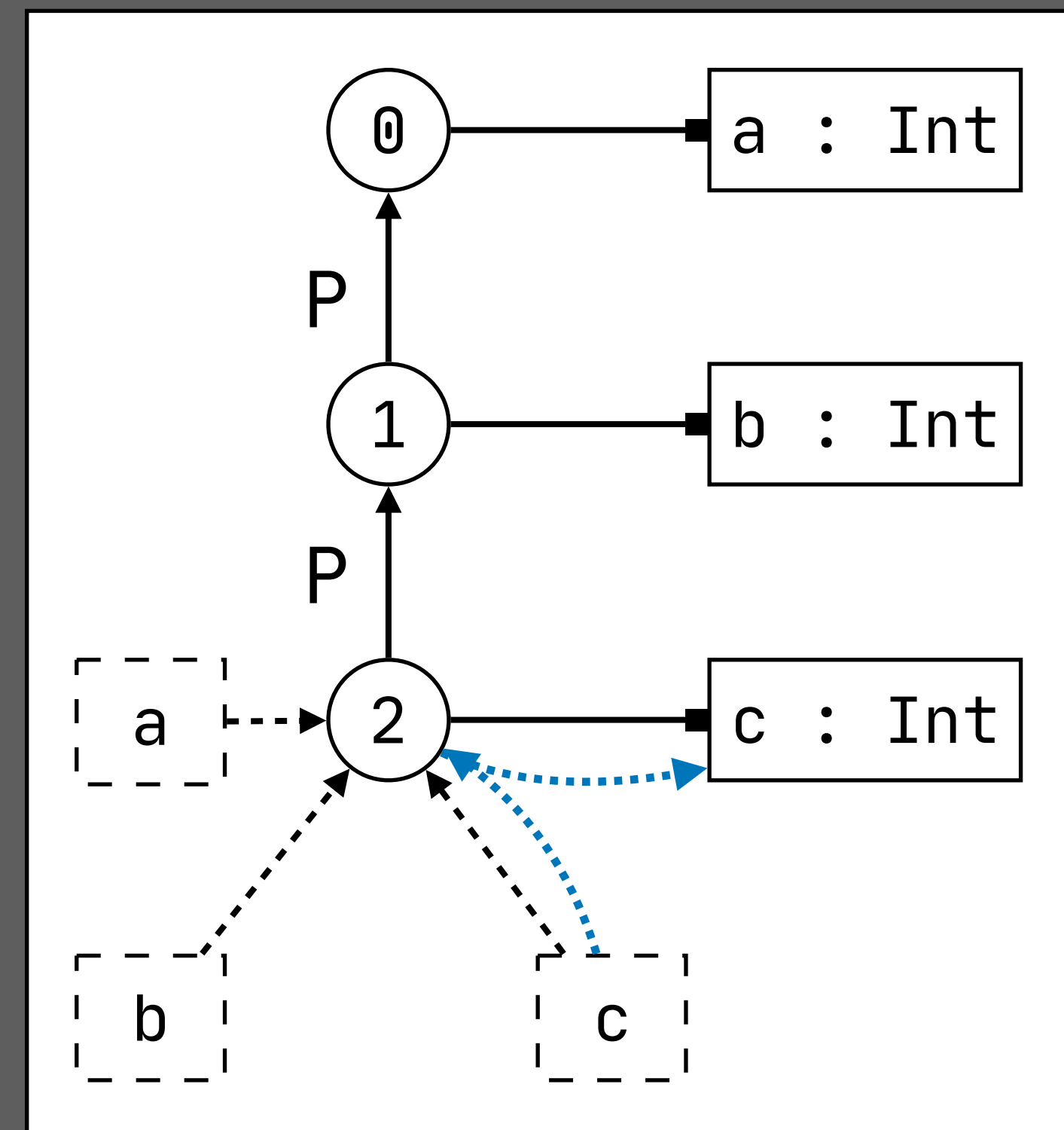
scope edge

```
resolveVar(s, x) = ps :-
  query var
  filter e and { x' :- x' = x }
  min /* */ and true
  in s  $\mapsto$  ps.
```

empty path **e** only allows resolution in 'this' scope

```
let a = 1 in
let b = 2 in
let c = 3 in
  a + b + c
```

```
let a = 1 in
let b = 2 in
let c = 3 in
  a + b + c
```



Path Wellformedness: Reachability

signature

constructors

Let : ID * Exp * Exp \rightarrow Exp

name-resolution

labels P

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let  $\xrightarrow{P}$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

scope edge

```
resolveVar(s, x) = ps :-
```

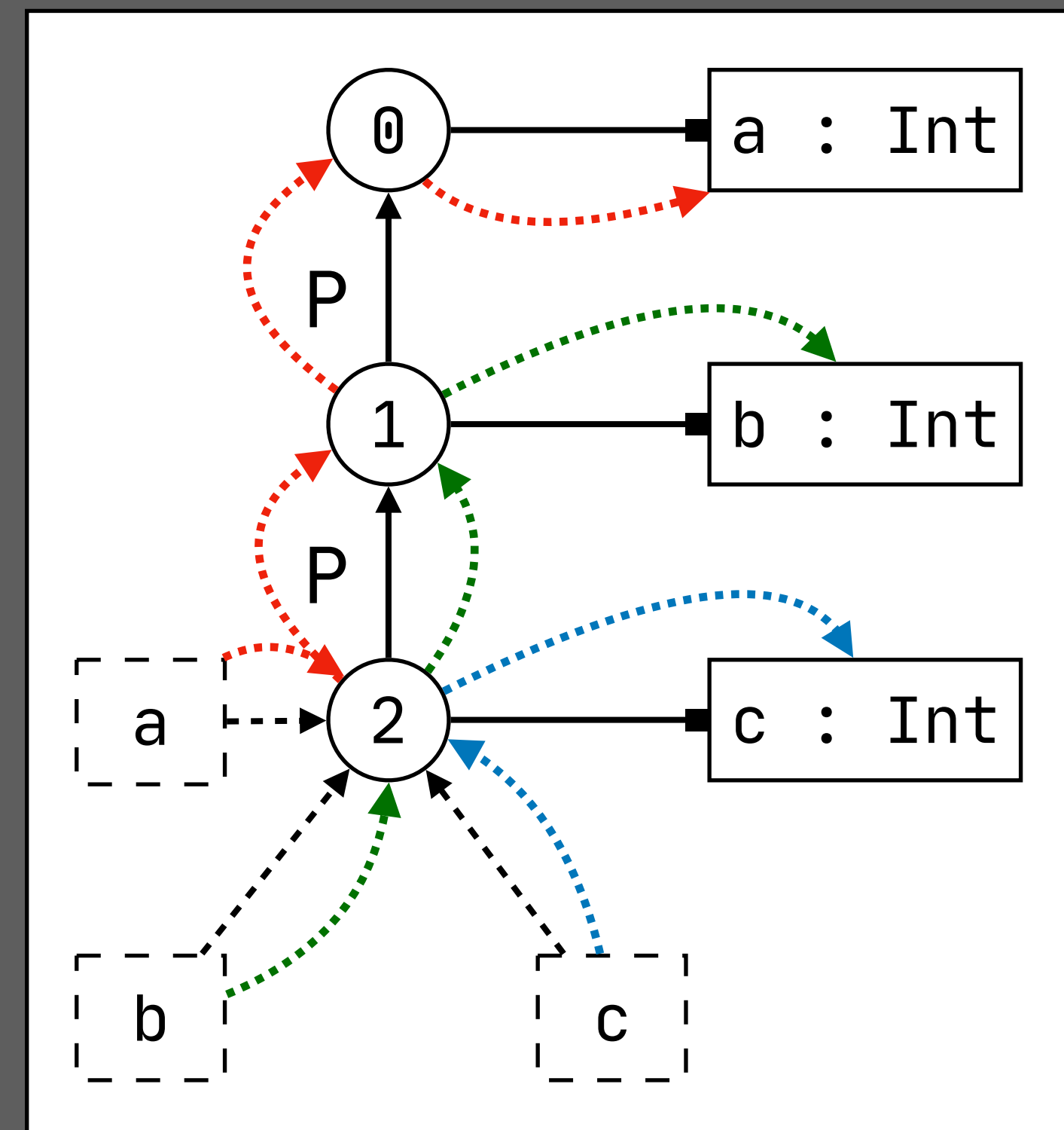
```
  query var
```

```
    filter P* and { x' :- x' = x }
```

```
      min /* */ and true
```

```
    in s  $\mapsto$  ps.
```

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



path P^* allows resolution through zero or more P edges

Duplicate Definitions Revisited

signature

constructors

Let : ID * Exp * Exp \rightarrow Exp

name-resolution

labels P

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let  $\xrightarrow{P}$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

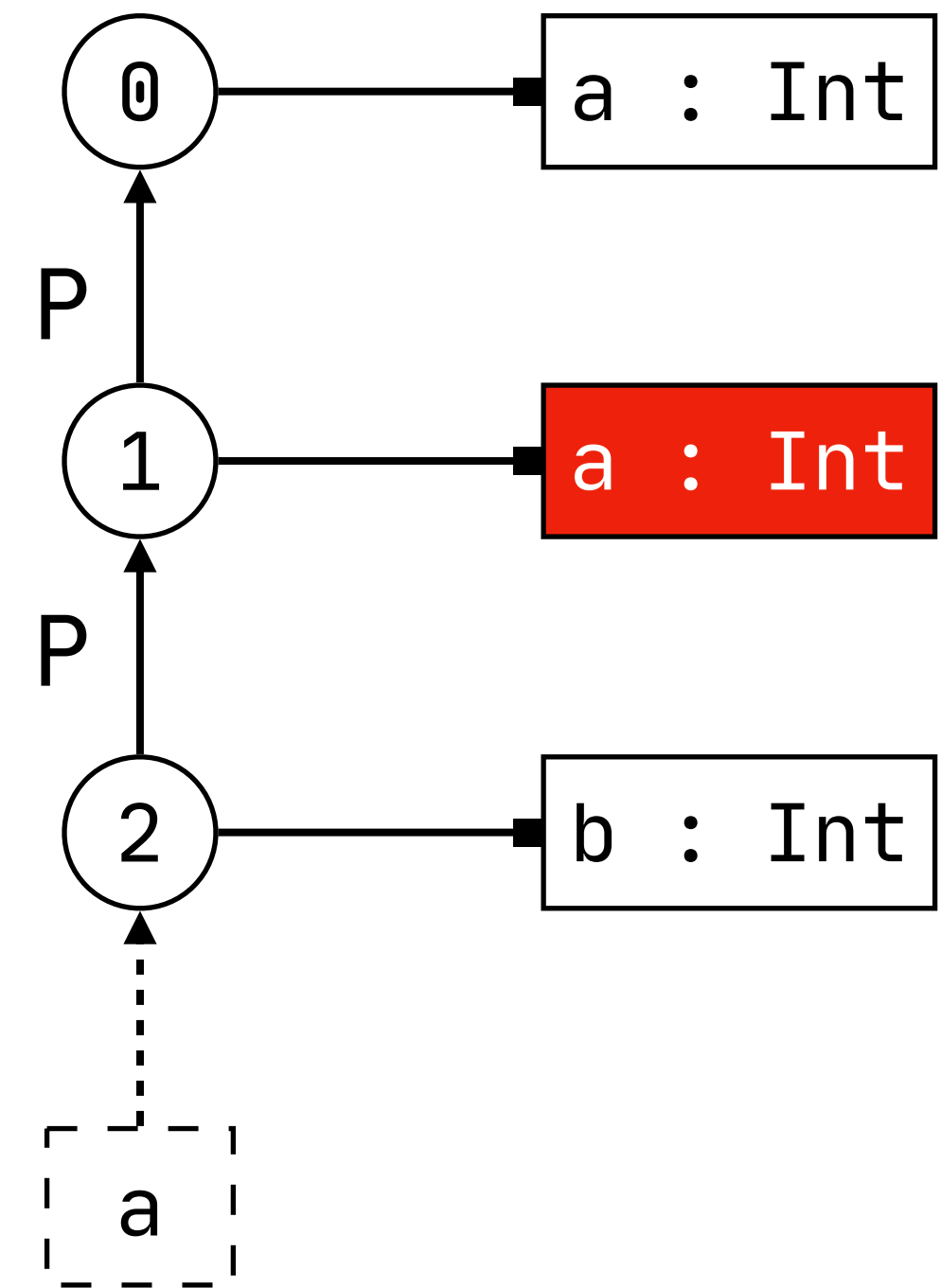
new scope

scope edge

```
resolveVar(s, x) = ps :-
  query var
  filter P* and { x' :- x' = x }
  min /* */ and true
  in s  $\mapsto$  ps.
```

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



path P^* allows resolution through zero or more P edges

Duplicate Definitions Revisited

signature

constructors

Let : ID * Exp * Exp → Exp

name-resolution

labels P

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P→ s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

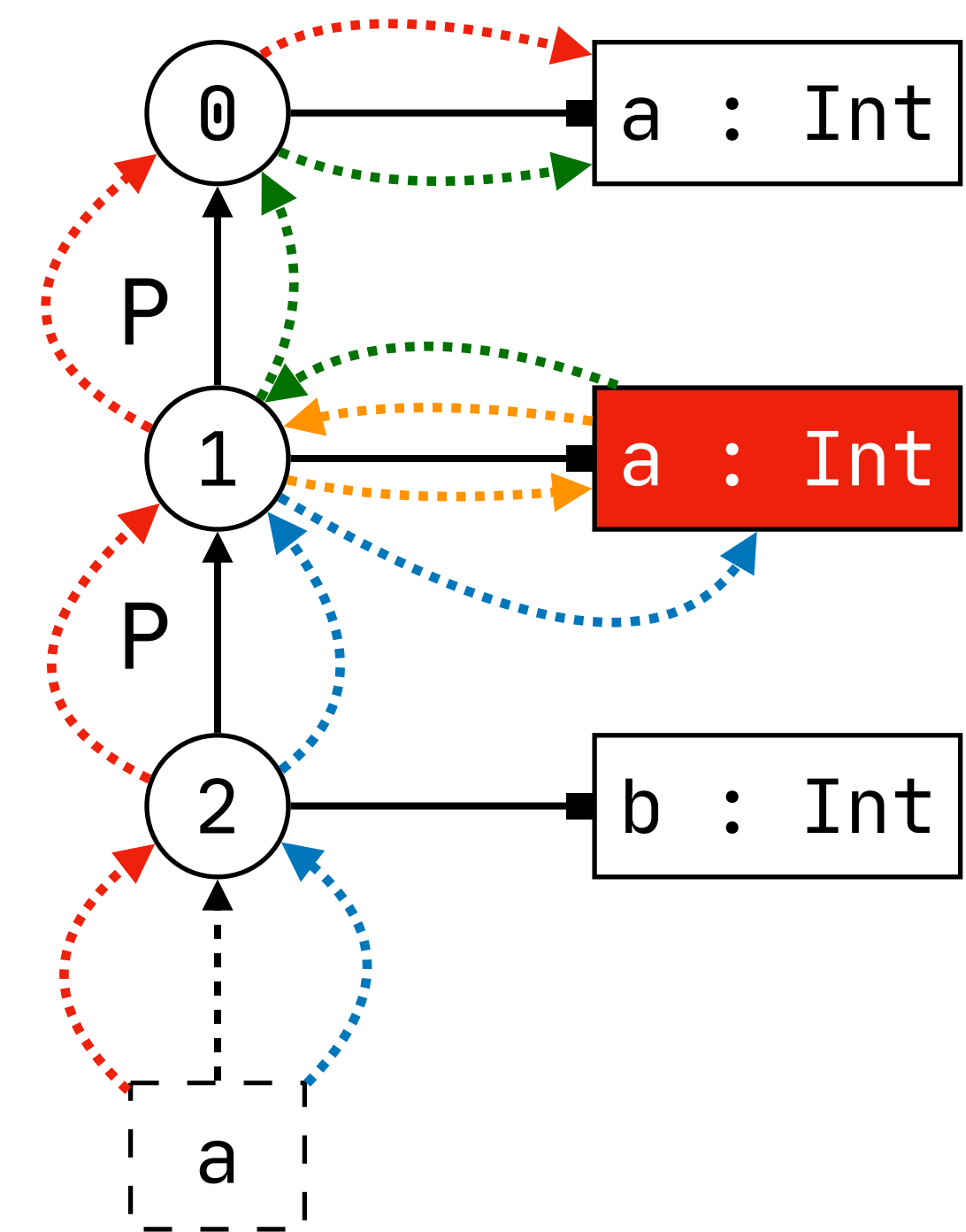
new scope

scope edge

```
resolveVar(s, x) = ps :-
  query var
  filter P* and { x' :- x' = x }
  min /* */ and true
  in s ↦ ps.
```

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



path P^* allows resolution through zero or more P edges

Path Specificity: Visibility (Shadowing)

signature
constructors
Let : ID * Exp * Exp → Exp
name-resolution
labels P

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P→ s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

scope edge

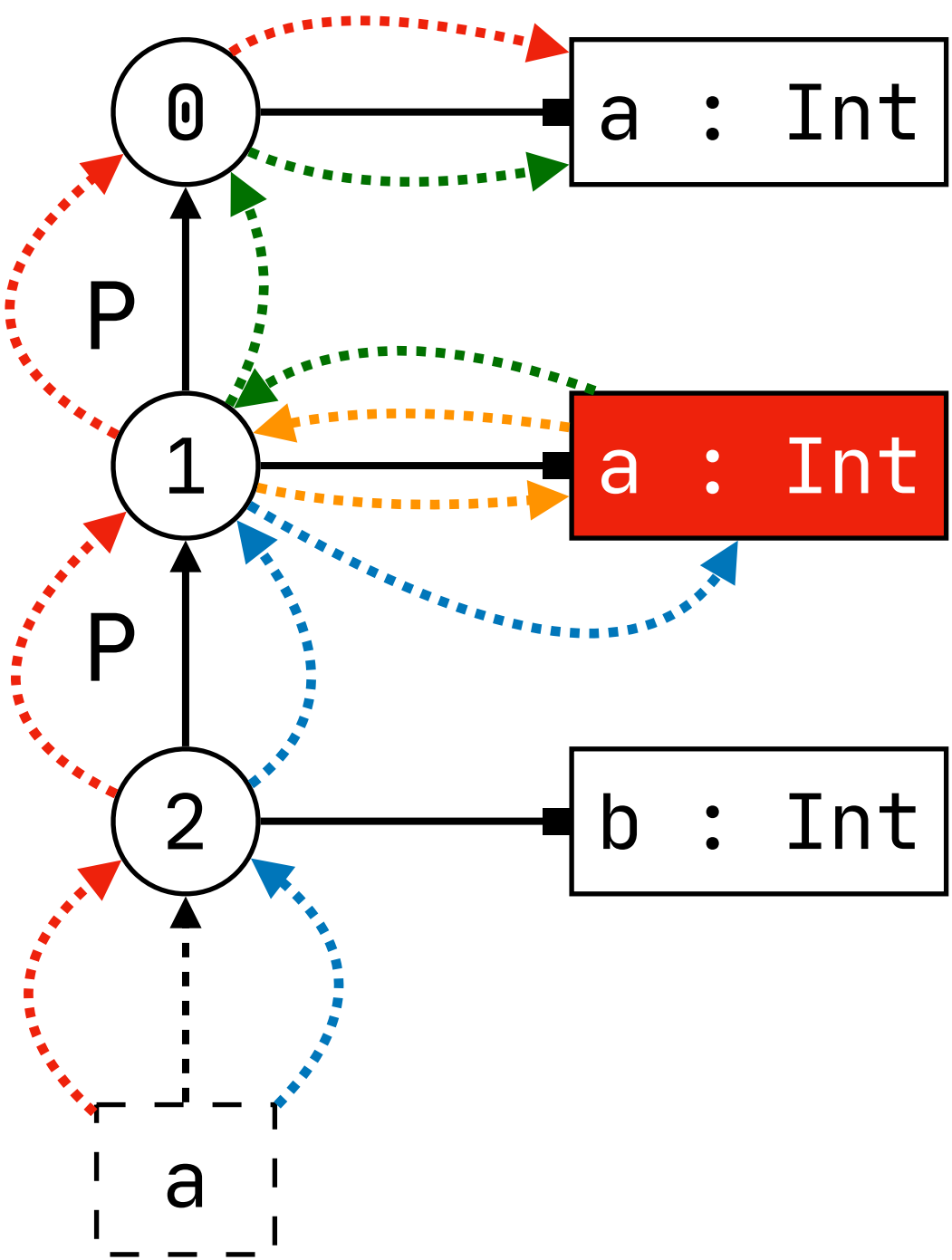
```
resolveVar(s, x) = ps :-
  query var
  filter P* and { x' :- x' = x }
  min /* */ and true
  in s ↦ ps.
```

path P^* allows resolution through zero or more P edges

prefer local scope (\$) over parent scope (P)

