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# MMÜ 305 FLUID MECHANICS I

# TERM PROJECT I

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# Relevant Data for Project

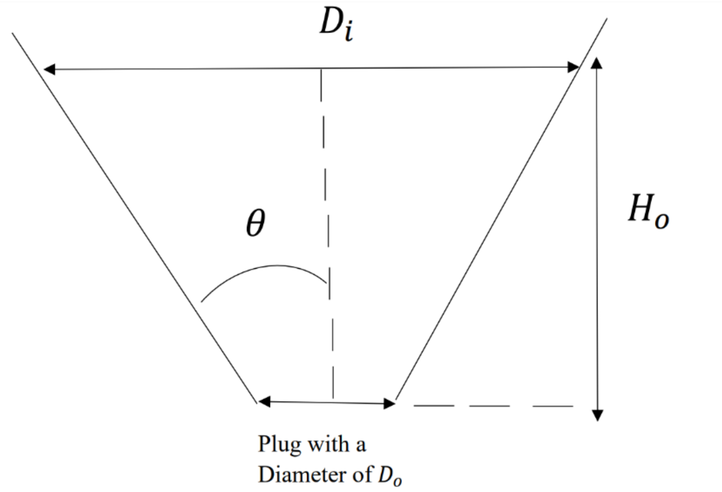
$Do$ → 0.209 Plug Diameter (cm)

$Df$ → 0.109 Plug Diameter (Part C) (cm)

$Ho$ → Initial Water Height (m)

$D$i → Instantaneous Diameter (cm)

$\theta$ → Funnel Angle (°)




## The formulas we will use in the project

- (Eq.1) The Bernoulli equation is the total pressure remains constant along a streamline. That is,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

- (Eq.2) The mass conservation equation. That is.

$$\rho A_1 V_1 = \rho A_2 V_2$$

- (Eq.3) The path-speed differential representation of the material in the opposite direction to the direction of motion. That is,

$$V = -\frac{dh}{dt}$$

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Solution of Bernoulli's equation for this system

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

In the first equation, we write the known ones and make the simplifications. (z1= h, z2= 0, p1= 0 gage, p2= 0,V1 ≌0). V1 was accepted as zero because of the large tank theory.

$$\gamma h = \frac{1}{2}\rho V_2^2$$

If we leave V2 alone; (Eq. 1.1)

$$V_2 = \sqrt{2gh}$$

Using the geometry of the figure, we write the magnitudes in terms of tangent theta. $D$i >> $Do$ since we use the large tank assumption. (Eq. 1.2)

$$Di = 2h\tan(\theta)$$

We simplify the Eq.2 and leave V2 alone (Eq. 1.3)

$$V_2 = \left(\frac{Di}{Do}\right)^2 V_1$$

If we substitute Eq.3, Eq.1.1 and Eq.1.2 in Eq.1.3.

$$\sqrt{2gh} = \left(\frac{4h^2\tan(\theta)^2}{Do^2}\right)\left(-\frac{dh}{dt}\right)$$

The resulting equation is made suitable for integration.

$$\int_h^{ho}\left(\frac{h^2}{\sqrt{2gh}}\right)dh = \left(-\frac{Do^2}{4\tan(\theta)^2}\right)\int_0^t dt$$

If we solve the integral and leave h(t) alone. We find the final form of the equation (Eq.4)

$$h(t) = \left[ho^{\frac{5}{2}} - \frac{5\sqrt{2g}Do^2 t}{8\tan(\theta)^2}\right]^{\frac{2}{5}}$$

According to the information given in the problem, the water will be completely drained from the funnel in 4 minutes. Therefore, we can find the ho value using Eq.4. (Eq.5)

