CENG213 Theory of Computation Final Exam January 21^{st}, 2020

Answer the questions in two hours.

1. **(30 points)** Let $L = \{x^R\#y : x, y \in \{0,1\}^*$ and x is a substring of y $\}$

    1. Show a context-free grammar for L.
    2. Show a natural PDA that accepts L.

2. **(30 points)** Construct a standard Turing machine to decide the following language:

    $L = \{w \in \{a, b, c, d\}^* : \#_b(w) \geq \#_c(w) \geq \#_d(w) \geq 0\}$

3. **(30 points)** Construct a DFA for the following language:

    $\{w \in \{a, b\}^* | w$ has exactly three a's and at least two b's$\}$

4. **(20 points)** What is the reflexive transitive closure R^* of the relation $R = \{(a, b), (a, c), (a, d), (d, c), (d, e)\}$? Draw a directed graph representing R^*.

Q1. Let $L = \{ x^R \# y : x, y \in \{0,1\}^* \text{ and } x \text{ is a substring of } y \}$

a. $S \rightarrow AB$

$A \rightarrow 0A0$

$A \rightarrow 1A1$

$A \rightarrow \# B$

$B \rightarrow 0B \mid 1B \mid e$

b. $(s, 0, e)\ (s, 0)$

$(s, 1, e)\ (s, 1)$

$(s, \#, e)\ (m, e)$

$(m, 0, e)\ (m, e)$

$(m, 1, e)\ (m, e)$

$(m, e, e)\ (r, e)$

$(r, 0, 0)\ (r, e)$

$(r, 1, 1)\ (r, e)$

$(r, e, e)\ (l, e)$

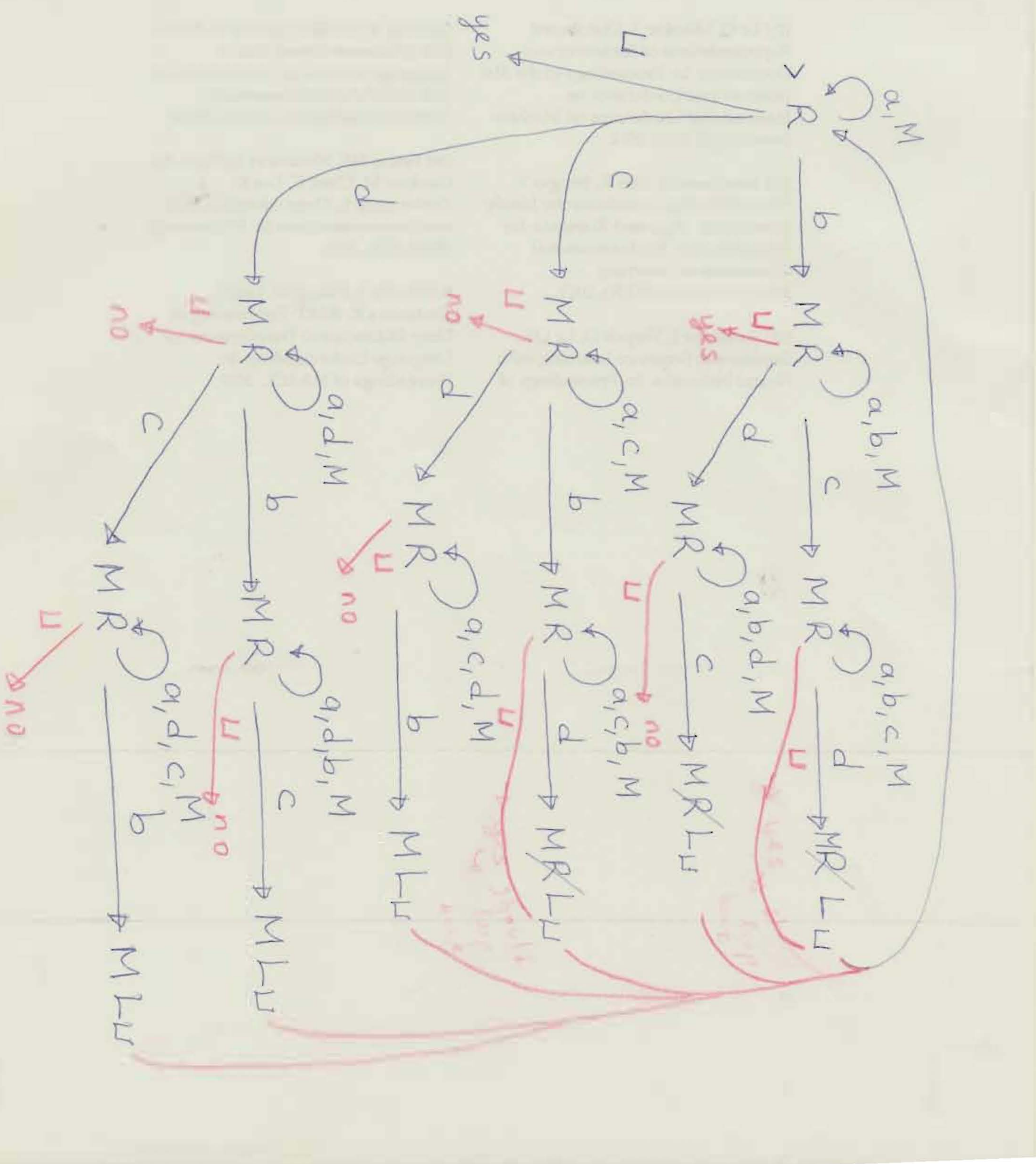
$(l, 0, e)\ (n, e)$

$(l, 1, e)\ (n, e)$

$(n, e, e)\ (\underline{f}, e)$

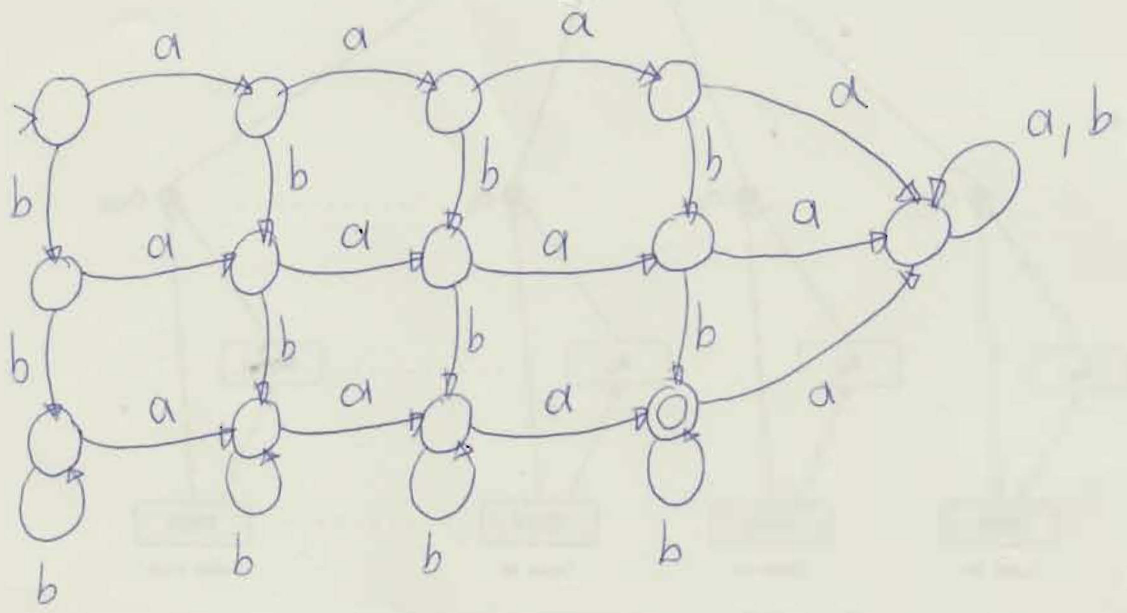
Q2.

$$L = \{ w \in \{a,b,c,d\}^* : \#_b(w) \geq \#_c(w) \geq \#_d(w) \geq 0 \}$$



Q3. Construct a DFA for the following language:

$\{ w \in \{a,b\}^* \mid w$ has exactly three a's and at least two b's $\}$



Q4. $R = \{(a,b), (a,c), (a,d), (d,c), (d,e)\}$

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

d as the connecting node.

(a,d), (d,e) add (a,e)

$$R^* = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

make it reflexive.

$$R^* = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$