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



prefer blue path over red path

prefer orange path over green path

Path Specificity: Visibility (Shadowing)

signature
constructors
Let : ID * Exp * Exp \rightarrow Exp
name-resolution
labels P

rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}  
typeOfExp(s, e1) = S,  
new s_let, s_let -P $\rightarrow$  s,  
declareVar(s_let, x, S),  
typeOfExp(s_let, e2) = T.
```

new scope

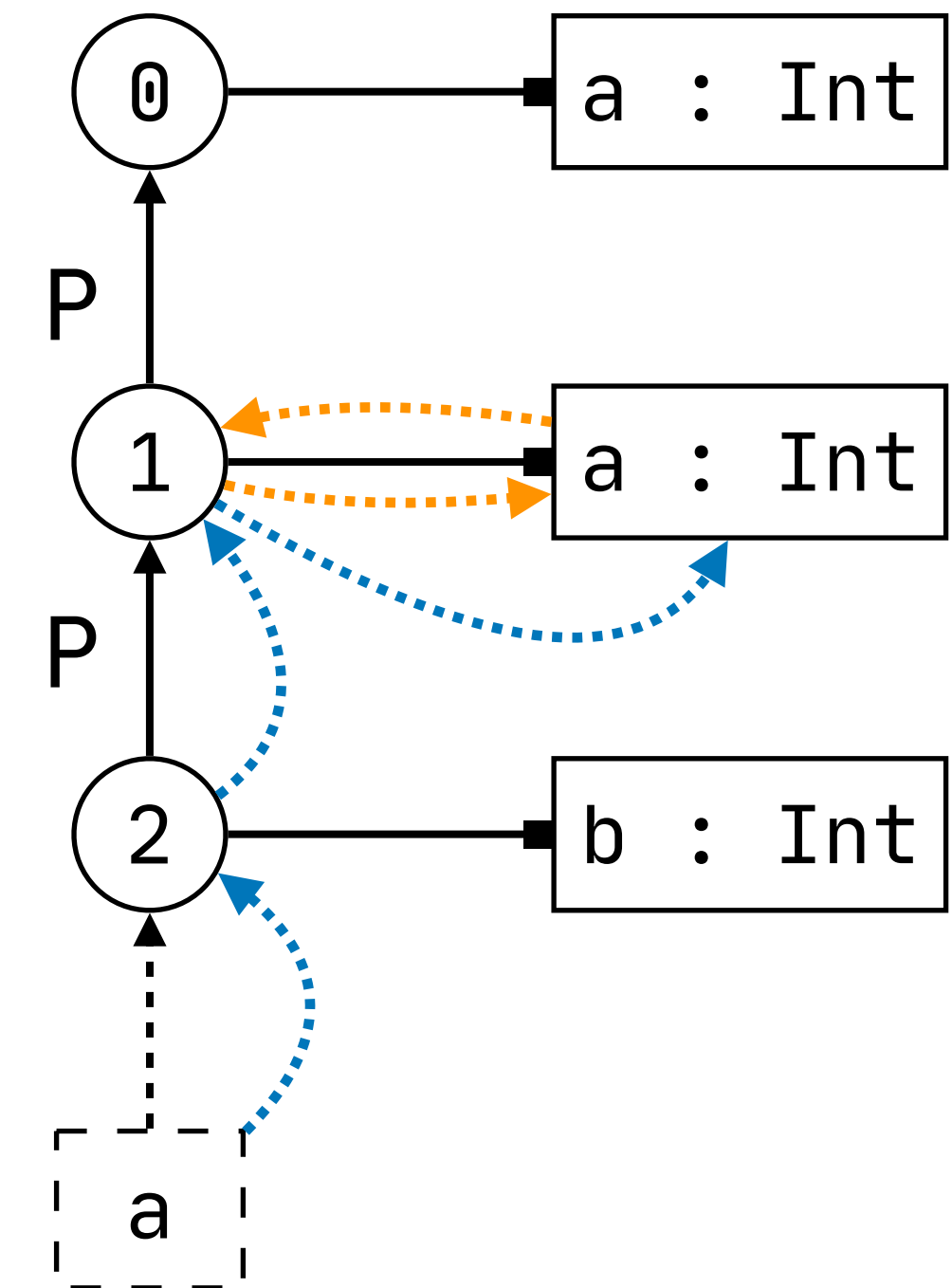
scope edge

```
resolveVar(s, x) = ps :-  
  query var  
  filter P* and { x' :- x' = x }  
  min $ < P and true  
  in s  $\mapsto$  ps.
```

path P^* allows resolution through zero or more P edges

prefer local scope (\$) over parent scope (P)

```
let a = 1 in  
let a = 2 in  
let b = 3 in  
a
```



How about non-lexical bindings?

Scopes as Types / Modules

Modules: Scopes as Types

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

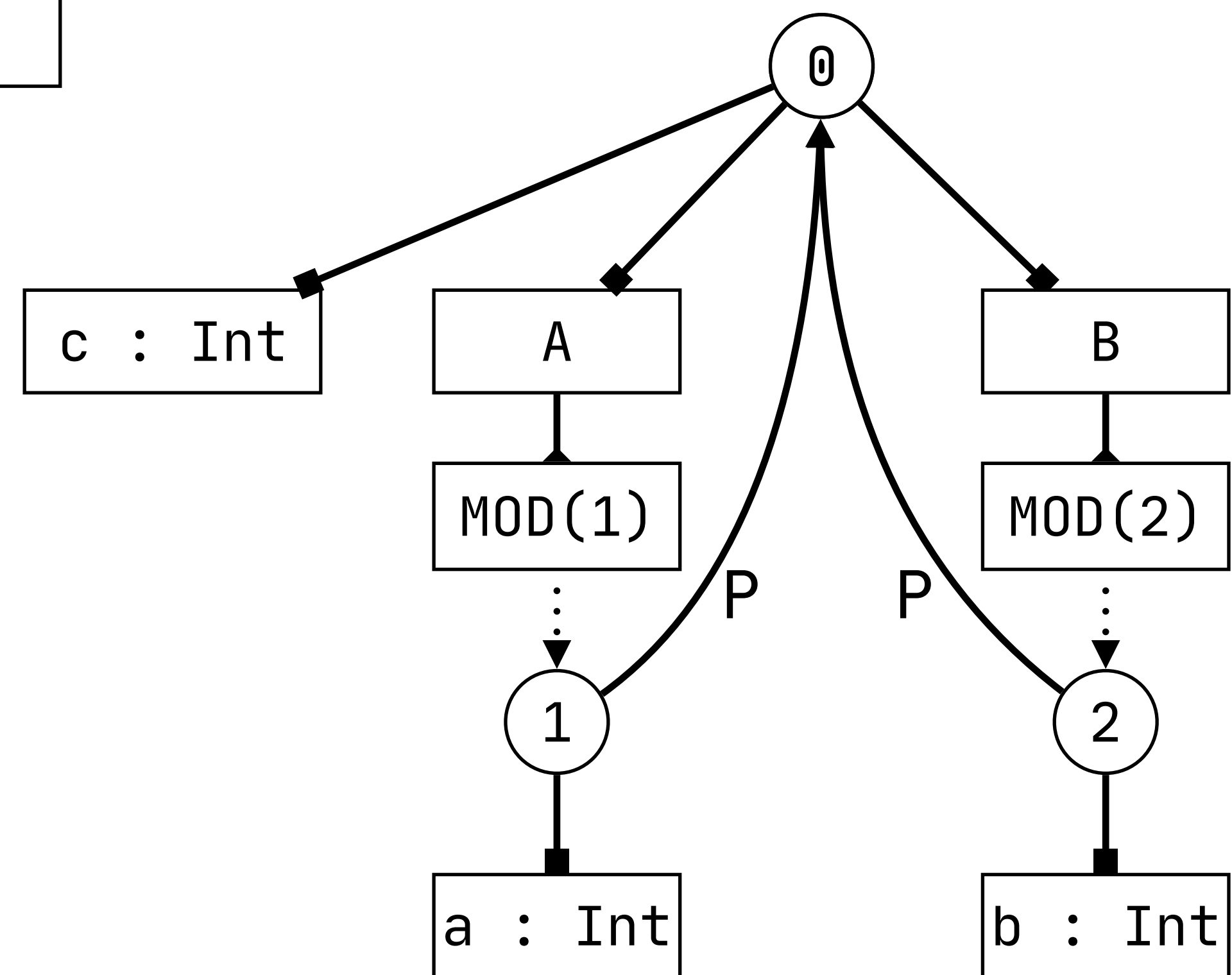
rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```



Resolving Import

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

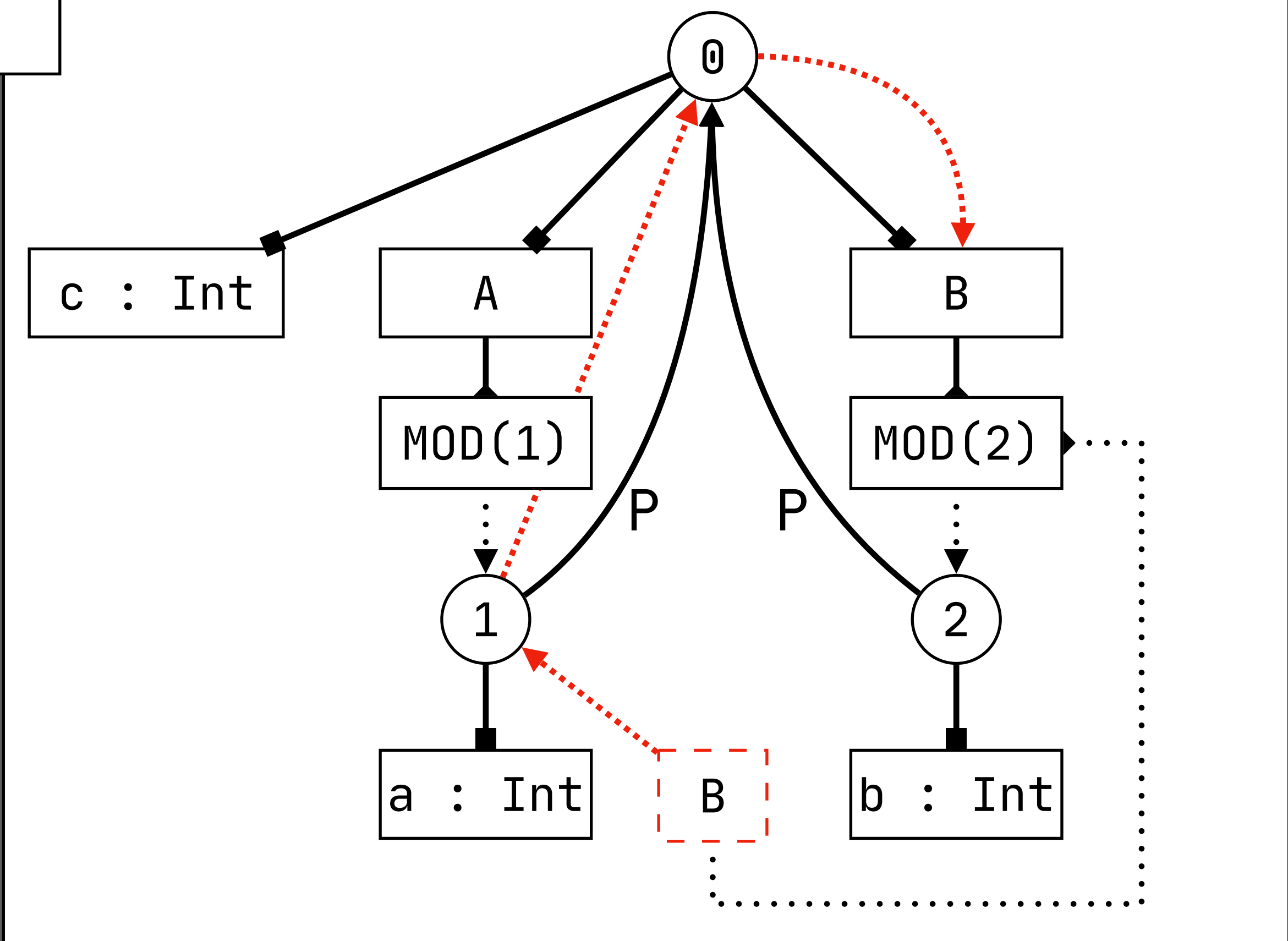
scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

```
resolveMod(s, x) = ps :-
  query mod
  filter P* and { x' :- x' = x }
  min $ < P and true
  in s ↦ ps.
```

resolve
module name



Import Edge

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

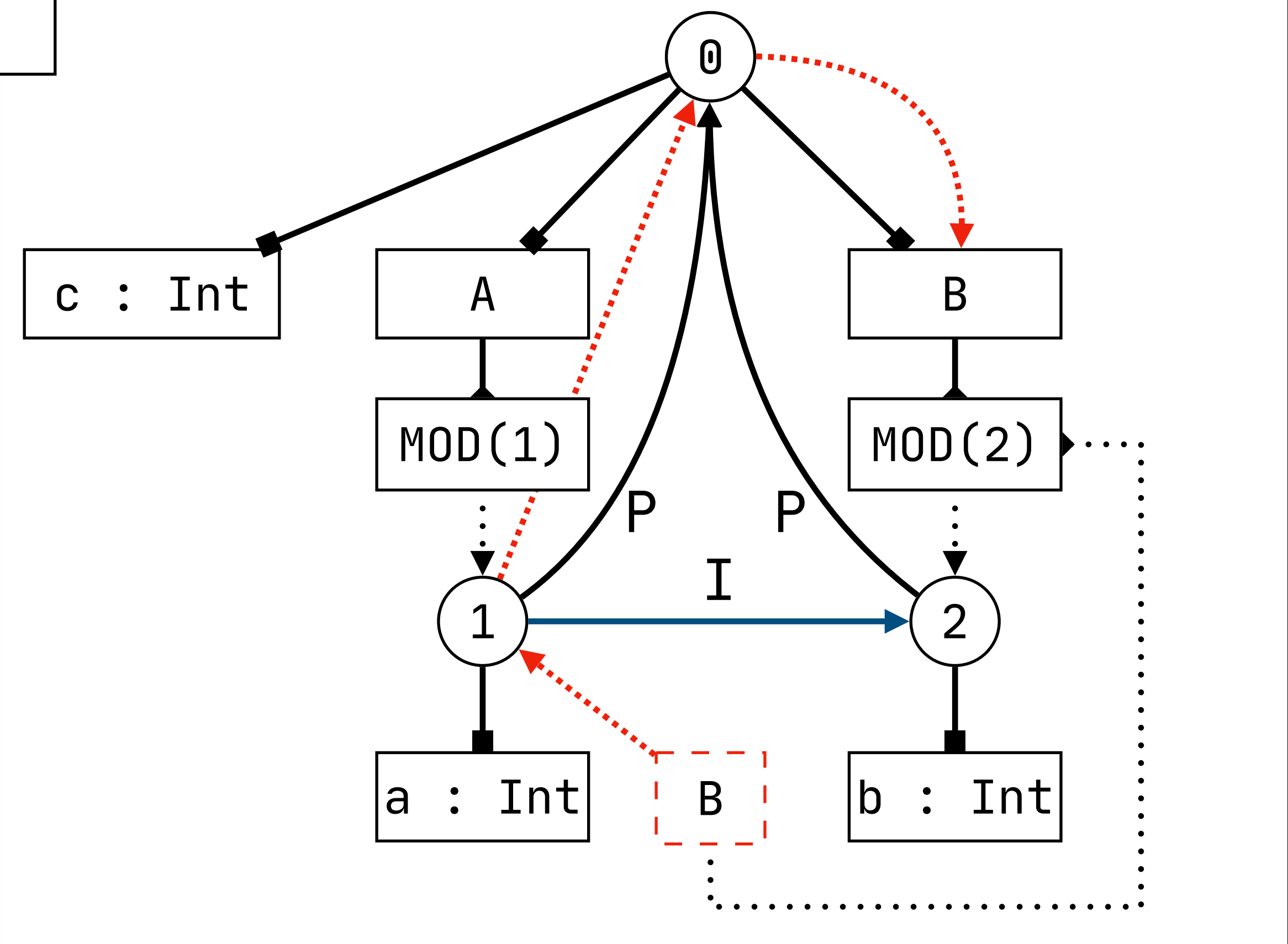
```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

import edge

```
resolveMod(s, x) = ps :-
  query mod
  filter P* and { x' :- x' = x }
  min $ < P and true
  in s ↦ ps.
```

resolve
module name



Resolving Variable through Import Edge

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

import edge

```
resolveVar(s, x) = ps :-
```

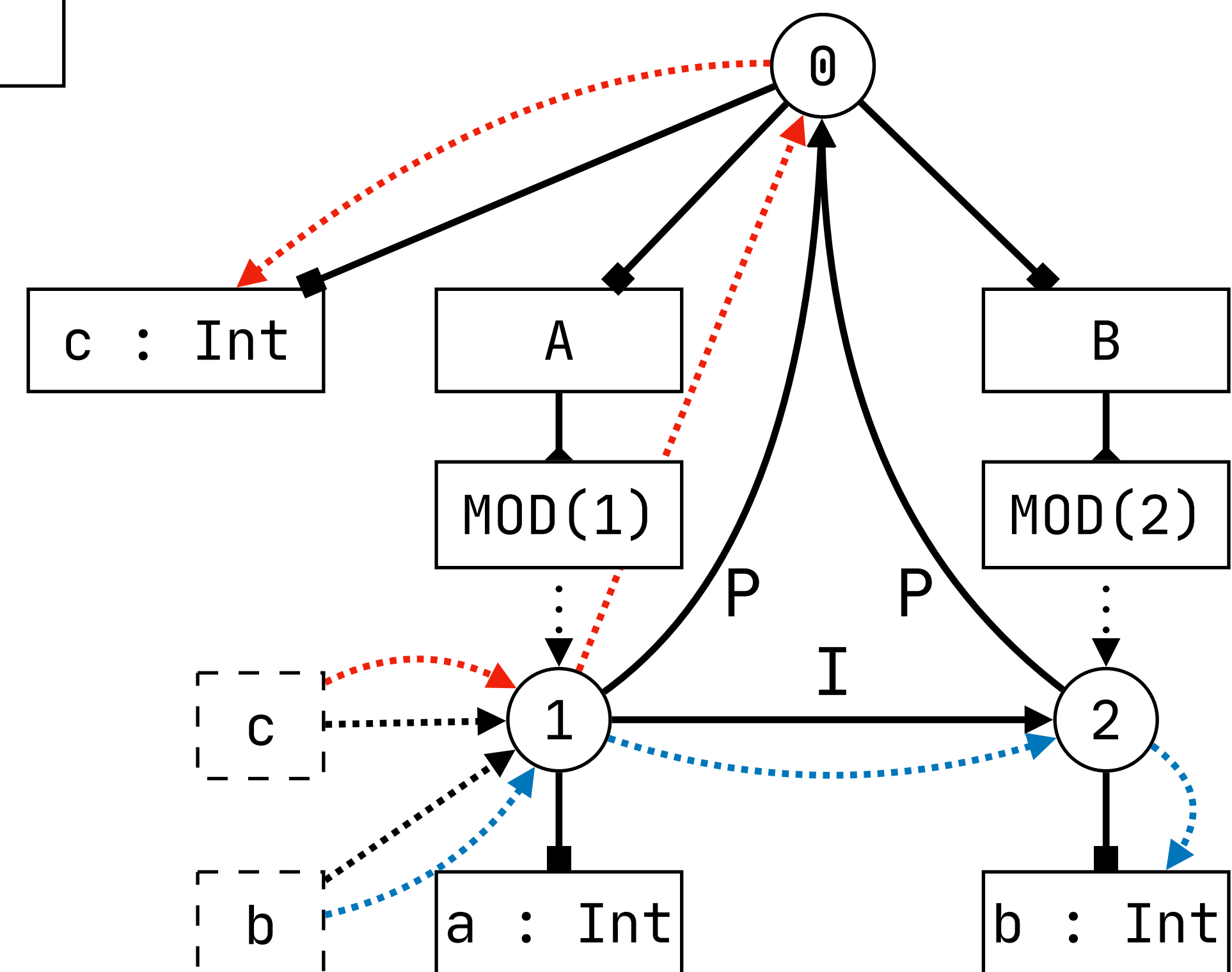
```
  query var
```

```
    filter P* I* and { x' :- x' = x }
```

```
    min $ < P, $ < I, I < P and true
```

```
    in s ↦ ps.
```

resolve variable through
import edges



Import Shadows Parent (Lexical Context)

signature

constructors

MOD : scope \rightarrow TYPE

```
Module : ID * list(Decl) → Decl
```

Import : $ID \rightarrow Decl$

scope as type

```
def b = 0
```

```
module A {
```

```
import B
```

```
def a = b
```

}

```
module B {
```

```
def b = 2
```

}

rules

```
dec10k(s, Module(m, dec1s)) :- {s_mod}
```

```
new s_mod, s_mod -P→ s,
```

```
declareMod(s, m, MOD(s_mod)),
```

```
declsOk(s_mod, decls).
```

lexical scope

scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
```

```
typeOfModRef(s, p) = MOD(s_mod),
```

$$s \stackrel{-I}{\rightarrow} s_{\text{mod.}}$$

resolve import

