CENG 113
Programming Basics

Problem Solving and Algorithms

G. Polya - How to solve it

The four-step approach:

- Understand the problem
- Choose a strategy
- Apply and solve
- Reflect (Look back)

https://math.berkeley.edu/~gmelvin/polya.pdf

Understand the problem

- Do you understand all the words used in stating the problem?
- What are you asked to find or to show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Understand the problem

- Determine the unknown
- Determine the data
- Determine the conditions
- Check if the conditions are sufficient to determine the unknown, or if they are insufficient, redundant or contradictory
- Draw a figure
- Devise suitable notation
- Separate the various parts of the conditions
- Write down the conditions

Choose a strategy

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious

Choose a strategy

- Find an easier related problem
- More general
- More restricted
- Solve part of the problem
- Simplify the conditions
- Change the data (do you need more, less?)
- Change the unknown
- Any notion missing in the statement?

Apply and solve

Carrying out your plan of the solution, check each step:

- Can you see clearly that the step is correct?
- Can you prove that it is correct?

Reflect - Look back

Examine the solution obtained:

- Can you check the result?
- Can you check the argument?
- Can you derive the solution differently?
- Can you see it at a glance?
- Can you use the result, or the method, for some other problem?

Problems in This Course

- Mainly **NOT** Artificial Intelligence but trial-error
- Some mathematical, some physical
- Some neither: abstract problem and solution is a creation (actually often problem itself is a creation)

An Example: Triangle and Square

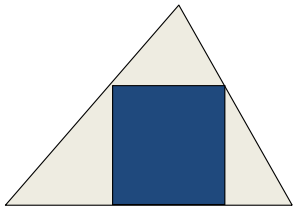
Inscribe a square in a given triangle. Two vertices of the square should be on the base of the triangle, the two other vertices of the square on the two other sides of the triangle, one on each.

- Unknown: a square
- Data: a triangle
- Conditions: positions of 4 corners of square

HOW TO SOLVE IT by Alain Fournier

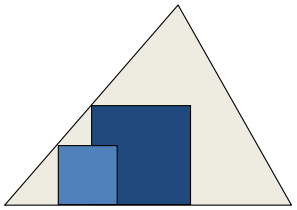
An Example: Triangle and Square

Draw a figure:



An Example: Triangle and Square

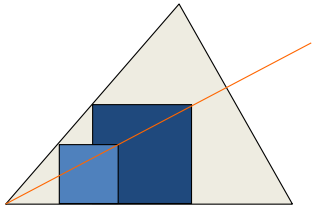
Relax the conditions:



We get more than one solution!

An Example: Triangle and Square

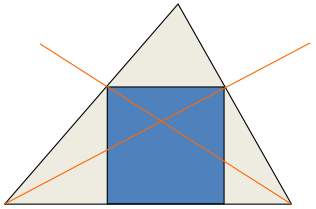
How does the solutions vary?



An Example: Triangle and Square

What if we restrict the solution again?

- Is it correct?
- Is it unique?



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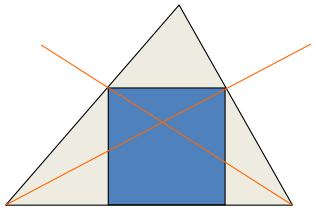
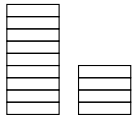


- Can we use the method for something else?

Another Example: Buckets

Get from the river exactly 6 liters of water when you have only a four liter bucket and a nine liter bucket to measure with.



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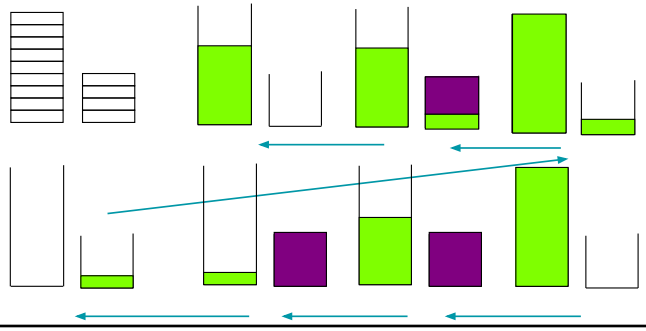


Another Example: Find the Number

Use the ten clues, in order, to find the mystery number:

1. It is a five-digit whole number.
2. It is divisible by 5.
3. It is divisible by 4.
4. The sum of its ten-thousands digit and thousands digit is 14.
5. The sum of its ten-thousands digit and its hundreds digit is 11.
6. The sum of its thousands digit and its tens digit is 8.
7. The sum of its hundreds digit and its units digit is 3.
8. The sum of its tens digit and its units digit is 2.
9. It is greater than 80,000.
10. Its thousands digit is 6.

