

Prolog and Logic Languages

Prolog

- Based on first-order predicate logic
- Original motivation: study of mechanical theorem proving
- Developed in 1970 by Colmerauer & Roussel (Marseilles) and Kowalski (Edinburgh) + others.
- Used in Artificial Intelligence, databases, expert systems.

Prolog Programs

- Program = a bunch of axioms
- Run your program by:
 - Enter a series of facts and declarations
 - Pose a query
 - System tries to prove your query by finding a series of inference steps
- “Philosophically” declarative
- Actual implementations are deterministic

Horn Clauses (Axioms)

- Axioms in logic languages are written:

$$H \leftarrow B1, B2, \dots, B3$$

Facts = clause with head and no body.

Rules = have both head and body.

Query – can be thought of as a clause with no body.

Terms

- H and B are terms.
- Terms =
 - Atoms - begin with lowercase letters: x, y, z, fred
 - Numbers: integers, reals
 - Variables - begin with capital letters: X, Y, Z, Alist
 - Structures: consist of an atom called a functor, and a list of arguments. ex. `edge(a,b)`. `line(1,2,4)`.

Lists

- `[]` % the empty list
- `[1]`
- `[1,2,3]`
- `[[1,2], 3]` % can be heterogeneous.

The `|` separates the head and tail of a list:
is `[a | [b,c]]`

Backward Chaining

START WITH THE GOAL and work backwards, attempting to decompose it into a set of (true) clauses.

This is what the Prolog interpreter does.

Forward Chaining

- START WITH EXISTING FACTS and clauses and work forward, trying to derive the goal.
- Unless the number of facts is very small and the number of rules is large, backward chaining will probably be faster.

Searching the database as a tree

- **DEPTH FIRST** - finds a complete sequence of propositions for the first subgoal before working on the others. (what Prolog uses)
- **BREADTH FIRST** - works on all subgoals in parallel.
- The implementers of Prolog chose depth first because it can be done with a stack (expected to use fewer memory resources than breadth first).

Unification

```
likes(bob,fondue) .  
likes(sue,fondue) .  
friends(X,Y) :-  
    likes(X, Something) ,  
    likes(Y, Something) .
```

% Y is a variable, find out who is friends with Sue.

```
?- friends(sue,Y) .
```

