Logic and Computer Design Fundamentals

Chapter 3 – Combinational Logic Design

Part 1 – Implementation Technology and Logic Design

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Overview

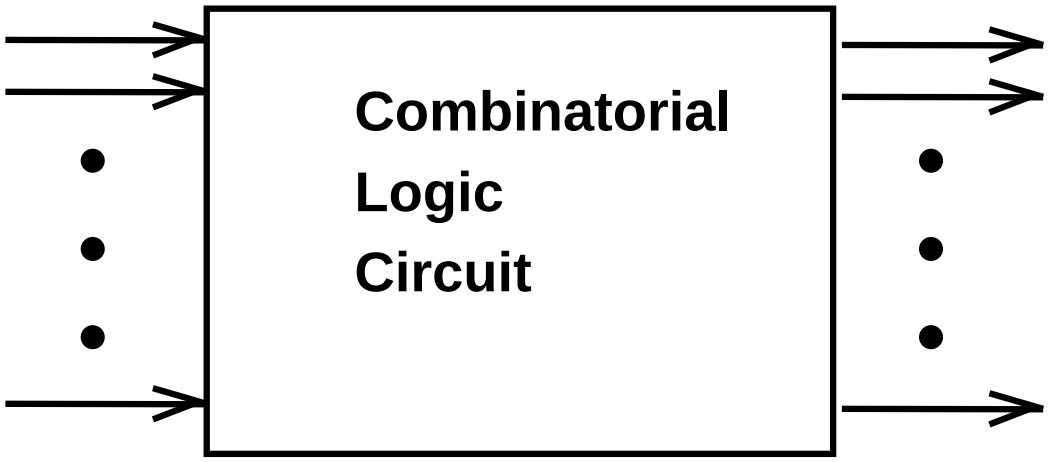
- **Part 1 – Design Procedure**
  - **Steps**
    - **Specification**
    - **Formulation**
    - **Optimization**
    - **Technology Mapping**
  - **Beginning Hierarchical Design**
  - **Technology Mapping - AND, OR, and NOT to NAND or NOR**
  - **Verification**
    - **Manual**
    - **Simulation**

Overview (continued)

- **Part 2 – Combinational Logic**
  - **Functions and functional blocks**
  - **Rudimentary logic functions**
  - **Decoding using Decoders**
    - **Implementing Combinational Functions with Decoders**
  - **Encoding using Encoders**
  - **Selecting using Multiplexers**
    - **Implementing Combinational Functions with Multiplexers**

Combinational Circuits

- **A combinational logic circuit has:**
  - **A set of *m* Boolean inputs,**
  - **A set of *n* Boolean outputs, and**
  - ***n* switching functions, each mapping the 2^m input combinations to an output such that the current output depends only on the current input values**
- **A block diagram:**



Design Procedure

1. **Specification**
   - **Write a specification for the circuit if one is not already available**
2. **Formulation**
   - **Derive a truth table or initial Boolean equations that define the required relationships between the inputs and outputs, if not in the specification**
   - **Apply hierarchical design if appropriate**
3. **Optimization**
   - **Apply 2-level and multiple-level optimization**
   - **Draw a logic diagram or provide a netlist for the resulting circuit using ANDs, ORs, and inverters**

Design Procedure

4. **Technology Mapping**
   - **Map the logic diagram or netlist to the implementation technology selected**
5. **Verification**
   - **Verify the correctness of the final design manually or using simulation**

Design Example

1. **Specification**
   - **BCD to Excess-3 code converter**
   - **Transforms BCD code for the decimal digits to Excess-3 code for the decimal digits**
   - **BCD code words for digits 0 through 9: 4-bit patterns 0000 to 1001, respectively**
   - **Excess-3 code words for digits 0 through 9: 4-bit patterns consisting of 3 (binary 0011) added to each BCD code word**
   - **Implementation:**
     - **multiple-level circuit**
     - **NAND gates (including inverters)**

Design Example (continued)

2. Formulation

- **Conversion of 4-bit codes can be most easily formulated by a truth table**
- **Variables**
  - BCD:
  A,B,C,D
- **Variables**
  - Excess-3
  W,X,Y,Z
- **Don't Cares**
  - BCD 1010
  to 1111

Input BCD A B C D	Output Excess-3 WXYZ
0 0 0 0	0 0 1 1
0 0 0 1	0 1 0 0
0 0 1 0	0 1 0 1
0 0 1 1	0 1 1 0
0 1 0 0	0 1 1 1
0 1 0 1	1 0 0 0
0 1 1 0	1 0 0 1
0 1 1 1	1 0 1 0
1 0 0 0	1 0 1 1
1 0 0 1	1 0 1 1

Design Example (continued)