HOW TO SOLVE IT by National Council of Teachers of Mathematics

Another Example: Find the Number

Understand the Problem:

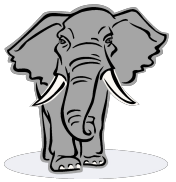
- Read the problem carefully.
  - Use clues in which order?
- Be sure you understand the terms
  - What is a “five-digit whole number”?
  - What does “divisible” mean?
  - What are units digits, tens digit, etc.?
- Yes, these may seem very easy… but that may not always be the case.

Another Example: Find the Number

Understand the Problem

- Break it into parts.
- Do you have all the tools you need?
  - Enough information
  - Appropriate skills
- Can you draw a picture?
  - Artistic skills not required.

How do you eat an elephant?



Another Example: Find the Number

Understand the Problem

- Introduce notation and variables.
  - Clue 1 tells us we have a 5-digit number. Without knowing anything else, we can write it as follows: abcde
  - Clue 2 tells us it is divisible by 5. So, e must be either 0 or 5.
  - Etc...

Another Example: Find the Number

Devise a Plan

- Follow the clues systematically
- Use rules of divisibility to limit possibilities
- Use algebraic equations to find digits
- If the problem is too hard, try doing an easier problem

Another Example: Find the Number

Carry Out the Plan

- Clue 1: Form of the number is abcde.
- Clue 2: Divisible by 5, therefore, e is 0 or 5.
- Clue 3: Divisible by 4, therefore, e should be even, then it is 0. The last two digits must be divisible by 4, so d is 0, 2, 4, 6, or 8.
- Clue 4: a + b = 14
- Clue 5: a + c = 11
- Clue 6: b + d = 8
- Clue 7: c + e = 3

Another Example: Find the Number

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Actually, we have enough clues now!

Another Example: Find the Number

Carry Out the Plan

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→ c = 3

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Another Example: Find the Number

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- Clue 4: a + b = 14
- Clue 5: a + c = 11 → b = 6
- Clue 6: b + d = 8 → a = 8
- Clue 7: c + e = 3 → d = 2

→ c = 3

Another Example: Find the Number

Carry Out the Plan

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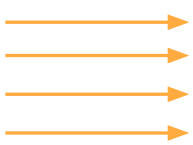


The answer: 86320

Another Example: Find the Number

Look Back

- Is it reasonable? Does it satisfy the clues… including the clues we didn't need?
  - (8) d + e = 2
  - (9) greater than 80,000
  - (10) thousands digit is 6
- We used 7 clues.
  - Could we have solved the problem with fewer?
- Did we answer the question?
  - Don't just give a, b, c, d, and e. The problem asks for a five-digit number.

Another Example: Find the Number

Look Back

- What did I learn from this problem?
  - Review simple algebra skills
  - Organized, logical thinking
- How can I use it to solve other problems?
- Could I have used another method?
  - Trial and error, maybe (but why???)

Another Example: Average Pay

Serge works at the supermarket as a cashier, a bagger, and a stocker. He earns $7.00 per hour as a cashier, $6.00 per hour as a bagger, and $5.00 per hour as a stocker. In a given week, Serge works 4 hours as a cashier, 9 hours as a bagger, and 7 hours as a stocker. What is Serge's average pay per hour?

Another Example: Average Pay

Understand the Problem

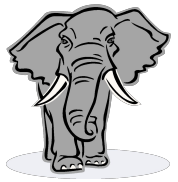
- Read it carefully. Note that you're asked for his average pay per hour.
- Be sure you understand the terms.
  - Does it matter what jobs he has, or only that he has three jobs?
  - Yes, this may seem very easy… but that may not always be the case.

Another Example: Average Pay

Understand the Problem

- Break it into parts.
- Do you have all the tools you need?
  - Enough information
  - Appropriate skills
- Can you draw a picture?
  - Artistic skills not required.

Remember: One bite at a time!



Another Example: Average Pay

Understand the Problem

- What information do we have?
- How can we organize it?

Cashier	$7.00 per hour	4 hours
Bagger	$6.00 per hour	9 hours
Stocker	$5.00 per hour	7 hours

Another Example: Average Pay

Devise a Plan

- Consider one job at a time.
- Find earnings for each job.
- Find total earnings and total hours.
- Find the average.

