# CENG 216 – Numerical Computation
# Midterm Exam

2018–19 Spring Semester
April 10, 2019

- This exam contains 5 questions on 6 pages.
- The exam duration is 100 minutes.
- Read the questions carefully before starting to solve the problems.
- No electronic devices are allowed during the exam except a single calculator.
- Good Luck!

| Question | Q1 | Q2 | Q3 | Q4 | Q5 | **Total** |
|---|---|---|---|---|---|---|
| Points | 20 | 25 | 20 | 20 | 15 | **100** |
| Grade | | | | | | |

## Q1 *(20 points)* Floating-Point Representation

IEEE single-precision floating numbers are 32 bits each, 1 bit for the sign bit, 8 bits for the exponent, and 23 bits for the mantissa.

i. Convert the number 4.2 to binary.

ii. What is the value of $4.2 - fl(4.2)$ in IEEE single-precision format assuming Round to Nearest rule is active?

iii. For $a, b, c \in \mathbb{R}$, $s = \frac{a+b+c}{2.0}$, list the conditions on $a, b, c$ so that the formula

$$\sqrt{s(s-a)(s-b)(s-c)}$$

will involve loss of significance (catastrophic cancellation).

*Hint:* Because the mantissa is 23 bits long, there are five groups of 4 bits and the last group is 3 bits long. Be careful when you are writing down the contents of the mantissa and the leftover bits to round.

## Q2 *(25 points)* Solving Equations

Calculate the $x$-coordinate of the intersection of the parabola $y = -x^2 + 4.0$ with the line $y = 4x - 1.0$ starting from an estimate of $x_0 = 1.5$ using

i. Ten iterations of the Fixed Point Iterations method.

ii. Three iterations of the Newton's method.

*Hint:* When writing down the iteration results, it is OK to write down only four decimal digits after the dot.

### Q3 *(20 points)* Linear Systems of Equations

We are given the following linear system of equations

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix}$$

i. Find the solution to this system using LU decomposition.

ii. For the approximate solution $\mathbf{x}_a = [-1.0, 1.0, 1.0]^\top$, calculate the relative forward, backward errors, and the error magnification factor.

## Q4 *(20 points)* Interpolation

i. Find the coefficients $a, b, c$ of the polynomial $P_2(x) = ax^2 + bx + c$ of degree two or less passing through the data points $(0, 1), (1, 3)$, and $(2, -1)$ using a method of your choice.

ii. Find the coefficients $a, b, c, d$ of the polynomial $P_3(x) = ax^3 + bx^2 + cx + d$ of degree three or less passing through the data points $(0, 1), (1, 3), (2, -1)$, and $(3, -3)$ using a method of your choice.

## Q5 *(15 points)* Ball Throwing Robot Arm

We want to design a ball throwing robot arm that will throw a ball with initial velocity $V_0$ at an angle $\theta$ to the ground plane. Assuming that the robot arm is at coordinates $x = 0$ the ball will hit the ground at

$$x_f = \frac{V_0^2 \sin 2\theta}{g}$$

where $g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$ is the gravitational constant.

We want to ensure that the ball hits the ground at $x_f = 0.1730861$ meters. Unfortunately, we are not free to choose the necessary $V_0$ and $\theta$ since the robot design constrains the speed to be a function of the angle as

$$V_0 = k(1 + \cos\theta)$$

where $k = 0.75$ is a design parameter.

i. Write down a single constraint in terms of $\theta$ that ensures that the ball hits the ground at the given value of $x_f$.

ii. Convert the solution of $\theta$ into a root finding problem, then solve this problem by searching for $\theta$ in the interval $[0^\circ, 40^\circ]$.

iii. Calculate the necessary $V_0$ for the $\theta$ you have found.

*Hint:* Although the $\theta$ range is given in degrees, perform the calculations in radians. Note that $360^\circ = 2\pi$ radians.