3. Optimization

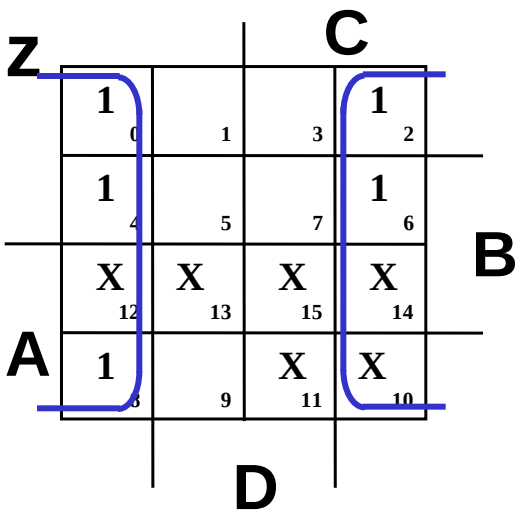
a. 2-level using K-maps

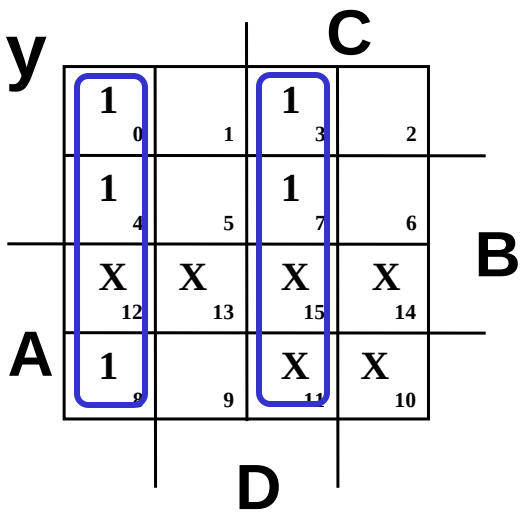
$W = A + BC + BD$

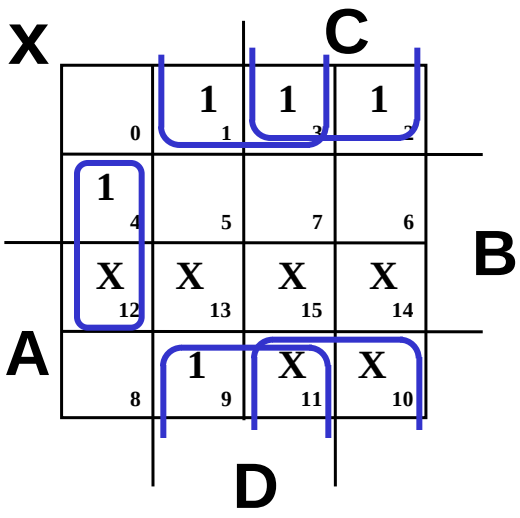
$X = \overline{B}C + \overline{B}D + B\overline{C}\,\overline{D}$

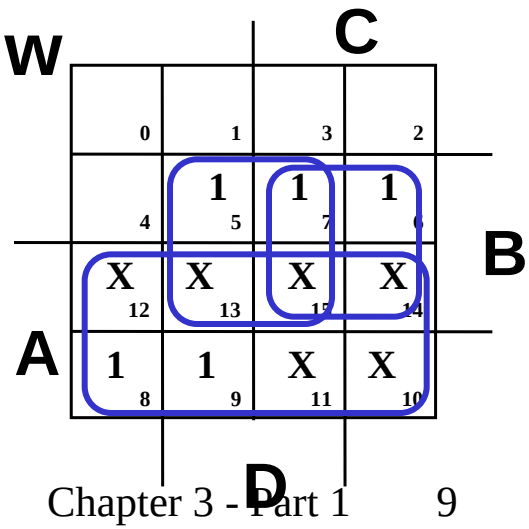
$Y = CD + \overline{C}\,\overline{D}$

$Z = \overline{D}$









Design Example (continued)

3. Optimization (continued)

b. Multiple-level using transformations

$W = A + BC + BD$

$X = \overline{B}C + \overline{B}D + B\overline{C}\,\overline{D}$

$Y = CD + \overline{C}\,\overline{D}$

$Z = \overline{D}$

$G = 7 + 10 + 6 + 0 = 23$

- **Perform extraction, finding factor:**

$T_1 = C + D$

$W = A + BT_1$

$X = \overline{B}T_1 + B\overline{C}\,\overline{D}$

$Y = CD + \overline{C}\,\overline{D}$

$Z = \overline{D}$

MORE?

$G = 2 + \cancel{4} + 4 + 7 + 6 + 0 = 19$

Design Example (continued)

3. Optimization (continued)

b. Multiple-level using transformations

$T_1 = C + D$

$W = A + BT_1$

$X = \overline{B}T_1 + B\overline{C}\,\overline{D}$

$Y = CD + \overline{C}\,\overline{D}$

$Z = \overline{D}$

G = 19

- **An additional extraction not shown in the text since it uses a Boolean transformation: ($\overline{C}\,\overline{D} = \overline{C + D} = \overline{T}_1$):**

