Logic and Computer Design Fundamentals

Chapter 1 – Digital Systems and Information

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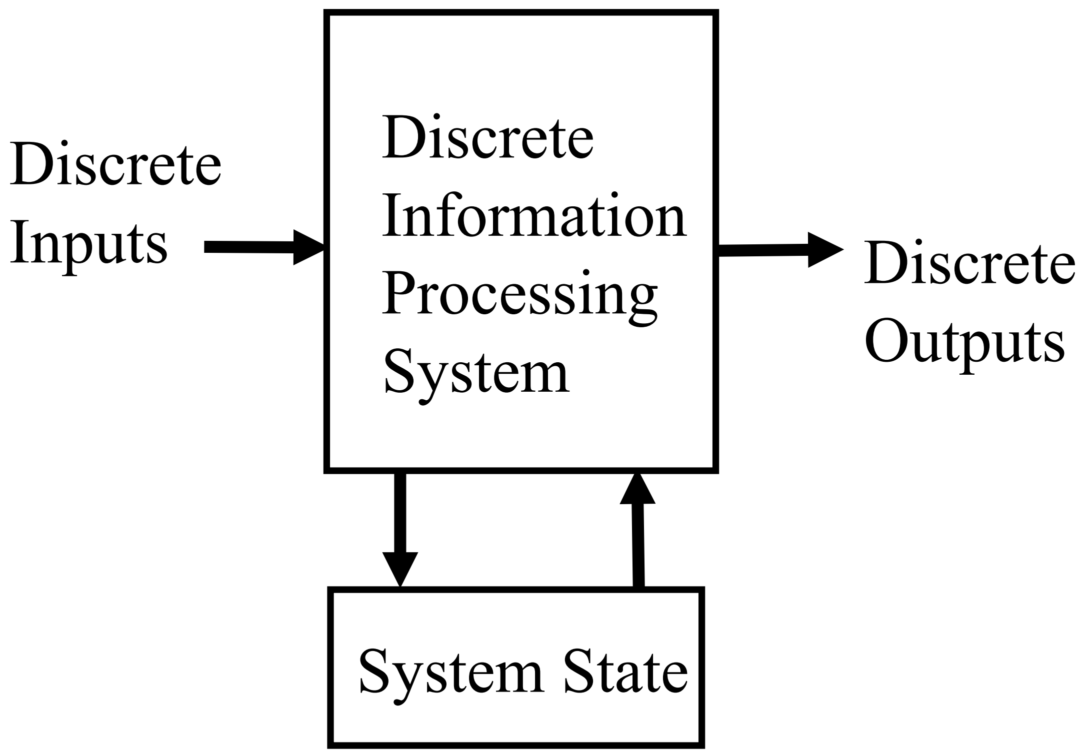
(Hyperlinks are active in View Show mode)

Overview

- **Digital Systems, Computers, and Beyond**
- **Information Representation**
- **Number Systems** [binary, octal and hexadecimal]
- **Arithmetic Operations**
- **Base Conversion**
- **Decimal Codes** [BCD (binary coded decimal)]
- **Alphanumeric Codes**
- **Parity Bit**
- **Gray Codes**

DIGITAL & COMPUTER SYSTEMS - Digital System

- **Takes a set of discrete information <u>inputs</u> and discrete internal information <u>(system state)</u> and generates a set of discrete information <u>outputs</u>.**



Types of Digital Systems

- No state present
  - Combinational Logic System
  - Output = Function(Input)
- State present
  - State updated at discrete times

    => Synchronous Sequential System
  - State updated at any time

    =>Asynchronous Sequential System
  - State = Function (State, Input)
  - Output = Function (State)
    or Function (State, Input)

Digital System Example:

A Digital Counter (e. g., odometer):

