Q1) HW2.1

Use mathematical induction to prove that, for all integers $n>0$,

$$\overset{2^0}{1}-\overset{2^1}{2}+\overset{2^2}{2^2}-2^3+\ldots+(-1)^n 2^n=\frac{2^{n+1}(-1)^n+1}{3}$$

Basis step: $P(1)=(-1)^0\cdot 2^0+(-1)^1\cdot 2^1=\frac{2^2(-1)+1}{3}=-1$

Inductive step: $P(k+1)=\underbrace{1-2+2^2\ldots\ldots(-1)^k\cdot 2^k}_{\text{Inductive hypothesis}}+(-1)^{k+1}\cdot 2^{k+1}=\frac{2^{k+1+1}(-1)^{k+1}+1}{3}$

$$=\frac{2^{k+1}(-1)^k+1}{3}+(-1)^{k+1}\cdot 2^{k+1}$$

$$=\frac{2^k\cdot 2\cdot(-1)^k+1-3\cdot(-1)^k\cdot 2^k\cdot 2}{3}$$

$$=\frac{2^k\cdot 2\cdot(-1)^k\cdot(-2)+1}{3}=\frac{2^k\cdot 2\cdot 2\cdot(-1)^k\cdot(-1)+1}{3}=\frac{2^{k+2}\cdot(-1)^{k+1}+1}{3}$$

Same

Q2) HW2.2

Each machine part made in a factory is stamped with a code of the form “letter-digit-digit-digit”, where the digits can be repeated. For instance, W065. At least how many parts should be produced to make sure that at least four of them have the same code stamped on them?

First part) $\underline{26}\ \underline{10}\ \underline{10}\ \underline{10} = 26000$ different codes can be stamped

Second part) $3 \cdot 26000 + 1$ Why? Because $\left\lceil \frac{78001}{26000} \right\rceil = 4$

Q3) HW2.3

How many different strings can be made by reordering the letters of CORRECT?

$$\frac{7!}{2!\cdot 2!}$$

Q4) HW2.4

How many different solutions are there to the equation:

$$x_1 + x_2 + x_3 + x_4 = 17 \qquad \text{where } x_i \geq 1$$

$$\begin{matrix} 1 & & 1 & & 1 & & 1 \end{matrix}$$

$$x_1 + 1 + x_2 + 1 + x_3 + 1 + x_4 + 1 = 17 \qquad x_i \geq 0$$

$$x_1 + x_2 + x_3 + x_4 = 13 \qquad x_i \geq 0$$

1) $C(13+4-1, 4-1) = C(16,3)$

2) xxx | xxxxx | xxxx | x $\qquad C(16,3)$

Q5) HW2.5

Solve the following recurrence relation:

$a_0=3 \quad a_1=4 \quad a_2=-35 \quad a_3=-450$

$$a_n = \overbrace{10}^{c_1}a_{n-1} - \overbrace{25}^{c_2}a_{n-2} \text{ where } a_0=3, \quad a_1=4.$$

Note: Here, solving means finding a closed-form (non-recursive) equation for a_n .

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$r^2 - c_1 \cdot r - c_2 = 0 \qquad r^2 - 10r + 25 = 0 \qquad (r-5)(r-5)=0$

$r=5$

$a_n = A \cdot r^n + B \cdot n \cdot r^n$

Equation 1) $a_0 = 3 = A \cdot 5^0 + B \cdot 0 \cdot 5^0 = A = 3$

Equation 2) $a_1 = 4 = A \cdot 5^1 + B \cdot 1 \cdot 5^1 = 5A + 5B = 4$

$15 + 5B = 4$

$\frac{5B}{5} = \frac{-11}{5}$

$a_n = 3 \cdot 5^n - \frac{11}{5} n \cdot 5^n$

closed-form solution

if there are two different roots:

$a_n = A \cdot r_1^n + B \cdot r_2^n$

Q6) For each of the given pairs below, find the greatest common divisor and tell if these numbers are relatively prime or not?