```
import edge
```

```
resolveVar(s, x) = ps :-
```

query var

filter $P^* \ I^*$ and $\{ x' \ :- \ x' = x \}$

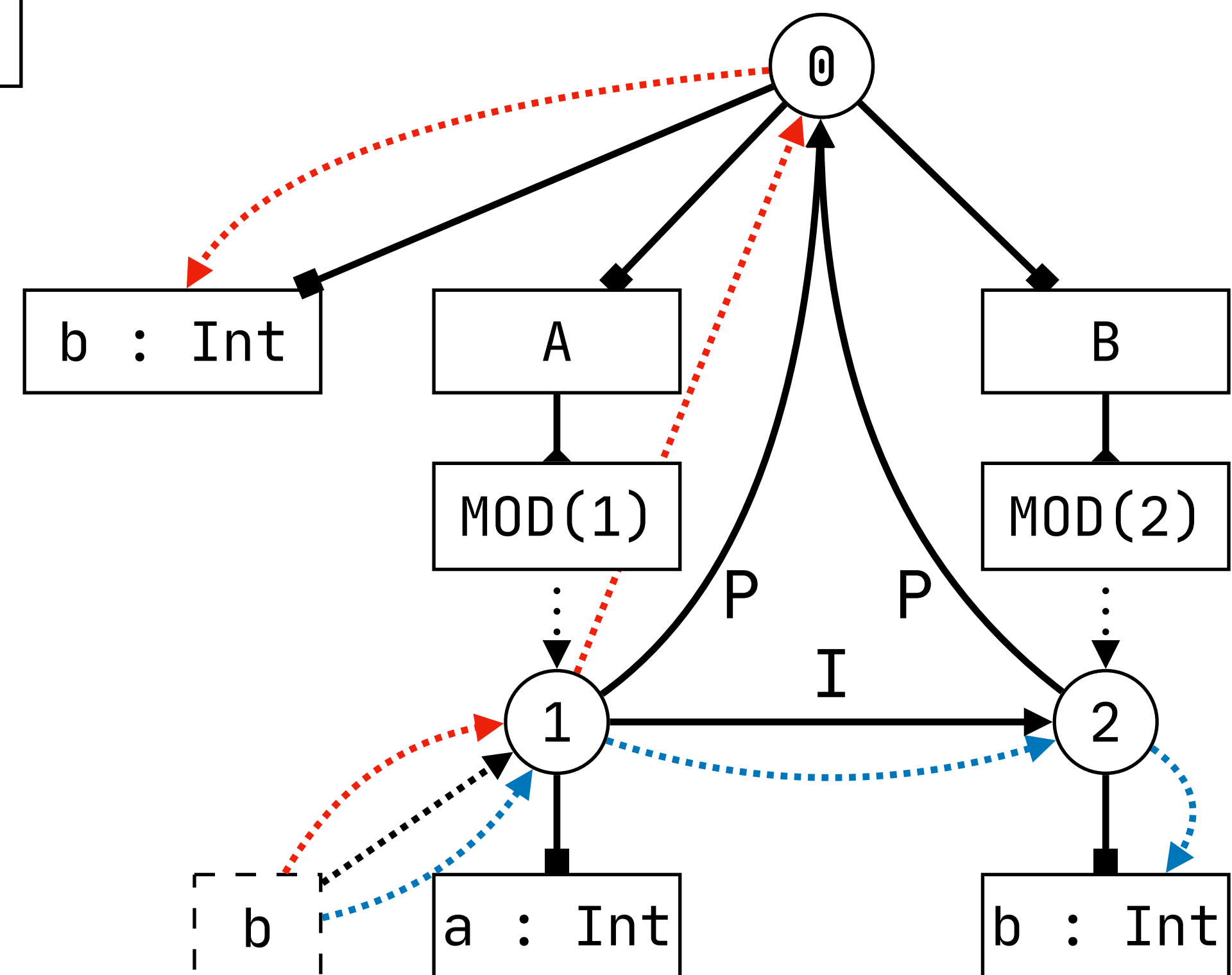
$\min \$ < P, \$ < I, I < P$ and true

$$\text{in } s \mapsto ps.$$

import after parent

prefer import

resolve variable through
import edges



Mutually Recursive Imports

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

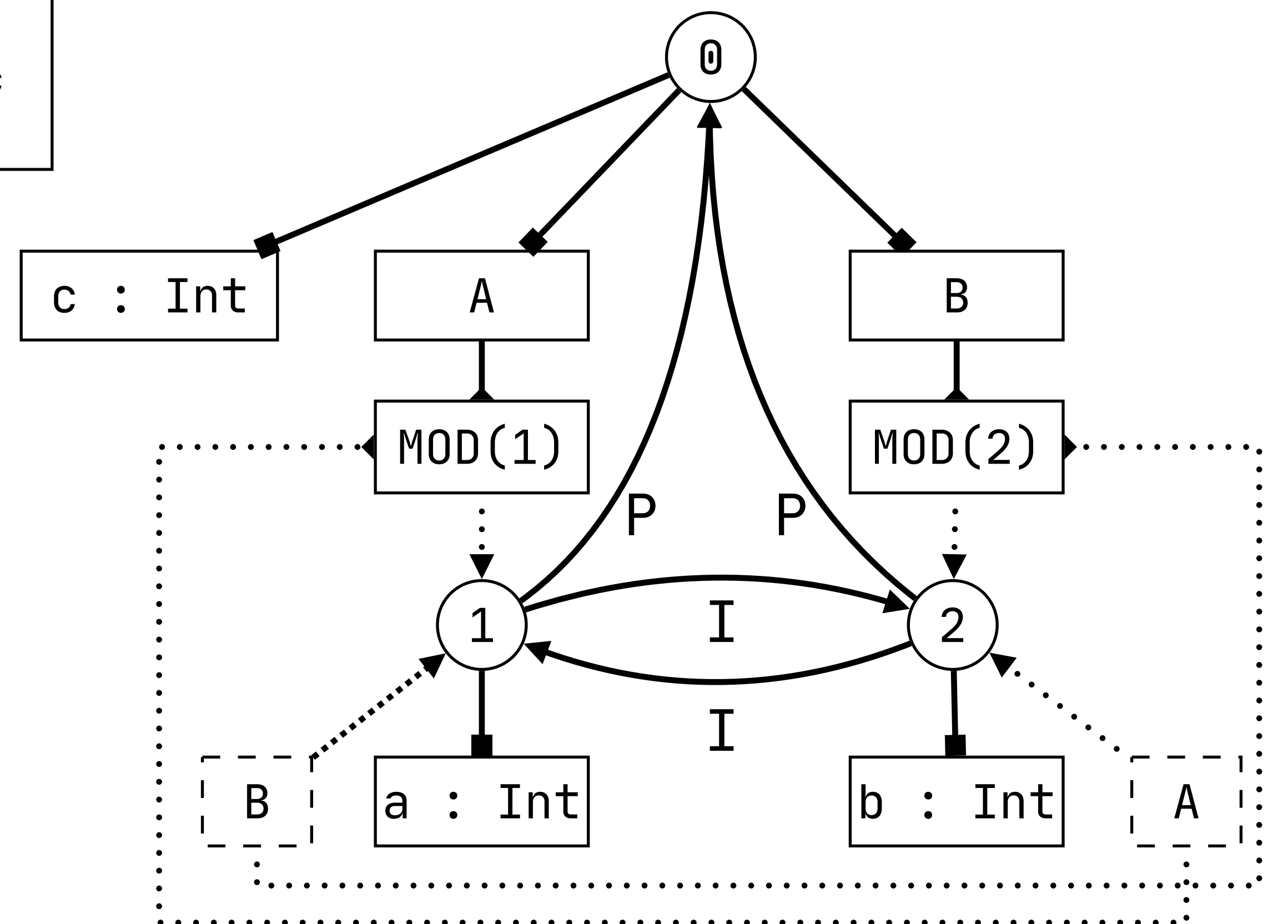
rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  import A
  def b = 2
  def d = a + c
}
```

```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s ↦ ps.
```



Mutually Recursive Imports

signature

constructors

MOD : scope \rightarrow TYPE
Module : ID * list(Decl) \rightarrow Decl
Import : ID \rightarrow Decl

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  import A
  def b = 2
  def d = a + c
}
```

rules

declOk(s, Module(m, decls)) :- {s_mod}
new s_mod, s_mod \rightarrow s,
declareMod(s, m, MOD(s_mod)),
declsOk(s_mod, decls).

scope as type

declOk(s, Import(p)) :- {s_mod s_end}
typeOfModRef(s, p) = MOD(s_mod),
s \rightarrow s_mod.

resolve import

import edge

resolveVar(s, x) = ps :-

query var

filter P* I* and { x' :- x' = x }

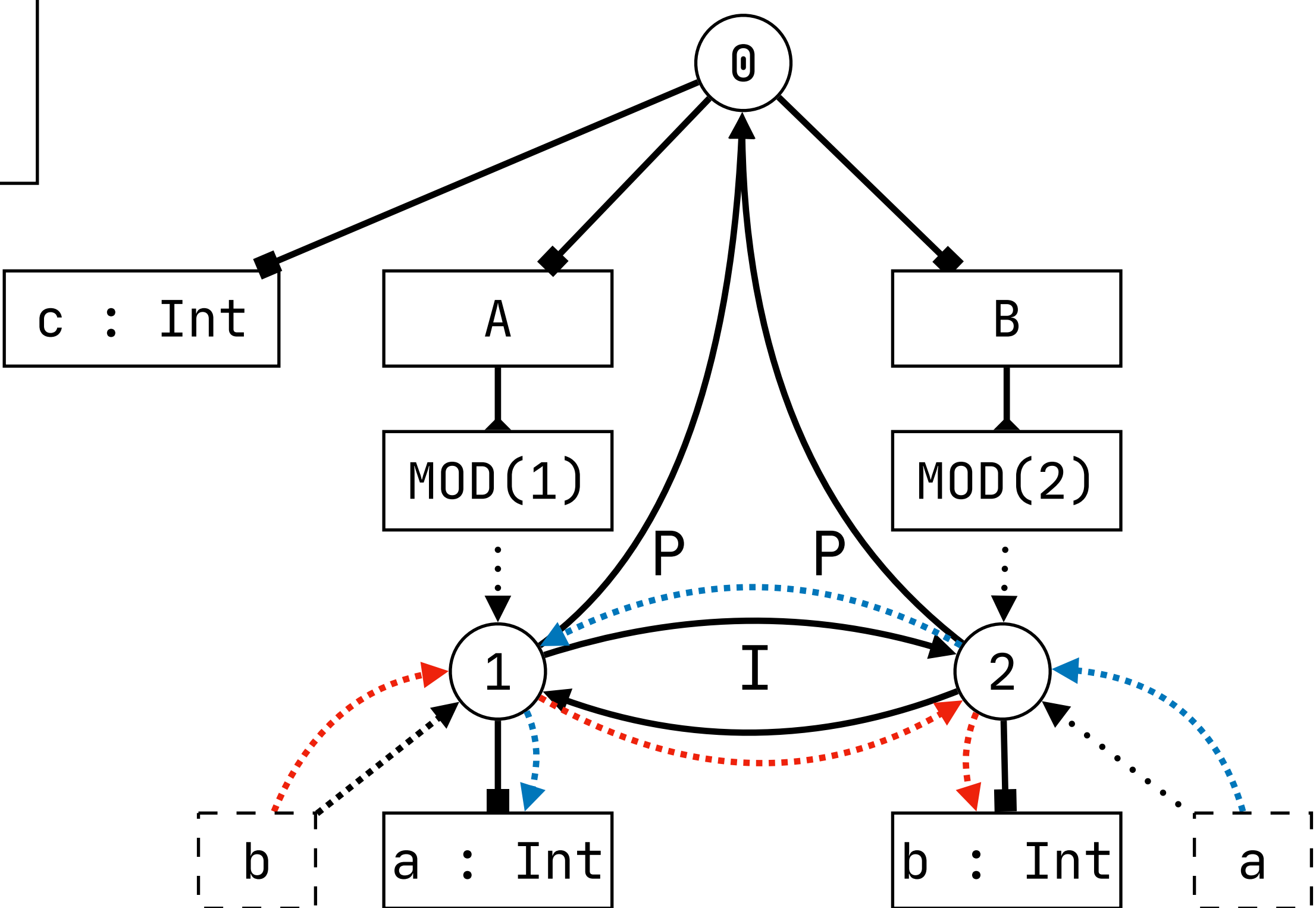
import after parent

min \$ < P, \$ < I, I < P and true

prefer import

in s \rightarrow ps.

resolve variable through
import edges



Transitive Import

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

rules

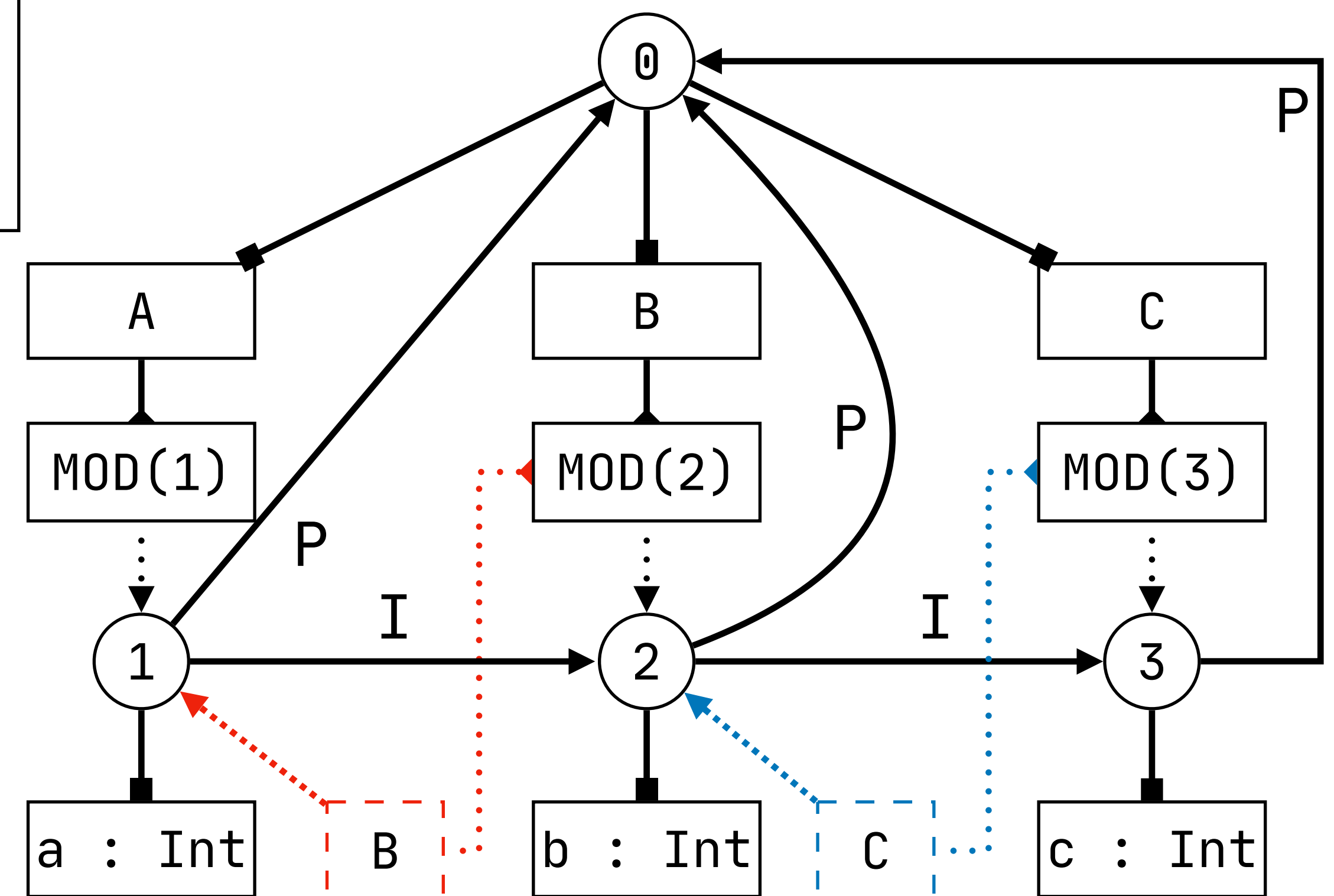
```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
```

```
  query var
    filter P* I* and { x' :- x' = x }
    min $ < P, $ < I, I < P and true
    in s ↦ ps.
```

```
module A {
  import B
  def a = b + c
}
module B {
  import C
  def b = c + 2
}
module C {
  def c = 1
}
```



Transitive Import

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

rules

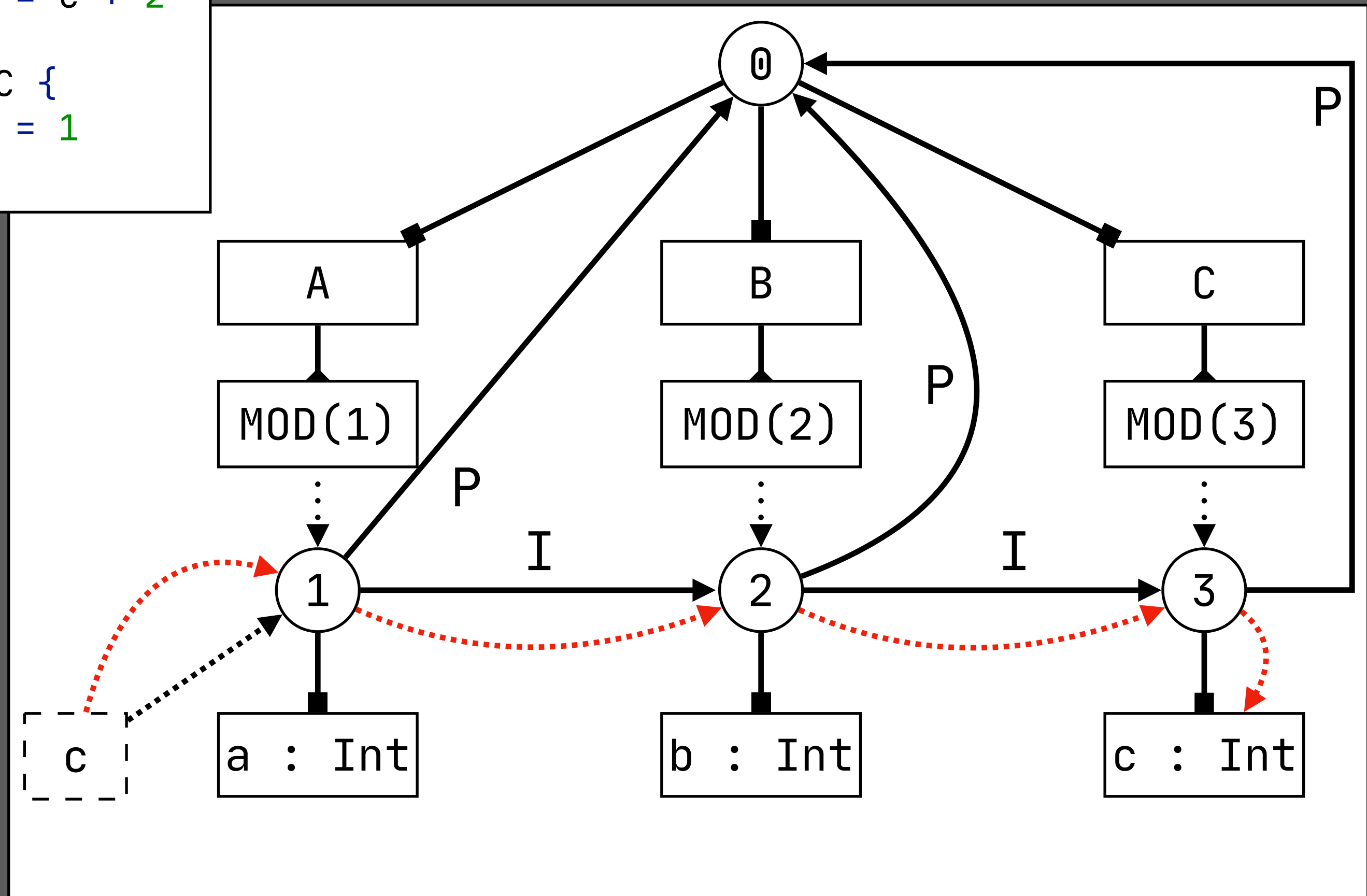
```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
```

```
  query var
    filter P* I* and { x' :- x' = x }
    min $ < P, $ < I, I < P and true
    in s ↦ ps.
```

