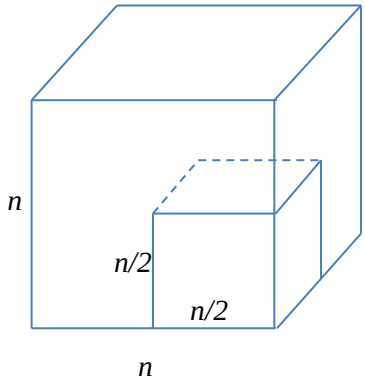
CENG 218 Spring 2023 Homework 1 Answer Sheet
Due date: 17th of April 2023, 23:59

Note: Submit photos or scans of your solutions via MS-Teams. Homeworks submitted after the due date will not be evaluated. Write all your explanations and comments in English! Text in Turkish will not be evaluated. Submit a single pdf file or a single zip file.

Q1 (30 points).



Imagine an algorithm that tries to compute a value regarding a 3D *n* x *n* x *n* volume. This algorithm is a recursive algorithm. At each step, first it checks the values of all the elements in this 3D volume one by one. After this, it divides the search space into 8 equal pieces of size *n*/2 x *n*/2 x *n*/2 and it eliminates one of these pieces. Algorithm recursively calls itself for the other 7 pieces.
a) Write the recurrence for this algorithm.
b) Find the time complexity (big-O or big-Theta estimate) of this algorithm by using any recurrence-solving method.

Q2 (20 points).

A divide-and-conquer algorithm to solve a problem divides the problem into four subproblems. For an n-sized problem, sizes of the subproblems are $n-1$, $n/3$, $n/5$ and $n/7$.
Also a constant time is spent to divide the problem and combine the solutions.

a) Write the recurrence relation for this algorithm.

b) Solve this recurrence with recurrence-tree method and determine an asymptotic upper bound.

Q3 (25 points).

Consider the following recurrence

$T(n) = T(n/2) + T(n/4) + T(n/8) + n^2$

Use substitution method to verify that this recurrence is $O(n^2)$.

Note: Do not forget to evaluate the base case for the substitution method solution.

Q4 (25 points).

We have seen in the lecture that we need to randomize Quicksort to avoid a complexity of $\Theta(n^2)$.

Let's assume we have a randomized version that guarantees to select a pivot such that, after partitioning, left and right subarrays would not exceed 4/5 of the original list size.

I.e. if the original list is *A*, and left and right subarrays are *L* and *G* respectively, then

$$|L| \leq \frac{4}{5}|A| \quad \text{and} \quad |G| \leq \frac{4}{5}|A|$$

Use a recursion-tree to determine lower and upper asymptotic bounds for the worst-case time complexity of this version of Quicksort.