(i) (210,13)

(ii) (49,154)

(i) $\gcd(210,13) = \gcd(210 \bmod 13, 13) = \gcd(13,2) = \gcd(2,1) = 1$

they are relatively prime.

(ii) $\gcd(49,154) = \gcd(154 \bmod 49, 49) = \gcd(49,7) = \gcd(7,0) = 7$

they are NOT relatively prime.

Q7) a) What is the inverse of 15 mod 26 ?

b) What is the inverse of 26 mod 15?

$\gcd(26,15) = 1$

Step	x = qy + r	x	y	gcd = ax+by
0	-	26	15	$1 = 3\cdot15 - 4(26-15) \Rightarrow 7\cdot15 - 4\cdot26 = 1$ $1 = 7\cdot15 - 4\cdot26$
1	$26 = 1\cdot15 + 11$	15	11	$1 = 3\cdot4 - 11$ $1 = 3(15-11) - 11$ $1 = 3\cdot15 - 4\cdot11$
2	$15 = 11\cdot1 + 4$	11	4	$1 = 4 - (11 - 2\cdot4)$ $= 4 - 11 + 2\cdot4$ $= 3\cdot4 - 11$
3	$11 = 2\cdot4 + 3$	4	3	$1 = 4 - 3\cdot1$
3	$4 = 3\cdot1 + 1$	3	$\boxed{1}$	Solve for r. Plug it in.

Inv of 15 mod 26

Inv of 26 mod 15 = -4 = 11

$7\cdot15 \bmod 26 = 1$

$26\cdot11 \bmod 15 = 1$

Q8) We want to guarantee that at least 3 people in a minibus were born in the same month of the year. To guarantee that at least how many people should be on that minibus?

smallest x

$$\left\lceil \frac{x}{12} \right\rceil = 3 \qquad \left\lceil \frac{25}{12} \right\rceil = 3 \qquad x = 25$$

Q9) Relation given in 0-1 matrix below, reflexive? symmetic? asymmetric? anti-symmetric?

$$\begin{bmatrix} 0 & 1 & \cancel{1} & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

reflexive? NO! it doesn't contain (a,a) pairs
diagonal is not all 1's.

symmetric? YES! All 1's have 1's on the opposite
side of the diagonal.

asymmetric? NO! Because both (a,b) and (b,a)
are elements of the relation.

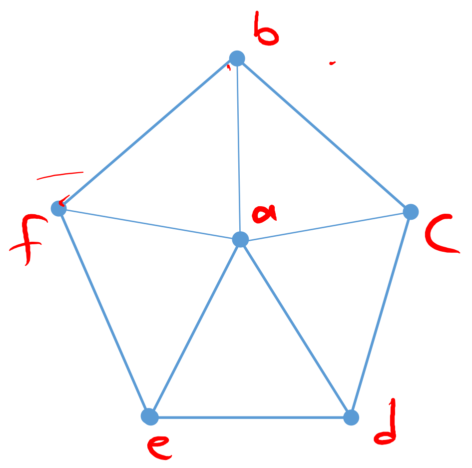
anti-symmetic? NO! 〃

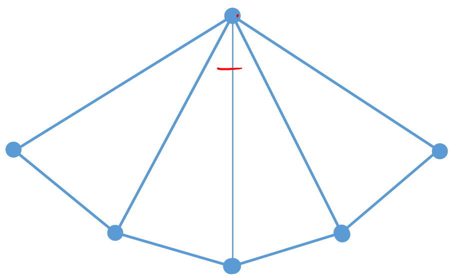
Q10) Assume the matrix given below is an adjacency matrix of an undirected graph G.