```
module A {
  import B
  def a = b + c
}
module B {
  import C
  def b = c + 2
}
module C {
  def c = 1
}
```



Changing Query Outcomes

(is not allowed)

Nested Modules

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

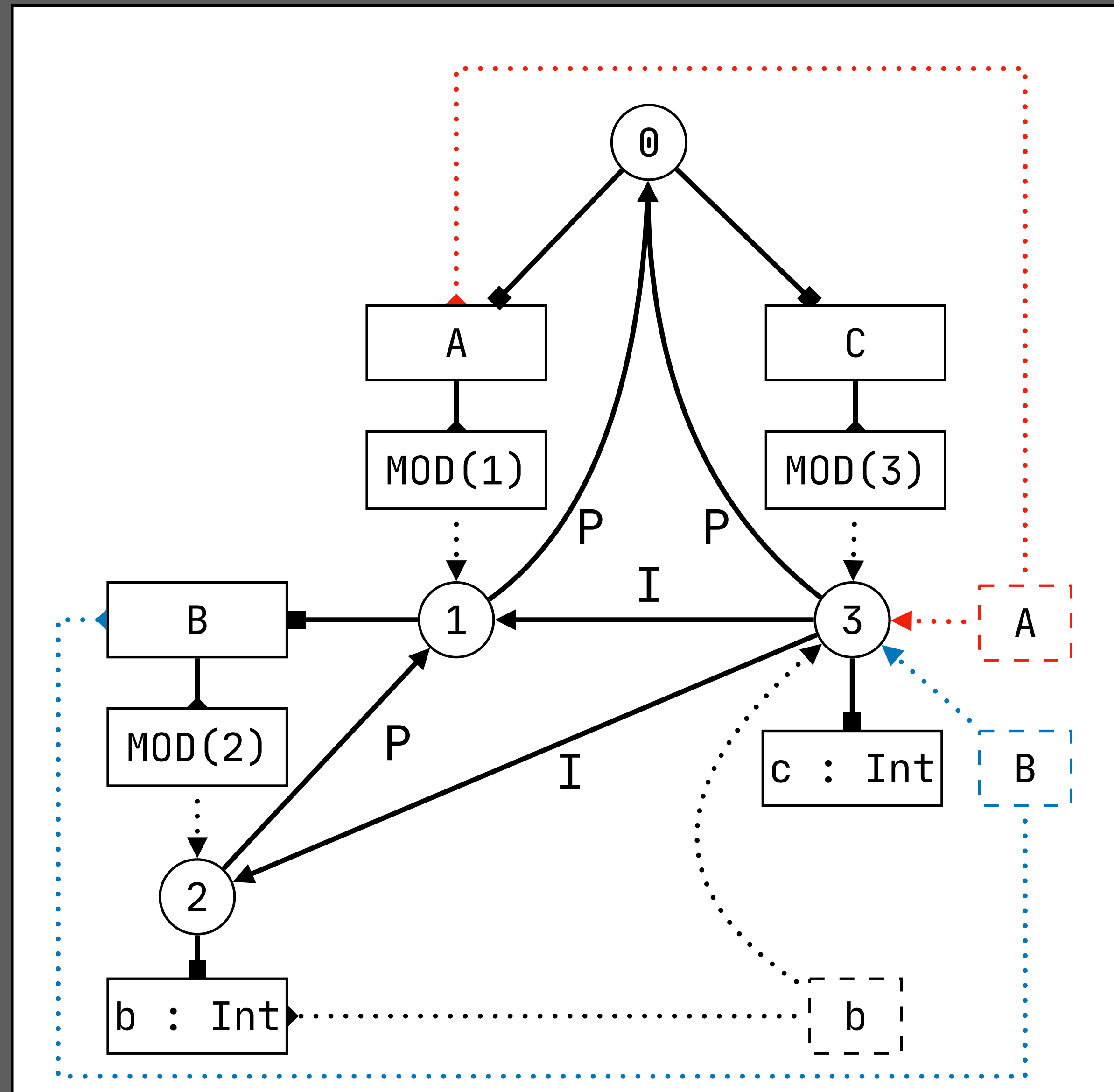
rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
  module B {
    def b = 1
  }
}
```

```
module C {
  import A
  import B
  def c = b
}
```



```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s ↦ ps.
```

Changing Result of Query

```
signature
constructors
  MOD      : scope → TYPE
  Module   : ID * list(Decl) → Decl
  Import   : ID → Decl
```

rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```

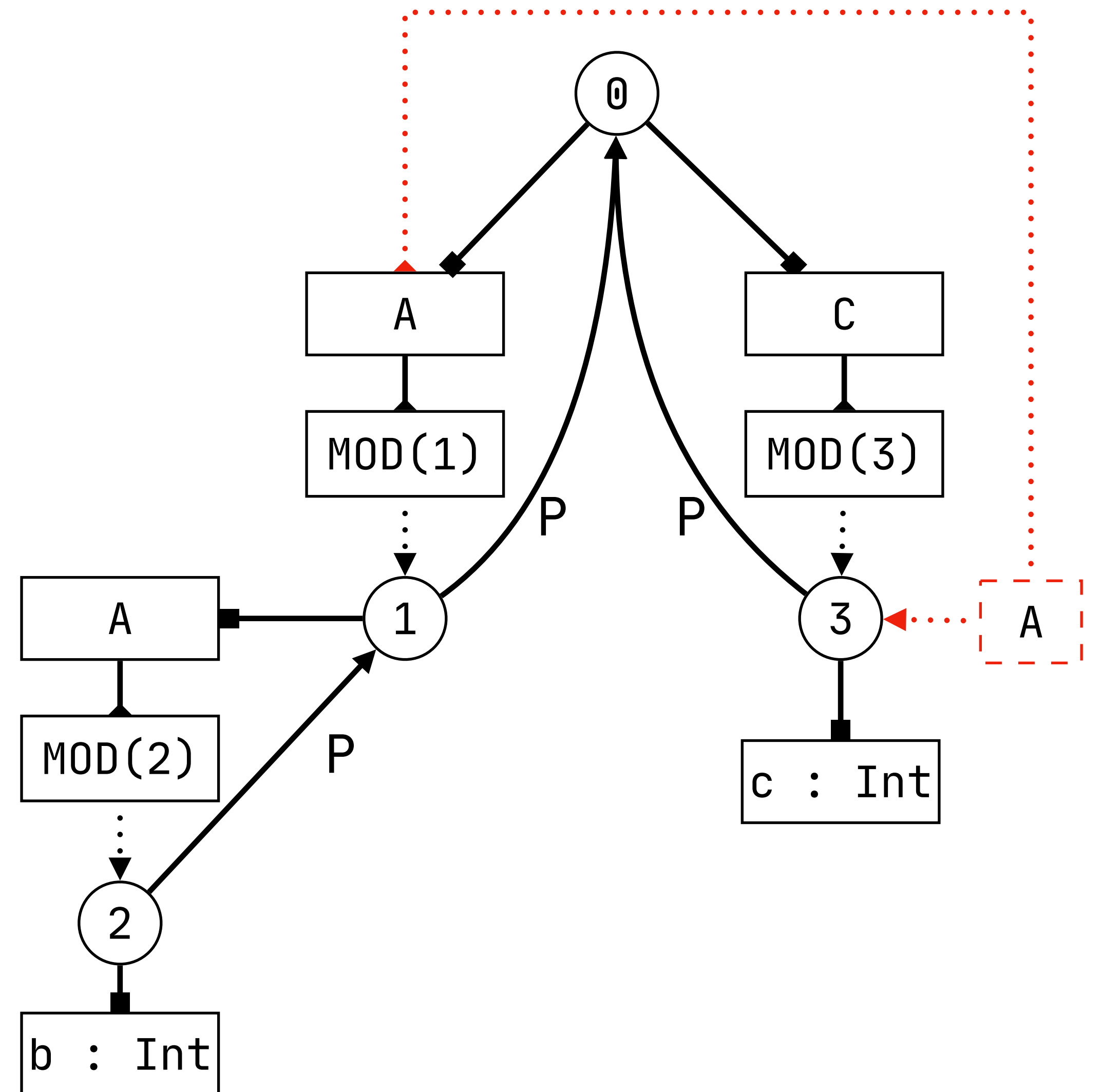
declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
    s -I→ s_mod.

```

```
resolveVar(s, x) = ps :-  
  query var  
    filter P* I* and { x' :- x' = x }  
    min $ < P, $ < I, I < P and true  
    in s  $\mapsto$  ps.
```

```
module A {  
  module A {  
    def b = 1  
  }  
}
```

```
module C {
    import A
    import A
    def c = b
}
```



Changing Result of Query

signature constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

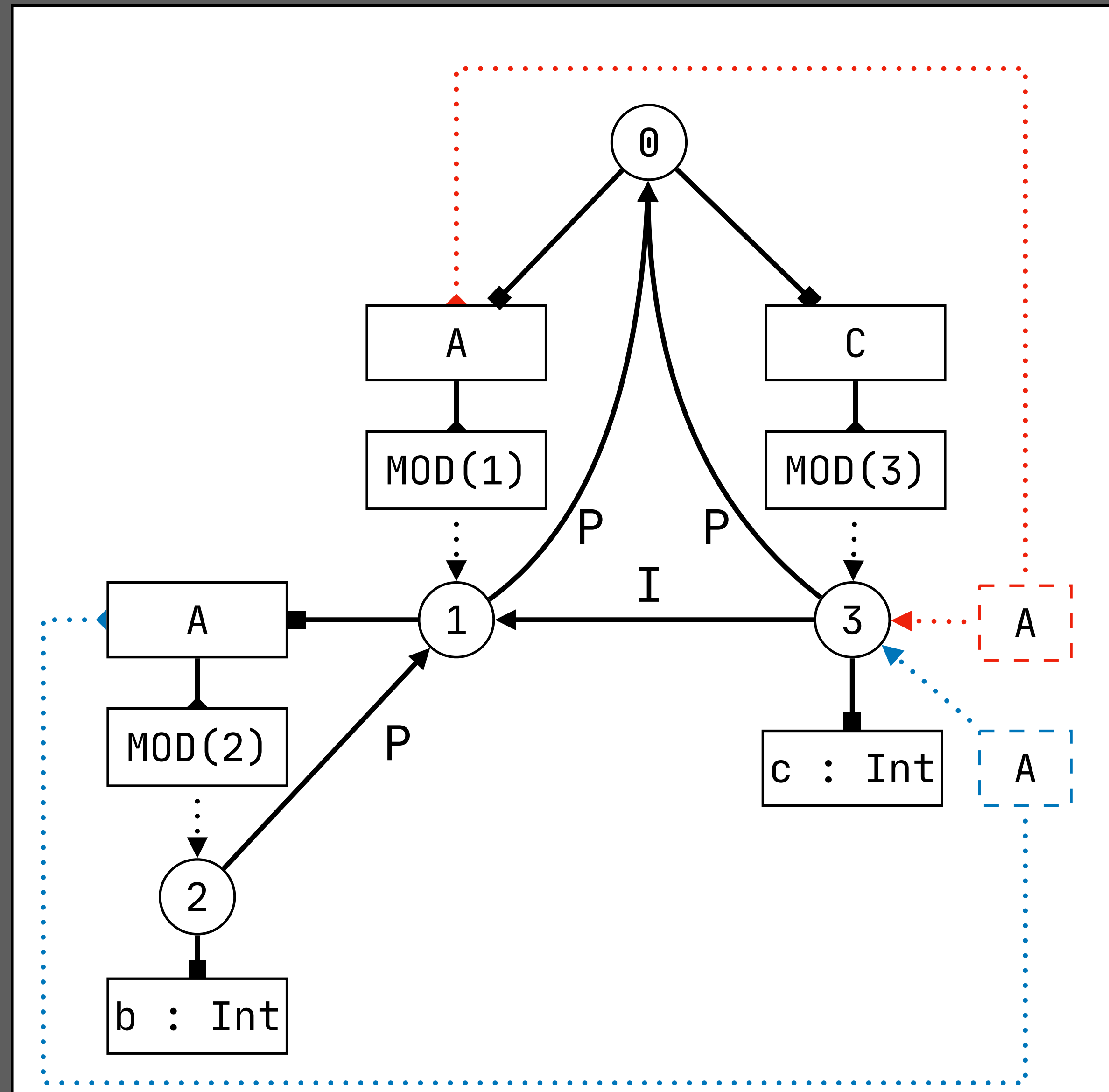
rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
  import A
  import A
  def c = b
}
```



```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s ↦ ps.
```


Changing Result of Query

signature constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

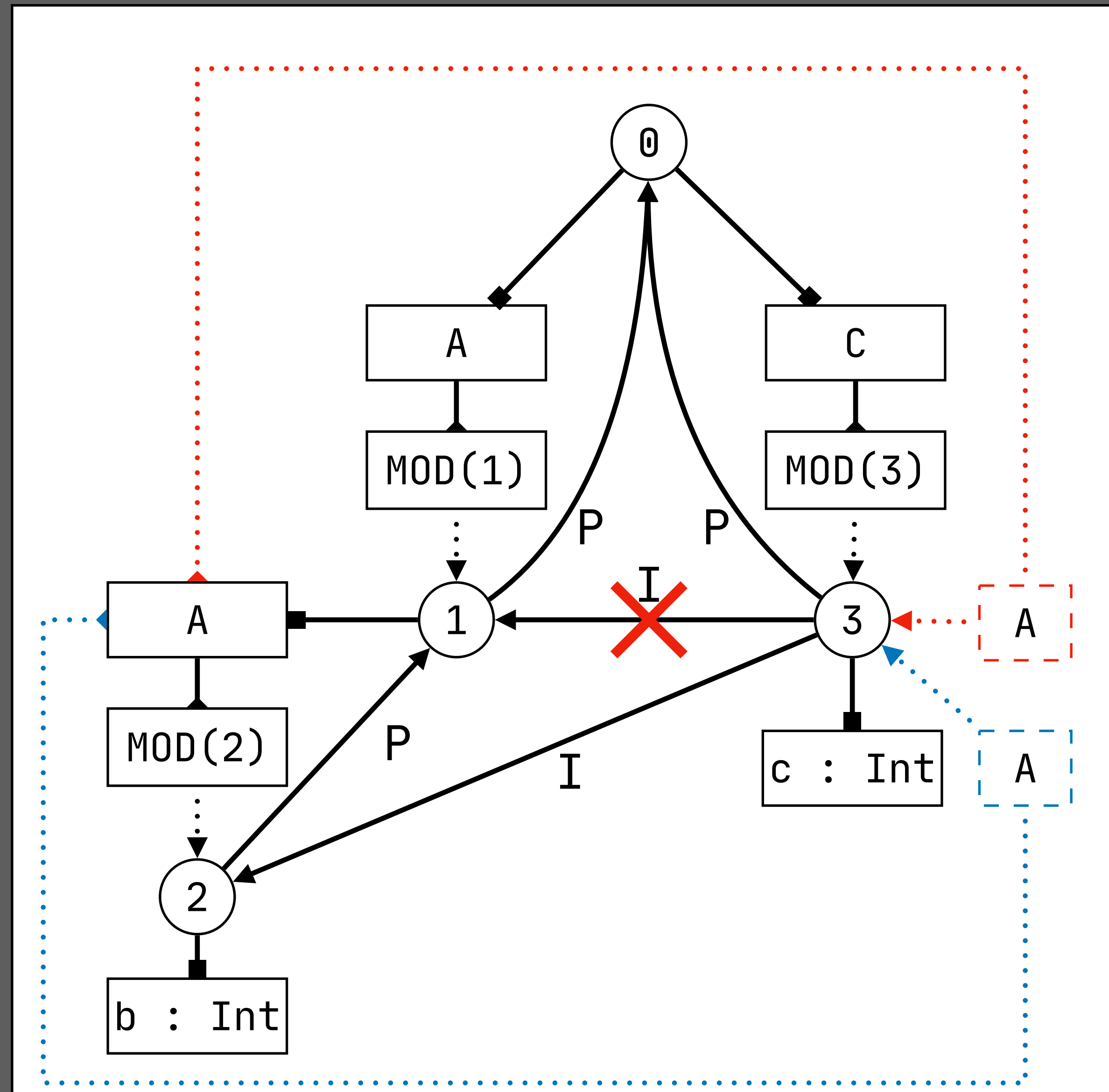
rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
  import A
  import A
  def c = b
}
```



Alternative Encoding: Scoped Imports

signature

sorts DecGroups

constructors

MOD : **scope** \rightarrow TYPE

Module : ID * DecGroups \rightarrow Decl

Import : ID \rightarrow Decl

ModRef : ID * ID \rightarrow Exp

Decs : **list**(Decl) \rightarrow DecGroups

Seq : **list**(Decl) * DecGroups
 \rightarrow DecGroups

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
  import A;
  import A
  def c = b
}
```

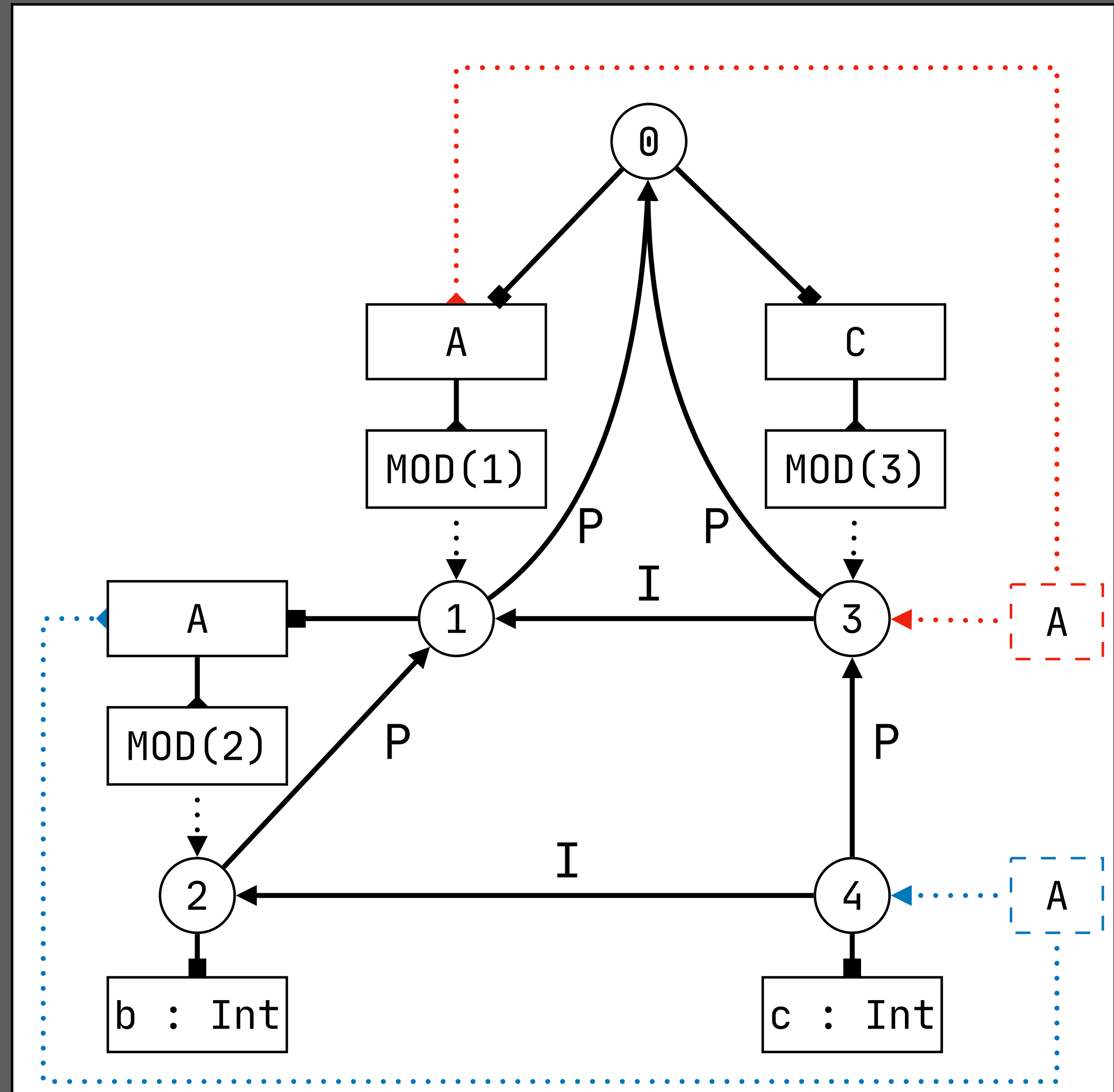
resolveVar(s, x) = ps :-

query var

filter P+ I* **and** { x' :- x' = x }

min \$ < P, \$ < I, I < P **and true**

in s \mapsto ps.