$W = A + BT_1$

$X = \overline{B}T_1 + B\overline{T}_1$

$Y = CD + \overline{T}_1$

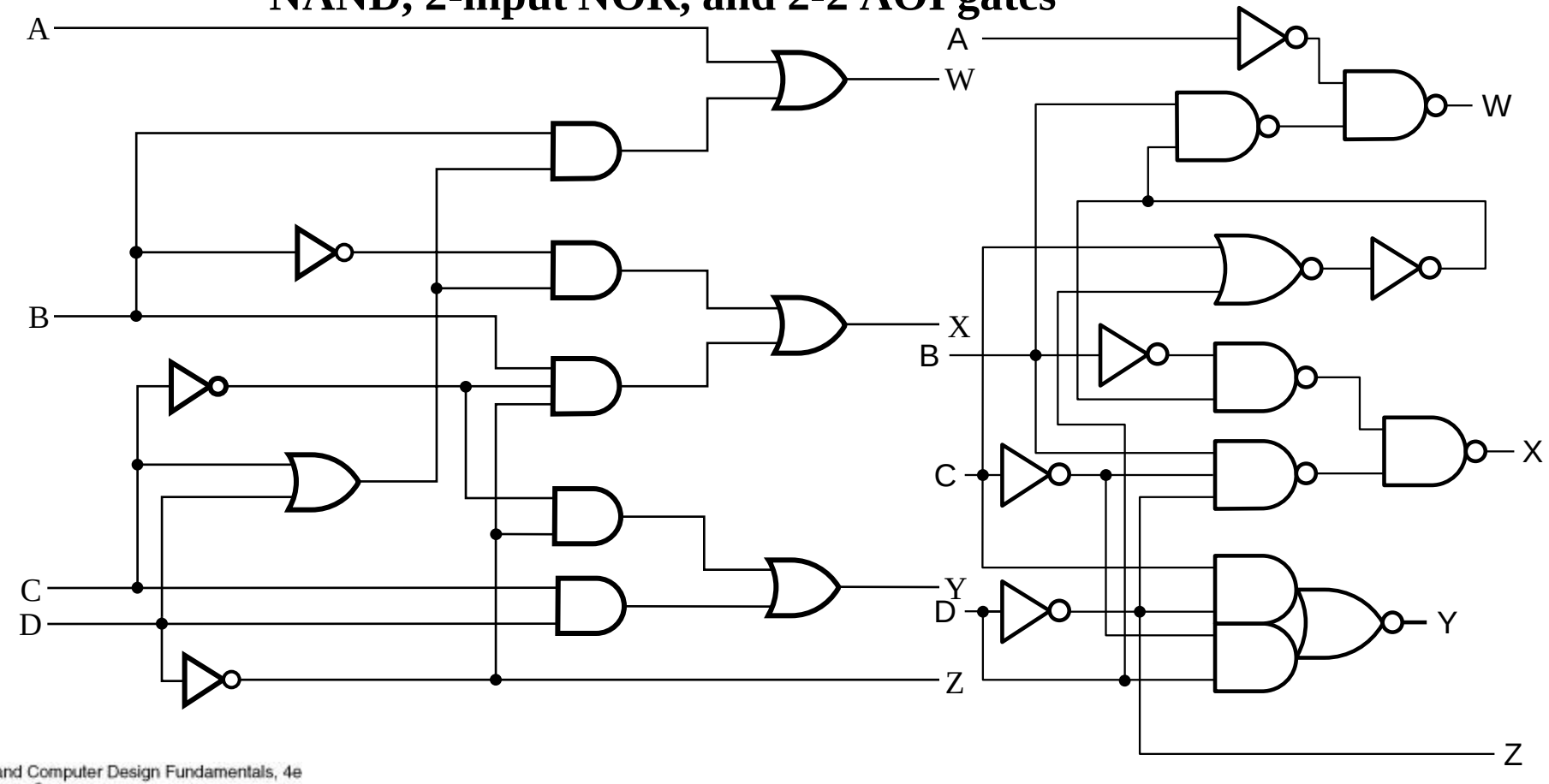
$Z = \overline{D}$

G = 2 + 4 + 6 + 4 + 0 = 16!

Design Example (continued)

4. Technology Mapping

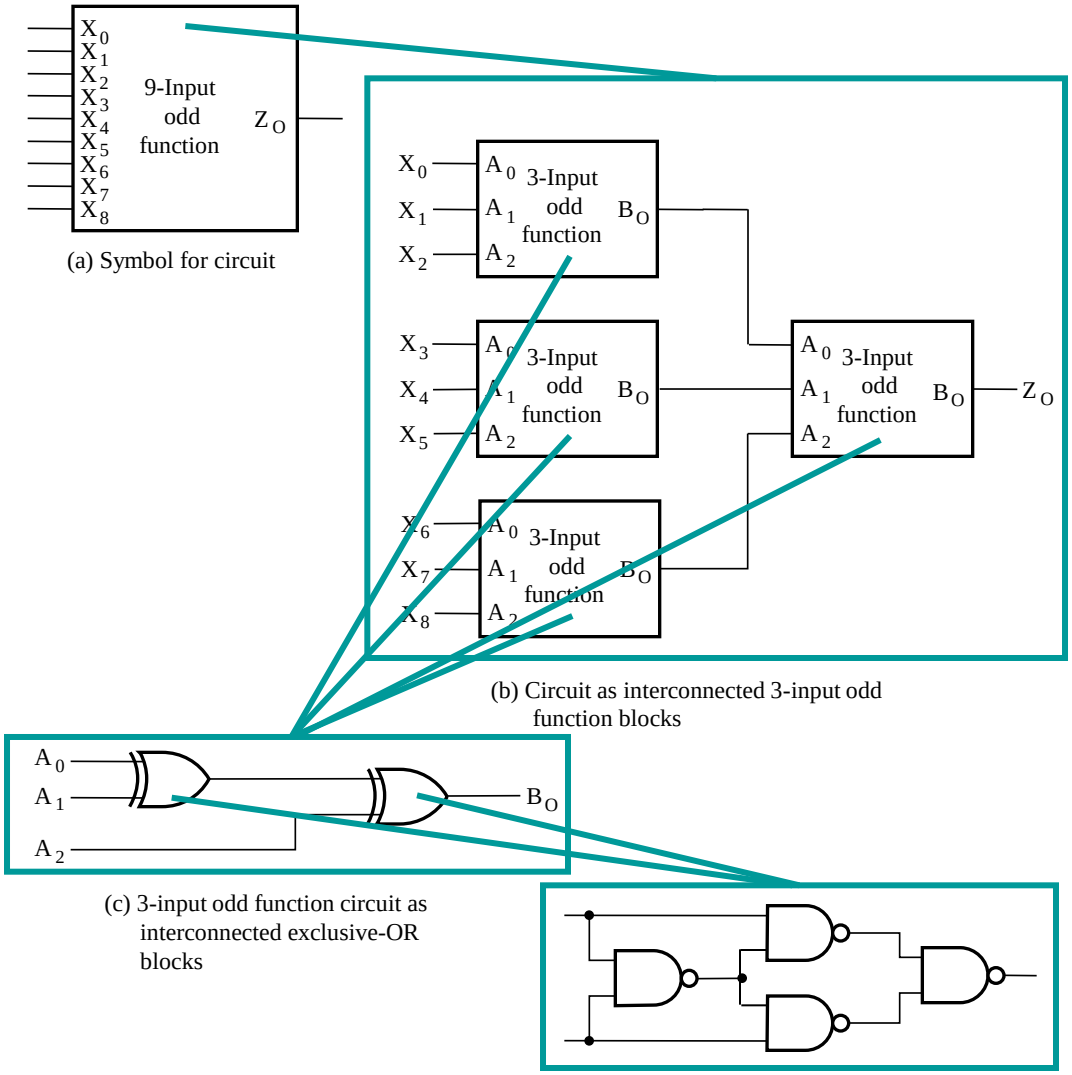
- **Mapping with a library containing inverters and 2-input NAND, 2-input NOR, and 2-2 AOI gates**