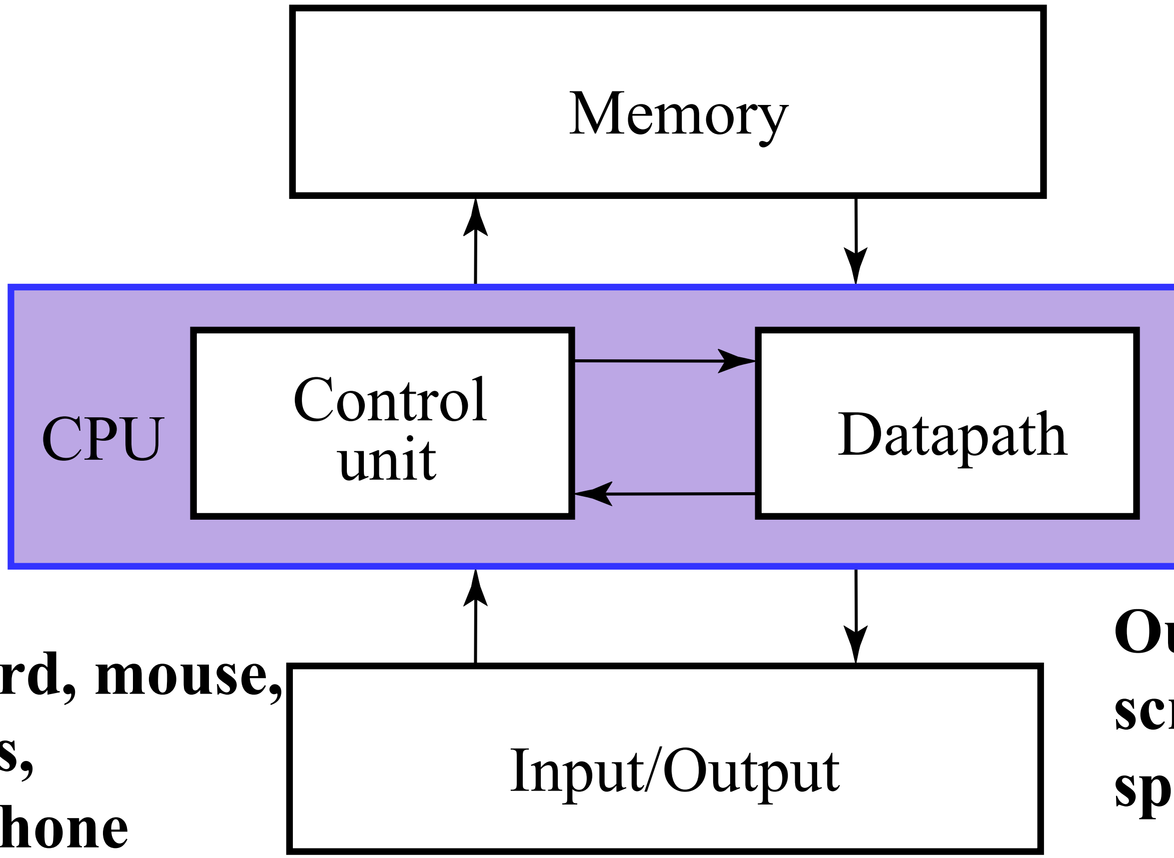
Count Up →
Reset →

0	0	1	3	5	6	4

Inputs: Count Up, Reset
Outputs: Visual Display
State: "Value" of stored digits

Synchronous or Asynchronous?

Digital Computer Example



Inputs: keyboard, mouse, wireless, microphone

Outputs: LCD screen, wireless, speakers

Synchronous or Asynchronous?

And Beyond – Embedded Systems

- Computers as integral parts of other products
- Examples of embedded computers
  - Microcomputers
  - Microcontrollers
  - Digital signal processors