Alternative Encoding: Scoped Imports — M Edge Label

signature

sorts DecGroups

constructors

MOD : scope \rightarrow TYPE

Module : ID * DecGroups \rightarrow Decl

Import : ID \rightarrow Decl

ModRef : ID * ID \rightarrow Exp

Decs : list(Decl) \rightarrow DecGroups

Seq : list(Decl) * DecGroups
 \rightarrow DecGroups

```
module A {  
  module A {  
    def b = 1  
  }  
}
```

```
module C {  
  import A;  
  import A  
  def c = b  
}
```

resolveVar(s, x) = ps :-

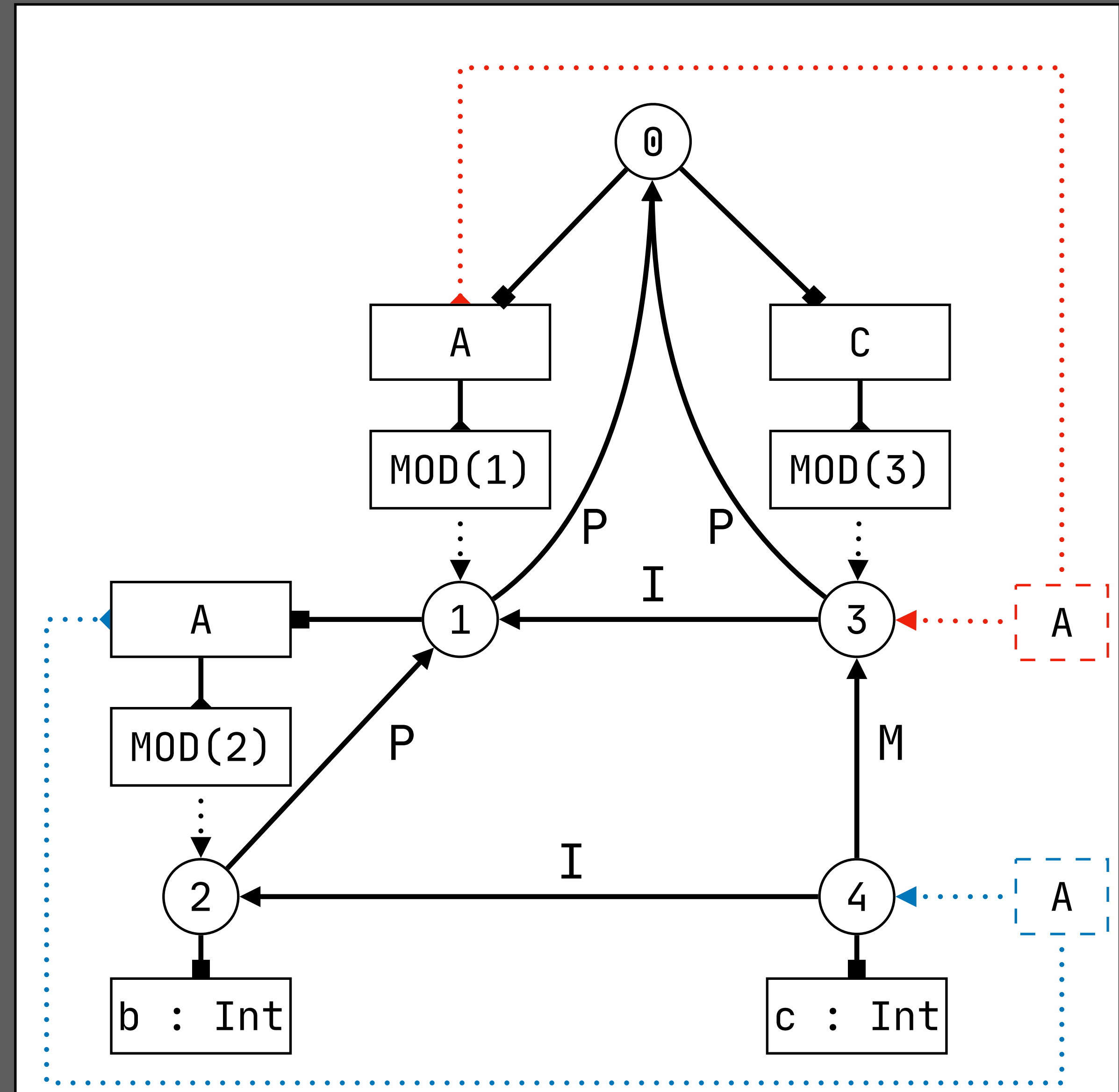
query var

filter (P | M) P* (I | M)*

and { x' :- x' = x }

min \$ < P, \$ < I, I < P and true

in s \mapsto ps.



Permission to Extend

Permission to Extend

signature

constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
ExtendRemote : ID * ID * Exp → Decl
```

```
module A {
  def a = b
}
module B {
  def A.b := 2
}
```

```
extend remote: def M.x := e
extend module M with declaration of x
```

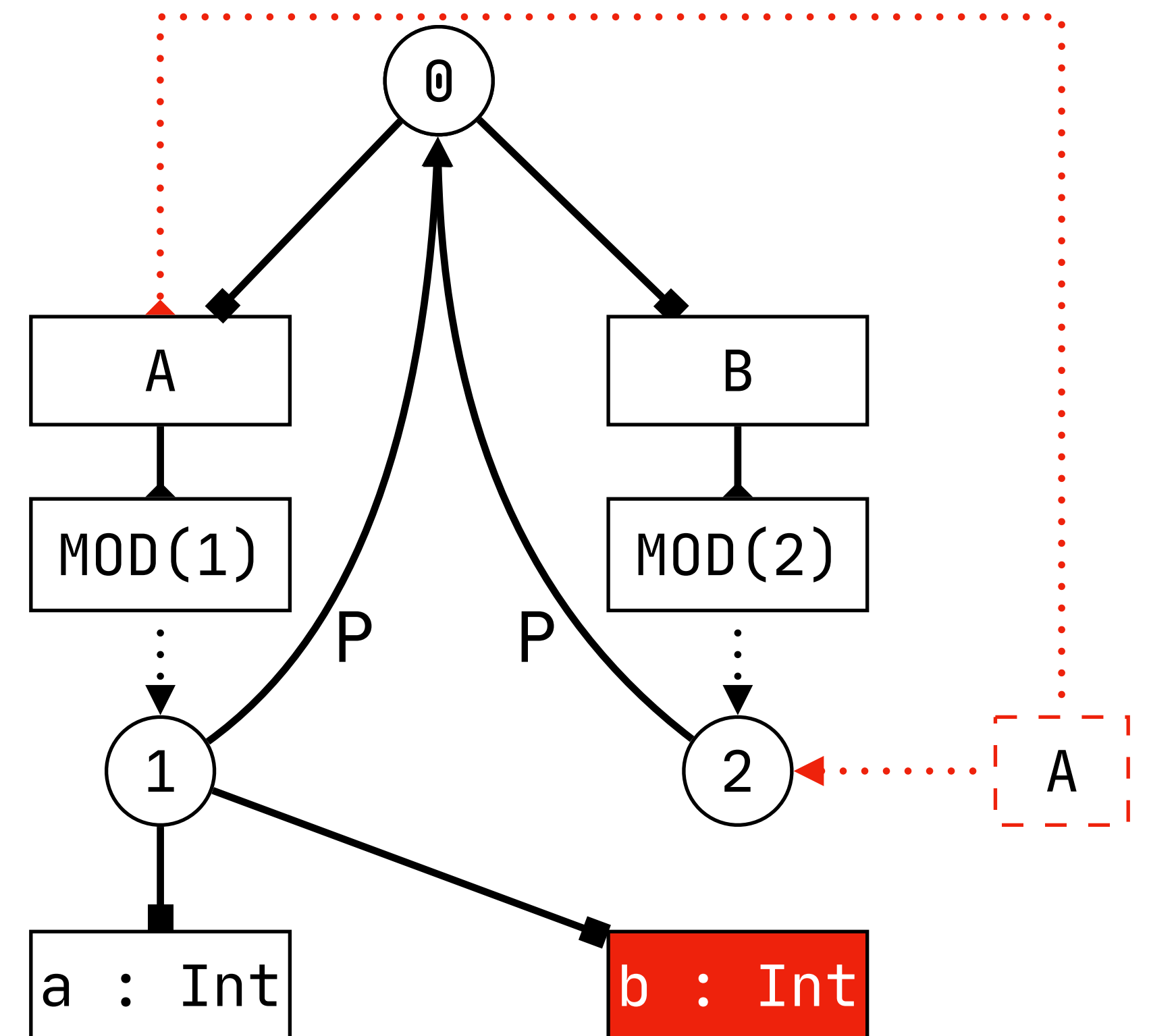
rules // extend remote

```
declOk(s, ExtendRemote(m, x, e)) :- {s_mod T}
  typeOfModRef(s, m) = MOD(s_mod),
  typeOfExp(s, e) = T,
  declareVar(s_mod, x, T).
// no permission to extend
```

Add declaration to scope
obtained through a query

This is not allowed in Statix

A predicate can only extend scopes over which it has
ownership, i.e. that it creates or gets passed down as an
argument



Type-Dependent Name Resolution / Records

Records

signature

constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```

Record Type: Scope as Type

signature constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

scope as type

rules // record type

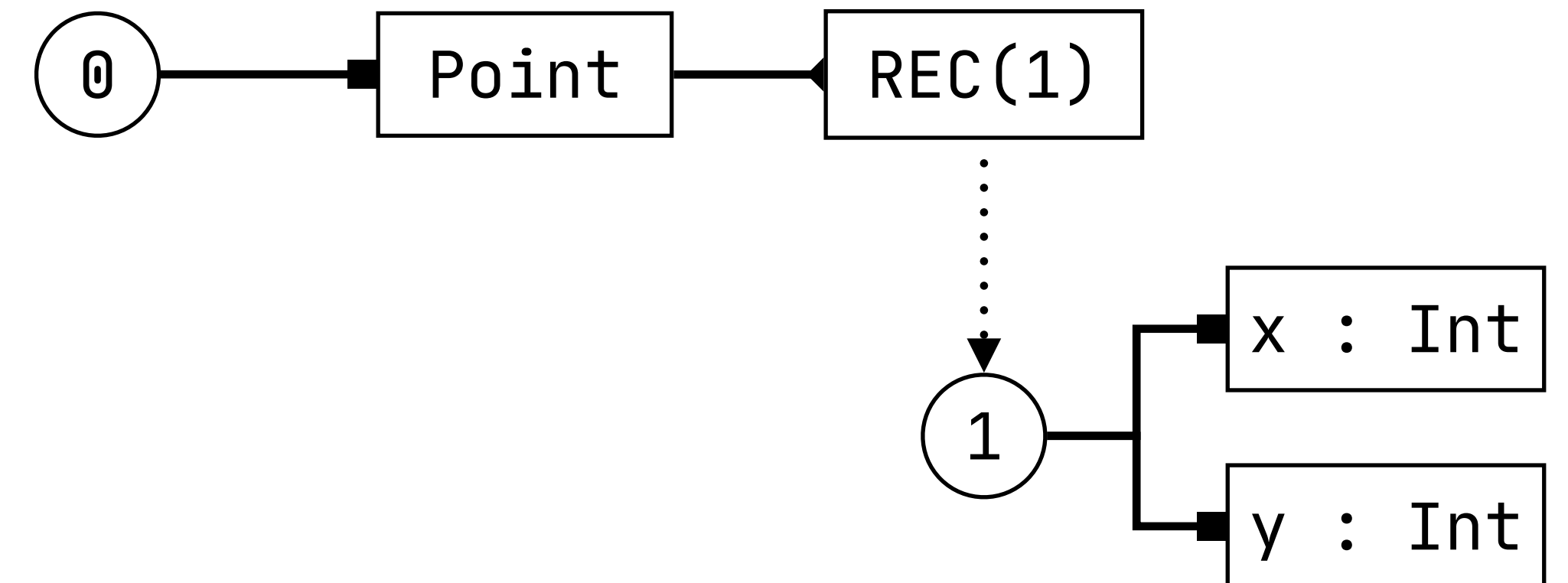
```
declOk(s, Record(x, fdecls)) :- {s_rec}
  new s_rec,
  fdeclsOk(s_rec, s, fdecls),
  declareType(s, x, REC(s_rec)).
```

```
fdeclOk(s_bnd, s_ctx, FDecl(x, t)) :- {T}
  typeOfType(s_ctx, t) = T,
  declareVar(s_bnd, x, T).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



Record Construction & Initialization

signature

constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

rules // record construction

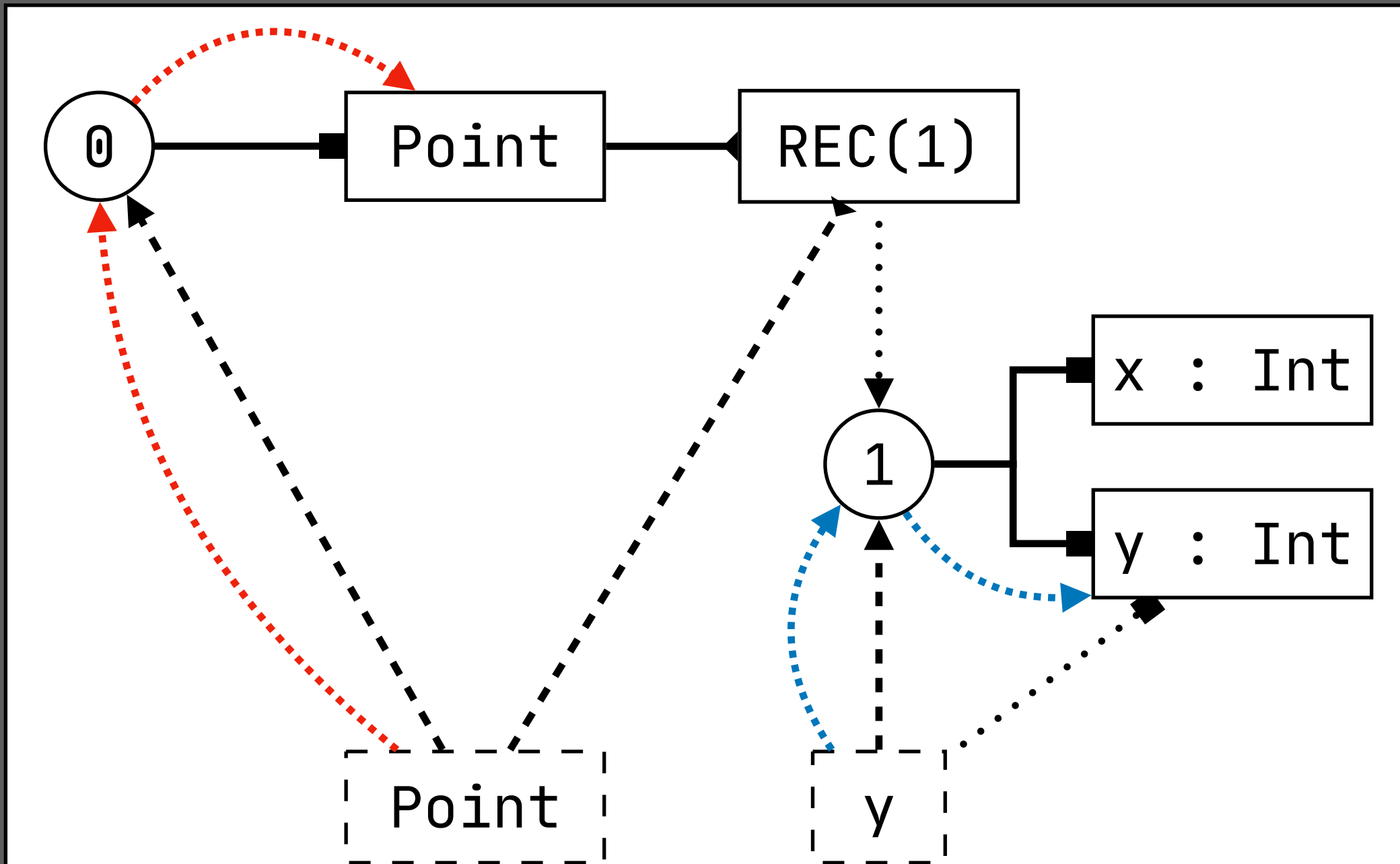
```
typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).
```

```
fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



Type-Dependent Name Resolution

signature

constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

rules // record construction

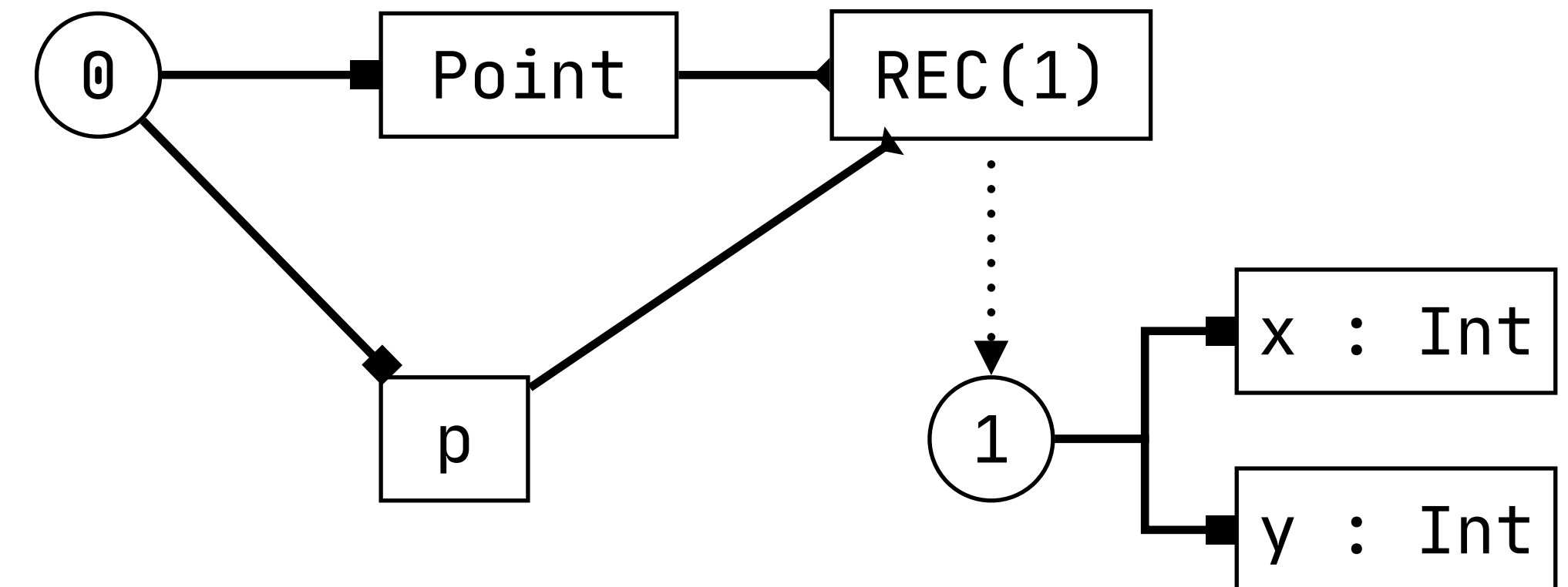
```
typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).
```

```
fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



Type-Dependent Name Resolution

signature

constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

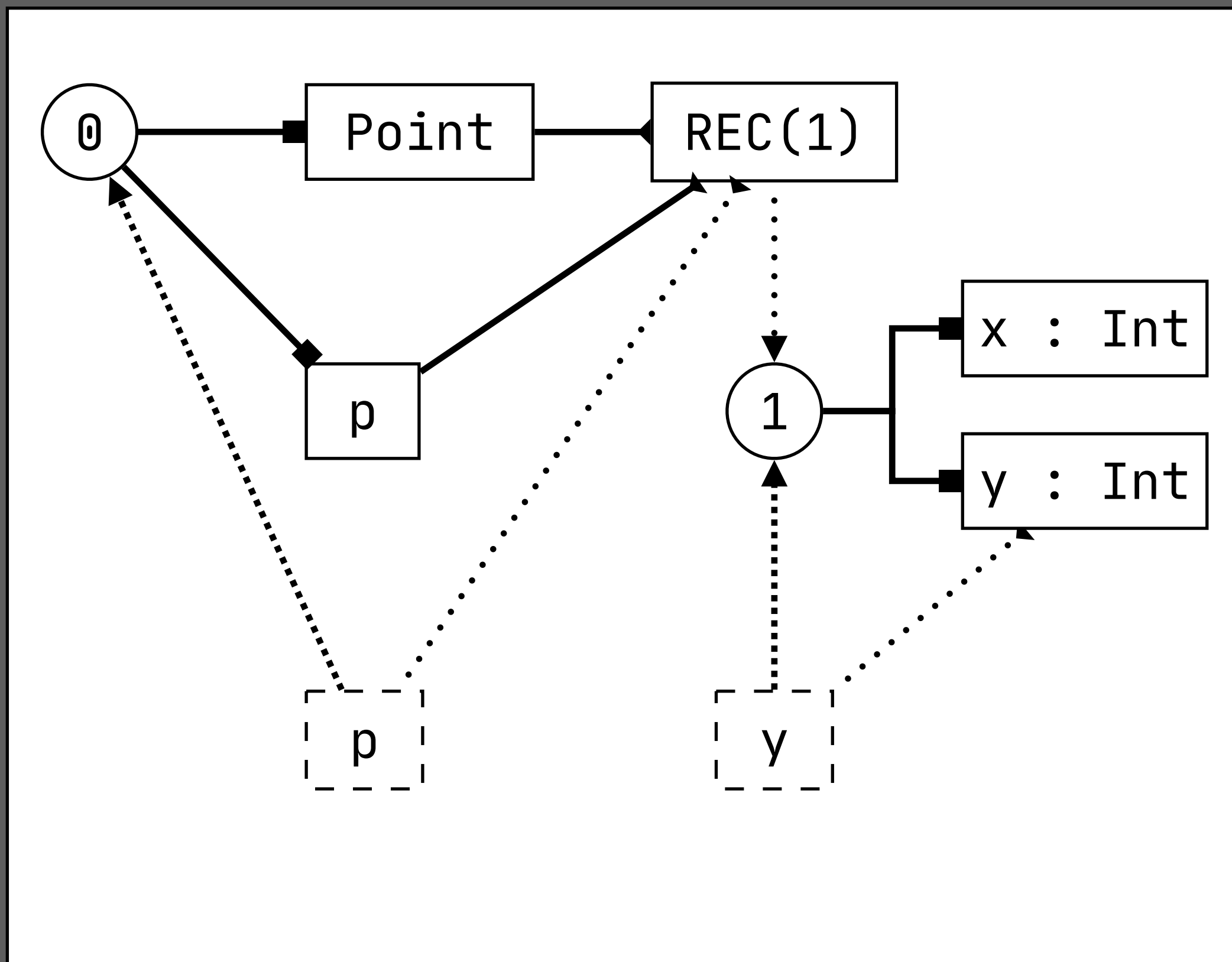
rules // record projection

```
typeOfExp(s, Proj(e, x)) = T :- {p d s_rec S}
  typeOfExp(s, e) = REC(s_rec),
  typeOfVar(s_rec, x) = T.
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



With

signature

constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

rules // with record value

