MMÜ 305 FLUID MECHANICS I

TERM PROJECT 2

Gökay KART

21832009

GÖKAY KART 06.01.2023

21832009

Relevant Data for Project

Diameter (D)→ 0.25 m

Elevation (static) head (z2-z1) → 15 m

Length (L)→ 12 E + 4 m

P1 = P2 = Patm

V1 = V2 = Vconstant

μ = 1.12 E – 3 N.s/m^2

ρ = 1000 kg/m^3

hL,minor → neglect

$\dot{\forall}$(lt/min)	0	100	200	300	400	500	600	700	800	900	1000
H_p (m)	28	28	27.9	27.4	26.5	25.2	23.4	20.8	17.6	13	6.8
η (%)	0	25	44	65	73	79	81	78	68	59	51

Table.1 System Data

The Formulas Used in the Project

Eq-1 (Bernoulli Equation, involves energy per unit weight)

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2.g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2.g}+z_2+h_L$$

Eq-2 (Flowrate Equation)

$$V=\frac{Q}{\pi\cdot\frac{D^2}{4}}$$

Eq-3 (Head loss due to major losses)

$$h_L=f\cdot\frac{L}{D}\cdot\frac{V^2}{2.g}=f.\frac{8.L.Q^2}{\pi^2.g.D^5}$$

Eq-4 (Reynolds Number)

$$Re=\rho\cdot V\cdot\frac{D}{\mu}=\frac{4\cdot\rho\cdot Q}{\pi\cdot\mu.D}$$

Eq-5 (Haaland Equation)

$$\frac{1}{\sqrt{f}}=-1.8\log\left[\left(\frac{\varepsilon}{D.(3.7)}\right)^{1.11}+\frac{6.9}{Re}\right]$$

Eq-6 (Power Equation)

$$P = \rho \cdot g \cdot Q \cdot \frac{H_p}{\eta}$$

Mathematical Solution of Systems

First, friction factor calculation is required for 3 systems. Eq-1 is simplified with the information given in the project. Values are substituted in the equation. The system equation is obtained.

$$h_p = z_2 - z_1 + \frac{f.8.\,L}{\pi^2.\,g.\,D^5}.\,Q^2$$

$$h_p = 15 + \frac{f.8.12x10^4}{\pi^2.\,(9.81).\,(0.25)^5}.\,Q^2$$

Eq-7 (System Equation)

$$h_p = 15 + (10153188.21).\,f.\,Q^2$$

The values given in the project in Eq-4 are written instead.

$$Re = \frac{4.1000}{\pi.\,\left(1.12x10^{-3}\right).\,(0.25)}.\,Q = (4547284.088).\,Q$$

Haaland formula (Eq.5) is used to leave the friction factor alone. The surface roughness coefficient is assumed to be zero (ε = 0). The simplified Reynolds value is written instead.

$$f = \left[-1.8\log\left[\frac{6.9}{(4547284.088).\,Q}\right]\right]^{-2}$$

To calculate the friction factor, 2nd order system curve is defined.

$$h_p = k_1 - k_2 \cdot Q^2$$

To determine the single pump system curve, the 2nd order pump head equation is determined by the boundary conditions. Boundary conditions are given in *Table.1*.

Boundary condition-1; Q = 0 → Hp = 28 m

$$h_p = 28 - k_2 \cdot Q^2$$

Boundary condition-2; Q= 1000 lt/min → Hp = 6.8 m

$$h_p = 28 - 76320 \cdot Q^2$$

The two equations equate to each other.

$$h_p = 28 - 76320 \cdot Q^2 = 15 + (10153188.21) \cdot f \cdot Q^2$$

Volumetric flowrate (Q) is left alone. The friction factor (f) value is calculated using the iteration method with the MATLAP code shown in *Appendix-1*. For the reliable iteration method, the initial value of the friction factor (f) is 0.02.

$$Q = \sqrt{\frac{13}{10153188.21 \cdot f + 76320}}$$

The above operations are performed for identical and parallel connected pumps and the f value is found.

$$Q = \sqrt{\frac{13}{10153188.21 \cdot f + 19233.6}}$$

The above operations are performed for identical and series connected pumps and the f value is found.

$$Q = \sqrt{\frac{41}{10153188.21 \cdot f + 38467}}$$

Equations are solved in MATLAP and values are shown in *Table.2*. Equations were iterated 10 times. Average friction factor (f = 0.0210) will be used for characteristic curve calculations.