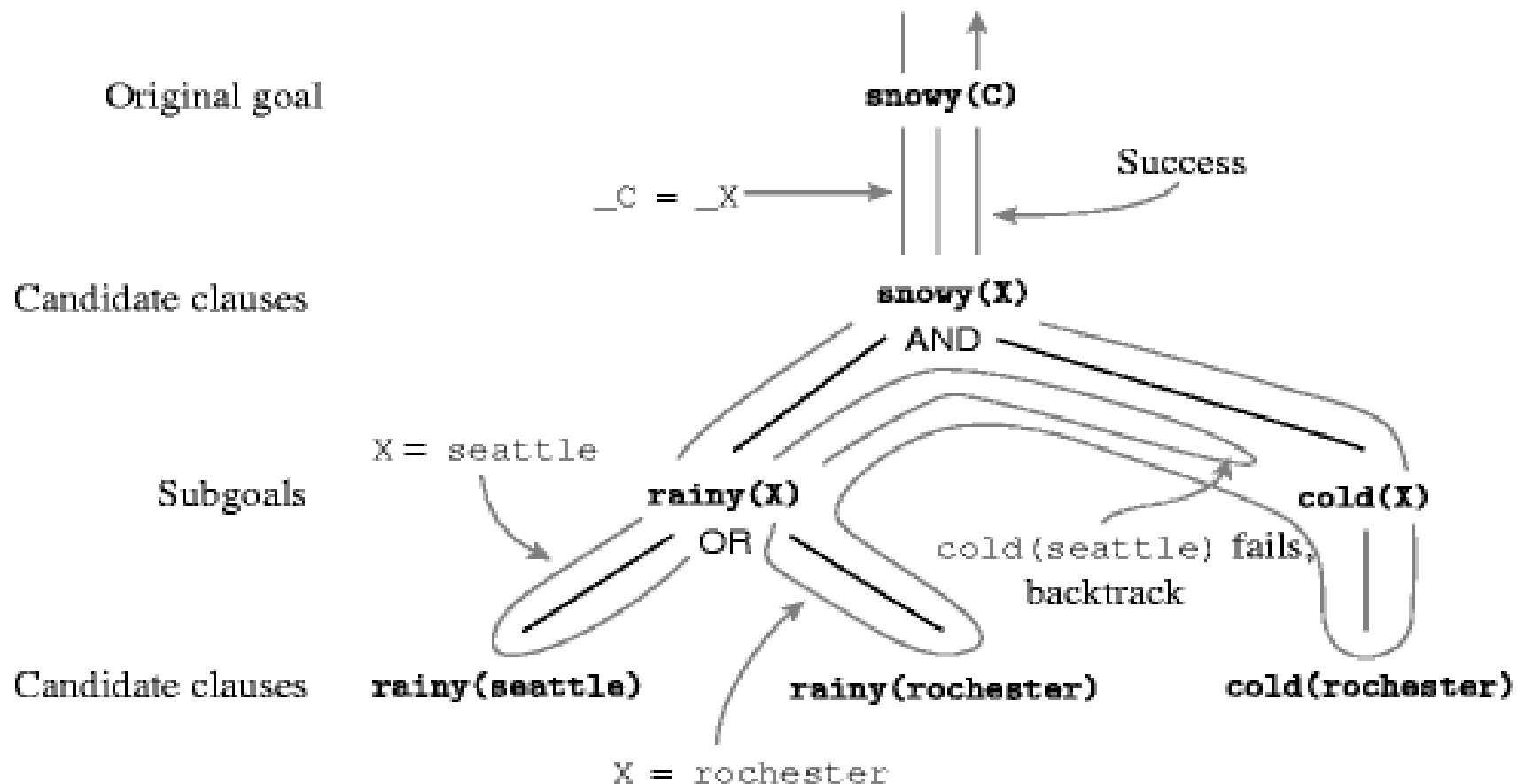
```
friends(sue,Y) :-    % replace X with sue in the clause  
    likes(sue, Something) ,  
    likes(Y, Something) .
```

We replace the 1st clause in friends with the empty body of the likes(sue,fondue) clause to get:

```
friends(sue,Y) :-  
    likes(Y, fondue). %- now we try to satisfy the second goal.  
(Finally we will return an answer to the original query like:Y=bob)
```

Backtracking search

```
rainy(seattle).  
rainy(rochester).  
cold(rochester).  
snowy(X) :- rainy(X), cold(X).
```



Order of Rule Evaluation

```
edge(a, b) . edge(b, c) . edge(c, d) .  
edge(d, e) . edge(b, e) . edge(d, f) .  
path(X, X) .  
path(X, Y) :- edge(Z, Y), path(X, Z) .
```

- The last two clauses tell us how to determine whether there is a path from node X to node Y.

Order of Rule Evaluation

```
edge(a, b) . edge(b, c) . edge(c, d) .  
edge(d, e) . edge(b, e) . edge(d, f) .  
path(X, X) .  
path(X, Y) :- edge(Z, Y) , path(X, Z) .
```

- The last two clauses tell us how to determine whether there is a path from node X to node Y.
- If we were to reverse the order of the terms on the right-hand side of the final clause?

```
edge(a, b) . edge(b, c) . edge(c, d) .  
edge(d, e) . edge(b, e) . edge(d, f) .  
path(X, X) .  
path(X, Y) :- path(X, Z) , edge(Z, Y) .
```