```
typeOfExp(s, With(e1, e2)) = T :- {s_with s_rec}
typeOfExp(s, e1) = REC(s_rec),
new s_with, s_with -P→ s, s_with -R→ s_rec,
typeOfExp(s_with, e2) = T.
```

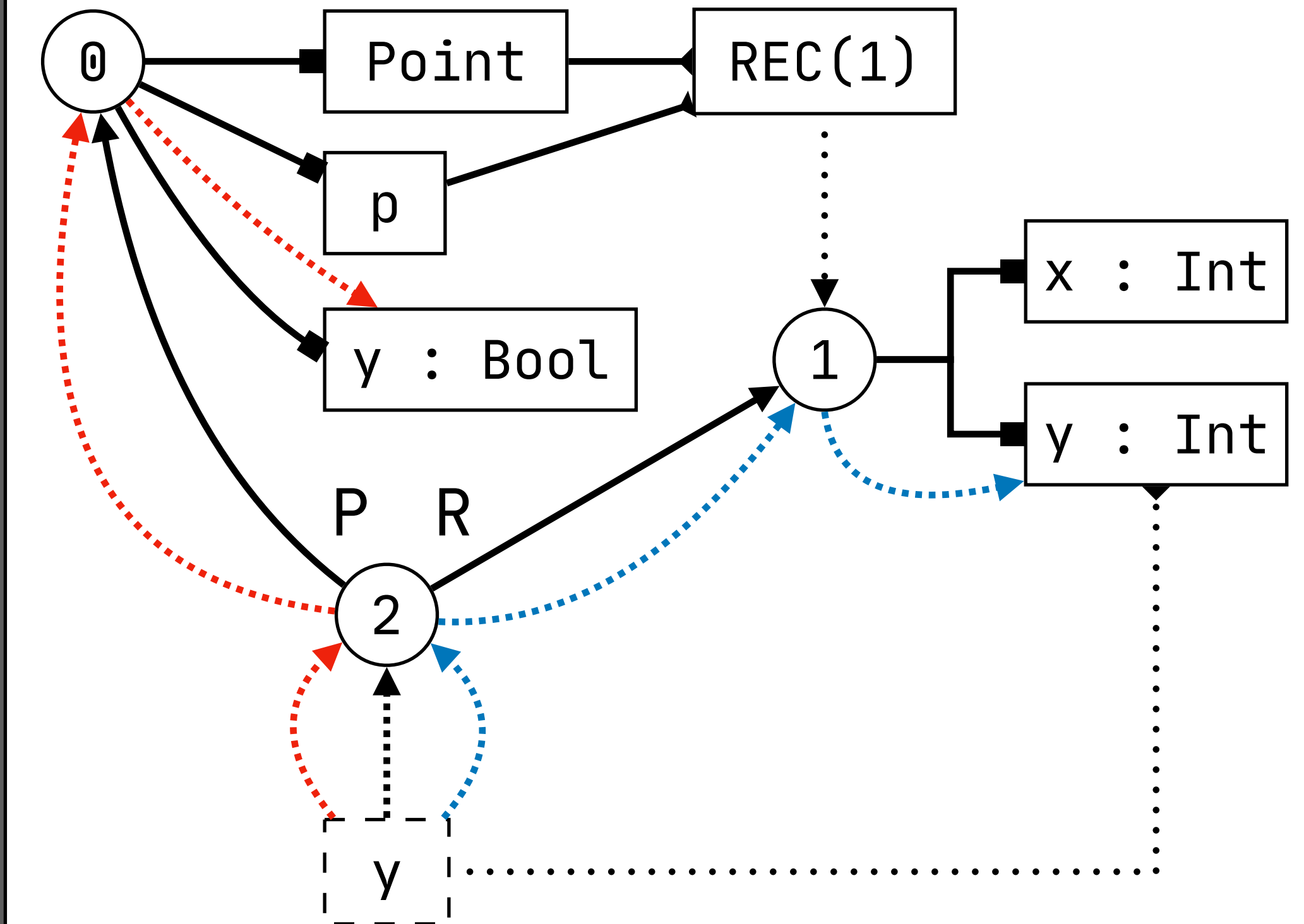
```
resolveVar(s, x) = ps :-
  query var
  filter P* R* and { x' :- x' = x }
  min $ < P, R < P and true
  in s ↦ ps.
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{x = 1, y = 2}
```

```
def y = true
```

```
> with p do y
```



Scheduling Constraint Resolution

Scheduling in Type Checkers

Type checker constructs scope graph

- Module, variable declarations
- Module imports
- Scopes

Type checker queries scope graph

- Type of variable reference

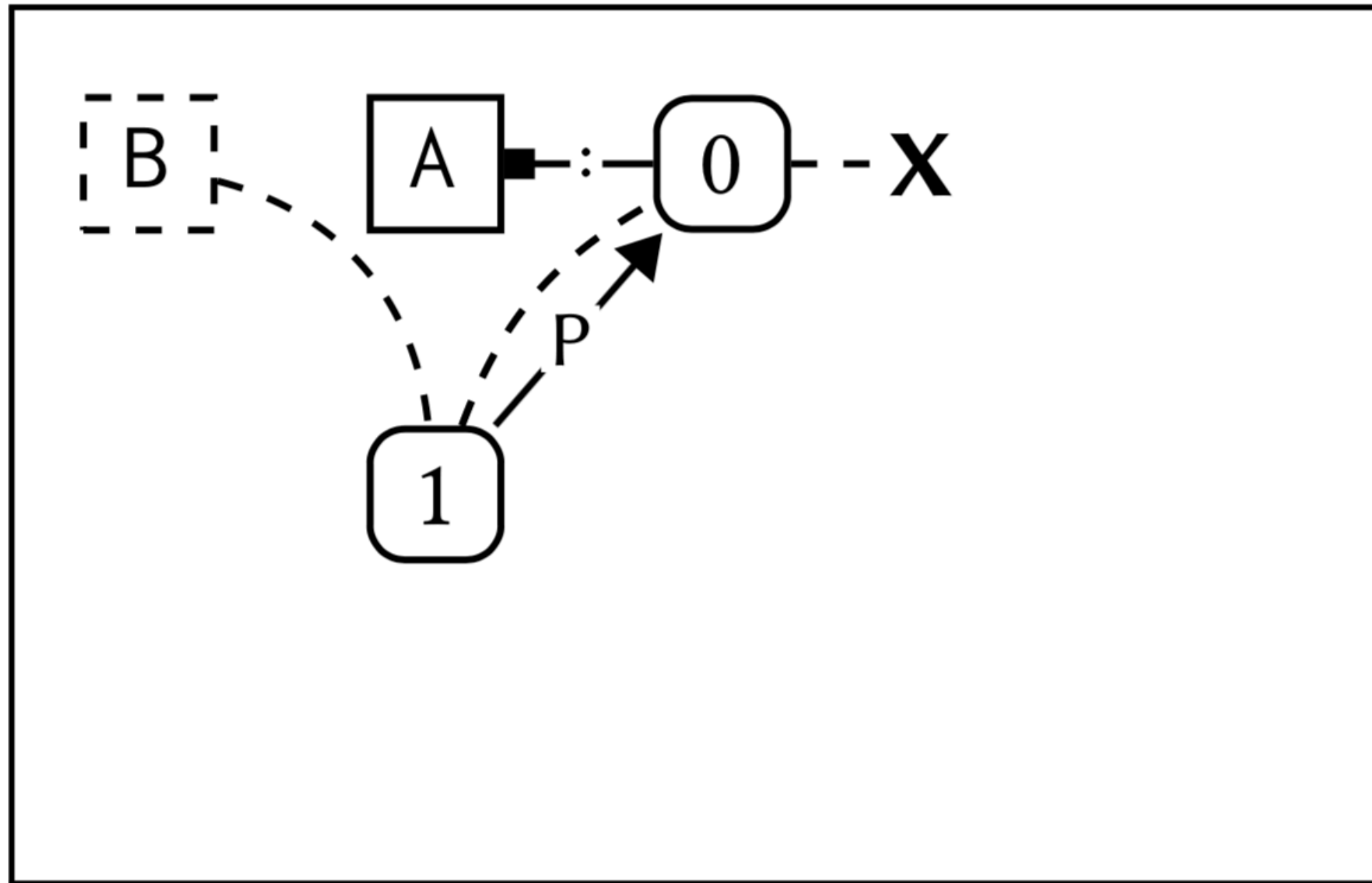
Scope graph construction depends on queries

- Imports require name resolution of module name

When is it safe to query the scope graph?

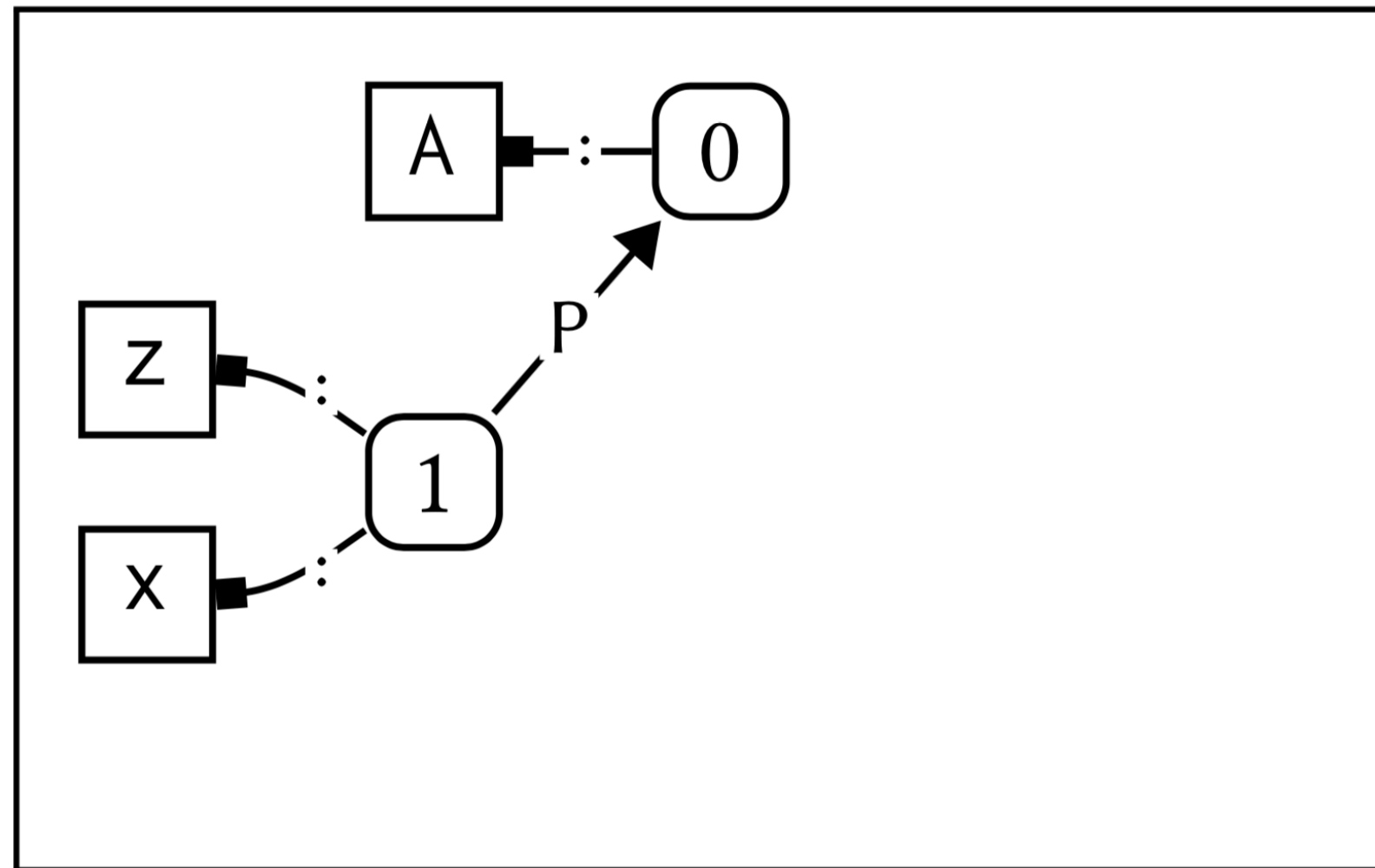
- In what order should type checker perform construction, querying?

A Single Stage Type Checker (Fails)

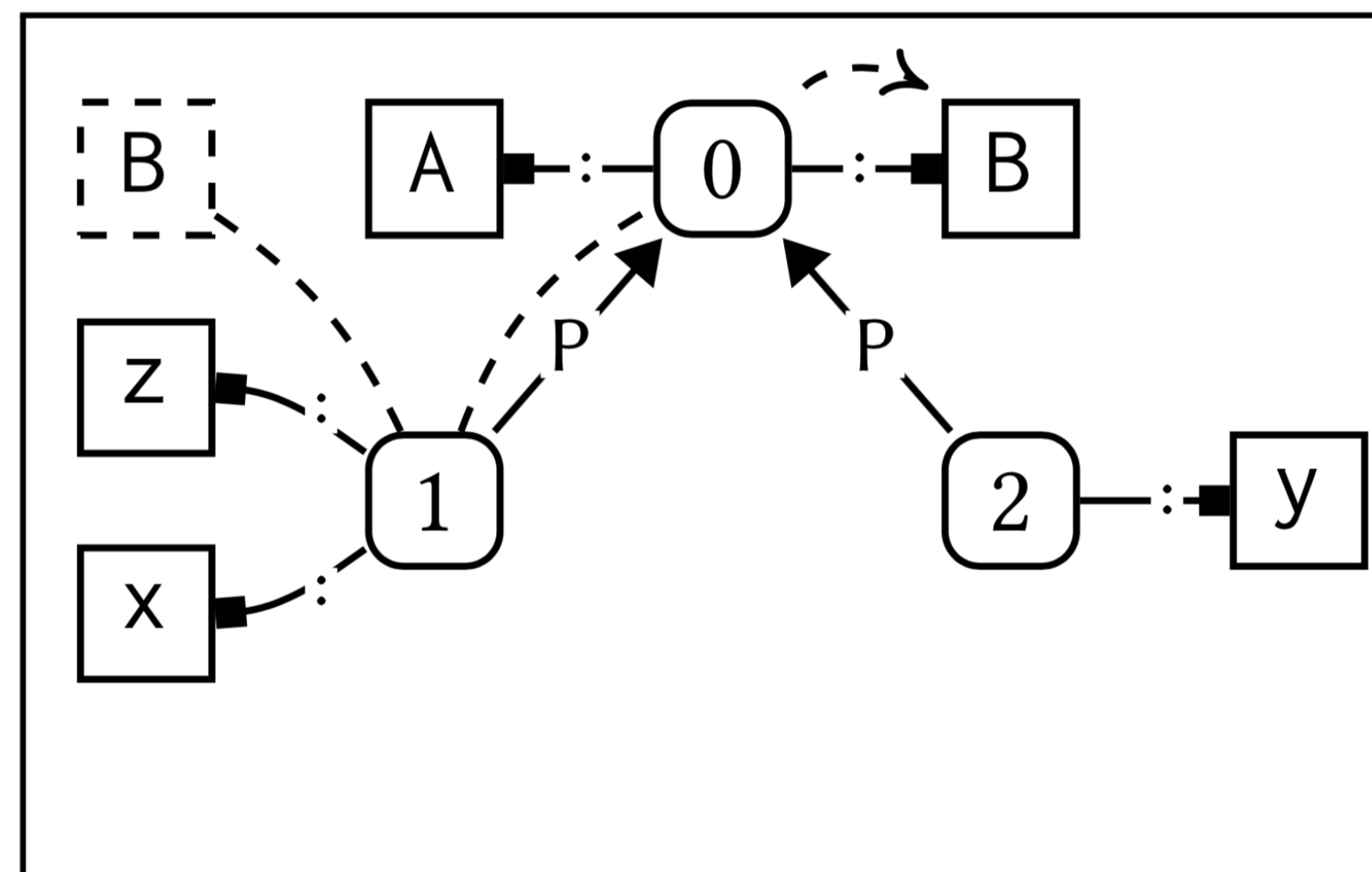


```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```

A Two Stage Type Checker: Stage 1 (Build Module Table)



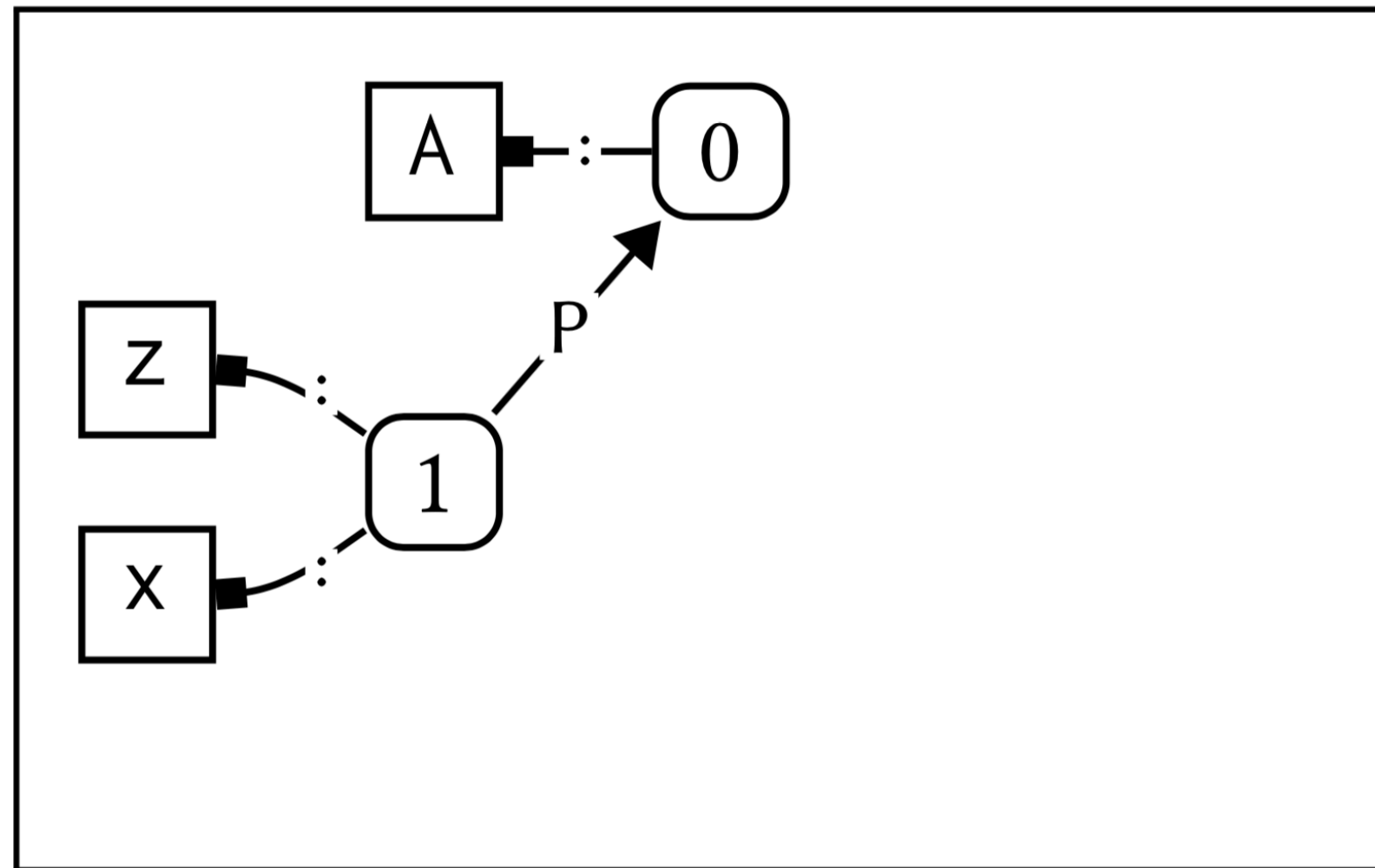
(1)