Friction factor (f)			
Single Pump, f	Parallel Pump, f	Series Pump. f	Average, f
0.0235	0.0192	0.0199	0.0210

Table.2 Average Friction Factor

The average friction factor (f=0.0210) value is written in Eq-7, the equation of the system curve is found. A system curve is created according to the volumetric flowrate (Q) values in *Table.1*.

Eq-8 (System Curve Equation)

$$h_p = 15 + 213216.95 \cdot Q^2$$

Using Head pump (Hp) and Efficiency (η) values in *Table.1*, Head pump equation and Efficiency equation are found in MATLAP Curve fitting application. These equations are used in the MATLAP codes shown in *Appendix-2*.

Equations were found for three separate curves. The MATLAP code shown in *Appendix-2* is run and the intersections of the equations and operating points are calculated.

Finally, the required power values (P) for the systems are calculated with the formula shown in Eq-6. These calculations are in the MATLAP codes shown in *Appendix-2*.

The flow rate and power requirement for each pump if:

a) A Single Pump

The values given in *Table.1* are used as for Single pump system. *Figure.1* is obtained when the code in *Appendix-2* is run. The intersection points of the created equations are shown in *Figure.1*. The point where the pump performance curve and the system curve intersect is the Operating Point.

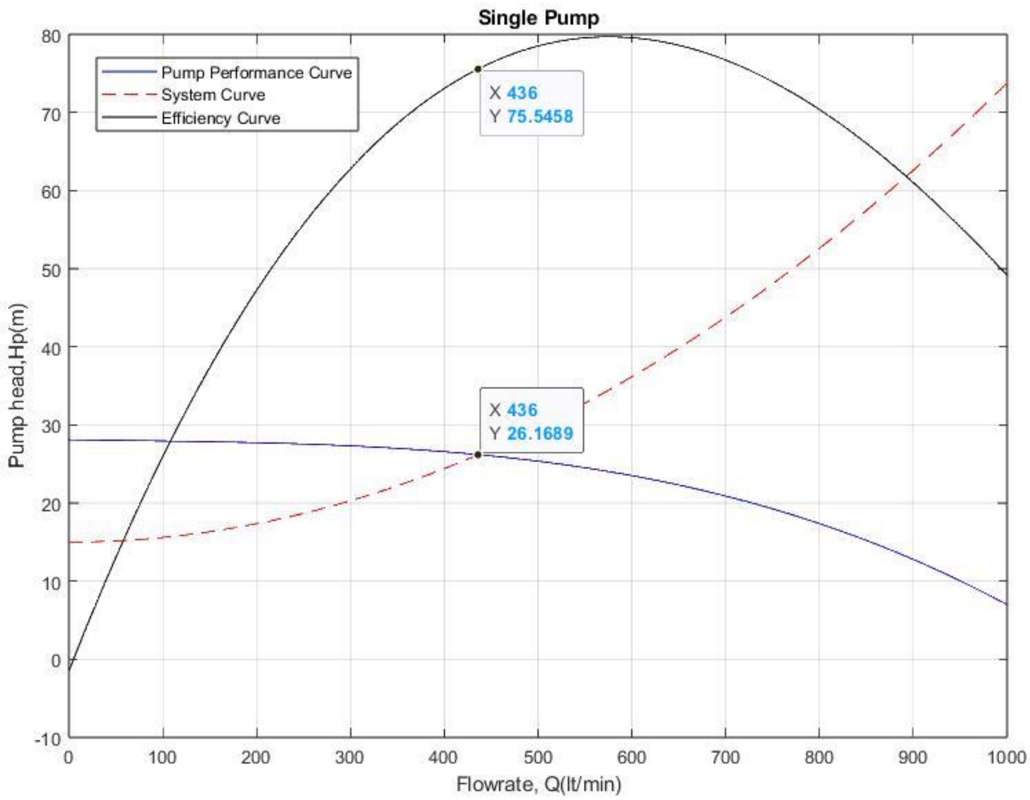

Figure.1 Single Pump Characteristics Curve

Data according to the operating point shown in the *Figure-1*

Flow Rate (Q)	436 lt/min
