CENG 216 HW2

1)

Given

parabola: $y = -x^2 + 4$

line: $y = 4x - 1$

$x_0 = 1.5$ x? → The other root is 1 because after the ten iteration of Fixed point it closes to 1

a) Ten iterations of Fixed Point iterations method.

$$4x - 1 = -x^2 + 4$$

$$4x = 5 - x^2$$

$$x = \frac{5 - x^2}{4}$$

$$x_1 = \frac{5 - x_0^2}{4} = \frac{5 - (1.5)^2}{4} = 0.6875$$

$$x_2 = \frac{5 - x_1^2}{4} = \frac{5 - (0.6875)^2}{4} = 1.131835$$

$$x_3 = \frac{5 - x_2^2}{4} = \frac{5 - (1.131835)^2}{4} = 0.929736$$

$$x_4 = \frac{5 - x_3^2}{4} = \frac{5 - (0.929736)^2}{4} = 1.033914$$

$$x_5 = \frac{5 - x_4^2}{4} = \frac{5 - (1.033914)^2}{4} = 0.982755$$

$$x_6 = \frac{5 - x_5^2}{4} = \frac{5 - (0.982755)^2}{4} = 1.008548$$

$$x_7 = \frac{5 - x_6^2}{4} = \frac{5 - (1.008548)^2}{4} = 0.995707$$

$$x_8 = \frac{5 - x_7^2}{4} = \frac{5 - (0.995707)^2}{4} = 1.002141$$

$$x_9 = \frac{5 - x_8^2}{4} = \frac{5 - (1.002141)^2}{4} = 0.998928$$

$$x_{10} = \frac{5 - x_9^2}{4} = \frac{5 - (0.998928)^2}{4} = 1.000535$$

4)

b) Three iterations of the Newton's Method.

Given

parabola : $y = -x^2 + 4$

line : $y = 4x - 1$

$x_0 = 1.5$ $x = ?$ → The other root is 1 because after the three iterations of Newtons method it close to 1

$$4x - 1 = -x^2 + 4$$

$$f(x) = x^2 + 4x - 5 = 0$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{(x_k^2 + 4x_k - 5)}{2x_k + 4}$$

$$= \frac{2x_k^2 + 4x_k - x_k^2 - 4x_k + 5}{2x_k + 4}$$

$$x_{k+1} = \frac{x_k^2 + 5}{2x_k + 4}$$

$$x_1 = \frac{x_0^2 + 5}{2x_0 + 4} = \frac{(1.5)^2 + 5}{2.(1.5) + 4} = \frac{7,25}{7} = 1.035714$$

$$x_2 = \frac{x_1^2 + 5}{2x_1 + 4} = \frac{(1.035714)^2 + 5}{2.(1.035714) + 4} = \frac{6,072404}{6,071428} = 1,000160$$

$$x_3 = \frac{x_2^2 + 5}{2x_2 + 4} = \frac{(1,000160)^2 + 5}{2.(1,000160) + 4} = \frac{6,000320}{6,00032} = 1$$

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2)

a)

L	U	
$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$	$\begin{bmatrix} 4 & 2 & 1 \\ 8 & 5 & 4 \\ -1 & -2 & -3 \end{bmatrix}$	
$\begin{bmatrix} 1 & ? & ? \\ 2 & ? & ? \\ ? & ? & ? \end{bmatrix}$	$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 2 \\ -1 & -2 & -3 \end{bmatrix}$	• Subtract $2 \times$ Row1 from Row 2
$\begin{bmatrix} 1 & ? & ? \\ 2 & ? & ? \\ -\frac{1}{4} & ? & ? \end{bmatrix}$	$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & -\frac{3}{2} & -\frac{11}{4} \end{bmatrix}$	• Subtract $-\frac{1}{4} \times$ Row1 from Row 3
$\begin{bmatrix} 1 & 0 & ? \\ 2 & 1 & ? \\ -\frac{1}{4} & -\frac{3}{2} & ? \end{bmatrix}$	$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$	• Subtract $-\frac{3}{2} \times$ Row2 from Row3

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -\frac{1}{4} & -\frac{3}{2} & 0 \end{bmatrix} \qquad U = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

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2)

a)

$$Ax = b$$

$$\Rightarrow LUx = b$$

$$\Rightarrow Lc = b \quad (c = Ux)$$

$*$ $Lc = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{-1}{4} & \frac{-3}{2} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\left.\begin{aligned} c_1 &= 3 \\ 2c_1 + c_2 &= 7 \\ -\frac{1}{4}c_1 - \frac{3}{2}c_2 + c_3 &= -1 \end{aligned}\right\}$$

