Answer the questions in two hours.

1. **(20 points)** Construct a pushdown automaton to accept the following language $L$:

   $L = \{b_i \# b_{i+1}^R : b_i \text{ is the binary representation of some integer } i, i \geq 0, \text{ without leading zeros}\}$.
   (For example $101\#011 \in L$.)

2. **(30 points)** Construct a Turing machine that converts a given number in unary notation into its binary equivalent.

   The unary string 111111 must be represented as 101 in binary.

3. **(20 points)** Give the equivalence classes under $\approx_L$ for these languages:

   1. $L = (ab \cup aba)^\star a$
   2. The language of balanced parentheses.

4. **(20 points)** Build a DFA that accepts only those words that have an even number of substrings $ab$. Write the regular expression for the same language.

5. **(20 points)** Give a context-free grammar generating the complement of the following language:

   $\{a^n b^n | n \geq 0\}$