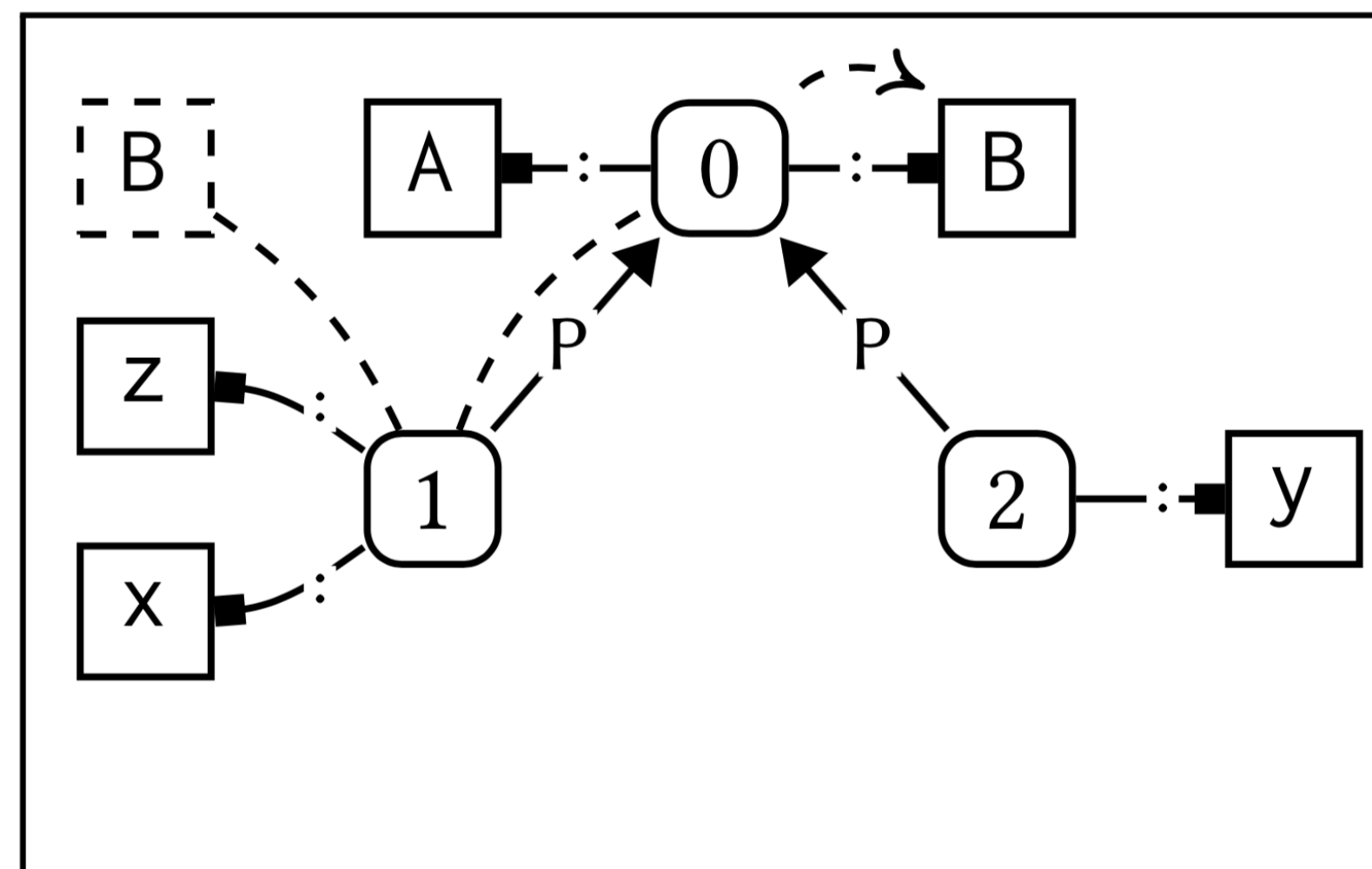
(2)

```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```

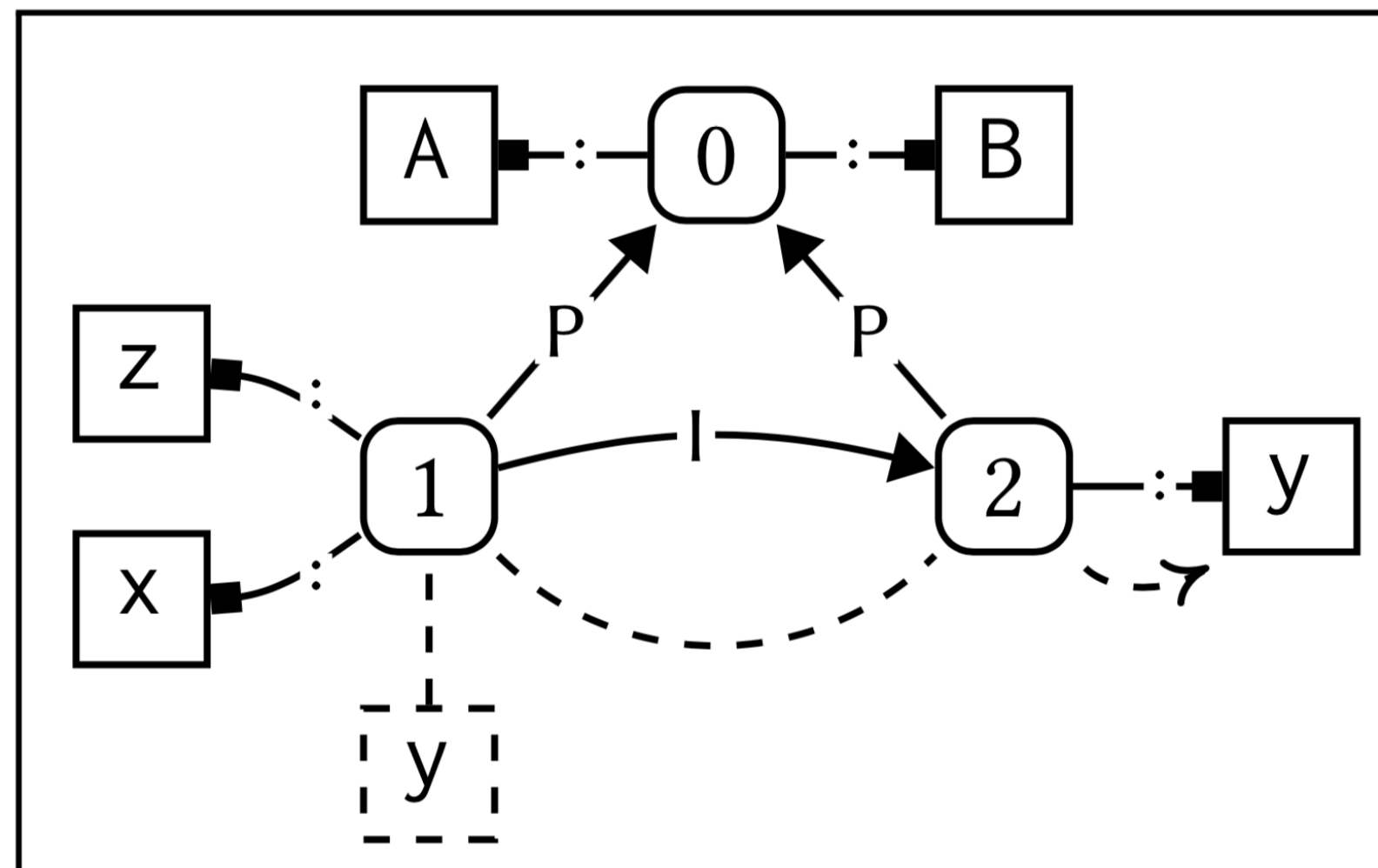
A Two Stage Type Checker: Stage 2 (Check Modules)



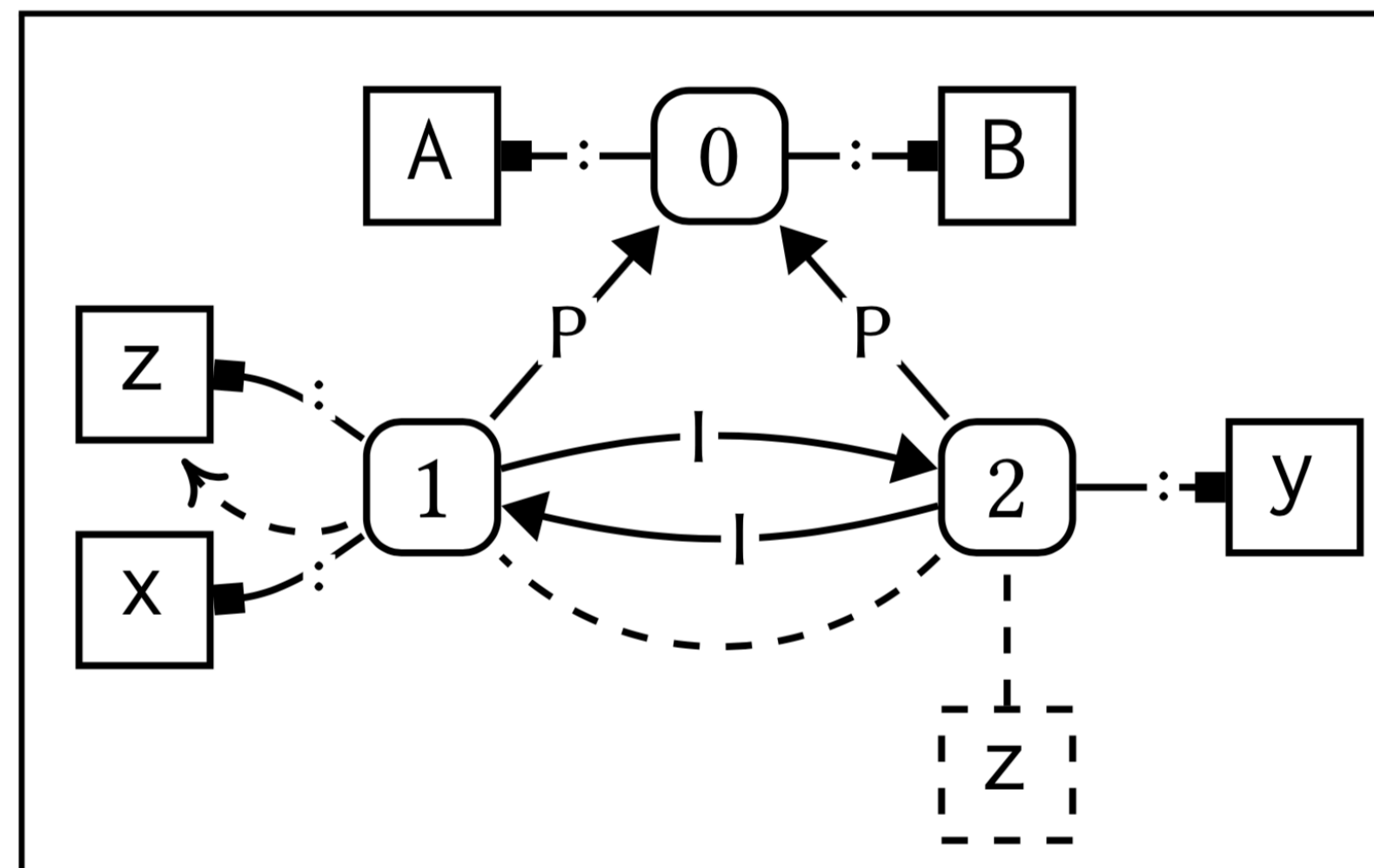
(1)



(2)



(3)

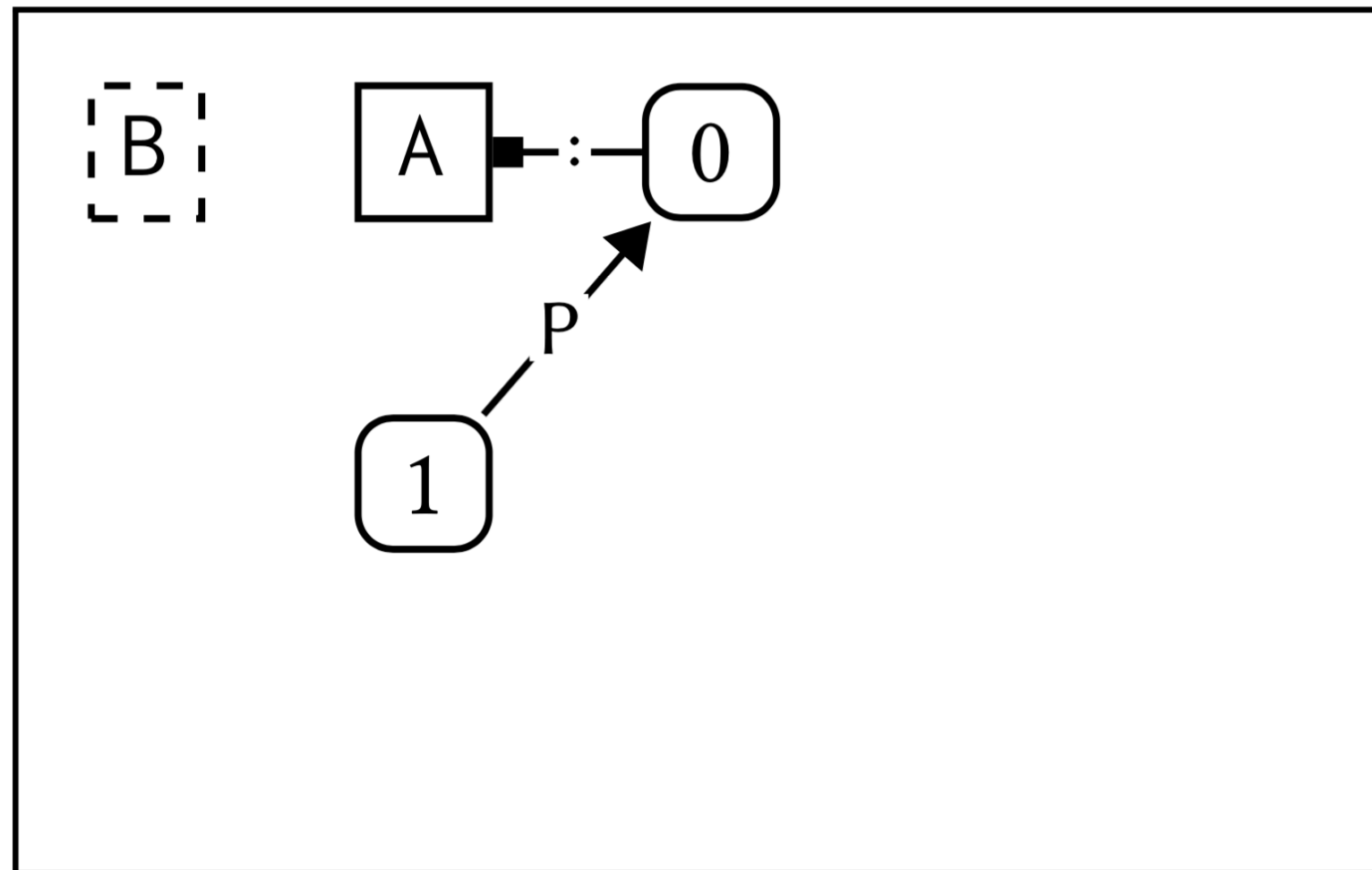


(4)

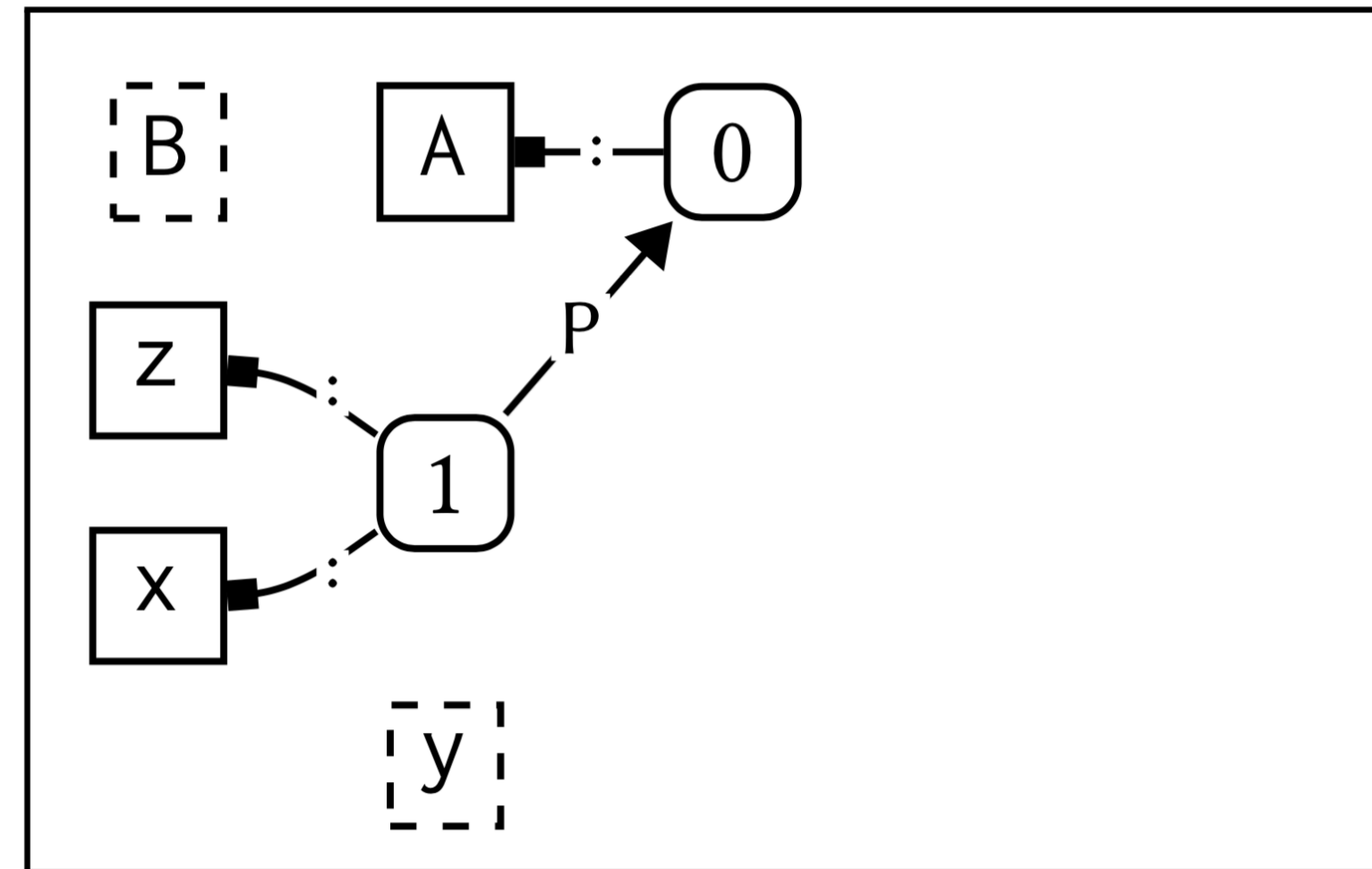
Requires that imports are resolved before variable references