Beginning Hierarchical Design

- **To control the complexity of the function mapping inputs to outputs:**
  - **Decompose the function into smaller pieces called *blocks***
  - **Decompose each block's function into smaller blocks, repeating as necessary until all blocks are small enough**
  - **Any block not decomposed is called a *primitive block***
  - **The collection of all blocks including the decomposed ones is a *hierarchy***
- **Example: 9-input parity tree (see next slide)**
  - **Top Level: 9 inputs, one output**
  - **2nd Level: Four 3-bit even parity trees in two levels**
  - **3rd Level: Two 2-bit exclusive-OR functions**
  - **Primitives: Four 2-input NAND gates**
  - **Design requires 4 X 2 X 4 = 32 2-input NAND gates**

Hierarchy for Parity Tree Example



(a) Symbol for circuit

(b) Circuit as interconnected 3-input odd function blocks

(c) 3-input odd function circuit as interconnected exclusive-OR blocks

(d) Exclusive-OR block as interconnected NANDs

Reusable Functions

- **Whenever possible, we try to decompose a complex design into common, *reusable* function blocks**
- **These blocks are**
  - **verified and well-documented**
  - **placed in libraries for future use**

Top-Down versus Bottom-Up

- **A *top-down design* proceeds from an abstract, high-level specification to a more and more detailed design by decomposition and successive refinement**
- **A *bottom-up design* starts with detailed primitive blocks and combines them into larger and more complex functional blocks**
- **Design usually proceeds top-down to known building blocks ranging from complete CPUs to primitive logic gates or electronic components.**
- **Much of the material in this chapter is devoted to learning about combinational blocks used in top-down design.**

Technology Mapping

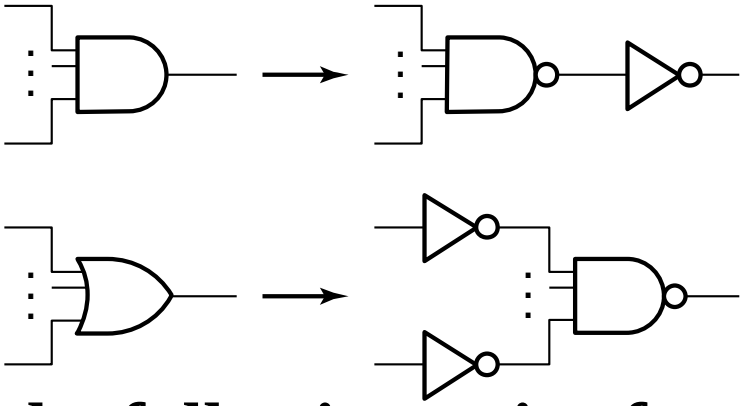
- **Mapping Procedures**
  - **To NAND gates**
  - **To NOR gates**
  - **Mapping to multiple types of logic blocks in covered in the reading supplement: Advanced Technology Mapping.**

Mapping to NAND gates

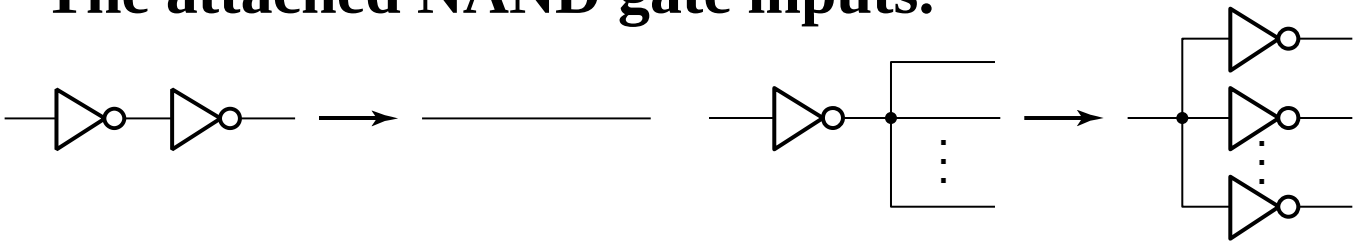
- **Assumptions:**
  - **Gate loading and delay are ignored**
  - **Cell library contains an inverter and *n*-input NAND gates, *n* = 2, 3, …**
  - **An AND, OR, inverter schematic for the circuit is available**
- **The mapping is accomplished by:**
  - **Replacing AND and OR symbols,**
  - **Pushing inverters through circuit fan-out points, and**
  - **Canceling inverter pairs**