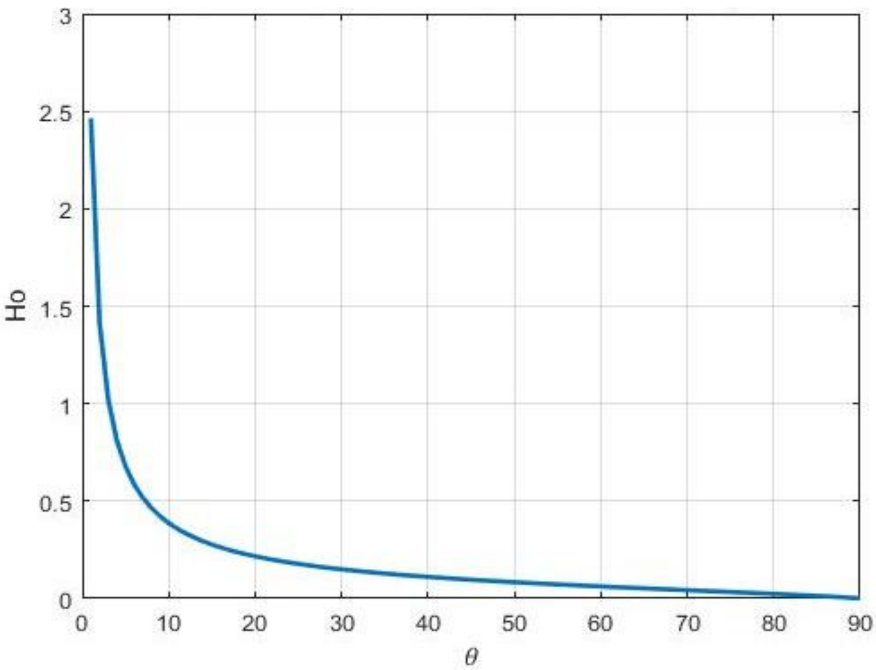
$$ho = \left( \frac{5\sqrt{2g}Do^2t}{8\tan(\theta)^2} \right)^{\frac{2}{5}}$$

## Part-A

After solving equation 5 in terms of expressions, known values are written in their place. (*Do, g and t*) Using MATLAB, solve Eq. 5 with theta variable increasing by 1 from 0° to 90°.

$$ho = \left( \frac{5\sqrt{2 \cdot 9,81}(0,00209)^2\,(4 \cdot 60)}{8\tan(\theta)^2} \right)^{\frac{2}{5}}$$

As seen in *Graph 1*, Ho values are found according to the theta variable. The MATLAP code is clearly found in *Appendix*. The operations were performed in the order described above and the graph was created.



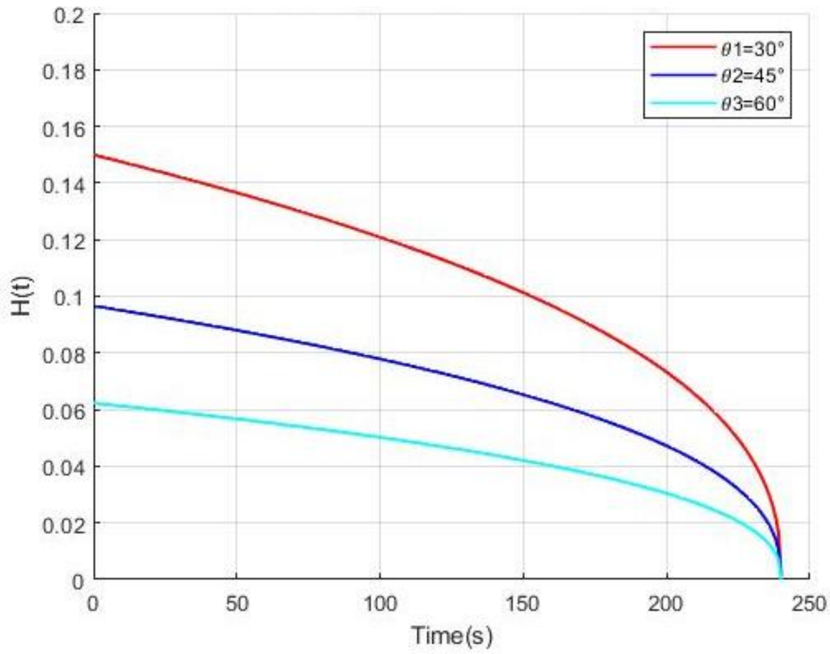

*Graph 1: The initial height with respect to funnel angle θ*

*Discussion Section of A*→ The effects of cone angle on initial height.

