

Answer the questions in two hours.

1. **(30 points)** Let $L = \{x^R \# y : x, y \in \{0, 1\}^* \text{ and } x \text{ is a substring of } y\}$
 1. Show a context-free grammar for L .
 2. Show a natural PDA that accepts L .
2. **(30 points)** Construct a standard Turing machine to decide the following language:
$$L = \{w \in \{a, b, c, d\}^* : \#_b(w) \geq \#_c(w) \geq \#_d(w) \geq 0\}$$
3. **(30 points)** Construct a DFA for the following language:
$$\{w \in \{a, b\}^* \mid w \text{ has exactly three } a\text{'s and at least two } b\text{'s}\}$$
4. **(20 points)** What is the reflexive transitive closure R^* of the relation $R = \{(a, b), (a, c), (a, d), (d, c), (d, e)\}$?
Draw a directed graph representing R^* .