NAND Mapping Algorithm

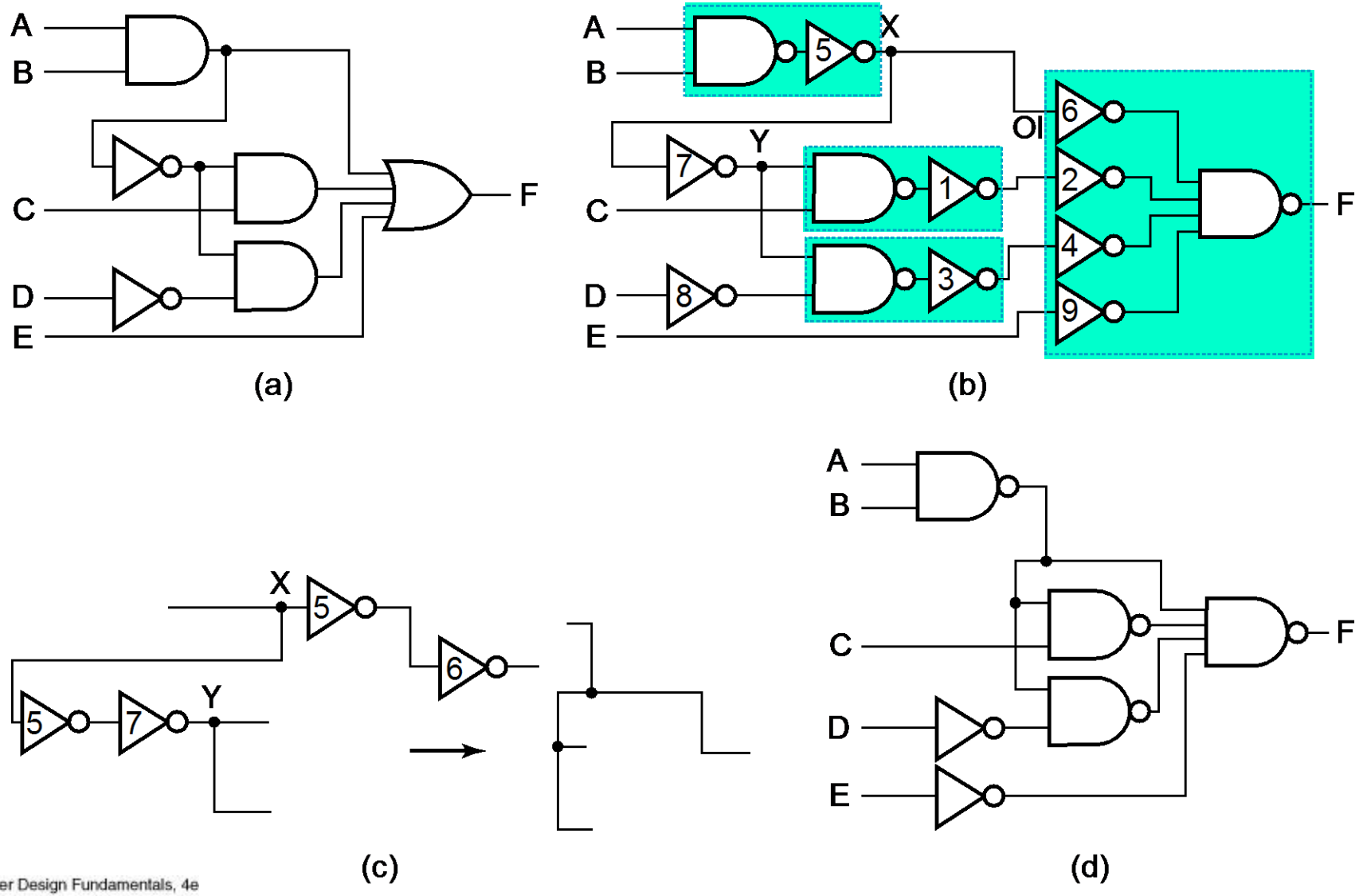
1. **Replace ANDs and ORs:**



2. **Repeat the following pair of actions until there is at most one inverter between :**
   a. **A circuit input or driving NAND gate output, and**
   b. **The attached NAND gate inputs.**



NAND Mapping Example



Mapping to NOR gates

- **Assumptions:**
  - **Gate loading and delay are ignored**
  - **Cell library contains an inverter and *n*-input NOR gates, *n* = 2, 3, …**
  - **An AND, OR, inverter schematic for the circuit is available**
- **The mapping is accomplished by:**
  - **Replacing AND and OR symbols,**
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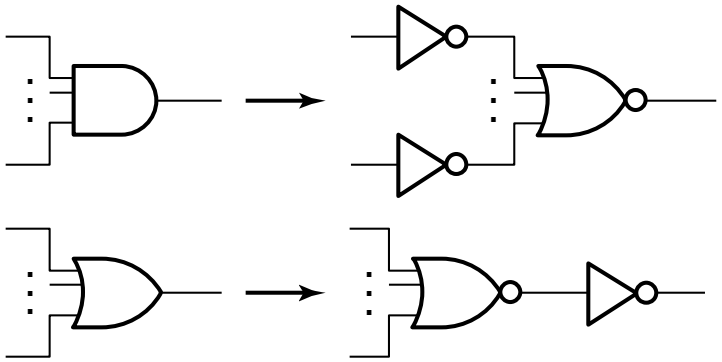
  - **Canceling inverter pairs**

