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Q1-)

a) $\boxed{T(n) = 7T(n/2) + n^3}$ recurrence for the algorithm

b) as the recurrence has the form

$T(n) = aT(n/b) + F(n)$, we can use master method to solve this recurrence

$a = 7$
$b = 2$

$n^{\log_b a}$, $F(n) = n^3$

$n^{\log_2 7} = n^{2,807}$

as $F(n) = \Omega(n^{\log_b a + \varepsilon})$

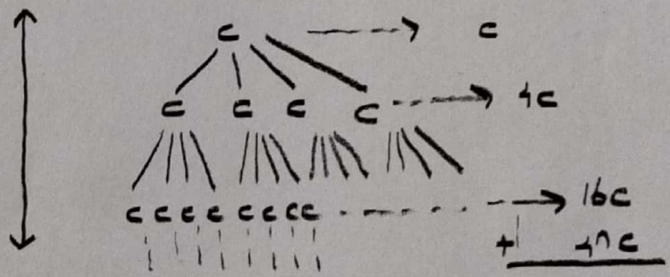
$$\boxed{T(n) = \Theta((F(n)) = \Theta(n^3)}$$

Q2-)

a) Recurrence relation for this algorithm is

$$\boxed{T(n) = T(n-1) + T(n/3) + T(n/5) + T(n/7) + \Theta(1)}$$

b) recursion tree for this recurrence is as follows



longer path to leaves determines heigth $h = n$

shortest path to leaves $= \log_7 n$

$\log_7 n < h < n$

$$c(\underbrace{1 + 4 + 4^2 + \dots + 4^{n-1}}_{\text{geometric series}})$$

$$c\left(\frac{4^{n-1+1} - 1}{4 - 1}\right) = c\left(\frac{4^n - 1}{3}\right)$$

$$\boxed{T(n) = O(4^n)}$$

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Q3-)

$$T(n) = T(n/2) + T(n/4) + T(n/8) + n^2$$

$T(n) \leq cn^2$

$T(n/2) \leq c\left(\frac{n}{2}\right)^2$

$T(n/4) \leq c\left(\frac{n}{4}\right)^2$

$T(n/8) \leq c\left(\frac{n}{8}\right)^2$

Inductive step

$$T(n) \leq c\left(\frac{n}{2}\right)^2 + c\left(\frac{n}{4}\right)^2 + c\left(\frac{n}{8}\right)^2 + n^2$$

$$\leq \frac{cn^2}{4} + \frac{cn^2}{16} + \frac{cn^2}{64} + n^2$$

$$\leq \frac{16cn^2 + 4cn^2 + cn^2}{64} + n^2$$

$$\leq \frac{21cn^2}{64} + n^2$$

$$\leq n^2\left(\frac{21}{64}c + 1\right)$$

$\frac{21}{64}c + 1 > 0$ $\qquad$ $\frac{21}{64}c > -1$

$c > -\frac{64}{21}$

basis step

we can take any $c > 0$, lets take $c = 1$, then

$T(n) \leq cn^2$

$T(1) \leq 1 . 1^2$

$= 1$

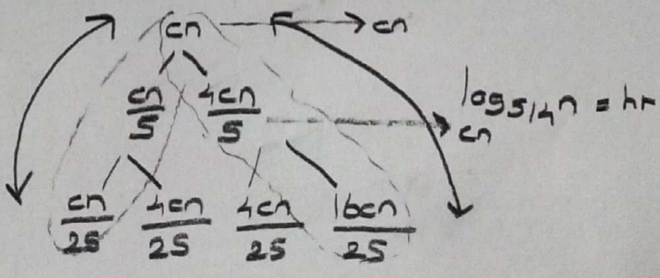
Q4-) If original list size is n, recurrence relation is as follows for worst-case

$$T(n) = T(n/5) + T(4n/5) + \Theta(n)$$

$h_l = \log_5 n$

$cn\log_5 n \rightarrow$ upper bound

$cn\log_{5/4} n \rightarrow$ lower bound



$$\boxed{cn\log_5 n \leq T(n) \leq cn\log_{5/4} n}$$