$c_1 = 3$

$c_2 = 7 - 2c_1 = 7 - 2(3) = 1$

$c_3 = -1 + \frac{1}{4}c_1 + \frac{3}{2}c_2 = -1 + \frac{1}{4}(3) + \frac{3}{2}(1) = \frac{5}{4}$

$$c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5/4 \end{bmatrix}$$

$*$ $Ux = c$

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5/4 \end{bmatrix}$$

$$\left.\begin{aligned} 4x_1 + 2x_2 + x_3 &= 3 \\ x_2 + 2x_3 &= 1 \\ \frac{1}{4}x_3 &= \frac{5}{4} \end{aligned}\right\}$$

$x_3 = 5$

$x_2 = 1 - 2x_3 = 1 - 2(5) = -9$

$x_1 = \dfrac{3 - x_3 - 2x_2}{4} = \dfrac{3 - (5) - 2.(-9)}{4} = \frac{16}{4} = 4$

$$\boxed{x_1 = 4 \qquad x_2 = -9 \qquad x_3 = 5}$$

4

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2)

b)

P	L	U	
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$	$\begin{bmatrix} 4 & 2 & 1 \\ 8 & 5 & 4 \\ -1 & -2 & -3 \end{bmatrix}$	
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$	$\begin{bmatrix} 8 & 5 & 4 \\ 4 & 2 & 1 \\ -1 & -2 & -3 \end{bmatrix}$	$R_1 \longleftrightarrow R_2$ (Exchange R_1 and R_2)
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & ? & ? \\ \frac{1}{2} & ? & ? \\ \frac{-1}{8} & ? & ? \end{bmatrix}$	$\begin{bmatrix} 8 & 5 & 4 \\ 0 & \frac{-1}{2} & -1 \\ 0 & \frac{-11}{8} & \frac{-5}{2} \end{bmatrix}$	• Subtract $\frac{1}{2} \times$ Row 1 from Row 2 • Subtract $\frac{-1}{8} \times$ Row 1 from Row 3
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & ? & ? \\ \frac{-1}{8} & ? & ? \\ \frac{1}{2} & ? & ? \end{bmatrix}$	$\begin{bmatrix} 8 & 5 & 4 \\ 0 & \frac{-11}{8} & \frac{-5}{2} \\ 0 & \frac{-1}{2} & -1 \end{bmatrix}$	$R_2 \longleftrightarrow R_3$ (Exchange R_2 and R_3)
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & ? \\ \frac{-1}{8} & 1 & ? \\ \frac{1}{2} & \frac{4}{11} & ? \end{bmatrix}$	$\begin{bmatrix} 8 & 5 & 4 \\ 0 & \frac{-11}{8} & \frac{-5}{2} \\ 0 & 0 & \frac{-1}{11} \end{bmatrix}$	• Subtract $\frac{4}{11} \times$ Row 2 from Row 3

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{8} & 1 & 0 \\ \frac{1}{2} & \frac{4}{11} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 8 & 5 & 4 \\ 0 & \frac{-11}{8} & \frac{-5}{2} \\ 0 & 0 & \frac{-1}{11} \end{bmatrix}$$

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2)

b)

$Ax = b$

$\Rightarrow PAx = Pb$

$\Rightarrow LUx = Pb \quad (PA = LU)$

$\Rightarrow Lc = Pb \quad (Ux = c)$

$*$ $Lc = Pb$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{8} & 1 & 0 \\ \frac{1}{2} & \frac{4}{11} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{8} & 1 & 0 \\ \frac{1}{2} & \frac{4}{11} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 3 \end{bmatrix}$$

$$\left.\begin{aligned} c_1 &= 7 \\ -\frac{1}{8}c_1 + c_2 &= -1 \\ +\frac{1}{2}c_1 + \frac{4}{11}c_2 + c_3 &= 3 \end{aligned}\right\}$$