Pump Heat (Hp)	26,17 m
Efficiency (η)	75.55%
Power Requirement (P)	2495.5W

b) Two pumps in parallel (Identical)

It is necessary to add the flow rates for the calculation of parallel connected pumps with the same characteristics. The total flow values are obtained by multiplying the flow value by 2 and a pump performance curve is created. Since the pumps are identical, the system curve and efficiency curve do not change. *Figure.2* is obtained when the code in *Appendix-2* is run. The intersection points of the created equations are shown in *Figure.2*. The intersection of the pump performance curve and the system curve is the Operating Point.

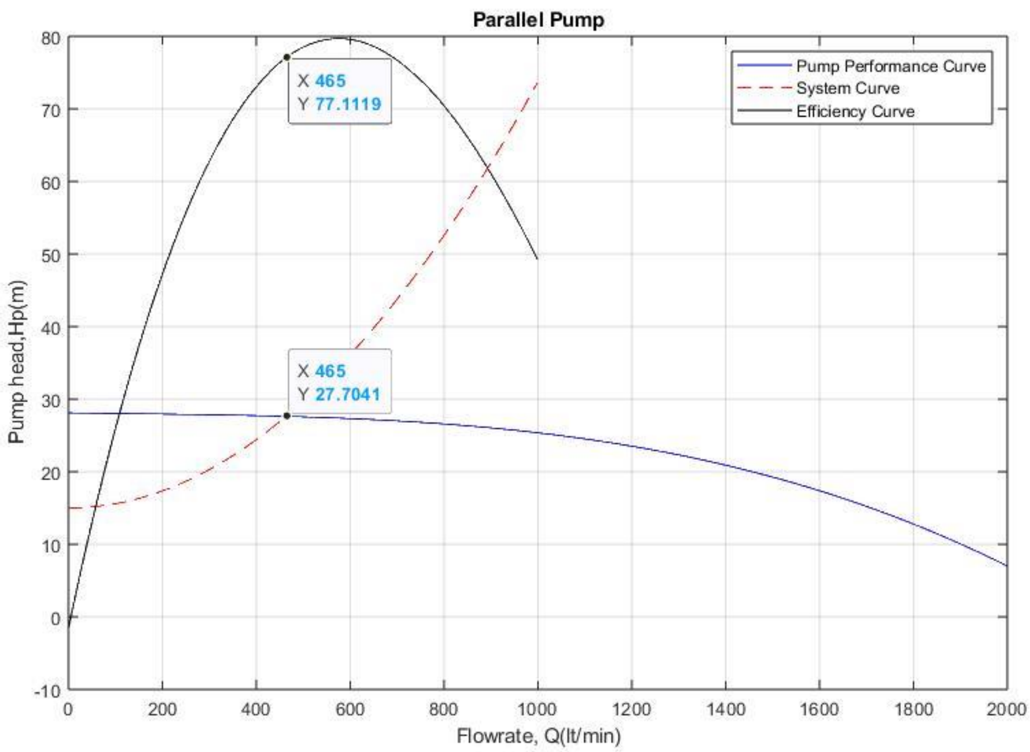

Figure.2 Parallel Pump Characteristics Curve

Data according to the operating point shown in the *Figure-2*

Flow Rate (Q)	465 lt/min
Pump Heat (Hp)	27.70 m
Efficiency (η)	77.11 %
Power Requirement (P)	2702.5W

c) Two pumps in series (Identical)

In order to calculate the pumps connected in series with the same specification, it is necessary to add the head of the pump. New values are obtained by multiplying the pump head (Hp) in *Table.1* by 2 and a pump performance curve is created. Since the pumps are identical, the system curve and efficiency curve do not change. *Figure.3* is obtained when the code in *Appendix-2* is run. The intersection points of the created equations are shown in *Figure.3*. The intersection of the pump performance curve and the system curve is the operating point.