```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```

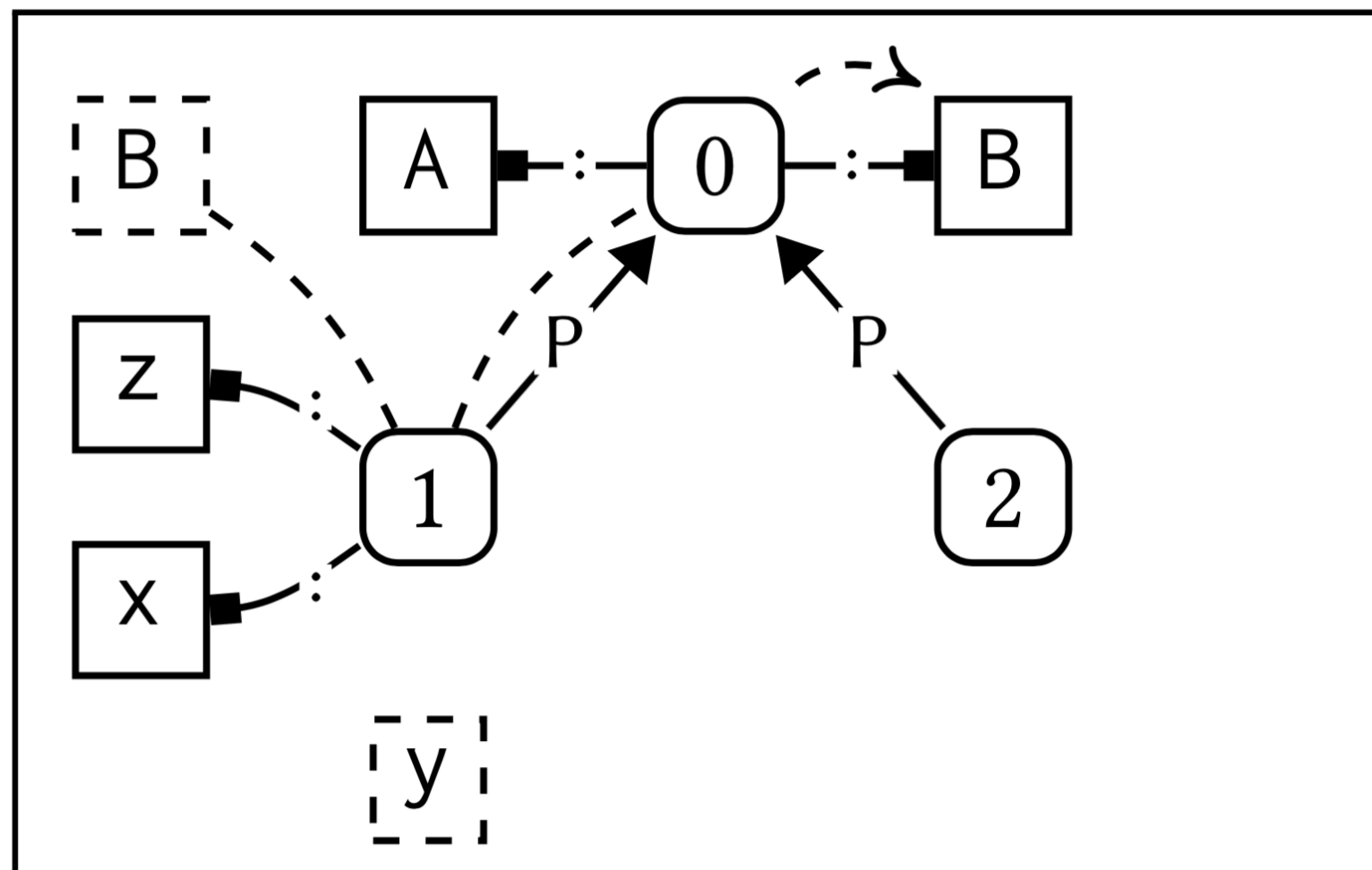

Dynamic



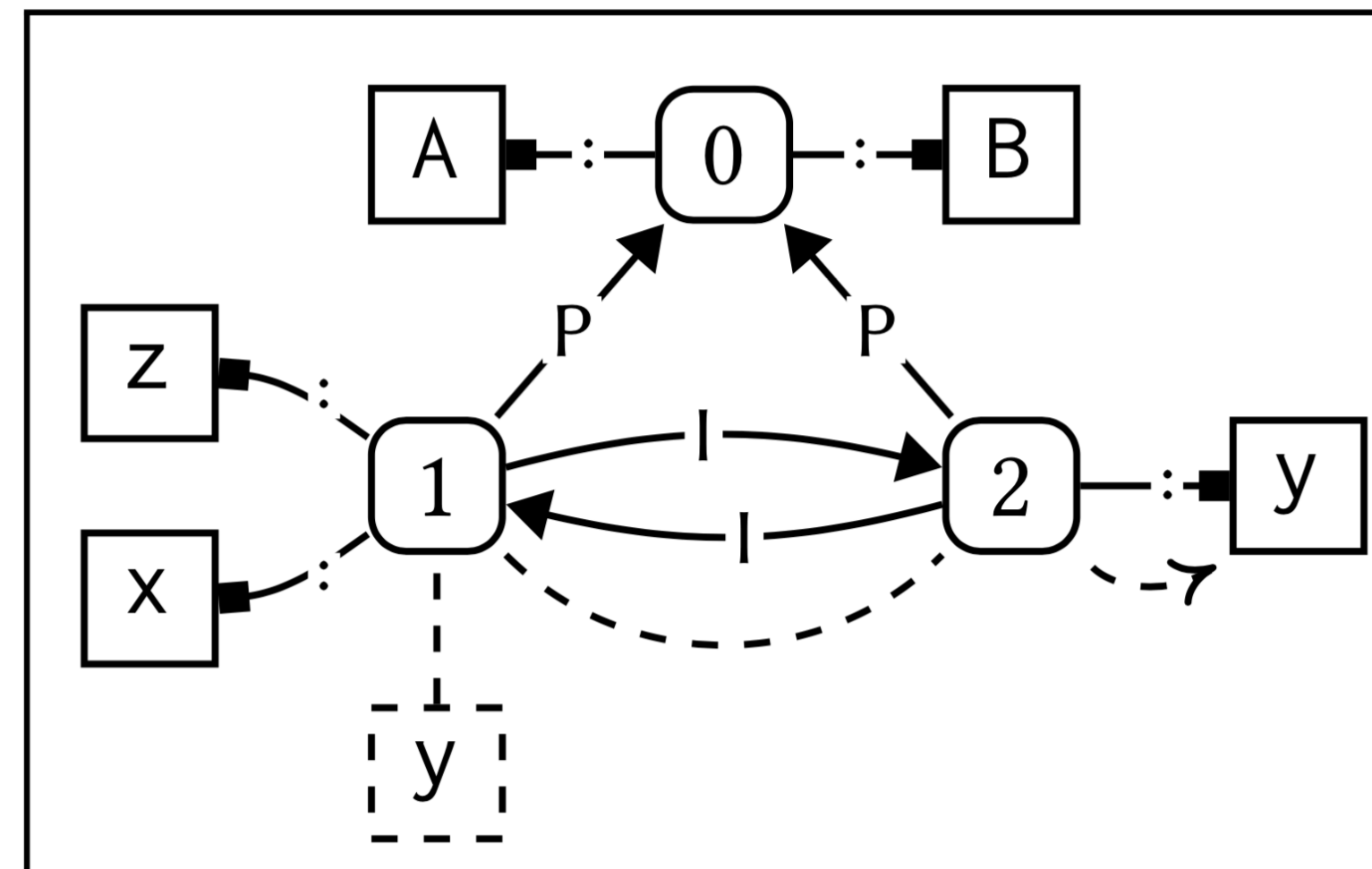
(1)



(2)



(3)

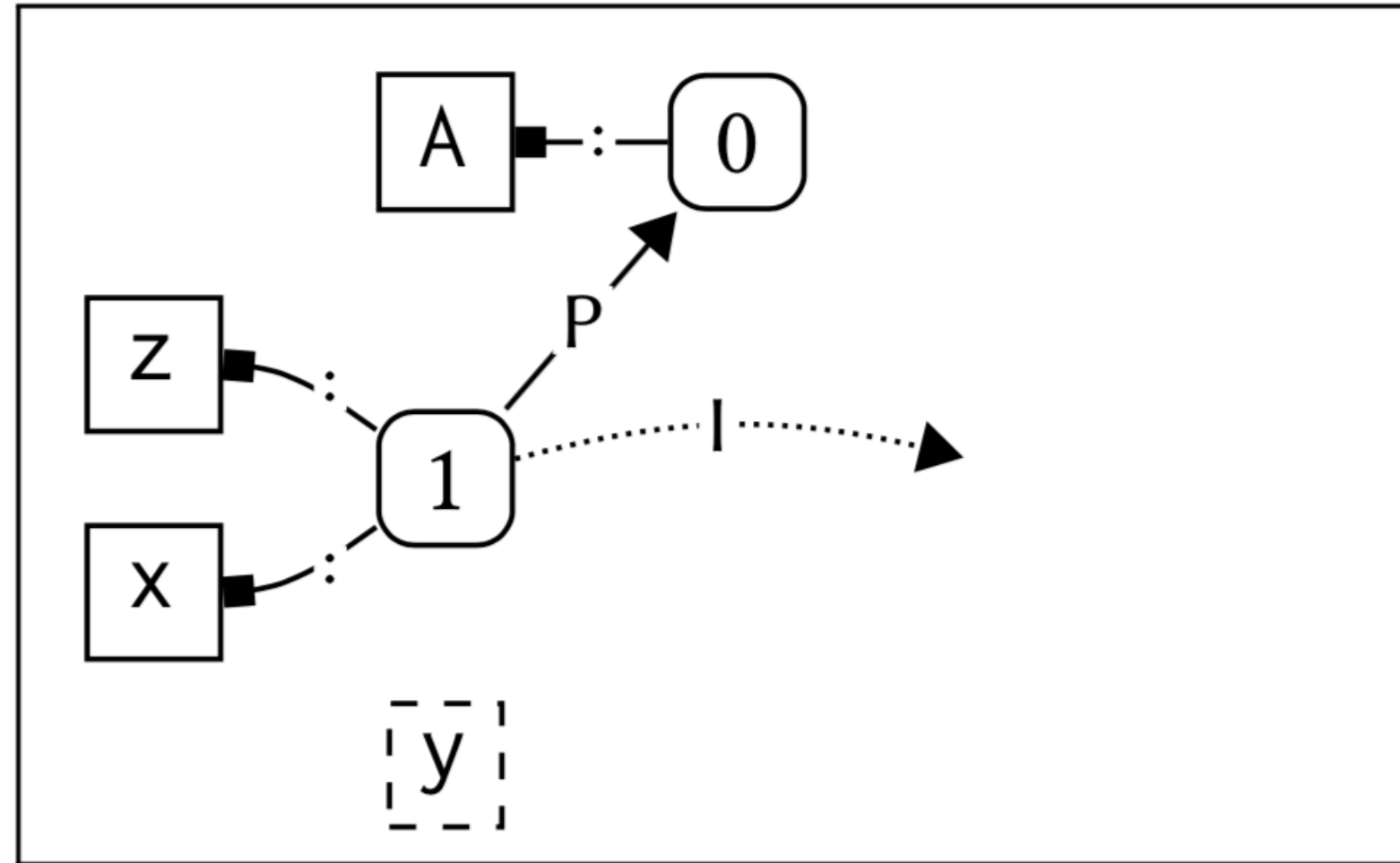


(4)

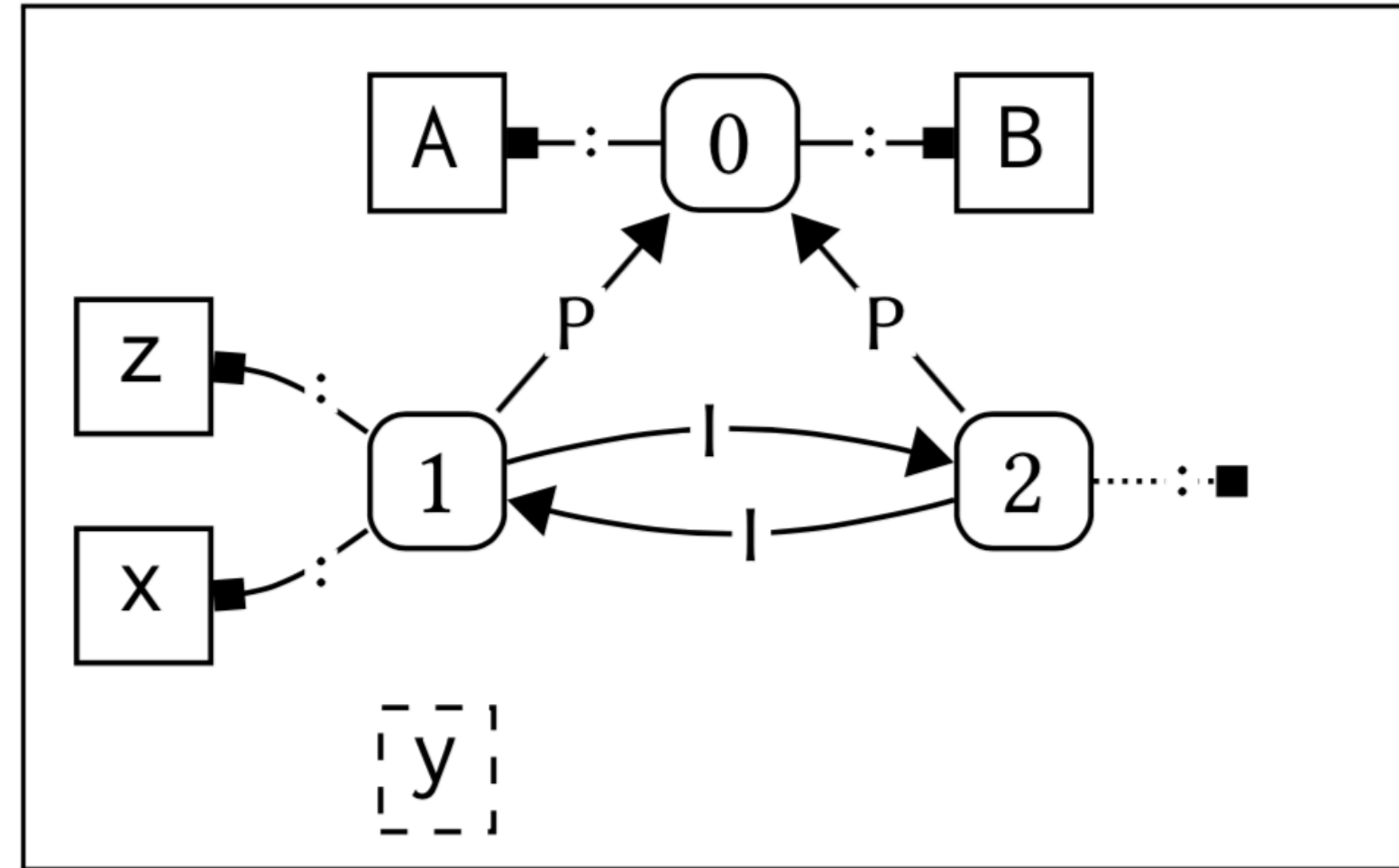
When do we have sufficient information to answer a query?

```
module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
```

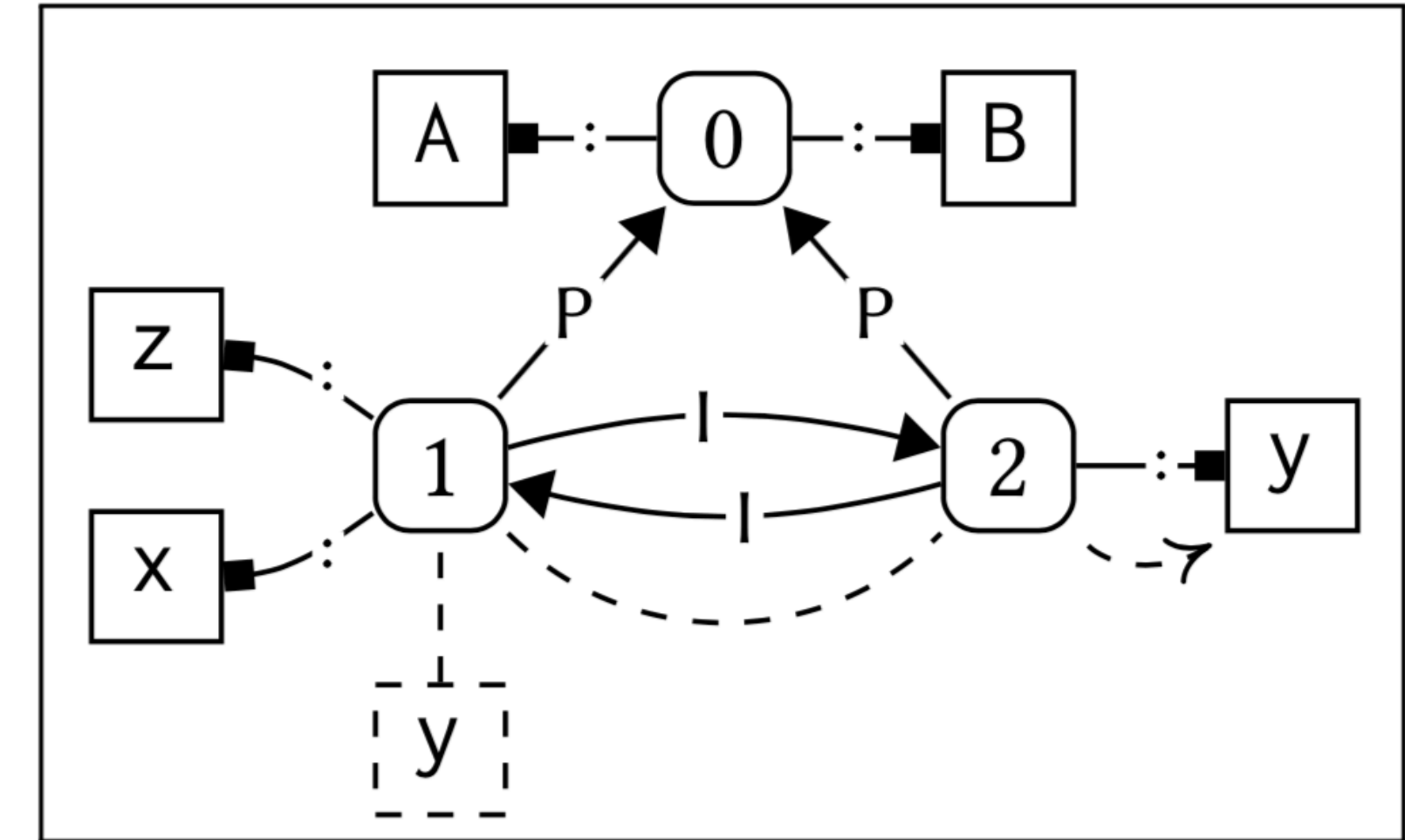
Critical Edges



(a) Intermediate scope graph



(b) Intermediate scope graph



(c) Final scope graph

```

module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
    
```

(Weakly) Critical Edges

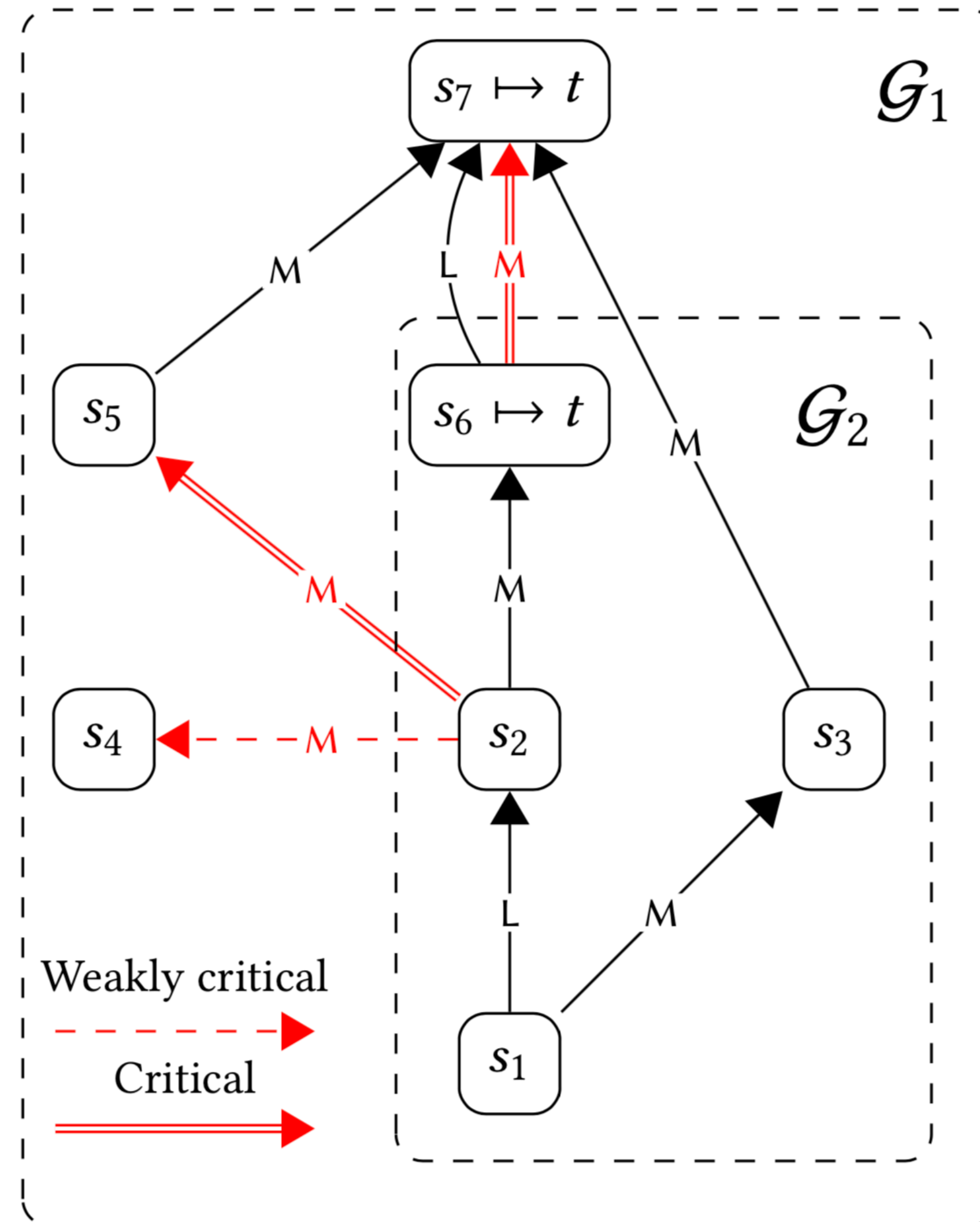


Fig. 11. (Weakly) critical edges for the query $s_1 \xrightarrow{LM^*} \gg_R D$, assuming $t \in D$

Scope graph represents context information

- Type checker constructs scope graph
- Type checker queries scope graph
- Scope graph construction depends on queries

When is it safe to query the scope graph?

- When there are no more critical edges *for this query*

Conclusion

Modeling Name Binding with Scope Graphs

- Scopes + declarations + edges (reachability)
- Queries to resolve references
- Visibility policies = path disambiguation
 - path well-formedness + path specificity
- Model wide range of name binding policies

Scheduling Constraint Resolution [OOPSLA'20]

- Declarative: no explicit scheduling / staging / stratification of traversal
- Only perform queries when outcome will not be changed (capture)
- Don't extend scopes 'remotely' (permission to extend)

Examples in this lecture: [ESOP'15] + [PEPM'16] in Statix

Scopes as Types [OOPSLA'18]

Applications

- Structural (sub)typing (records)
- Parametric polymorphism (System F)
- Nominal subtyping (FJ)
- Generic classes (FGJ)

Under investigation

- Make those encodings less clunky
- Hindley-Milner: inference supported, but how to generalize?

Ongoing Work

Incremental multi-file analysis

- Given a change, which files need to be reanalyzed?

Code completion [vision: ECOOP 2019]

- Given a hole, what can be filled in?
- Expressions, but also declarations, ...

Refactoring

- Renaming, inlining, ...

Other editor services

- Quick fixes, ...

Random term generation

- Generate program that is well-typed and well-bound