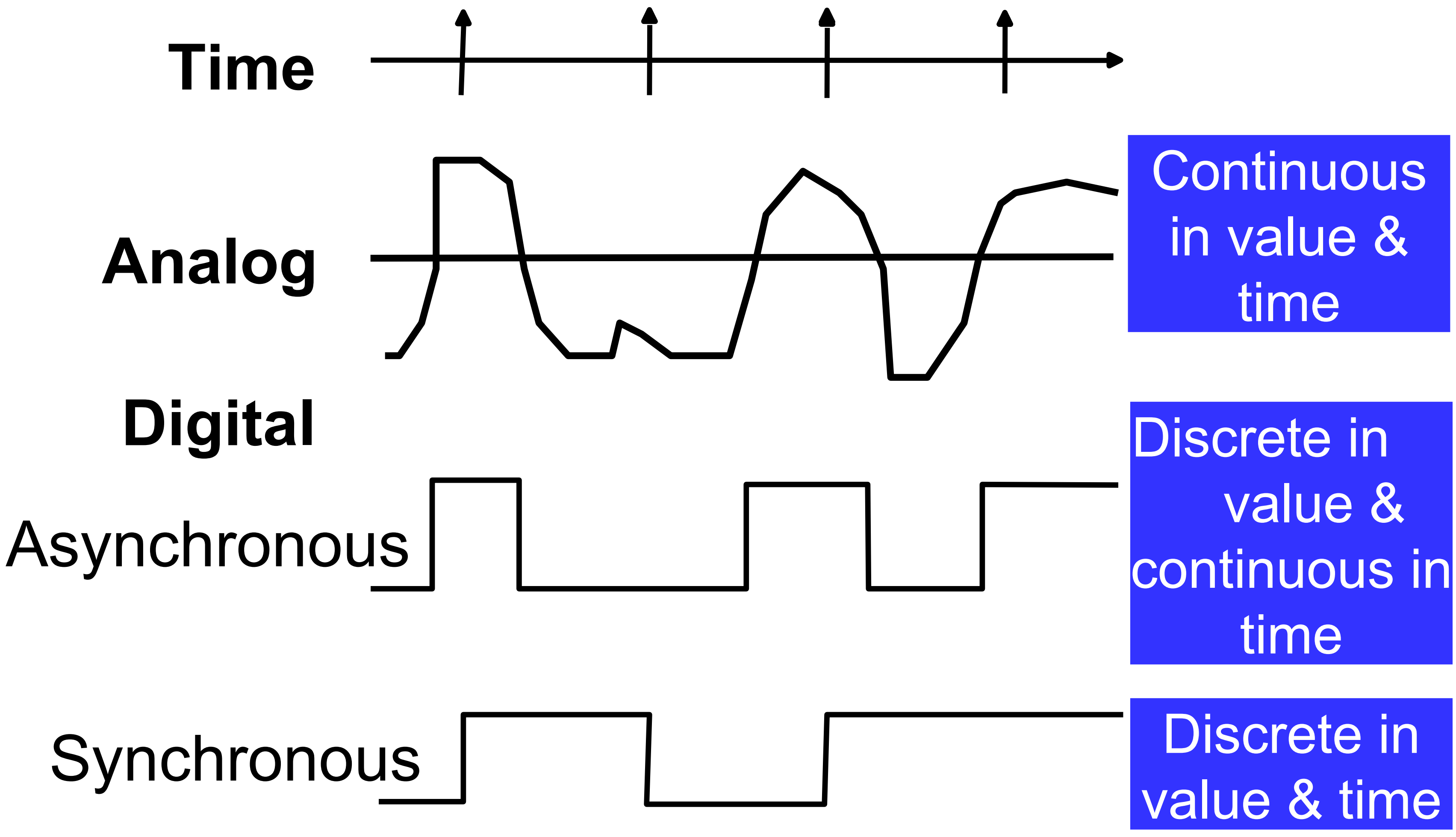
Embedded Systems

- Examples of Embedded Systems Applications
  - Cell phones
  - Automobiles
  - Video games
  - Copiers
  - Dishwashers
  - Flat Panel TVs
  - Global Positioning Systems