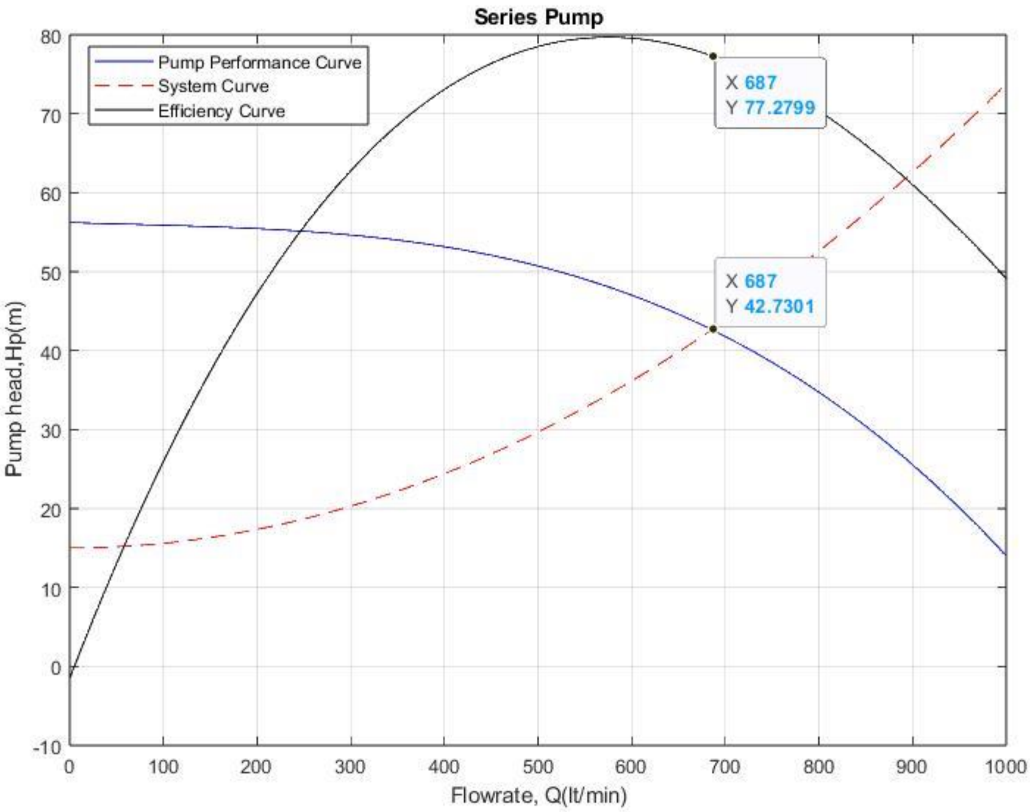

Figure.3 Series Pump Characteristics Curve

Data according to the operating point shown in the *Figure-3*

Flow Rate (Q)	687 lt/min
Pump Heat (Hp)	42.73 m
Efficiency (η)	77.28%
Power Requirement (P)	6185.9 W

d) The graphs that we compare Single Pump, Two pumps (identical, parallel) and Two pumps (identical, series) systems with each other are shown below. The intersection points of the system curve and the pump performance curve give operating points.

When the graphs are examined, the lowest power requirement is the system with single pump (P= 2495.5 Watt), and the highest power requirement is the system with 2 pumps connected in series (P= 6185.9 Watt).

When we look at *Figure.4*, the pump heads (Hp) of the single pump system and the 2-pump system connected in parallel are close to each other.

When we look at *Figure.5*, there is a significant difference between the pump head (Hp) values of the single-pump system and the 2-pump system connected in series.

