

CENG216 – Numerical Computation

Homework 2

April 04, 2022

Due Date: April 25, 2022

Exercise 1. Root Finding (30 pts)

Calculate the x-coordinate of the intersection of the parabola $y = -x^2 + 4.0$ with the line $y = 4x - 1.0$ starting from an estimate of $x_0 = 1.5$ using

- (a) Ten iterations of the Fixed Point Iterations method.
- (b) Three iterations of the Newton's method.

When writing down the iteration results, it is OK to write down only six decimal digits after the dot.

Exercise 2. Solving Linear Systems (30 pts)

- (a) Solve the following system of equations by finding the LU decomposition of the matrix and applying two step back-substitution.

$$\begin{bmatrix} 4 & 2 & 1 \\ 8 & 5 & 4 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

- (b) Solve the system above again, this time by finding the $PA = LU$ decomposition.
- (c) Find the forward and backward errors and the error magnification factor of the system $x_1 + x_2 = 2$, $2x_1 + 2.01x_2 = 4$ for the approximate solution $[2, 1]$.

Exercise 3. Vaccination Rate (40 pts)

We want to decide the vaccination rate for a population in order to achieve herd immunity. The ratio of individuals in a population who have antibodies to an infectious agent is called seroprevalence. Assume that the following equation which involves seroprevalence and vaccination rate is true:

$$S_{true} = 1 - (1 - S_{observed}) * v^5$$

where $S_{observed}$ will be assumed to be a function of v :

$$S_{observed} = 0.1 + 0.7\sqrt[4]{v}$$

We want to ensure that $S_{true} = 0.9$.

- a. Write down a single constraint in terms of v that ensures that the true seroprevalence rate reached to the desired value.
- b. Convert the solution of v into a root finding problem, then solve this problem by searching for v in the interval $[0,1]$.
- c. Calculate the necessary $S_{observed}$ for the v you have found.

Important Notes

1. Assignments must be submitted by **teams of two students**. Once you form a team and start working on the homework, you cannot change your team. Otherwise, two different teams may end up with similar submissions, which will be considered cheating.
2. One of the team members should submit a single PDF file named as CENG216_HW2_StdId1_StdId2.pdf.