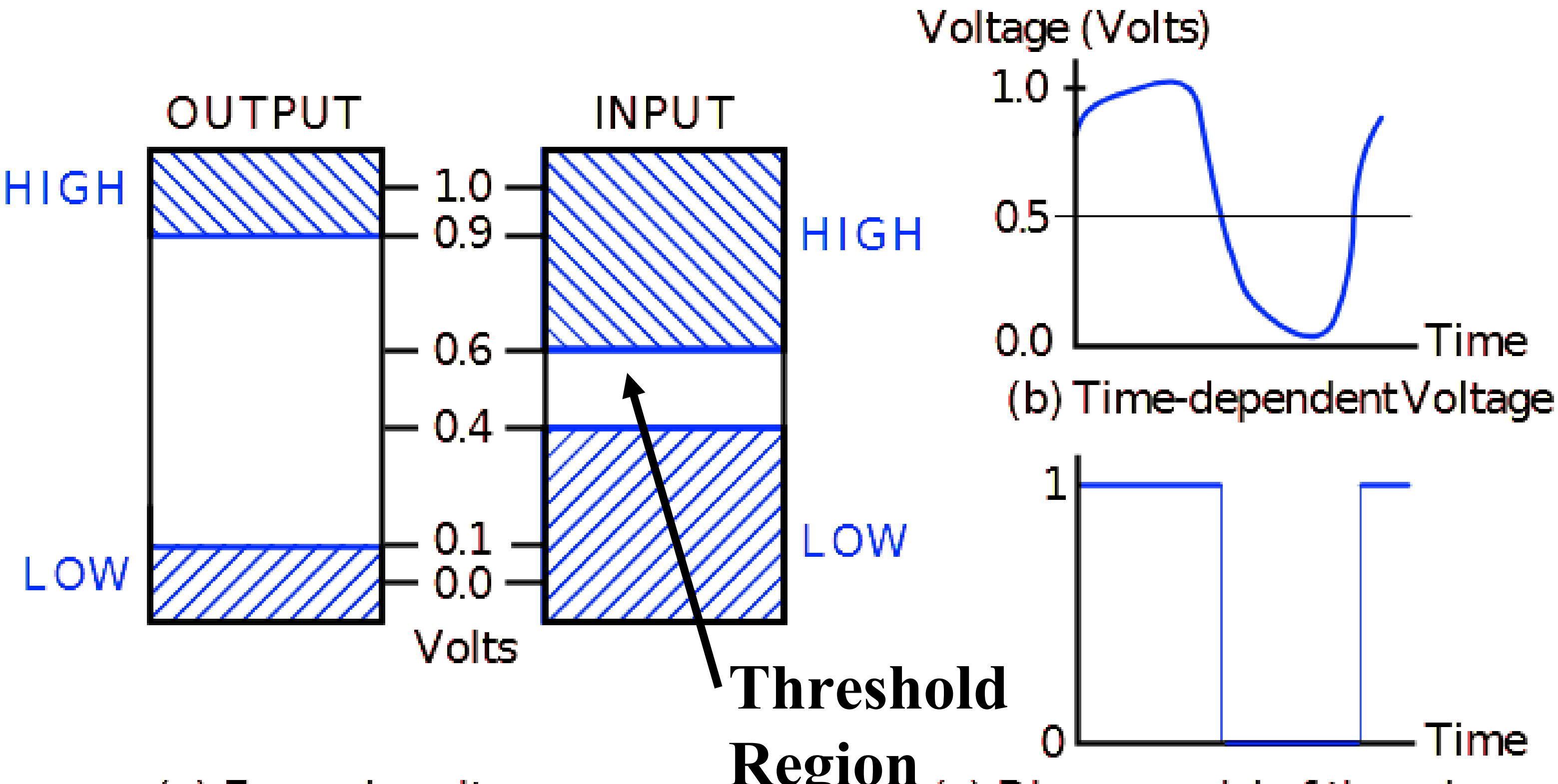
INFORMATION REPRESENTATION - Signals

- **Information variables represented by physical quantities.**
- **For digital systems, the variables take on discrete values.**
- **Two level, or binary values are the most prevalent values in digital systems.**
- **Binary values are represented abstractly by:**
  - **digits 0 and 1**
  - **words (symbols) False (F) and True (T)**
  - **words (symbols) Low (L) and High (H)**
  - **and words On and Off.**
- **Binary values are represented by values or ranges of values of physical quantities**

Signal Examples Over Time



Signal Example – Physical Quantity: Voltage



(a) Example voltage ranges

(b) Time-dependent Voltage

(c) Binary model of time-dependent voltage

Binary Values: Other Physical Quantities

- **What are other physical quantities represent 0 and 1?**
  - **CPU** **Voltage**
  - **Disk** **Magnetic Field Direction**
  - **CD** **Surface Pits/Light**
  - **Dynamic RAM** **Electrical Charge**

NUMBER SYSTEMS – Representation

- Positive radix, positional number systems
- A number with *radix* ***r*** is represented by a string of digits:

  $$A_{n-1}A_{n-2}\ \ldots\ A_1A_0\ .\ A_{-1}A_{-2}\ \ldots\ A_{-m+1}A_{-m}$$

  in which $\mathbf{0} \le A_i < r$ and **.** is the *radix point*.
- The string of digits represents the power series:

$$(\text{Number})_r = \left(\sum_{i=0}^{i=n-1} A_i \cdot r^i\right) + \left(\sum_{j=-m}^{j=-1} A_j \cdot r^j\right)$$

(Integer Portion) + (Fraction Portion)

Number Systems – Examples

	General	Decimal	Binary
Radix (Base)	r	10	2
Digits	0 => r - 1	0 => 9	0 => 1
0	r^0	1	1
1	r^1	10	2
2	r^2	100	4
3	r^3	1000	8
Powers of 4	r^4	10,000	16
	r^5	100,000	32
Radix 5	r^{-1}	0.1	0.5
-1	r^{-2}	0.01	0.25
-2	r^{-3}	0.001	0.125
-3	r^{-4}	0.0001	0.0625
-4	r^{-5}	0.00001	0.03125
-5			

Special Powers of 2

- 2^{10} (1024) is Kilo, denoted "K"
- 2^{20} (1,048,576) is Mega, denoted "M"
- 2^{30} (1,073, 741,824)is Giga, denoted "G"
- 2^{40} (1,099,511,627,776) is Tera, denoted “T"

ARITHMETIC OPERATIONS - Binary Arithmetic

- **Single Bit Addition with Carry**
- **Multiple Bit Addition**
- **Single Bit Subtraction with Borrow**
- **Multiple Bit Subtraction**
- **Multiplication**
- **BCD Addition**

Single Bit Binary Addition with Carry

Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):

Carry in (Z) of 0:

4 different examples!