When we look at *Figure.6*, the efficiency values are quite close for systems with serial and parallel pumps.

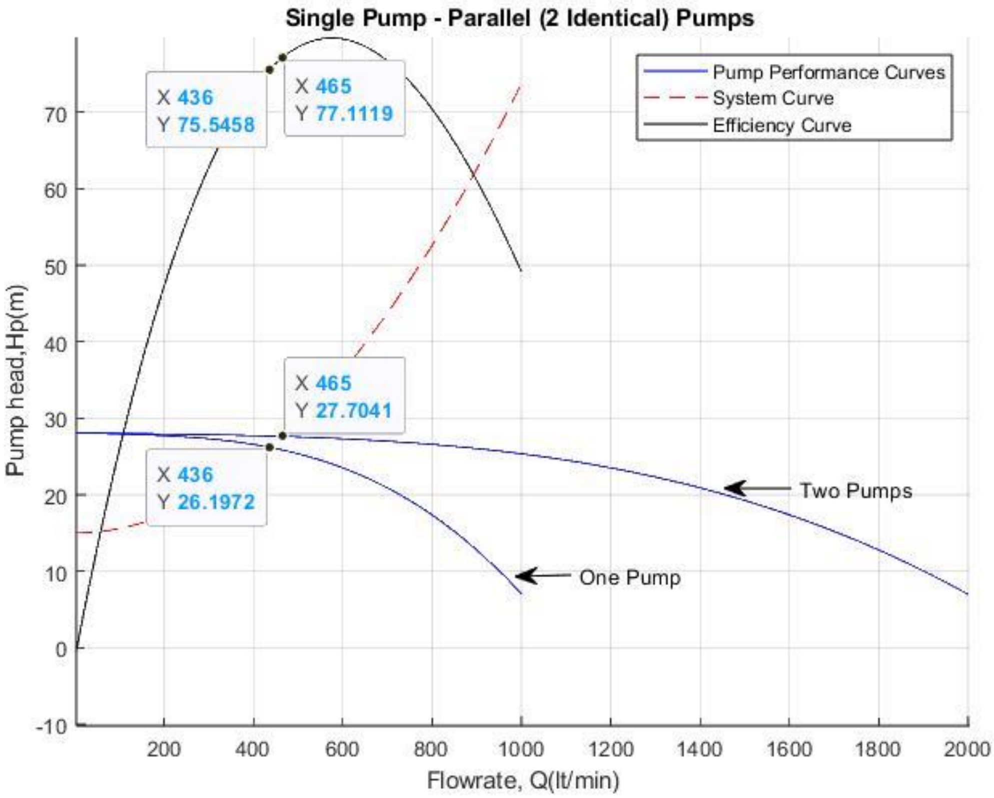

Figure.4 Single-Parallel Pump Characteristics Curve

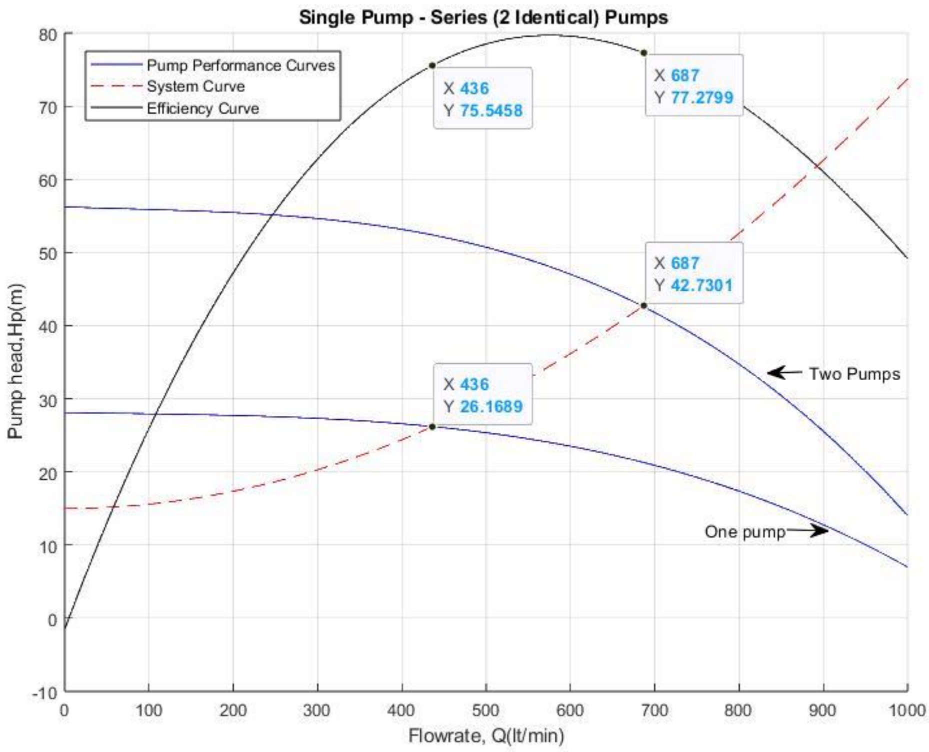

Figure.5 Single - Series Pump Characteristics Curve

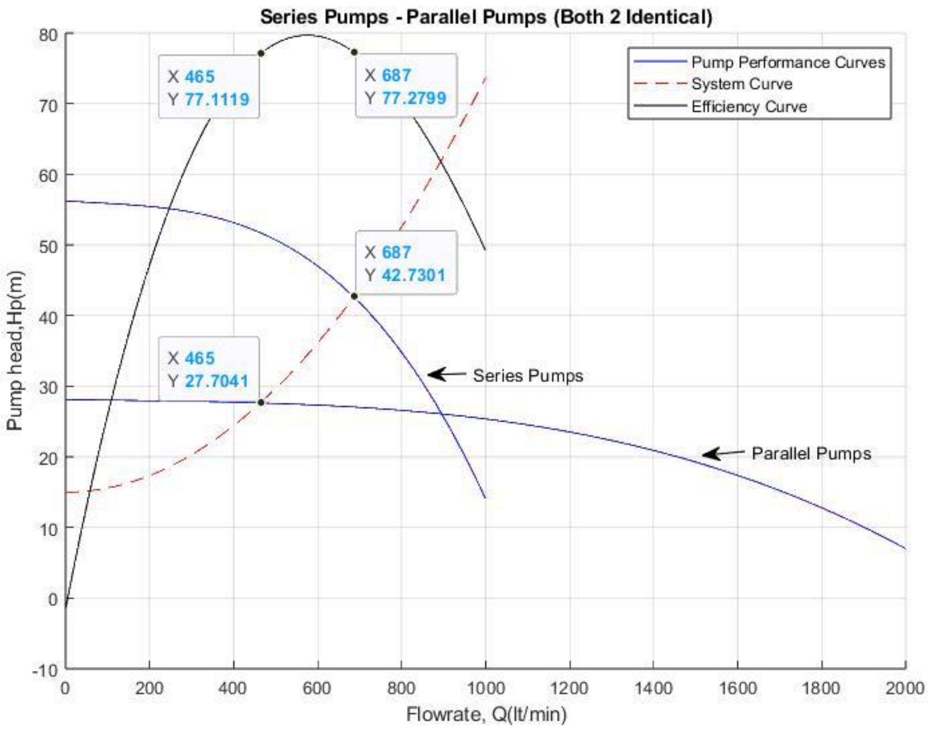

Figure.6 Series – Parallel Pump Characteristics Curve

e) Discussion

"Centrifugal pumps in parallel are used to overcome larger volume flows than one pump can handle alone."(*Reference, Link-2*). In *Figure.7*, Point 1 is the place where the system operates when single pump is running. It is a common misconception that if you run the second pump, the flow will double to reach point 3. In real life this would never happen because the system curve cannot be a horizontal curve. Point 2 is where the system operates with both pumps running. If we consider a system with a theoretically infinite number of pumps, the head tends to have a constant value regardless of the system curve.