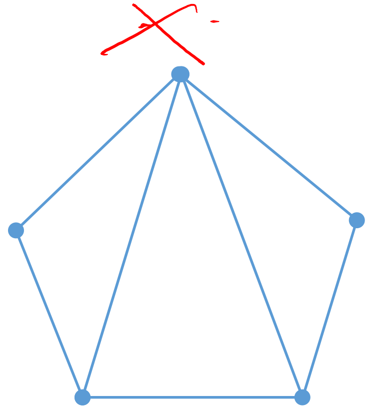
$$
\begin{array}{c c}
 & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{array}{c} \deg(a)=5 \quad a \\ \deg(b)=3 \quad b \\ (c)=3 \quad c \\ (d)=3 \quad d \\ (e)=3 \quad e \\ (f)=3 \quad f \end{array} &
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Hamilton vertex

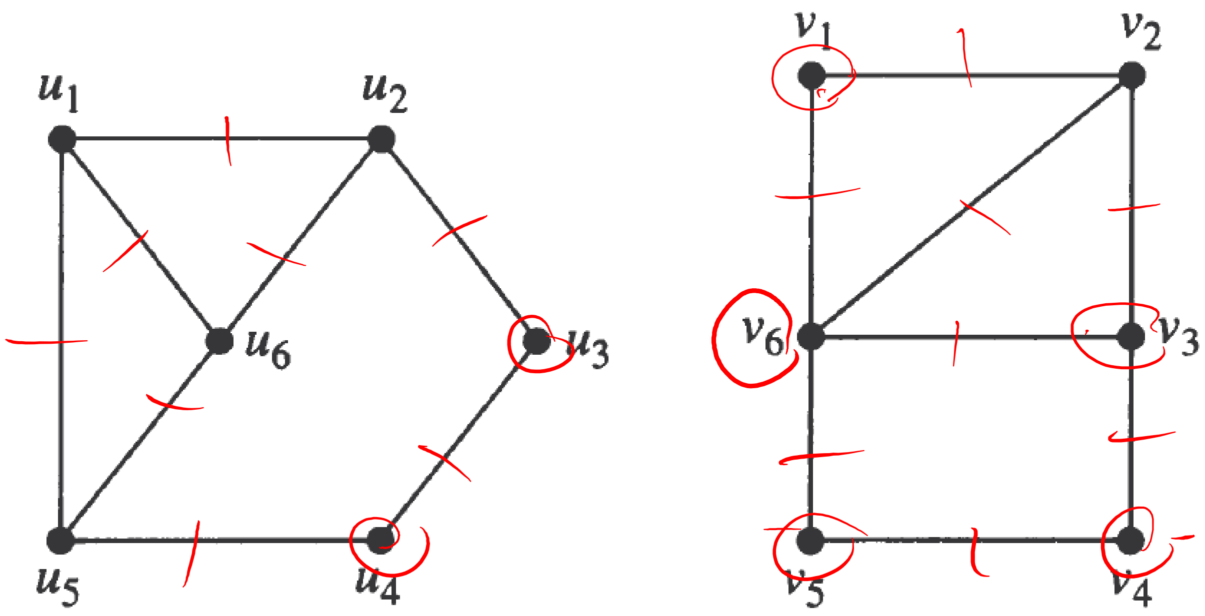
Which one of the following may be G? Does G have a Euler path or circuit?







Q11) For the pair given below, determine if the given two graphs are isomorphic? If your answer is yes, then exhibit an isomorphism (which vertex is which). If your answer is no, provide a valid argument that no isomorphism can exists.



in graph-2 $\deg(v_6) = 4$

no vertex in graph-1 has degree 4.

So, isomorphism can not exist.

$u_1 \to v_4$
$u_2 \to v_5$
$u_3 \to v_6$
$u_4 \to v_1$
$u_5 \to v_2$
$u_6 \to v_3$

adjacency list

u_1	u_2 u_5 u_6
u_2	u_1 u_6 u_3
u_3	u_2 u_4
u_4	u_3 u_5
u_5	u_1 u_6 u_4
u_6	u_1 u_2 u_5

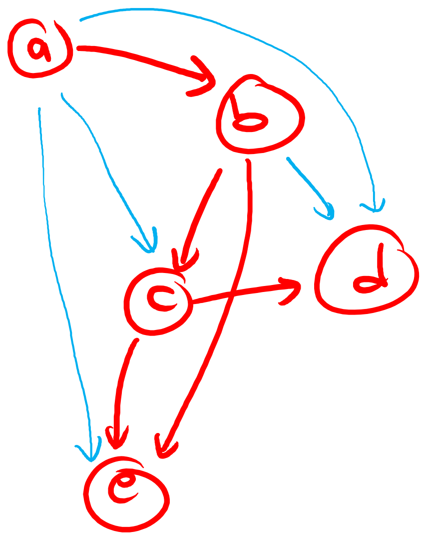
adj.

v_4 v_5 v_2 v_3

Q12) If the relation given below is not transitive, give its transitive closure as a 0-1 matrix.