Z	0	0	0	0
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 0	0 1	0 1	1 0

Carry in (Z) of 1:

4 different examples!

Z	1	1	1	1
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 1	1 0	1 0	1 1

Multiple Bit Binary Addition

- **Extending this to two multiple bit examples:**

Carries	**0**	**0**
Augend	**01100**	**10110**
Addend	**+10001**	**+10111**
Sum		

- **Note: The 0 is the default Carry-In to the least significant bit.**

Single Bit Binary Subtraction with Borrow

- Given two binary digits (X,Y), a borrow in (Z) we get the following difference (S) and borrow (B):
- Borrow in (Z) of 0:

4 different examples!

Z	0	0	0	0
X	0	0	1	1
- Y	-0	-1	-0	-1
BS	0 0	1 1	0 1	0 0

- Borrow in (Z) of 1:

4 different examples!

Z	1	1	1	1
X	0	0	1	1
- Y	-0	-1	-0	-1
BS	11	1 0	0 0	1 1

Multiple Bit Binary Subtraction

- Extending this to two multiple bit examples:

Borrows	0	0
Minuend	10110	10110
Subtrahend	- 10010	- 10011
Difference		

- Notes: The 0 is a Borrow-In to the least significant bit. If the Subtrahend > the Minuend, interchange and append a – to the result.

Binary Multiplication

The binary multiplication table is simple:

0 ∗ 0 = 0 | 1 ∗ 0 = 0 | 0 ∗ 1 = 0 | 1 ∗ 1 = 1

Extending multiplication to multiple digits:

Multiplicand	**1011**
Multiplier	**x 101**
Partial Products	**1011**
	0000 -
	1011 - -
Product	**110111**

BASE CONVERSION - Positive Powers of 2

- Useful for Base Conversion

Exponent	Value
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Exponent	Value
11	2,048
12	4,096
13	8,192
14	16,384
15	32,768
16	65,536
17	131,072
18	262,144
19	524,288
20	1,048,576
21	2,097,152

Converting Binary to Decimal

- To convert to decimal, use decimal arithmetic to form Σ (digit × respective power of 2).
- Example:Convert 11010_2 to N_{10}:

Converting Decimal to Binary

- **Method 1**
  - **Subtract the largest power of 2 (see slide 14) that gives a positive remainder and reduce the power.**
  - **Repeat, subtracting from the prior remainder and reducing the power, until the remainder is zero.**
  - **Place 1's in the positions in the binary result corresponding to the powers recorded; in all other positions place 0's.**
- **Example: Convert 625_{10} to N_2**

Commonly Occurring Bases

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

- The six letters (in addition to the 10 integers) in hexadecimal represent:

Numbers in Different Bases

- **Good idea to memorize!**

Decimal (Base 10)	Binary (Base 2)	Octal (Base 8)	Hexadecimal (Base 16)
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	01011	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10

Conversion Between Bases

- **Method 2**
- **To convert from one base to another:**

  1) Convert the Integer Part

  2) Convert the Fraction Part

  3) Join the two results with a radix point

Conversion Details

- **To Convert the Integer Part:**

  Repeatedly divide the number by the new radix and save the remainders. The digits for the new radix are the remainders in *reverse order* of their computation. If the new radix is > 10, then convert all remainders > 10 to digits A, B, …

- **To Convert the Fractional Part:**

  Repeatedly multiply the fraction by the new radix and save the integer digits that result. The digits for the new radix are the integer digits in *order* of their computation. If the new radix is > 10, then convert all integers > 10 to digits A, B, …

Example: Convert 46.6875_{10} To Base 2

- **Convert 46 to Base 2**

- **Convert 0.6875 to Base 2:**

- **Join the results together with the radix point:**

Additional Issue - Fractional Part

- Note that in this conversion, the fractional part can become 0 as a result of the repeated multiplications.
- In general, it may take many bits to get this to happen or it may never happen.
- Example Problem: Convert 0.65_{10} to N_2
  - 0.65 = 0.1010011001001 …
  - The fractional part begins repeating every 4 steps yielding repeating 1001 forever!
- Solution: Specify number of bits to right of radix point and round or truncate to this number.