NOR Mapping Algorithm

1. **Replace ANDs and ORs:**



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   a. **A circuit input or driving NAND gate output, and**
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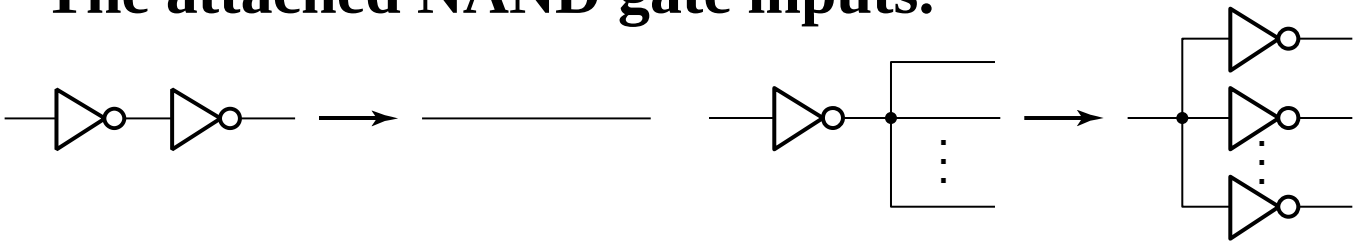
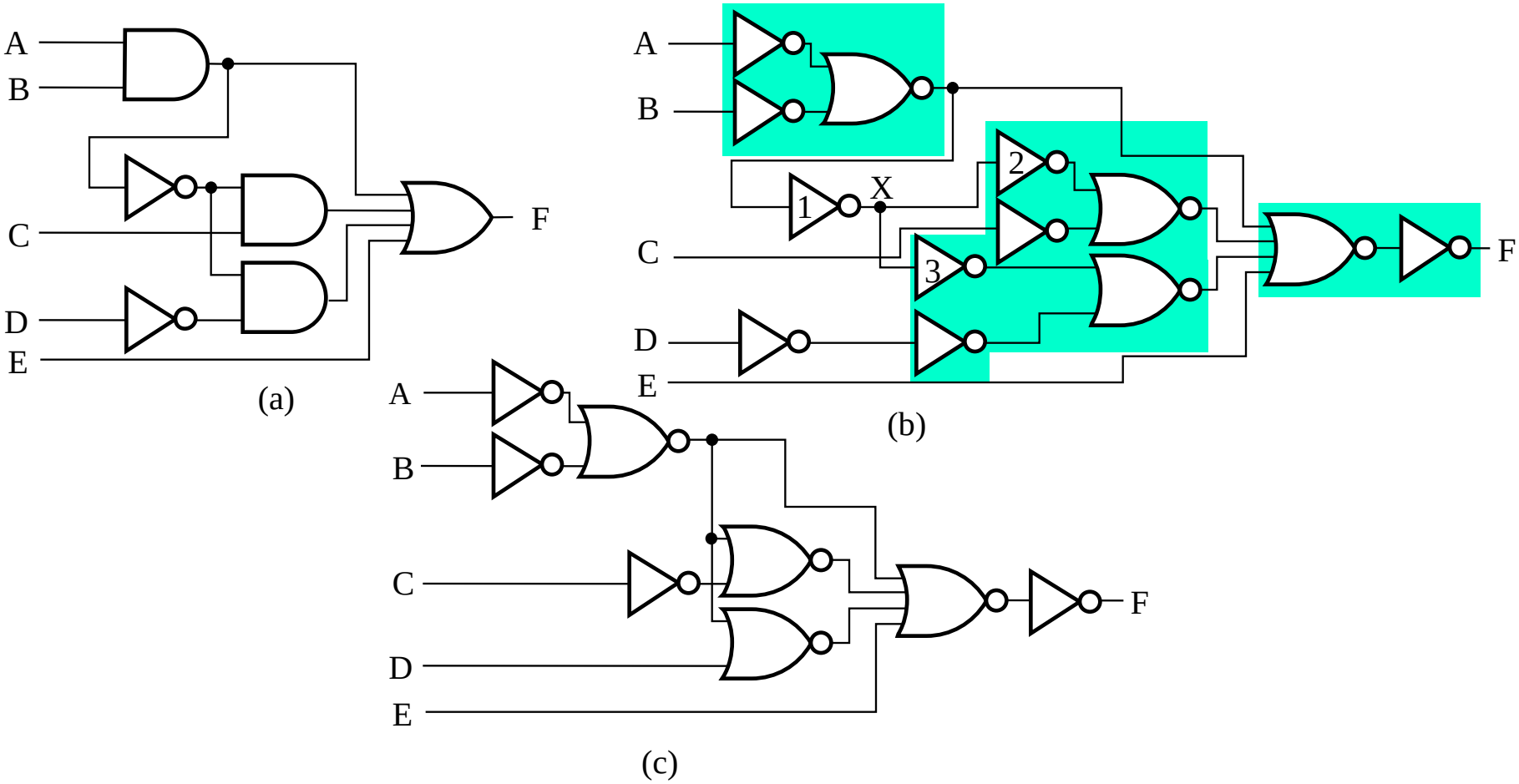


NOR Mapping Example



Verification

- **Verification - show that the final circuit designed implements the original specification**
- **Simple specifications are:**
  - **truth tables**
  - **Boolean equations**
  - **HDL code**
- **If the above result from formulation and are not the original specification, it is critical that the formulation process be flawless for the verification to be valid!**

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- **Simple specifications are:**
  - **truth tables**
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  - **HDL code**
- **If the above result from formulation and are not the original specification, it is critical that the formulation process be flawless for the verification to be valid! (WHY?)**

Basic Verification Methods

- **Manual Logic Analysis**
  - **Find the truth table or Boolean equations for the final circuit**
  - **Compare the final circuit truth table with the specified truth table, or**
  - **Show that the Boolean equations for the final circuit are equal to the specified Boolean equations**
- **Simulation**
  - **Simulate the final circuit (or its netlist, possibly written as an HDL) and the specified truth table, equations, or HDL description using test input values that fully validate correctness.**
  - **The obvious test for a combinational circuit is application of all possible "care" input combinations from the specification**

Verification Example: Manual Analysis

- **BCD-to-Excess 3 Code Converter**
  - **Find the SOP Boolean equations from the final circuit.**
  - **Find the truth table from these equations**
  - **Compare to the formulation truth table**
- **Finding the Boolean Equations:**

$$T_1 = \overline{\overline{C + D}} = C + D$$

$$W = \overline{\overline{A}\,(\overline{T_1\,B})} = A + B\,T_1$$

$$X = (T_1\,B)\,(B\,\overline{C}\,\overline{D}) = \overline{B}\,T_1 + B\,\overline{C}\,\overline{D}$$

$$Y = \overline{C\,\overline{D} + \overline{C}\,D} = C\,D + \overline{C}\,\overline{D}$$

Verification Example: Manual Analysis

- **Find the circuit truth table from the equations and compare to specification truth table:**

Input BCD A B C D	Output Excess-3 W X Y Z
0 0 0 0	0 0 1 1
0 0 0 1	0 1 0 0
0 0 1 0	0 1 0 1
0 0 1 1	0 1 1 0
0 1 0 0	0 1 1 1
0 1 0 1	1 0 0 0
0 1 1 0	1 0 0 1
0 1 1 1	1 0 1 0
1 0 0 0	1 0 1 1
1 0 0 1	1 0 1 1

The tables match!

