Answer the questions in 100 minutes.

1. **(30 points)** Construct a deterministic finite automaton (DFA) to recognize the following language:

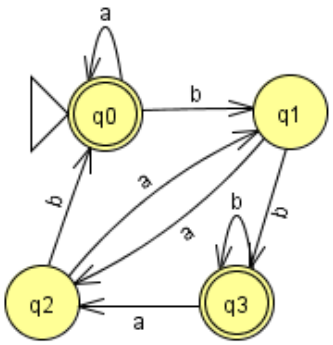
   $L = \{w \in \{a, b\}^* : w$ contains an a in the third position from the end (e.g. $bbbabb \in L$, $bba \notin L$.)

2. **(25 points)**

   1. Write a regular expression for the following language:

      The set of strings of 0's and 1's with at most one pair of consecutive 1's.

   2. Convert the following DFA into an equivalent regular expression:



3. **(25 points)**

   1. If $A = \{1, 2, 3, 4, 5\}$ and R is the equivalence relation on A that induces the partition $A = \{1, 2\} \cup \{3, 4\} \cup \{5\}$, what is R?

   2. For $A = R^2$, define R on A by $(x_1, y_1)R(x_2, y_2)$ if $x_1 = x_2$.
      (a) Verify that R is an equivalence relation on A.
      (b) Describe geometrically the equivalence classes and partition of A induced by R.

4. **(20 points)** Consider any function $f(L_1) = L_2$, where L_1 and L_2 are both languages over the alphabet $\Sigma = \{0, 1\}$. A function f is **nice** iff whenever L_2 is regular, L_1 is regular. For the following functions, f, state whether or not it is nice and prove your answer.

   1. $f(L) = \{w : w$ is formed by taking a string in L and replacing all 1's with 0's and leaving the 0's unchanged$\}$
   2. $f(L) = \{w : w$ is formed by taking a string in L and replacing all 1's with 0's and all 0's with 1's (simultaneously)$\}$