Checking the Conversion

- To convert back, sum the digits times their respective powers of r.
- From the prior conversion of 46.6875_{10}

$$101110_2 = 1 \cdot 32 + 0 \cdot 16 + 1 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 0 \cdot 1$$
$$= 32 + 8 + 4 + 2$$
$$= 46$$

$$0.1011_2 = 1/2 + 1/8 + 1/16$$
$$= 0.5000 + 0.1250 + 0.0625$$
$$= 0.6875$$

Octal (Hexadecimal) to Binary and Back

- **Octal (Hexadecimal) to Binary:**
  - **Restate the octal (hexadecimal) as three (four) binary digits starting at the radix point and going both ways.**
- **Binary to Octal (Hexadecimal):**
  - **Group the binary digits into three (four) bit groups starting at the radix point and going both ways, padding with zeros as needed in the fractional part.**
  - **Convert each group of three bits to an octal (hexadecimal) digit.**

Octal to Hexadecimal via Binary

- Convert octal to binary.
- Use groups of four bits and convert as above to hexadecimal digits.
- Example: Octal to Binary to Hexadecimal

$$6 \quad 3 \quad 5 \;.\; 1 \quad 7 \quad 7_{8}$$

- Why do these conversions work?

A Final Conversion Note

- You can use arithmetic in other bases if you are careful:
- Example: Convert 101110_2 to Base 10 using binary arithmetic:

Step 1 101110 / 1010 = 100 r 0110

Step 2 100 / 1010 = 0 r 0100

Converted Digits are 0100_2 | 0110_2

or 4 6 $_{10}$

Binary Numbers and Binary Coding

- Flexibility of representation
  - Within constraints below, can assign any binary combination (called a code word) to any data as long as data is uniquely encoded.
- Information Types
  - Numeric
    - Must represent range of data needed
    - Very desirable to represent data such that simple, straightforward computation for common arithmetic operations permitted
    - Tight relation to binary numbers
  - Non-numeric
    - Greater flexibility since arithmetic operations not applied.
    - Not tied to binary numbers

Non-numeric Binary Codes

- Given n binary digits (called <u>bits</u>), a <u>binary code</u> is a mapping from a set of <u>represented elements</u> to a subset of the 2^n binary numbers.
- Example: A binary code for the seven colors of the rainbow
- Code 100 is not used

Color	Binary Number
Red	000
Orange	001
Yellow	010
Green	011
Blue	101
Indigo	110
Violet	111

Number of Bits Required

- **Given M elements to be represented by a binary code, the minimum number of bits, *n*, needed, satisfies the following relationships:**

  $$2^n \geq M > 2^{(n-1)}$$

  $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the *ceiling function*, is the integer greater than or equal to x.

- **Example: How many bits are required to represent <u>decimal digits</u> with a binary code?**

Number of Elements Represented

- Given n digits in radix r, there are r^n distinct elements that can be represented.
- But, you can represent m elements, m < r^n
- Examples:
  - You can represent 4 elements in radix $r = 2$ with $n = 2$ digits: (00, 01, 10, 11).
  - You can represent 4 elements in radix $r = 2$ with $n = 4$ digits: (0001, 0010, 0100, 1000).
  - This second code is called a "one hot" code.

DECIMAL CODES - Binary Codes for Decimal Digits

- There are over 8,000 ways that you can chose 10 elements from the 16 binary numbers of 4 bits. A few are useful:

Decimal	Binary	Gray	Decimal of Gray
0	0000	0000	0
1	0001	0001	1
2	0010	0011	3
3	0011	0010	2
4	0100	0110	6
5	0101	0111	7
6	0110	0101	5
7	0111	0100	4
8	1000	1100	12
9	1001	1101	13
10	1010	1111	15
11	1011	1110	14
12	1100	1010	10
13	1101	1011	11
14	1110	1001	9
15	1111	1000	8

Decimal	8,4,2,1	Excess3	8,4,-2,-1	Gray
0	0000	0011	0000	0000
1	0001	0100	0111	0100
2	0010	0101	0110	0101
3	0011	0110	0101	0111
4	0100	0111	0100	0110
5	0101	1000	1011	0010
6	0110	1001	1010	0011
7	0111	1010	1001	0001
8	1000	1011	1000	1001
9	1001	1100	1111	1000

reflected binary code a.k.a. gray code

Binary Coded Decimal (BCD)

- The BCD code is the 8,4,2,1 code.
- 8, 4, 2, and 1 are weights
- BCD is a *weighted* code
- This code is the simplest, most intuitive binary code for decimal digits and uses the same powers of 2 as a binary number, but only encodes the first ten values from 0 to 9.
- Example: 1001 (9) = 1000 (8) + 0001 (1)
- How many "invalid" code words are there?
- What are the "invalid" code words?

