

Answer the questions in two hours.

1. (30 points) Let $L = \{w \in \{a, b\}^* : \text{the first, middle, and last characters of } w \text{ are identical}\}$.
 1. Show a context-free grammar for L .
 2. Show a natural PDA that accepts L .
 3. Prove that L is not regular.
2. (30 points) Construct a standard Turing machine that computes congruence in modulo 3 for an input that has a binary encoding. For example, 111 is congruent to 1 in modulo 3.
3. (20 points) Describe the equivalence classes \approx_L for the following language:
 $L = \{w \in \{a, b\}^* : \text{the number of } a\text{'s is equal to the number of } b\text{'s and the length of } w \text{ is at most } 10\}$.
4. (20 points)
 1. Find the leftmost derivation for the word $abba$ in the grammar:
 $S \rightarrow AA, A \rightarrow aB, B \rightarrow bB|e$
 2. Given a CFG in Chomsky Normal Form and restricting all derivations of words to being leftmost derivation, is it still possible that some word w has two nonidentical derivation trees? In other words, is it still possible that the grammar is ambiguous?

$S \Rightarrow AA \Rightarrow aBA \Rightarrow abBA \Rightarrow abbBA \Rightarrow abbA \Rightarrow abbaB$
 $\Rightarrow abba$