

# **Database Theory for Relational Databases:**

**Multivalued Dependencies  
4<sup>th</sup> Normal Form**

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**CENG315 INFORMATION MANAGEMENT**

# Relational Design by Decomposition

- “Mega relations” + properties of data
- The system decomposes those on based on the properties that are specified
- The final set of decomposed relations satisfy normal form
  - Normal forms are “good” relations
    - No anomalies
    - No lost information
- The properties themselves are defined either as
  - a) Functional dependencies → Boyce-Codd Normal Form (BCNF)
  - b) Multivalued dependencies → Fourth Normal Form

# Example

- Apply (SSN, cName, hobby)
- FD's?
- Keys?
- BCNF?
- Good design?

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- Good design? No.

# Definition of Multivalued Dependency

- A *multivalued dependency (MVD)* on  $R$ ,  $X \twoheadrightarrow Y$ , says that if two tuples of  $R$  agree on all the attributes of  $X$ , then their components in  $Y$  may be swapped, and the result will be two tuples that are also in the relation.
- i.e., for each value of  $X$ , the values of  $Y$  are independent of the values of  $R-X-Y$ .
- MVD's can cause redundancy in a relation.



# Example: MVD

Customers (name, addr, phones, drinksLiked)

- A customer may have several phone numbers and may like several drinks.
- Those are independent facts about the customer.
- A customer's phones are independent of the drinks they like.
  - $\text{name} \twoheadrightarrow \text{phones}$  and  $\text{name} \twoheadrightarrow \text{drinksLiked}$
- Thus, each of a customer's phones appears with each of the drinks they like in all combinations.
- This repetition is unlike FD redundancy.
  - $\text{name} \rightarrow \text{addr}$  is the only FD.

## Tuples Implied by **name** $\rightarrow\rightarrow$ **phones**

- If we have tuples:

name	addr	phones	drinksLiked
sue	a	p1	d1
sue	a	p2	d2

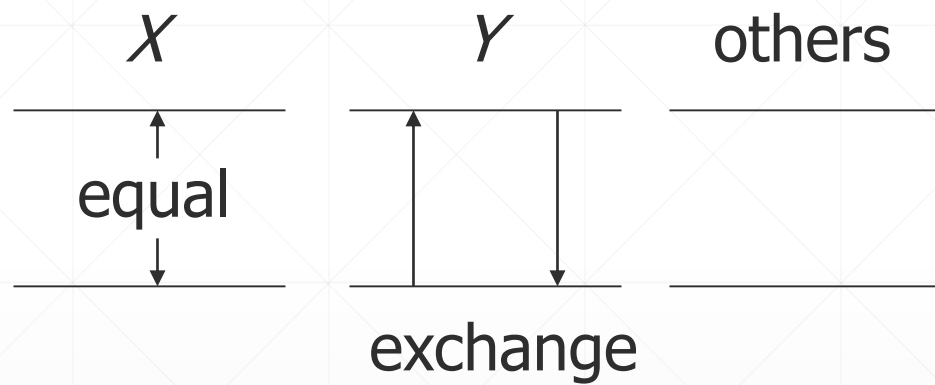
## Tuples Implied by **name** $\rightarrow\rightarrow$ **phones**

- If we have tuples:

name	addr	phones	drinksLiked
sue	a	p1	d1
sue	a	p2	d2
sue	a	p2	d1
sue	a	p1	d2

- Then these tuples must also be in the relation.

## Picture of MVD $X \rightarrow \rightarrow Y$



# MVD Rules

- Every FD is an MVD.
  - If  $X \rightarrow Y$ , then swapping  $Y$ 's between two tuples that agree on  $X$  doesn't change the tuples.
  - Therefore, the “new” tuples are surely in the relation, and we know  $X \twoheadrightarrow Y$ .
- *Complementation:* If  $X \twoheadrightarrow Y$ , and  $Z$  is all the other attributes, then  $X \twoheadrightarrow Z$ .

# Splitting Doesn't Hold

- Like FD's, we cannot generally split the left side of an MVD.
- But unlike FD's, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

# Example: Multiattribute Right Sides

Customers (name, areaCode, phone, drinksLiked, manf)

- A customer can have several phones, with the number divided between areaCode and phone (last 7 digits).
- A customer can like several drinks, each with its own manufacturer.