Excess 3 Code and 8, 4, –2, –1 Code

Decimal	Excess 3	8, 4, –2, –1
0	0011	0000
1	0100	0111
2	0101	0110
3	0110	0101
4	0111	0100
5	1000	1011
6	1001	1010
7	1010	1001
8	1011	1000
9	1100	1111

- What interesting property is common to these two codes?

Warning: Conversion or Coding?

- **Do NOT mix up conversion of a decimal number to a binary number with coding a decimal number with a BINARY CODE.**
- **$13_{10} = 1101_2$ (This is conversion)**
- **13 $\Leftrightarrow$ 0001|0011 (This is coding)**

BCD Arithmetic

- Given a BCD code, we use binary arithmetic to add the digits:

8	1000	Eight
+5	+0101	Plus 5
13	1101	is 13 (> 9)

- Note that the result is MORE THAN 9, so must be represented by two digits!
- To correct the digit, subtract 10 by adding 6 modulo 16.

8	1000	Eight
+5	+0101	Plus 5
13	1101	is 13 (> 9)
	+0110	so add 6
	carry = 1 0011	leaving 3 + cy
	0001 | 0011	Final answer (two digits)

a.k.a. decimal adjust

- If the digit sum is > 9, add one to the next significant digit

BCD Addition Example

- **Add 2905_{BCD} to 1897_{BCD} showing carries and digit corrections.**

$$
\begin{array}{rcccc}
 & & & & 0 \\
 & 0001 & 1000 & 1001 & 0111 \\
+ & \underline{0010} & \underline{1001} & \underline{0000} & \underline{0101} \\
 & & & & \\
 & \underline{} & \underline{} & \underline{} & \underline{}
\end{array}
$$

ALPHANUMERIC CODES - ASCII Character Codes

- American Standard Code for Information Interchange (Refer to Table 1-4 in the text)
- This code is a popular code used to represent information sent as character-based data. It uses 7-bits to represent:
  - 94 Graphic printing characters.
  - 34 Non-printing characters
- Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return)
- Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).

Bits b_7 b_6 b_5					0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
b_4 ↓	b_3 ↓	b_2 ↓	b_1 ↓	Column → Row ↓	0	1	2	3	4	5	6	7
0	0	0	0	0	NUL	DLE	SP	0	@	P	`	p
0	0	0	1	1	SOH	DC1	!	1	A	Q	a	q
0	0	1	0	2	STX	DC2	"	2	B	R	b	r
0	0	1	1	3	ETX	DC3	#	3	C	S	c	s
0	1	0	0	4	EOT	DC4	$	4	D	T	d	t
0	1	0	1	5	ENQ	NAK	%	5	E	U	e	u
0	1	1	0	6	ACK	SYN	&	6	F	V	f	v
0	1	1	1	7	BEL	ETB	'	7	G	W	g	w
1	0	0	0	8	BS	CAN	(	8	H	X	h	x
1	0	0	1	9	HT	EM	)	9	I	Y	i	y
1	0	1	0	10	LF	SUB	*	:	J	Z	j	z
1	0	1	1	11	VT	ESC	+	;	K	[	k	{
1	1	0	0	12	FF	FC	,	<	L	\	l	|
1	1	0	1	13	CR	GS	-	=	M	]	m	}
1	1	1	0	14	SO	RS	.	>	N	^	n	~
1	1	1	1	15	SI	US	/	?	O	_	o	DEL

ASCII Properties

ASCII has some interesting properties:

- Digits 0 to 9 span Hexadecimal values 30_{16} to 39_{16} .
- Upper case A-Z span 41_{16} to $5A_{16}$.
- Lower case a-z span 61_{16} to $7A_{16}$.
  - Lower to upper case translation (and vice versa) occurs by flipping bit 6.
- Delete (DEL) is all bits set, a carryover from when punched paper tape was used to store messages.
- Punching all holes in a row erased a mistake!

PARITY BIT Error-Detection Codes

- Redundancy (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.

- A simple form of redundancy is parity, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.
- A code word has even parity if the number of 1's in the code word is even.
- A code word has odd parity if the number of 1's in the code word is odd.