When *Graph-1* is examined, it is observed that the cone angle is a very effective parameter for the initial water height. It is observed that the cone angle has very effective changes in the first degrees. For example, when the angle is 1 degree, the initial water height is 2.46 meters. However, the fact that the angle is 2 degrees reduces the first water height to 1.41 m. Contrary to what is thought, this value is quite high. The reason for this event is in accordance with the *tan(x)* trigonometry graph. As seen in the graph, as the cone angle increases, the initial water height decreases curvilinearly.

## Part – B

Assuming that water is completely discharged from the funnel in Eq. 5 in 4 minutes, we find the ho values for *θ1=30°, θ2=45°* and *θ3=60°*. By substituting the Ho values in the 4th equation, we obtained the time dependent h(t) equations. We calculated the h(t) equation with respect to time using MATLAB. As seen in *Graph 2*, we have obtained curves according to different funnel angles. The MATLAP code is clearly found in *Appendix*. The operations were performed in the order described above and the graph was created.




*Graph 2: The Water Height Relative to Fixed Angle θ*

*Discussion Section of B*→ The effects on funnel angle on water removal rate.

As seen in *Graph-2*, the effect was examined from 3 different levels. It is seen that the biggest difference is in the initial water heights. The angles then follow a similar curve. When the slopes in the graph are examined, while the $\theta$ angle decreases, the slopes increase in the negative direction. As a result, due to the geometry of the funnel, the water height changes a lot at low funnel angles. In addition, 15-20 seconds before the end of water discharge, the change in water heights increases significantly.

## Part-C

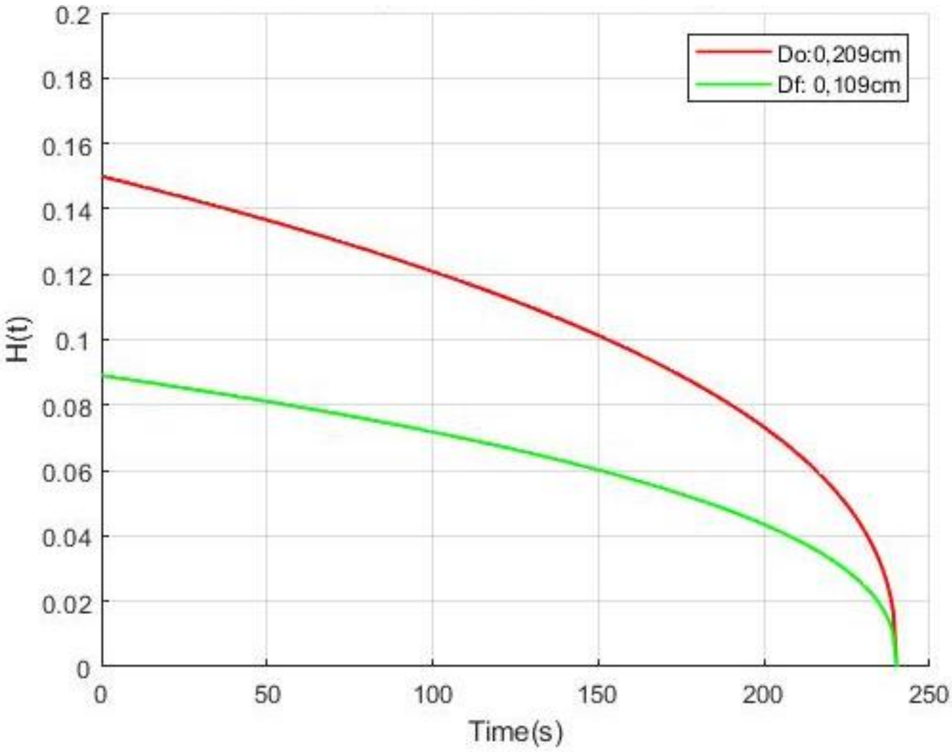
In the Eq.5, the value of Do is replaced by the values of Df. The Ho value is calculated for 30 degrees.