Verification Example: Simulation

- **Simulation procedure:**
  - **Use a schematic editor or text editor to enter a gate level representation of the final circuit**
  - **Use a waveform editor or text editor to enter a test consisting of a sequence of input combinations to be applied to the circuit**
    - **This test should guarantee the correctness of the circuit if the simulated responses to it are correct**
    - **Short of applying all possible "care" input combinations, generation of such a test can be difficult**

Verification Example: Simulation

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  - **Use a schematic editor or text editor to enter a gate level representation of the final circuit**
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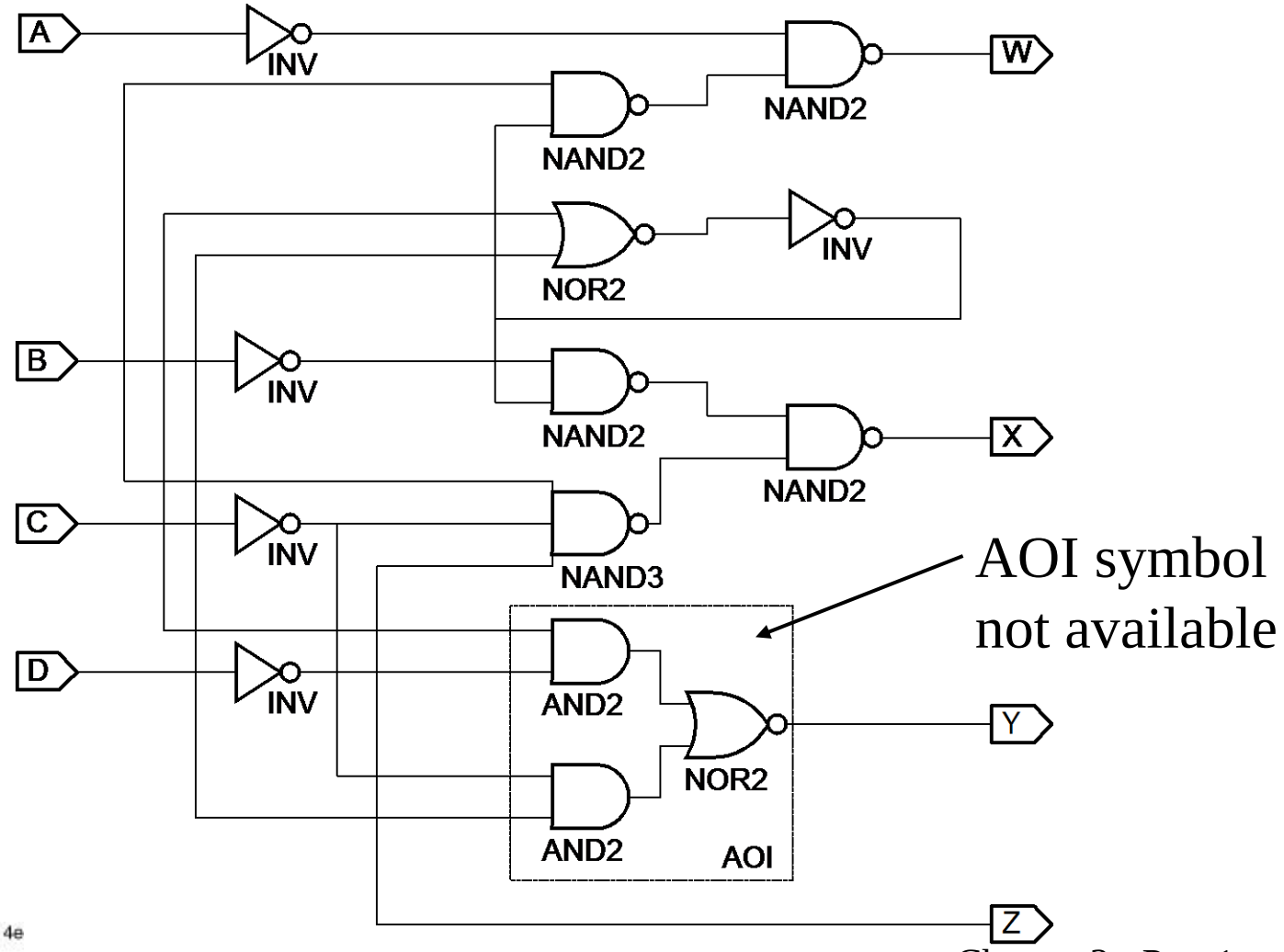
    - **This test should guarantee the correctness of the circuit if the simulated responses to it are correct**
    - **Short of applying all possible "care" input combinations, generation of such a test can be difficult (WHY?)**

