CENG 115 Fall 2023 Homework 2
Due date: 28th of December, 9:45

Note: You are expected to bring your handwritten answers to the class (your own section) on 28th December. Assignments submitted afterwards will not be evaluated.

1. **(25 points)**
   Use *mathematical induction* to prove that

   $$1 - 2 + 2^2 - 2^3 + \ldots + (-1)^n 2^n = \frac{2^{n+1}(-1)^n + 1}{3} \quad \text{where } \forall n \in \mathbf{Z}^+.$$

   Hint: Do not forget to define and prove basis step and inductive step separately.

2. **(20 points)**
   Each machine part made in a factory is stamped with a code of the form "letter-digit-digit-digit", where the digits can be repeated. For instance, W065. At least how many parts should be produced to make sure that *at least four of them* have the same code stamped on them?

3. **(15 points)**
   How many *different* strings can be made by reordering the letters of CORRECT?

4. **(15 points)**
   How many *different* solutions are there to the equation:

   $$x_1 + x_2 + x_3 + x_4 = 17 \qquad \text{where } x_i \geq 1$$

5. **(25 points)**
   Solve the following *recurrence* relation:

   $$a_n = 10a_{n-1} - 25a_{n-2} \text{ where } a_0 = 3, \ a_1 = 4.$$

   Note: Here, solving means finding a closed-form (non-recursive) equation for a_n.