$$ho = \left(\frac{5\sqrt{2\cdot 9,81}(0,00109)^2\,(4\cdot 60)}{8\tan{(30)}^2}\right)^{\frac{2}{5}}$$

In the Eq.4, the value of Do is replaced by the values of Df. For 30 degrees, the change function of h(t) value with respect to time is formed.

$$h\left(t\right) = \left[\left(\frac{5\sqrt{2\cdot 9,81}(0,00109)^2\,(4\cdot 60)}{8\tan{(30)}^2}\right)^{\frac{5}{2}} - \frac{5\sqrt{2\cdot 9,81}(0,00109)^2\cdot t}{8\tan{(30)}^2}\right]^{\frac{2}{5}}$$

The created equation is calculated in MATLAB from 0 to 240 seconds. As seen in *Graph 3*, it is compared with the value in Part-B in the same chart. The MATLAP code is clearly found in *Appendix*. The operations were performed in the order described above and the graph was created.




*Graph 3: The Water Relative to fixed Angle θ1=30°*

## Part – D

We do not use the large tank assumption, so we have to examine it as in *Figure 1*. Let's rebuild Eq 1.2. (Eq.6)

$$Di = 2 \cdot h \cdot \tan(\theta) + Do$$

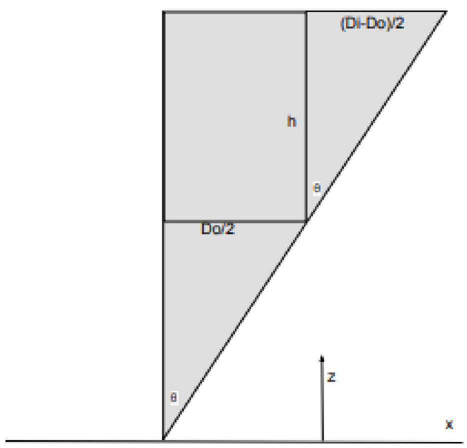



*Figure 1: The large tank assumption is not valid.*

As Equation 1.2 changes, Bernoulli's solution also changes. Since we do not use the big tank theorem, we cannot neglect V1. In the Bernoulli equation, known values are substituted and Eq.7 is found. The new equation is formed as follows.

$$\gamma h + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$

We simplify the Eq.7 and leave V2 alone. (Eq. 8)

$$V_2 = \sqrt{2 \cdot g \cdot h + V_1^2}$$

If we substitute Eq.3, Eq.6 and Eq.8 in Eq.1.3.

$$\sqrt{2 \cdot g \cdot h + V_1^2} = \left(\frac{(Do + 2 \cdot h \cdot \tan(\theta))}{Do}\right)^2 \cdot V_1$$

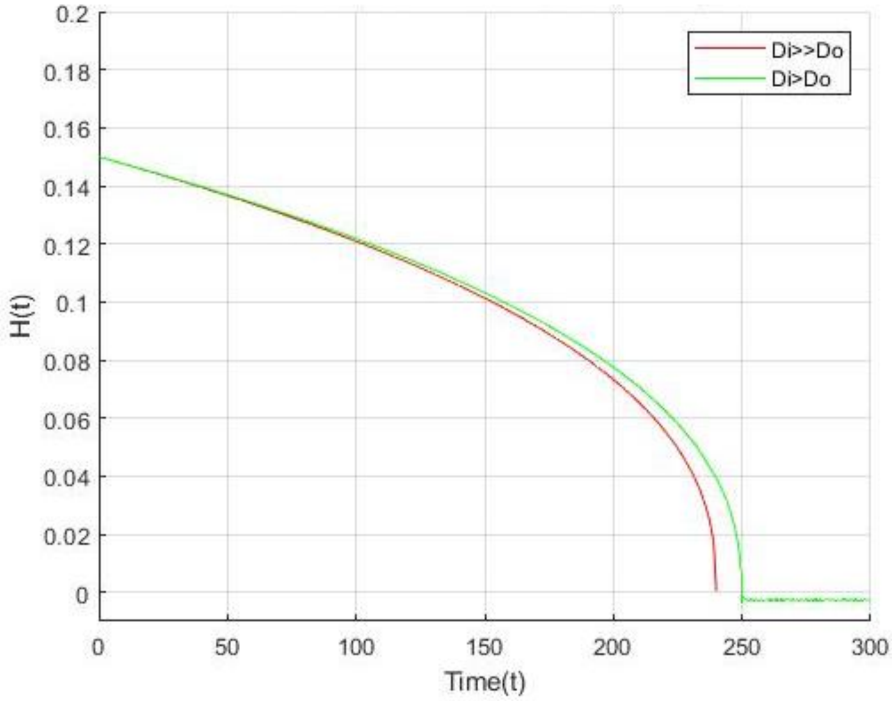
Square the two sides. V1 is left alone.