4-Bit Parity Code Example

- **Fill in the even and odd parity bits:**

Even Parity Message - Parity	Odd Parity Message - Parity
000 _	000 _
001 _	001 _
010 _	010 _
011 _	011 _
100 _	100 _
101 _	101 _
110 _	110 _
111 _	111 _

- **The codeword "1111" has <u>even parity</u> and the codeword "1110" has <u>odd parity</u>. Both can be used to represent 3-bit data.**

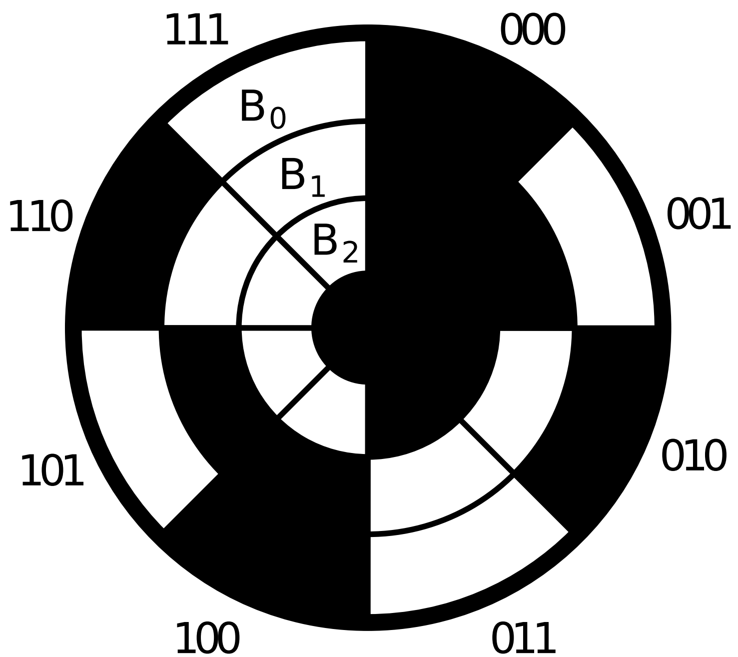
GRAY CODE – Decimal

Decimal	8,4,2,1	Gray
0	0000	0000
1	0001	0100
2	0010	0101
3	0011	0111
4	0100	0110
5	0101	0010
6	0110	0011
7	0111	0001
8	1000	1001
9	1001	1000

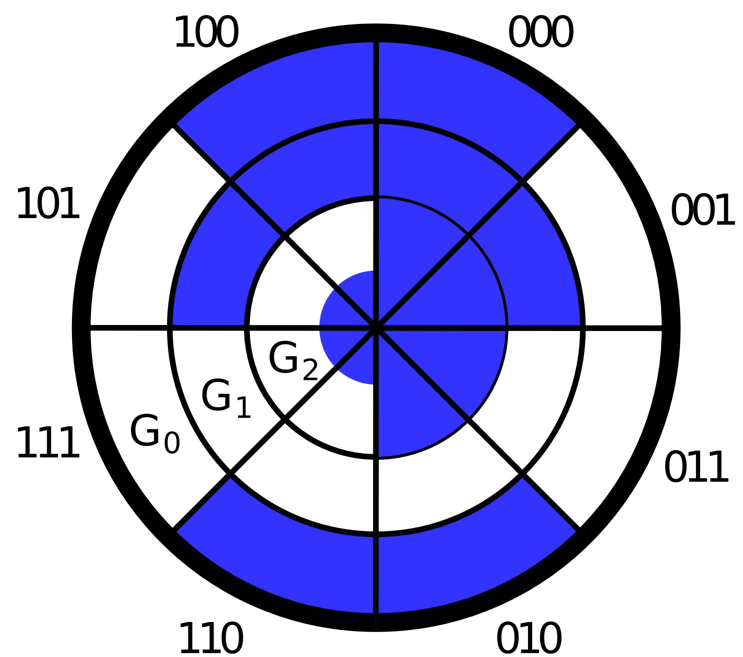
- **What special property does the Gray code have in relation to adjacent decimal digits?**

Optical Shaft Encoder

- Does this special Gray code property have any value?
- An Example: Optical Shaft Encoder



(a) Binary Code for Positions 0 through 7



(b) Gray Code for Positions 0 through 7

Shaft Encoder (Continued)

- **How does the shaft encoder work?**
- **For the binary code, what codes may be produced if the shaft position lies between codes for 3 and 4 (011 and 100)?**
- **Is this a problem?**

Shaft Encoder (Continued)

- **For the Gray code, what codes may be produced if the shaft position lies between codes for 3 and 4 (010 and 110)?**
- **Is this a problem?**
- **Does the Gray code function correctly for these borderline shaft positions for all cases encountered in octal counting?**

UNICODE

- **UNICODE extends ASCII to 65,536 universal characters codes**
  - **For encoding characters in world languages**
  - **Available in many modern applications**
  - **2 byte (16-bit) code words**
  - **See Reading Supplement – Unicode on the Companion Website** http://www.prenhall.com/mano

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