Answer the questions in 90 minutes.

1. **(30 points)** Construct a deterministic finite automaton (DFA) to recognize the following set of strings of $0s$ and $1s$:

    The set of strings containing a string of $1s$ such that the number of $1s$ equals 2 modulo 3, followed by an even number of $0s$.

2. **(30 points)** Write a regular expression to describe each of the following languages:

    1. $L_1 - L_2$, where $L_1 = a^*b^*c^*$ and $L_2 = c^*b^*a^*$
    2. $\{w \in \{a, b\}^* : \#_a(w) \equiv 0 \pmod 3\}$

3. **(20 points)** Describe the equivalence classes $\approx_L$ for the following language:

    L : The set of all strings of $0s$ and $1s$ with at least two consecutive $0s$ or three consecutive $1s$.

4. **(20 points)** The reversal of a language L, written L^R, is the language consisting of the reversals of all its strings. For instance, if $L = \{001, 10, 111\}$, then $L^R = \{100, 01, 111\}$. Show that if L is a regular language, then so is its reversal language.