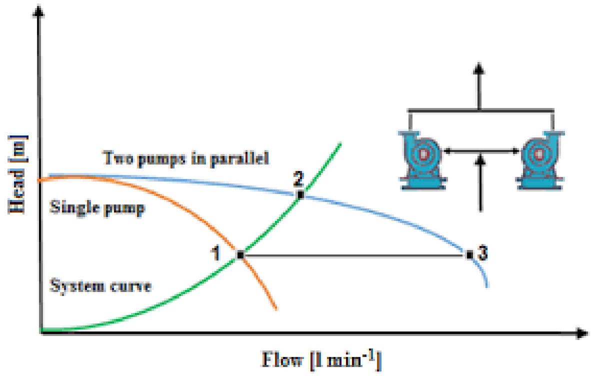

Figure.7 Single – Parallel Pump Characteristics Curve (Reference, Link-3)

"Centrifugal pumps in series are used to overcome larger system head loss than one pump can handle alone." (*Reference, Link-2*). In *Figure.8*, Point 1 is where the system operates while a pump is running. It is a common misconception that if you turn on the second pump, the heat pump will double up to reach point 2. In real life this would never happen because the system curve cannot be a vertical curve. Point 3 is where the system operates with both pumps running. If we used a theoretically infinite number of pumps in the system, the effect of the head pump on the system would be lost.

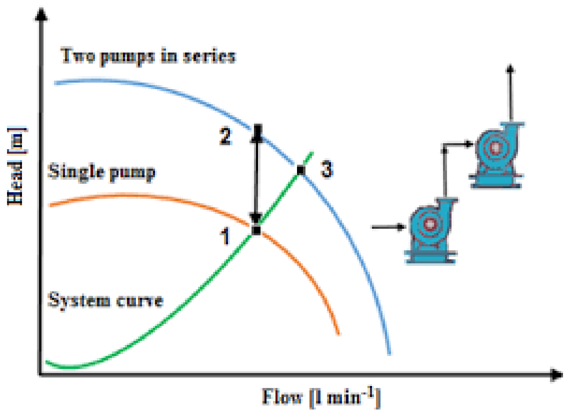

Figure.8 Single – Series Pump Characteristics Curve (Reference, Link-3)

In the Discussion section, *References, Link-1* is used in general terms.

Appendix-1

Calculate Friction Factor (f)

```
clc
clear
clear all
format short
%Single Pump
f1 = 0.02
for i = 1:10
    Q = (13/((10153188.21*f1)+76320))^0.5;
    Re = (4*1000.*Q)/(1.12*10^-3*pi*0.25);
    x1 = (1./(-1.8*log10((6.9./Re)))).^2;
    Hp =15+(8*120*10^3.*Q.^2.*f1)/(pi^2*9.81*0.25^5);
    f1 = x1
end
%Parallel Pump
f2 = 0.02
for i = 1:10
    Q = ((13/((10153188.21*f2)+19233.56075))^0.5)*2;
    Re = (4*1000.*Q)/(1.12*10^-3*pi*0.25);
    x2 = (1./(-1.8*log10((6.9./Re)))).^2;
    Hp =15+(8*120*10^3.*Q.^2.*f2)/(pi^2*9.81*0.25^5);
    f2 = x2
end
%Series Pump
f3 = 0.02
for i = 1:10
    Q = (41/((10153188.21*f3)+38467))^0.5;
    Re = (4*1000.*Q)/(1.12*10^-3*pi*0.25);
    x3 = (1./(-1.8*log10((6.9./Re)))).^2;
    Hp =(15+(8*120*10^3.*Q.^2.*f3)/(pi^2*9.81*0.25^5))*2;
    f3 = x3
end
```