$c_1 = 7$

$c_2 = -1 + \frac{1}{8}(7) = -\frac{1}{8}$

$c_3 = 3 - \frac{1}{2}c_1 - \frac{4}{11}c_2 = 3 - \frac{7}{2} + \frac{1}{22} = -\frac{5}{11}$

$$c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -\frac{1}{8} \\ -\frac{5}{11} \end{bmatrix}$$

$$Ux = \begin{bmatrix} 7 \\ -\frac{1}{8} \\ -\frac{5}{11} \end{bmatrix}$$

$*$ $Ux = c$

$$\begin{bmatrix} 8 & 5 & 4 \\ 0 & -\frac{11}{8} & -\frac{5}{2} \\ 0 & 0 & -\frac{1}{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -\frac{1}{8} \\ -\frac{5}{11} \end{bmatrix}$$

$$\left.\begin{aligned} 8x_1 + 5x_2 + 4x_3 &= 7 \\ -\frac{11}{8}x_2 - \frac{5}{2}x_3 &= -\frac{1}{8} \\ -\frac{1}{11}x_3 &= -\frac{5}{11} \end{aligned}\right\}$$

$x_3 = 5$

$x_2 = -\frac{8}{11}\left[-\frac{1}{8} + \frac{5}{2}x_3\right] = -\frac{8}{11}\left[-\frac{1}{8} + \frac{25}{2}\right] = -9$

$x_1 = \frac{1}{8}\left[7 - 5x_2 - 4x_3\right] = \frac{1}{8}\left[7 + 45 - 20\right] = 4$

$$\boxed{x_1 = 4, \quad x_2 = -9 \quad x_3 = 5}$$

6

2)

c)

Given

$$\left.\begin{array}{l} x_1 + x_2 = 2 \\ 2x_1 + 2.01x_2 = 4 \end{array}\right\} \text{ systems}$$

$[2, 1] \rightarrow$ Approximate solution

$$\begin{bmatrix} 1 & 1 \\ 2 & 2.01 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad x_a = [2, 1]^T$$

- Backward Error: $\|r\|_\infty = \|b - Ax_a\|_\infty$

$$\left\| \begin{bmatrix} 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 2.01 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\|_\infty = \left\| \begin{bmatrix} -1 \\ -2.01 \end{bmatrix} \right\|_\infty = 2.01,$$

- Forward Error: $\|x_a - x\|_\infty$

$$\left\| \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\|_\infty = \left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\|_\infty = 1$$

- Error Magnification Factor

$$\frac{\dfrac{\|x_a - x\|_\infty}{\|x\|_\infty}}{\dfrac{\|r\|_\infty}{\|b\|_\infty}} = \frac{\dfrac{1}{1}}{\dfrac{2.01}{4}} \approx 1.990049$$

7

3)

Given

$$S_{True} = 1 - (1 - S_{ob}) * v^S$$

$$S_{ob} = 0.1 + 0.7\sqrt[4]{v}$$

We want to ensure that $S_{True} = 0.9$

a)

$$S_{True} = 1 - (1 - (0.1 + 0.7\sqrt[4]{v})) * v^S$$

b)

$$0.9 = 1 - (1 - (0.1 + 0.7\sqrt[4]{v})) * v^S$$

$$0.1 = (1 - (0.1 + 0.7\sqrt[4]{v})) * v^S$$

$$0.1 = (0.9 - 0.7\sqrt[4]{v}) * v^S$$

$$1 = (9 - 7\sqrt[4]{v}) * v^S$$

$$v^S = \frac{1}{9 - 7\sqrt[4]{v}}$$

$$v = \sqrt[S]{\frac{1}{9 - 7\sqrt[4]{v}}}$$

3)

5) We will use fixed-point iteration to find root (V)

Our initial guess for at V=0 where $\sqrt[5]{\frac{1}{9-7\sqrt[4]{V}}}$

$x_0 = 0$

$x_1 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_0}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0}}} = 0.644439$

$x_2 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_1}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.644439}}} = 0.818129$

$x_3 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_2}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.818129}}} = 0.843450$

$x_4 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_3}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.843450..}}} = 0.843450$

$x_5 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_4}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.843450}}} = 0.847166$

$x_6 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_5}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.847166}}} = 0.847712$

$x_7 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_6}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.847793}}} = 0.847793$

$x_8 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_7}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.847793}}} = 0.847805$

$x_9 = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_8}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.847805}}} = 0.847806$

$x_{10} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{x_9}}} = \sqrt[5]{\frac{1}{9-7\sqrt[4]{0.847806}}} = 0.847807$

After ten iterations of the fixed point iterations method $V = 0.8478$

9

3)

c)

$$S_{observed} = 0.1 + 0.7\sqrt[4]{v}$$

$$= 0.1 + 0.7\sqrt[4]{0.3472}$$

$$= 0.771635$$

10