CENG 216 – Numerical Computation
Final Exam

2019–20 Spring Semester
June 30, 2020

- This exam contains 4 questions on 2 pages.
- The exam duration is **75 minutes**.
- Use your **own handwriting**, no typesetting is allowed.
- Make sure to **write your name/student ID** on each page you submit.
- Make sure to **sign** each page.
- All values for angles are given in **radians**.
- Good Luck!

Q1 *(25 points)* Householder Reflectors and Orthogonality

i. Compute the Householder reflector that transforms the vector $\begin{bmatrix}2\\1\end{bmatrix}$ to the vector $\begin{bmatrix}-1\\2\end{bmatrix}$.

ii. Show that the matrix

$$\mathtt{Q} = \begin{bmatrix}0.2524571 & -0.2599396 & 0.9320391\\ 0.9340050 & -0.1861786 & -0.3049136\\ 0.2527848 & 0.9475067 & 0.1957827\end{bmatrix}$$

is almost orthogonal (when computations are rounded to four figures after the dot position).

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Q2 *(25 points)* Least Squares and Curve Fitting

The following temperature measurements have been obtained during a day at Gülbahçe, İzmir:

$$(t = 10, T = 28), (t = 12, T = 30), (t = 14, T = 35), \text{ and } (t = 16, T = 33).$$

i. Assuming that the temperature changes continuously as $T(t) = a_2t^2 + a_1t + a_0$ what are the values of the coefficients $a_i, i = 0, 1, 2$ if we fit to the data in the least square sense?

ii. What will be the value of the peak temperature $T_{\max}$ and the time that it will take place $t_{\max}$? *Hint:* Differentiate the continuous curve to find the formula for the peak location. Use the calculated coefficients to find the required values.

Q3 *(25 points)* Newton's Method with Numerical Derivatives

We are given the following function

$$f(x) = e^{\sin(e^x \cos(x) \ln(x))} - 1$$

with approximate root location $x_0 = 0.6$ and $f(x_0) = -0.5695990729412462$.

Apply three iterations of Newton's method to find the approximate root x_r **using numerically approximated derivatives**. You may choose any valid approximation formula and step size.

Hint: Please use the given function value $f(x_0)$ to check that you are evaluating the correct function.

Q4 *(25 points)* Numerical Integration

i. Compute an approximation of the integral $\int_0^1 x \cos x \, dx$ using the Simpson's rule.

ii. Compute an approximation of the integral $\int_0^1 x \cos x \, dx$ using the composite Trapezoid rule by partitioning the interval $[0, 1]$ into **two** sub-intervals.