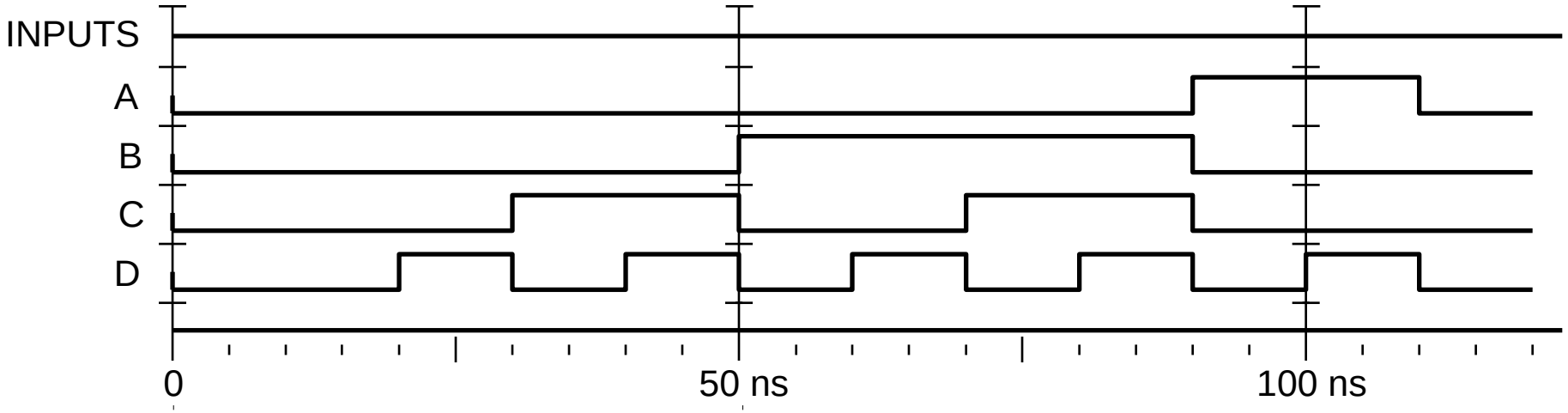
Verification Example: Simulation

- **Enter BCD-to-Excess-3 Code Converter Circuit Schematic**



Verification Example: Simulation

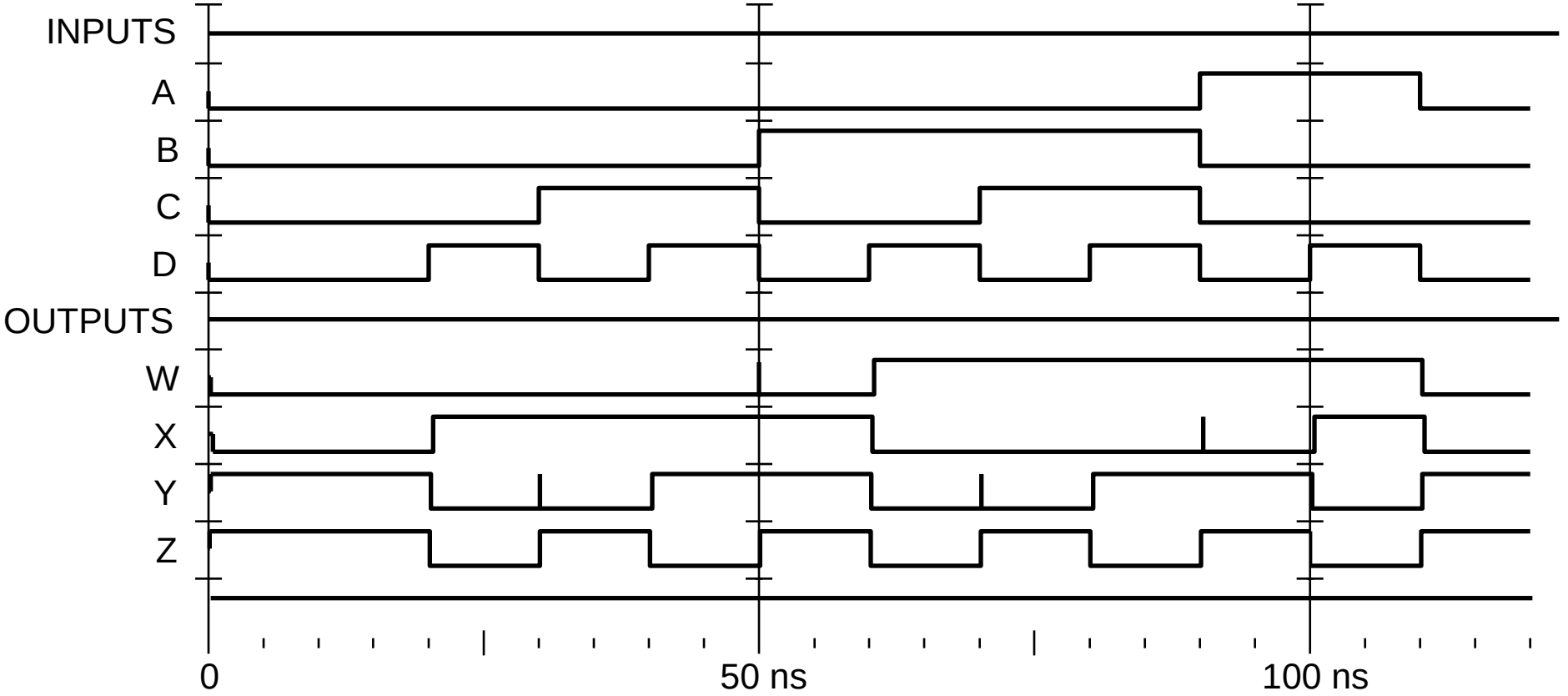
- **Enter waveform that applies all possible input combinations:**



- **Are all BCD input combinations present? (Low is a 0 and high is a one)**

Verification Example: Simulation

- **Run the simulation of the circuit for 120 ns**



- **Do the simulation output combinations match the original truth table?**

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