Another Example: Average Pay

Carry Out the Plan

- Jobs
  - Cashier: 4 hr × $7/hr = $28
  - Bagger: 9 hr × $6/hr = $54
  - Stocker: 7 hr × $5/hr = $35
- Total pay: $28 + $54 + $35 = $117
- Total hours: 4 + 9 + 7 = 20 hr
- Average pay: $117 ÷ 20 hr = **$5.85/hr**

Another Example: Average Pay

Look Back

- Is it reasonable?
- What did I learn from this problem?
  - The answer is **not** $6.00
- How can I use it to solve other problems?
- How else could I have solved the problem?
  - Weighted average

Another Example: The Farmyard

There are some cows and chickens in the farmyard. A worm counts there are 15 animals and 48 legs. How many cows are there?

Another Example: The Farmyard

Understand the Problem

- Given (that means **Input**)
  - animals = 15
  - legs = 48
- Question (that means **Output**)
  - cows = ?

Another Example: The Farmyard

Devise a Plan

- Known (that may mean Input / Knowledge)
  - legs = cows*4 + chicken*2
  - animals = cows + chicken
- Solution (that means Process)
  - legs = cows*4 + (animals - cows)*2

Another Example: The Farmyard

Carry Out the Plan

- **Solution** (that means finding the Process)
  - legs = cows*4 + (animals - cows)*2
  - legs = cows*4 + animals*2 - cows*2
  - legs = 4*cows + 2*animals – 2*cows
  - legs = 2*cows + 2*animals
  - cows = (legs – 2*animals)/2
- **Input**: legs = 48, animals = 15
- **Process**: cows = (legs – 2*animals)/2
- **Output**: print cows

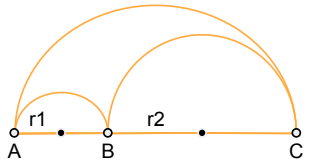
Another Example: The Farmyard

Look Back

- Verify your formula on an example:
  - cows = 1, chicken = 2, animals = 3, legs = 8
  - cows = (8 – 2 * 3) / 2
  - cows = (8 – 6) / 2
  - cows = 1

Another Example: Arc Length

Is the circular arc from A to C longer, the same or shorter than the two arcs from A to B then B to C?



Your turn...

What is an algorithm?

- **The idea behind any computer program**
- Stays the same **independent** of
  - Which kind of hardware it is running on
  - Which programming language it is written in
- Solves a **well-specified** problem in a general way
- Is specified by
  - Describing the set of instances (input) it must work on
  - Describing the desired properties of the output

What is an algorithm?

Muhammad ibn Musa al-Khwarizmi

- **The Compendious Book on Calculation by Completion and Balancing**
- **Al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala**

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Algorithm

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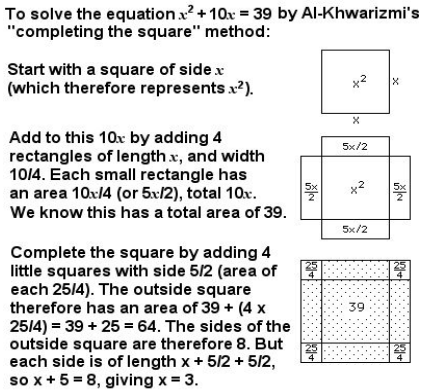
Algorithm

Algebra

What is an algorithm?



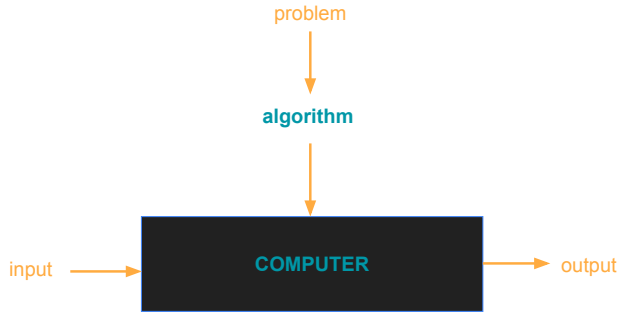
What is an algorithm?

- Before a computer can perform a task, it must have an algorithm that tells it what to do.
- Informally: “An algorithm is a set of steps that define how a task is performed.”

What is an algorithm?

- Formally: “An algorithm is an ordered set of unambiguous executable steps, defining a terminating process.”
  - Ordered set of steps: structure!
  - Executable steps: doable!
  - Unambiguous steps: follow the directions!
  - Terminating: must have an end!

What is an algorithm?



