CENG 115 Fall 2023 Homework 1
Due date: 23th of November, 9:45 AM

Note: You are expected to bring <u>hardcopies</u> of your answers to the class (your own section) on 23rd Nov (Recitation hour). Homeworks submitted afterwards will not be evaluated.

1. **(25 points)** A detective has interviewed three witnesses to a crime. These are the cook, the gardener, and the handyman. From the stories of the witnesses the detective has concluded that
   - the cook and the gardener cannot both be telling the truth;
   - the gardener and the handyman are not both lying (i.e., if one of them is lying the other is not)
   - if the handyman is telling the truth; then, the cook is lying.

   Can the detective determine whether the cook is telling the truth or lying? **Explain** your *reasoning*.

2. **(25 points)** Determine whether '**the set of positive integers that are divisible by 5 but not by 6**' is countable or uncountable. If this set is *countable*, **prove it by proposing a bijection** (one-to-one correspondence) from the set of positive integers to this set.
   Hint: In addition to the arithmetic operators, you are allowed to use *floor* and *ceiling* functions.

3. (20 points)

a) **Prove** that $f(x) = 2x^2+5x \log_2 x$ is $O(x^2)$ by finding a pair of (c,k) for the inequality in the big-O definition.

b) $\sum_{k=1}^{n}(3k^2-1)$ is $\Theta(?)$.

4. (30 points)

a) Suppose you are given a list of positive integers $\{ a_1, a_2, \ldots a_n\}$. **Describe** an algorithm (write-down its **pseudocode**) that goes through the elements in the list *one by one* and finds the index of the first element that is *greater* than the sum of all previous elements in the list. If there is no such element, the index should be returned as zero. **Attention**: Your pseudo-code should not include functions/structures that are specific to C, C++, Java or Python

b) Define the **worst-case** time complexity of the algorithm you described in (a) with its exact order of growth, i.e. Θ. You can **assume** that all the lines in your code take *a fixed amount of time*, *t*. **Explain** your answer shortly.