$$V_1^2 = \frac{(2 \cdot g \cdot h)}{\left(\frac{(Do+2 \cdot h \cdot \tan(\theta))}{Do}\right)^4 - 1}$$

Final adjustments are made and both sides are enclosed in the square root.

$$\frac{dh}{dt} = -\frac{\left(Do^2 \cdot \sqrt{2 \cdot g \cdot h}\right)}{\sqrt{(Do + 2 \cdot h \cdot \tan(\theta))^4 - Do^4}}$$

I tried to calculate the newly formed equation numerically, but I could not reach the final. Then I solved the equation using MATLAB ODE45. Then I chose *$\theta 1=30°$* from the values in Part-B. As seen in *Graph 4*, I showed the variation of the height of the water with time using two different assumptions. The MATLAP code is clearly found in *Appendix*. The operations were performed in the order described above and the graph was created.



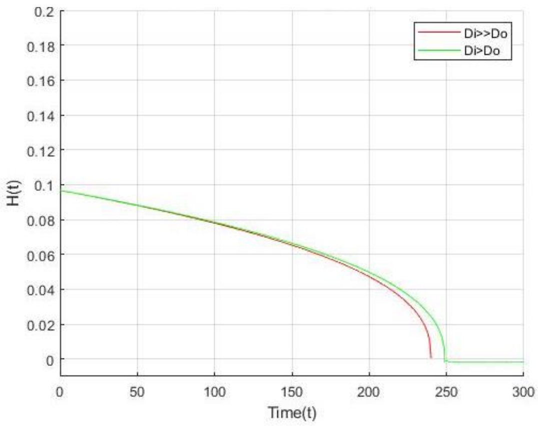

*Graph 4: The Large Tank Assumption θ1=30°*

*Discussion Section of D*→ The Effects of Large Tank Assumption.

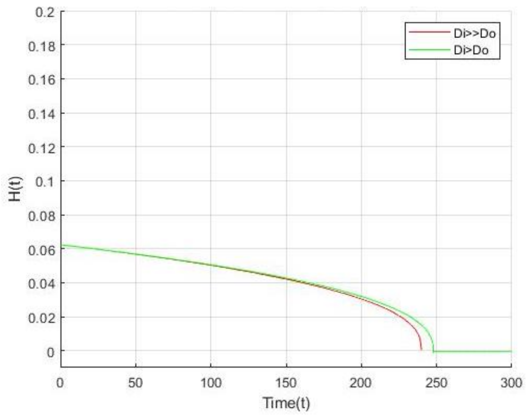
The reason for using this assumption is that Do has little effect in the Bernoulli equation. For example, in a funnel with a Di / Do ratio of 10, the V2/V1 ratio becomes 100 due to the mass conservation equation (Eq.1.3). Since this ratio is used by squaring in Bernoulli's equation, the ratio increases 10,000 times. Even with a Di / Do ratio of 10, the effect on the equation is very small. As a result, the higher this ratio, the less effect Do will have on the equation.