## Example: Multiattribute Right Sides (Cont.)

Customers (name, areaCode, phone, drinksLiked, manf)

- Since the areaCode-phone combinations for a customer are independent of the drinksLiked-manf combinations, we expect that the following MVD's hold:

name  $\twoheadrightarrow$  areaCode phone

name  $\twoheadrightarrow$  drinksLiked manf



## Example Data

- Here is possible data satisfying these MVD's:

name	areaCode	phone	drinksLiked	manf
Sue	650	555-1111	Coke	C.C.
Sue	650	555-1111	IceTea	Lipton
Sue	415	55-9999	Coke	C.C.
Sue	415	55-9999	IceTea	Lipton

- But we cannot swap area codes or phones by themselves.
  - That is, neither  $\text{name} \twoheadrightarrow \text{areaCode}$  nor  $\text{name} \twoheadrightarrow \text{phone}$  holds for this relation.

# Fourth Normal Form (4NF)

- The separation of independent facts is what 4NF is about.
- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.
- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

# 4NF Definition

- A relation  $R$  is in **4NF** if: whenever  $X \twoheadrightarrow Y$  is a nontrivial MVD, then  $X$  is a superkey.
  - **Nontrivial MVD** means that:
    1.  $Y$  is not a subset of  $X$ , and
    2.  $X$  and  $Y$  are not, together, all the attributes.
  - Note that the definition of “superkey” still depends on FD’s only.

# BCNF Versus 4NF

- Remember that every FD  $X \rightarrow Y$  is also an MVD,  $X \twoheadrightarrow Y$ .
- Thus, if  $R$  is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation (after conversion to an MVD).
- But  $R$  could be in BCNF and not 4NF, because MVD's are “invisible” to BCNF.

# Decomposition and 4NF

- If  $X \twoheadrightarrow Y$  is a 4NF violation for relation  $R$ , we can decompose  $R$  using the same technique as for BCNF.
  1.  $XY$  is one of the decomposed relations.
  2. All but  $Y - X$  is the other.  $(R - (Y - X))$

# 4NF Decomposition Algorithm

- **Input:** Relation  $R$ , FD's for  $R$ , MVD's for  $R$
- **Output:** Decomposition of  $R$  into 4NF relations with “lossless join”
- Compute keys for  $R$
- Repeat until all relations are in 4NF:
  - Pick any  $R'$  with nontrivial  $X \twoheadrightarrow Y$  that violates 4NF
  - Decompose  $R'$  into  $R_1(X, Y)$  and  $R_2(X, \text{rest})$
  - Compute FD's and MVD's for  $R_1$  and  $R_2$
  - Compute keys for  $R_1$  and  $R_2$

# Example: 4NF Decomposition

Apply (SSN, cName, hobby)

MVD's:  $SSN \twoheadrightarrow cName$

$SSN \twoheadrightarrow hobby$

- Key is {SSN, cName, hobby}.
- All dependencies violate 4NF.

## Example: 4NF Decomposition (Cont.)

- Decompose using  $SSN \twoheadrightarrow cName$ :

### 1. Apply1 (SSN, cName)

- No FD's
- No MVD's
- In 4NF

### 2. Apply2 (SSN, hobby)

- No FD's
- No MVD's
- In 4NF



## Example: 4NF Decomposition (Cont.)

Customers (name, addr, phones, drinksLiked)

FD:                     $\text{name} \rightarrow \text{addr}$

MVD's:                 $\text{name} \twoheadrightarrow \text{phones}$   
                          $\text{name} \twoheadrightarrow \text{drinksLiked}$

- Key is {name, phones, drinksLiked}.
- All dependencies violate 4NF.

## Example: 4NF Decomposition (Cont.)

- Decompose using  $\text{name} \rightarrow \text{addr}$ :
  1. Customers1 (name, addr)
    - In 4NF; only dependency is  $\text{name} \rightarrow \text{addr}$ .
  2. Customers2 (name, phones, drinksLiked)
    - Not in 4NF. MVD's  $\text{name} \twoheadrightarrow \text{phones}$  and  $\text{name} \twoheadrightarrow \text{drinksLiked}$  apply. No FD's, so all three attributes form the key.

## Example: 4NF Decomposition (Cont.)

- Either MVD  $\text{name} \twoheadrightarrow \text{phones}$  or  $\text{name} \twoheadrightarrow \text{drinksLiked}$  tells us to decompose to:
  - Customers3 (name, phones)
  - Customers4 (name, drinksLiked)

# References

- Jeffrey D. Ullman and Jennifer Widom, “A First Course in Database Systems”, 3<sup>rd</sup> Ed., 2007.
  - <http://infolab.stanford.edu/~ullman/fcdb/aut07/slides/mvds.pdf>
- Stanford DB Class