Hint: You can use a directed graph to understand and solve the question.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



$R = \{ (a,b)\ (b,c)\ (b,e)\ (c,d)\ (c,e) \}$

$R^* = \{ (a,c)\ (a,e)\ (a,d)\ (b,d)$
$(a,b)\ (b,c)\ (b,e)\ (c,d)\ (c,e) \}$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q13) Let S be the set of all people in the world and relation R is defined for ordered pairs I.e. $(a,b) \in R$ where a and b are people. If R means 'a weighs more than b', then is (S,R) a partially ordered set? (poset)

Explain your answer by discussing the properties of a partially ordered set.

Properties of poset:

1) Reflexive: NO! Because R doesn't contain (a,a) pairs.

2) Anti-symmetric: YES! Because if $(a,b) \in R$ then $(b,a) \notin R$.

3) Transitive: YES! If $(a,b) \in R \wedge (b,c) \in R$ then $(a,c) \in R$.

Since it's not reflexive, R is not a poset.

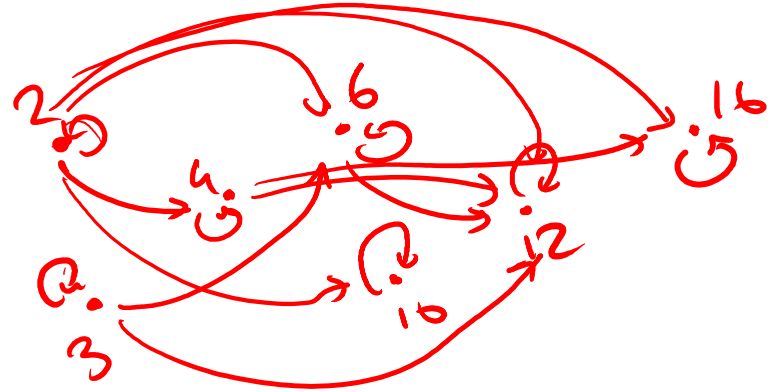
If R was "a weighs equal or more than b" $a \geq b$ then R would be poset.

Reflex. Anti-sym. Transitive

Q14)
Draw the Hasse diagram for the relation R on $A = \{2,3,4,6,10,12,16\}$ where aRb means $a \mid b$.

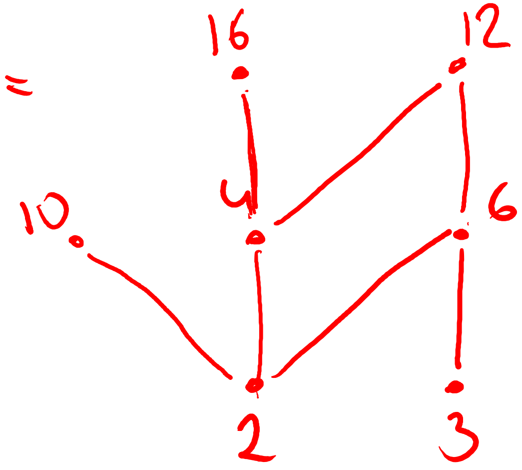
$R = \{(2,4)(2,6)(2,10)(2,12)(2,16)(3,6)(3,12)(4,12)(4,16)(6,12)(2,2)(3,3)(4,4)(6,6)(10,10)(12,12)(16,16)\}$

$G=$



anti-symmetric
reflexive
transitive

$H=$



- no self loop
- no arrow heads (assume all arrows upward)
- no transitive arcs