Appendix-2

Calculate Characteristic Curve (Single - Series - Parallel)

```
%Single Pump
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06  % Heat Pump Equation
p3 =    -0.00197
p4 =       28.11
hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);

a1 =   7.517e-08
a2 =  -0.0003315  % Efficiency Equation
a3 =      0.3069
a4 =      -1.476
n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4

m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))  %System Equation
```

```
figure(2)
plot(x,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Single Pump')
legend('Pump Performance Curve','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')

P = (9.81)*(1000)*(26.1689)*(436)*(1.66*(10^(-5)))/(0.755458) %Power Req.

%Parallel Pumps
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
z=[0:2:2000]
p1 =  -2.413e-08
p2 =   4.977e-06  % Heat Pump Equation
p3 =    -0.00197
p4 =       28.11

hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);

a1 =   7.517e-08
a2 =  -0.0003315   % Efficiency Equation
a3 =      0.3069
a4 =      -1.476
n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4

m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))%System Equation

figure(3)
plot(z,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Parallel Pump')
legend('Pump Performance Curve','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')

P = (9.81)*(1000)*(27.7041)*(465)*(1.66*(10^(-5)))/(0.771119) % Power Req.

% Series Pumps
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06  % Heat Pump Equation
p3 =    -0.00197
p4 =       28.11
hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4)*2;

a1 =   7.517e-08
a2 =  -0.0003315   % Efficiency Equation
a3 =      0.3069
a4 =      -1.476
n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4

m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))%System Equation
```

```
figure(1)
plot(x,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Series Pump')
legend('Pump Performance Curve','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')

P = (9.81)*(1000)*(42.7301)*(687)*(1.66*(10^(-5)))/(0.772799) % Power Req.
```

Appendix-3

Comparing Curves with each other (Single-Parallel, Single-Series, Parallel-Series)

```
%Single Pump - Parallel (2 Identical) Pumps
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11

hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);

a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476

n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
hold on
plot(x,hp,'b',x,m,'--r',x,n,'k')

clc
clear all
x=[0:1:1000]
b=[0:1:1000]
z=[0:2:2000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11

hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);

a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476

n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
```

```
plot(z,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Single Pump - Parallel (2 Identical) Pumps ')
legend('Pump Performance Curves','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')




%Single Pump - Series (2 Identical) Pumps

clc
clear all
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11

hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);

a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476

n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
hold on
plot(x,hp,'b',x,m,'--r',x,n,'k')
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11
hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4)*2;

a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476

n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
plot(x,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Single Pump - Series (2 Identical) Pumps ')
legend('Pump Performance Curves','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')

%Series Pumps - Parallel Pumps (Both 2 Identical)
clc
clear all
```

```
x=[0:1:1000]
b=[0:1:1000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11
hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4)*2;
a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476
n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
hold on
plot(x,hp,'b',x,m,'--r',x,n,'k')
clc
clear all
x=[0:1:1000]
b=[0:1:1000]
z=[0:2:2000]
p1 =  -2.413e-08
p2 =   4.977e-06
p3 =    -0.00197
p4 =       28.11
hp=(p1.*b.^3 + p2.*b.^2 + p3.*b + p4);
a1 =   7.517e-08
a2 =  -0.0003315
a3 =      0.3069
a4 =      -1.476
n = a1.*b.^3 + a2.*b.^2 + a3.*b + a4
m= 15+(10153188.21.*0.0210.*((x.*1.66*10^-5).^2))
plot(z,hp,'b',x,m,'--r',x,n,'k')
grid on
title('Series Pumps - Parallel Pumps (Both 2 Identical)')
legend('Pump Performance Curves','System Curve','Efficiency Curve')
xlabel('Flowrate, Q(lt/min)')
ylabel('Pump head,Hp(m)')
```

References

Link-1) "What You Need to Know About Parallel Pump Operation"; https://www.pumpsandsystems.com/pumps/what-you-need-know-about-parallel-pump-operation

Link-2)" Pumps - Parallel vs. Serial Arrangement"; https://www.engineeringtoolbox.com/pumps-parallel-serial-d_636.html

Link-3) "Reliability of parallel and serial centrifugal pumps for dewatering

in mining process"; https://actamont.tuke.sk/pdf/2018/n2/3qazizada.pdf

Book-1) Fundemental of Fluid Mechanics/Munson/Section-12.4.4