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1. Question 1-)

a)  $\bar{x}\bar{y} + \bar{x}y + xy = \bar{x} + y$

1.  $\bar{x}(\bar{y} + y) + xy$  (by distributive laws)
2.  $\bar{x} + xy$  (by  $\bar{y} + y = 1$ )
3.  $(\bar{x} + x)(\bar{x} + y)$  (by distributive laws)
4.  $\bar{x} + y$  (by  $\bar{x} + x = 1$ )

b)  $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$

1.  $\bar{A}B + AB + \bar{B}\bar{C} + \bar{B}C$  (by commutativity)
2.  $B(\bar{A} + A) + \bar{B}(\bar{C} + C)$  (by distributive laws)
3.  $B + \bar{B}$  (by  $\bar{x} + x = 1$ )
4.  $1$  (by  $\bar{x} + x = 1$ )

c)  $y + \bar{x}z + x\bar{y} = x + y + z$

1.  $y + x\bar{y} + \bar{x}z$  (by commutativity)
2.  $(y + x)(y + \bar{y}) + \bar{x}z$  (by distributive laws)
3.  $(y + x) + \bar{x}z$  (by  $y + \bar{y} = 1$ )
4.  $(y + x + \bar{x})(y + x + z)$  (by distributive laws)
5.  $(y + 1)(y + x + z)$  (by  $x + \bar{x} = 1$ )
6.  $(y + x + z)$  (by  $y + 1 = 1$ )
7.  $x + y + z$  (by commutativity)

$$d) \quad \bar{x}\bar{y} + \bar{y}z + xz + xy + y\bar{z} = \bar{x}\bar{y} + xz + y\bar{z} \quad (2)$$

$$1. \quad \bar{x}\bar{y} + \bar{y}z + xz + y\bar{z} \quad (\text{by consensus theorem})$$

$$(\bar{x}\bar{y} + xz + y\bar{z} = \bar{x}\bar{y} + xz)$$

$$2. \quad \bar{x}\bar{y} + xz + y\bar{z} \quad (\text{by consensus theorem})$$

$$(xz + y\bar{z} + x\bar{y} = xz + y\bar{z})$$

## 2. Question - 2)

$$a) \quad \bar{x}\bar{y} + xy\bar{z} + \bar{x}y \quad (\text{to three literals})$$

$$\bar{x}(\bar{y} + y) + xy\bar{z}$$

$$\bar{x} + xy\bar{z} = (\bar{x} + x)(\bar{x} + y\bar{z}) = \boxed{\bar{x} + y\bar{z}}$$

↓  
reduced boolean expression

$$b) \quad x + y(z + \overline{x+z}) \quad \text{to two literals}$$

$$1. \quad x + y(z + \bar{x}\bar{z}) \quad (\text{de morgan law})$$

$$2. \quad x + y((z + \bar{z})(z + \bar{x})) \quad (\text{by distributive law})$$

$$3. \quad x + y(z + \bar{x}) \quad (\text{by } x + \bar{x} = 1)$$

$$4. \quad x + yz + y\bar{x} \quad (\text{by distributive law})$$

$$5. \quad (x + x')(x + y) + yz \quad (\text{by distributive law})$$

$$6. \quad (x + y) + yz \quad (\text{by } x + x' = 1)$$

$$7. \quad \boxed{x + y} \quad (\text{by simplification})$$

$$(x + xy = x)$$

↓  
reduced boolean expression



c)  $\bar{w}x(z + \bar{z} + yz) + x(w + \bar{w}yz)$  to one literal (3)

1.  $\bar{w}x\bar{z} + \bar{w}xy\bar{z} + wx + \bar{w}xyz$  (by distributive law)
2.  $\bar{w}xz(\underbrace{y + \bar{y}}_1) + wx + \bar{w}x\bar{z}$  (by  $x + x' = 1$ )
3.  $\bar{w}xz + wx + \bar{w}x\bar{z}$
4.  $w'x(z + \bar{z}) + wx$  (by distributive law)
5.  $w'x + wx$  (by  $x + x' = 1$ )
6.  $x(\underbrace{w' + w}_1)$  (by  $x + x' = 1$ )
7.  $\boxed{x}$   
 $\downarrow$   
 reduced boolean expression

d)  $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + \bar{A}C$  to four literals

1.  $AB\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}C$  (by distributive law)
2.  $AB\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A} + \bar{C}$  (by demorgan law)
3.  $\bar{A} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{C} + AB\bar{C}\bar{D} + ABCD$  (by commutative law)
4.  $\bar{A}(1 + \bar{B}\bar{C}\bar{D} + \bar{B}CD) + \bar{C}(1 + AB\bar{D}) + ABCD$  (by distributive law)
5.  $\bar{A} + \bar{C} + ABCD$  (by  $x + y + 1 = 1$ )
6.  $\bar{C} + \bar{A} + ABCD$  (by commutative law)
7.  $\bar{C} + (\bar{A} + AB)(\bar{A} + CD)$  (by distributive law)
8.  $\bar{C} + (\bar{A} + A)(\bar{A} + B)(\bar{A} + CD)$  (by distributive law)
9.  $\bar{C} + (\bar{A} + B)(\bar{A} + CD)$  (by  $x + \bar{x} = 1$ )

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10.  $\bar{C} + \bar{A} + \bar{A}CD + B\bar{A} + BCD$  (by distributive law)

11.  $\bar{C} + \bar{A} + B\bar{A} + BCD$  (by simplification)

12.  $\bar{C} + \bar{A} + BCD$  (by simplification)

13.  $\bar{A} + (\bar{C} + C)(\bar{C} + B)$  (by distributive law)

14.  $\bar{A} + \bar{C} + BD$  (by  $x + x' = 1$ )

reduced boolean  
expression