Answer the questions in two hours.

1. **(30 points)** Let $L = \{w \in \{a,b\}^* :$ the first, middle, and last characters of w are identical$\}$.
   1. Show a context-free grammar for L.
   2. Show a natural PDA that accepts L.
   3. Prove that L is not regular.
2. **(30 points)** Construct a standard Turing machine that computes congruence in modulo 3 for an input that has a binary encoding. For example, 111 is congruent to 1 in modulo 3.
3. **(20 points)** Describe the equivalence classes $\approx_L$ for the following language:
   $L = \{w \in \{a,b\}^* :$ the number of a's is equal to the number of b's and the length of w is at most $10\}$.
4. **(20 points)**
   1. Find the leftmost derivation for the word *abba* in the grammar:
      $S \to AA, A \to aB, B \to bB|e$
   2. Given a CFG in Chomsky Normal Form and restricting all derivations of words to being leftmost derivation, is it still possible that some word w has two nonidentical derivation trees? In other words, is it still possible that the grammar is ambiguous?

$S \to AA \Rightarrow aBA \Rightarrow abBA \Rightarrow abbBA \Rightarrow abbA \Rightarrow abbaB$

$\Rightarrow abba$

Q1 $L = \{ w \in \{a, b\}^* :$ the first, middle, and last characters of w are identical$\}$

a)

$S \rightarrow aAa$

$S \rightarrow bBb$

$A \rightarrow aAa$

$A \rightarrow aAb$

$A \rightarrow bAa$

$A \rightarrow bAb$

$B \rightarrow aBa$

$B \rightarrow aBb$

$B \rightarrow bBa$

$B \rightarrow bBb$

$A \rightarrow a$

$B \rightarrow b$

$S \rightarrow e$

b) Show a natural PDA that accepts L.

(s, a, e) (p, a)

(s, b, e) (q, b)

(p, a, e), (p, a)

(p, b, e) (p, a)

(p, a, e) (r, e)

(r, a, a) (r, e)

(r, b, a) (r, e)

(q, a, e), (q, a)

(q, b, e), (q, a)

(q, b, e), (s, e)

(s, a, a), (s, e)

(s, b, a), (s, e)

s and r are final states.

Q1.

c) Prove that L is not regular.

[e] : member.

[a] : concaterate with aa.

concaterate with aaaa.

concaterate with aaba.

concaterate with baaa.

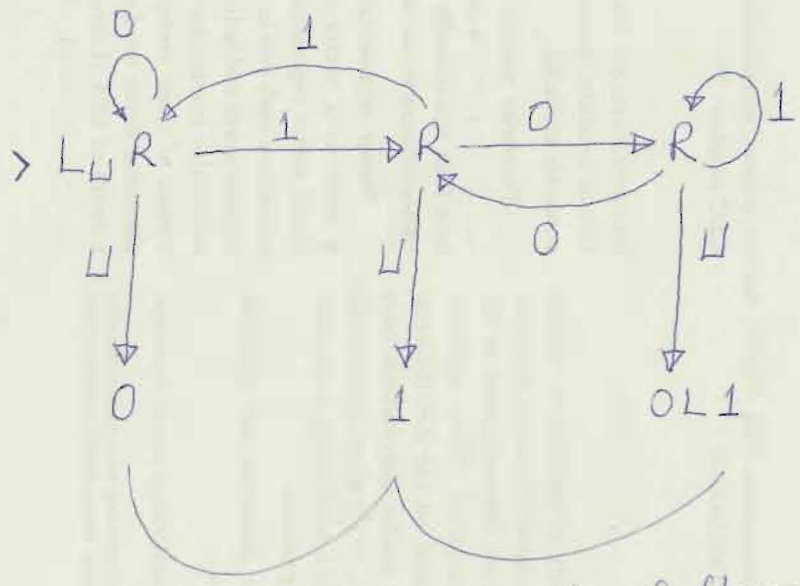
concaterate with baba.

lexicographic order

⋮

infinitely many equivalence classes.

Q2. Turing machine that computes congruence in modulo 3 for an input that has a binary encoding.



You can clean the leftmost part.

Q3. Describe the equivalence classes $\approx_L$ for the following language:

$L = \{ w \in \{a,b\}^* :$ the number of a's is equal to the number of b's and the length of w is at most $10\}$.

[e] : member

[a] : concatenate with a b.

all different

[a] : concatenate with abb.
[a] : concatenate with bab.
[a] : concatenate with bba.

} length 4.

[a] : concatenate with aabbb.
[a] : concatenate with ababb.
[a] : concatenate with abbab.
[a] : concatenate with abbba.
[a] : concatenate with baabb.
[a] : concatenate with babab.
[a] : concatenate with babba.

} length 6.

$2 \times 3 + 1 = 7$

[a] : concatenate with aaabbb

⋮

} length 8

$4 \times 3 + 1 = 13$

} length 10

$5 \times 4 + 1 = 21$

Q3 (continued).

An identical scenario for [b].

[aaaaaa]: nonmember and no possibility to become a member.

Q4.

a) Leftmost derivation for the word abba in the grammar:

$S \to AA$, $A \to aB$, $B \to bB | e$.

$S \to AA \Rightarrow aBA \Rightarrow abBA \Rightarrow abbBA$
$\Rightarrow abbA \Rightarrow abbaB \Rightarrow abba$

b) We can convert any CFG into Chomsky Normal Form. Restricting all derivations to the leftmost does not remove ambiguity from the grammar because two different parse trees can have two different leftmost derivations.