**Di >> *Do*, A1 >>A2, V2>>V1**

If we evaluate the MATLAP solution for *θ1=30°* as seen in *Graph-4*, it is seen that the curves are quite similar. The *0.2%* deviation of the green line after about 250 seconds is related to the MATLAB application rounding the given values. In addition, according to *Graph-5* and *Graph-6*, increasing the funnel angle increases the similarity. It makes sense to apply this principle except in applications of high importance in daily life.




*Graph 5: The Large Tank Assumption θ2=45°*




*Graph 6: The Large Tank Assumption θ3=60°*

# Appendix

```
clc,clear,close all;

a=1:1:90;       %θ
g =9.81         %gravity
t =4*60         %time
do=0.00209      %Do

ho=((5* ((2*g)^0.5)* do^2* t)./(8* (tand(a).^2))).^(2/5) %Ho

%Section- A
%The Initial Height With Respect To Funnel Angle θ
figure;
plot(a,ho,'LineW',1.2);
grid on;
title('The Initial Height With Respect To Funnel Angle θ')
xlabel('θ');
ylabel('Ho');
axis([0 90 0 3]);

%Section- B
%The Water Height Relative to Fixed Angle θ1, θ2, θ3
a1= 30    %θ1
ho1=((5* ((2*g)^0.5)* do^2* t)./(8* (tand(a1).^2))).^(2/5) %Ho1
a2= 45    %θ2
ho2=((5* ((2*g)^0.5)* do^2* t)./(8* (tand(a2).^2))).^(2/5) %Ho2
a3= 60    %θ3
ho3=((5* ((2*g)^0.5)* do^2* t)./(8* (tand(a3).^2))).^(2/5) %Ho3

T = 0:240;  %time
h1 =  ( ho1^(5/2) - (5*((2*g)^0.5)* do^2* T)/(8* (tand(a1)^2))).^(2/5)
%H1(t)
h2 =  ( ho2^(5/2) - (5*((2*g)^0.5)* do^2* T)/(8* (tand(a2)^2))).^(2/5)
%H2(t)
h3 =  ( ho3^(5/2) - (5*((2*g)^0.5)* do^2* T)/(8* (tand(a3)^2))).^(2/5)
%H3(t)
```

```
figure;
hold on;
plot(T,h1,'r','LineW',1.2);  %θ1=30°
plot(T,h2,'b','LineW',1.2);  %θ2=45°
plot(T,h3,'c','LineW',1.2);  %θ3=60°

legend('θ1=30°','θ2=45°','θ3=60°');
title('The Water Height Relative to Fixed Angle ?')
xlabel('Time(s)');
ylabel('H(t)');
grid on;
axis([0 250 0 0.2]);

%Section- C
%The diameter of Do is changed to Df
Df= 0.00109

hof=((5* ((2*g)^0.5)*Df^2* t)./(8* (tand(a1).^2))).^(2/5)  % We know a1=30°
hf= ( hof^(5/2) - (5*((2*g)^0.5)* Df^2* T)/(8* (tand(a1)^2))).^(2/5)

figure;
hold on;
plot(T,h1,'r','LineW',1.2);   %H1(t) with Do
plot(T,hf,'g','LineW',1.2);   %H1(t) with Df
xlabel('Time(s)');
ylabel('H(t)');
title('The Water Height Relative to Fixed Angle θ1=30°')
legend('Do:0,209cm','Df: 0,109cm');
grid on;
axis([0 250 0 0.2]);

%Section- D
% The Large Tank Assumption Is Not Valid

f = @(t,h) - (do^2 * (2*g*h)^(0.5))/  ((do + 2*h*tand(a1))^4- do^4)^(0.5);
%Dif. Eq. (We know a1,g,do)

tfinal =300;      %time
[t,h] = ode45(f, [0 tfinal], ho1); % we calculated ho1 on Part-B

figure;
hold on;
plot(T,h1,'r');
plot(t,h,'g');
title('The Large Tank Assumption (θ1=30°)');
xlabel('Time(t)');
ylabel('H(t)');
legend('Di>>Do','Di>Do');
axis([0 300 -0.01 0.2]);
grid on;

%All attachment must be run together for the MATLAB code to work correctly.
%Appendix does not include the solutions in Graph 5 and Graph 6.
```