CENG 216 – Numerical Computation
Final Exam

2018–19 Spring Semester
June 19, 2019

- This exam contains 4 questions on 6 pages.
- The exam duration is 100 minutes.
- Read the questions carefully before starting to solve the problems.
- Check your answers whenever you can by verifying simple truths.
- No electronic devices are allowed during the exam except a single calculator.
- Good Luck!

Question	Q1	Q2	Q3	Q4	**Total**
Points	25	25	25	25	**100**
Grade					

Q1 *(25 points)* Interpolation

Find the first endpoint, two control points and the last endpoint for the one-piece Bézier curve

$$x(t) = 1 + 6t^2 + 2t^3,$$
$$y(t) = 1 - t + t^3.$$

Q2 *(25 points)* Linear Least-Squares and QR

Solve the following system using QR decomposition via Householder reflections such that the error vector $\mathtt{A}\mathbf{x}_{LS} - \mathbf{b}$ has the least norm.

$$\mathtt{A}\mathbf{x} = \begin{bmatrix} 0 & 3 \\ 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} = \mathbf{b}$$

First decompose $\mathtt{A} = [\mathbf{a}_1 \mathbf{a}_2] = \mathtt{QR}$ by picking $\mathbf{v}_1 = \mathbf{a}_1 + \|\mathbf{a}_1\| \, \mathbf{e}_1$ and $\mathtt{H}_1 = \mathtt{I} - 2\frac{\mathbf{v}_1 \mathbf{v}_1^\top}{\mathbf{v}_1^\top \mathbf{v}_1}$ and applying a similar transformation to $\mathtt{H}_1\mathtt{A}$ such that $\mathtt{H}_2\mathtt{H}_1\mathtt{A} = \mathtt{R}$. Then solve the system

$$\mathtt{Q}^\top \mathtt{A}\mathbf{x} = \mathtt{Q}^\top \mathbf{b}$$

and obtain $\mathbf{x}_{LS}$.

Q3 *(25 points)* Nonlinear Least Squares

We are given distance measurements to three base stations as follows:

Base Station	Coordinates	Distance
1	$(0, 0)$	0.44
2	$(1, 0)$	0.63
3	$(0, 1)$	0.89

Find the location of the point in two dimensions that best matches to these measurements using the Gauss-Newton method, starting at the initial estimate $(0.5, 0.5)$, and performing **two** iterations. *Hint:* Point-point distance in two dimensions is given as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Q4 *(25 points)* Numerical Differentiation and Integration

Simpson's rule is given as

$$\int_{x_0}^{x_2} f(x)\,dx \approx \frac{h}{3}(y_0 + 4y_1 + y_2),$$

where $h = x_2 - x_1 = x_1 = x_0$ and $y_i = f(x_i)$.

i. Compute the derivative of $f(x) = e^x \cos x$ at $x = 1.0$ using the centered difference formula.

ii. Compute an approximation of the integral $\int_0^3 e^x \cos x\,dx$ using the Simpson's rule.

iii. Compute an approximation of the integral $\int_0^3 e^x \cos x\,dx$ using the composite Trapezoid rule by partitioning the interval $[0, 3]$ into **two** subintervals.