Order of Rule Evaluation

```
edge(a, b) . edge(b, c) . edge(c, d) .  
edge(d, e) . edge(b, e) . edge(d, f) .  
path(X, X) .  
path(X, Y) :- path(X, Z) , edge(Z, Y) .
```

- The Prolog interpreter would search for a node Z that is reachable from X before checking to see whether there is an edge from Z to Y.
- The program would still work, but it would not be as efficient.
- What would happen if in addition we were to reverse the order the last two clauses?

```
edge(a, b) . edge(b, c) . edge(c, d) .  
edge(d, e) . edge(b, e) . edge(d, f) .  
path(X, Y) :- path(X, Z) , edge(Z, Y) .  
path(X, X) .
```

Improperly Ordered Declarations

```

edge(a, b). edge(b, c). edge(c, d).
edge(d, e). edge(b, e). edge(d, f).
path(X, Y) :- path(X, Z), edge(Z, Y).
path(X, X).
    
```

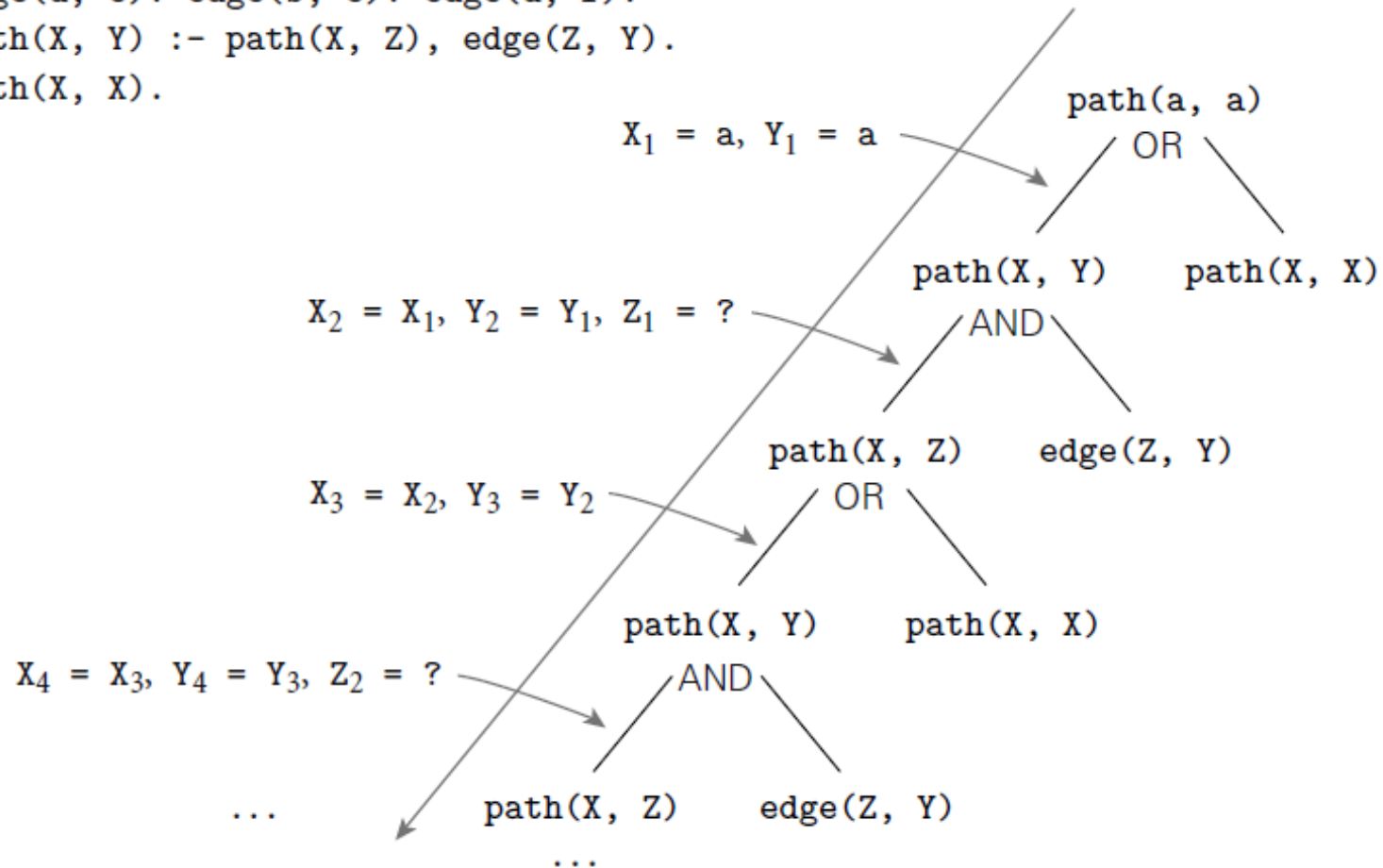


Figure 12.2 Infinite regression in Prolog. In this figure even a simple query like `?- path(a, a)` will never terminate: the interpreter will never find the trivial branch.

Backtracking

- Consider a piece-wise function
 - if $x < 3$, then $y = 0$
 - if $x \geq 3$ and $x < 6$, then $y = 2$
 - if $x \geq 6$, then $y = 4$
- Let's encode this in Prolog
 - $f(x, 0) \text{ :- } x < 3.$
 - $f(x, 2) \text{ :- } 3 \leq x, x < 6.$
 - $f(x, 4) \text{ :- } 6 \leq x.$

Backtracking

- Let's encode this in Prolog

```
f(X,0) :- X < 3.  
f(X,2) :- 3 =< X, X < 6.  
f(X,4) :- 6 =< X.
```
- Consider

```
?- f(1,Y), 2<Y.
```
- This matches the $f(X,0)$ predicate, which succeeds
 - Y is then instantiated to 0