Important Properties of Algorithms

- Correct
  - Always return the desired output for all legal instances of the problem.
- Unambiguous
- Precise
- Efficient
  - Can be measured in terms of
    - Time (tends to be more important)
    - Space

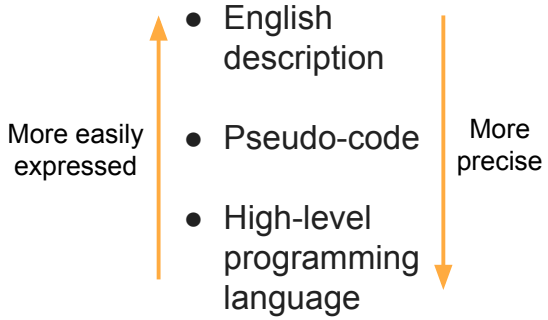
Representation of Algorithms

- A single algorithm can be represented in many ways:
  - Formulas: F = (9/5)C + 32
  - Words: Multiply the Celsius by 9/5 and add 32
  - Flow Charts
  - Pseudo-code
- In each case, the **algorithm** stays the same; the **implementation** differs!

Representation of Algorithms

- A **program** is a representation of an algorithm designed for computer applications.
- **Process** is the activity of executing a program, or execute the algorithm represented by the program.

Representation of Algorithms



Pseudocode

- Pseudocode is like a programming language but its rules are less stringent.
- Written as a combination of English and programming constructs
  - Based on selection (if, switch) and iteration (while, repeat) constructs in high-level programming languages
- Design using these high level primitives
  - Independent of actual programming language

Pseudocode

An example: A recipe for Hollandaise sauce

Never-Fail Blender Hollandaise

1 cup butter
4 egg yolks
1/4 teaspoon salt
1/4 teaspoon sugar
1/4 teaspoon Tabasco
1/4 teaspoon dry mustard
2 tablespoons lemon juice

Heat butter until bubbling. Combine all other ingredients in blender. With blender turned on, pour butter into yolk mixture in slow stream until all is added. Turn blender off. Keeps well in refrigerator for several days. When reheating, heat over hot—not boiling—water in double boiler. Makes about 1-1/4 cups sauce.

Pseudocode

```
If concerned about cholesterol
      Put butter substitute in a pot
Else
      Put butter in a pot
Turn on burner
Put pot on the burner
While (NOT bubbling)
      Leave pot on the burner
Put other ingredients in the blender
Turn on blender
While (more in pot)
      Pour contents into lender in slow steam
Turn off blender
```

Pseudocode

A more scientific example: The sequential search algorithm in pseudocode

```
procedure Search (List, TargetValue)
if (List empty)
    then
      (Declare search a failure)
    else
      (Select the first entry in List to be TestEntry;
       while (TargetValue > TestEntry and
                there remain entries to be considered)
            do (Select the next entry in List as TestEntry.);
       if (TargetValue = TestEntry)
            then (Declare search a success.)
            else (Declare search a failure.)
      ) end if
```

Pseudocode

Another example: Algorithm to Convert base-10 number to other bases

```
While ( the quotient is not zero )
    Divide the decimal number by the new base
    Make the remainder the next digit to the left in the answer
    Replace the original decimal number with
```

What is 93 in base 8?

93/8 gives 11 remainder 5
11/6 gives 1 remainder 3
1/8 gives 0 remainder 1

Answer: 1 3 5

Pseudocode

Another example: Algorithm to Convert base-10 number to other bases

```
Write "Enter the new base"
Read newBase
Write "Enter the number to be converted"
Read decimalNumber
Set quotient to 1
While (quotient is not zero)
        Set quotient to decimalNumber DIV newBase
        Set remainder to decimalNumber REM newBase
        Make the remainder the next digit to the left in the answer
        Set decimalNumber to quotient
Write "The answer is "
Write answer
```

Devising an Algorithm

- To discover an algorithm is to solve the problem!
  - Next step, implementation, is the **EASY** part!
- Problem solving techniques are not unique to Computer Science.
- Problem solving remains an art!
  - Ideally, there should be an algorithm to find/develop algorithms. However, this is not the case as some problems do not have algorithmic solutions.

Problem Solving Strategies

- Working backwards
  - Reverse-engineer
  - Once you know it can be done, it is much easier to do
- Look for a related problem that has been solved before
  - Design patterns
  - Sort a particular list such as: David, Alice, Carol and Bob to find a general sorting algorithm

Problem Solving Strategies

- Stepwise Refinement
  - Break the problem into several sub-problems
  - Solve each subproblem separately
  - Produces a modular structure
- K.I.S.S. = Keep It Simple Stupid!

Stepwise Refinement

- A top-down methodology that progresses from the general to the specific
  - Bottom-up methodologies progress from the specific to the general.
  - These two approaches complement each other
- Solutions posses a natural modular structure
  - ...hence its popularity in algorithmic design.