 - The second part ($2 < Y$) causes this query to fail
- Prolog then backtracks and tries the other predicates
 - But if the first one succeeds, the others will always fail!
 - This, the extra backtracking is unnecessary

Backtracking

- We want to tell Prolog that if the first one succeeds, there is no need to try the others
- We do this with a cut:
 $f(x,0) \text{ :- } x < 3, !.$
 $f(x,2) \text{ :- } 3 \leq x, x < 6, !.$
 $f(x,4) \text{ :- } 6 \leq x.$
- The cut (!) prevents Prolog from backtracking backwards through the cut

Backtracking

- New Prolog code:
 $f(x,0) \text{ :- } x < 3, !.$
 $f(x,2) \text{ :- } 3 \leq x, x < 6, !.$
 $f(x,4) \text{ :- } 6 \leq x.$
- Note that if the first predicate fails, we know that $x \geq 3$
 - Thus, we don't have to check it in the second one.
 - Similarly with $x \geq 6$ for the second and third predicates
- Revised Prolog code:
 $f(x,0) \text{ :- } x < 3, !.$
 $f(x,2) \text{ :- } x < 6, !.$
 $f(x,4).$

Backtracking

- What if we removed the cuts:
 $f(x, 0) \text{ :- } x < 3.$
 $f(x, 2) \text{ :- } x < 6.$
 $f(x, 4).$
- Then the following query:
 $?- f(1, x).$
- Will produce three answers (0, 2, 4)

Example

- Maximum of two values without a cut:

$\text{max}(X, Y, X) \text{ :- } X \geq Y.$

$\text{max}(X, Y, Y) \text{ :- } X < Y.$

- Maximum of two values with a cut:

$\text{max}(X, Y, X) \text{ :- } X \geq Y, \text{ !}.$

$\text{max}(X, Y, Y).$

Example

- Exercise 1.3.

References

- M. L. Scott, Programming Language Pragmatics (4th Edition), Morgan Kaufmann, 2016.
- Lecture notes of Aaron Bloomfield, CS 415.