Topic 4.2 Practice Problems

1. For each statement below, determine whether it violates any of the four properties of equality. If it does not violate them, identify which property (or properties) were used.

   (a) $6t = 5 \;\rightarrow\; \dfrac{6t}{3} = \dfrac{5}{3}$

   (b) $5 = t \;\rightarrow\; 5 \cdot 4 = 4 \cdot t$

   (c) $3 = 3 \;\rightarrow\; 3 - x = 3 - x$

   (d) $8s = 24 \;\rightarrow\; \dfrac{8s}{8} = 24 \cdot 8$

   (e) $\dfrac{x}{5} = 10 \;\rightarrow\; 5\left(\dfrac{x}{5}\right) = 10 \cdot 4.5$

   (f) $3x = 2x \;\rightarrow\; 3x - 2x = 2x - 2x$

   (g) $x + 3 = 6 \;\rightarrow\; 2(x + 3) + 5 - 3 = 2(6) + 5 - 3$

   (h) $a = b \;\rightarrow\; 2\left(\dfrac{a}{8} - 4\right) + 3 = 2\left(\dfrac{b}{8} - 4\right) + 3$

2. Using the properties of equality, prove that $m = c$ given that

$$\frac{2(m-1)+6}{8} = \frac{2(c-1)+6}{8}.$$

   Show all algebraic steps clearly.

3. A balance scale has green blocks on the left side and blue blocks on the right side. Each green block weighs 3 grams, and each blue block weighs 2 grams. Assume there are 24 green blocks on the left side.

   (a) For the scale to be balanced, how many blue blocks must be on the right side?

   (b) Ahmed removes 18 grams from the right side. How many grams must be removed from the left side to keep the scale balanced? How many blue blocks is this?

   (c) Ahmed removes half of the green blocks from the left side. How many grams are removed from the left side? What fraction of the original blue blocks must be removed from the right side to keep the scale balanced?

   (d) Ali adds five times the original number of green blocks to the left side. How many blue blocks must be added to the right side to keep the scale balanced?

   (e) Repeat parts (a)–(d), assuming now that each green block weighs 1.75 grams (blue blocks still weigh 2 grams).

4. **Equation Encryption.** To encrypt an equation, perform the following operations in order:

   (1) multiply both sides by 5, (2) add 3 to both sides, (3) divide both sides by 8.

(a) Encrypt each equation:

$$1 = 1, \qquad x = 3, \qquad x = y, \qquad 4x + 3 = 1.20.$$

(b) List the properties of equality required to *decrypt* the equations.

(c) Decrypt the following equations:

$$6 = 6, \qquad \frac{15x + 3}{8} = \frac{103}{8}, \qquad \frac{5ab + 3}{8} = \frac{5xy + 3}{8}, \qquad \frac{7.5x + 13}{8} = 1.4375.$$

5. Below are three algebraic proofs. If a mistake occurs, identify the first incorrect step, correct it, and then complete the solution from that point onward.

(a)

Step 1) $15x + 30 = 40$
Step 2) $15x + 30 - 40 = 40 - 40$
Step 3) $15x - 10 = 0$
Step 4) $15x - 10 - 15x = 0 - 15x$
Step 5) $-10 = -15x$
Step 6) $\frac{-10}{15} = \frac{-15x}{-15}$
Step 7) $-\frac{2}{3} = x$

(b)

Step 1) $4s + 3 = 5$
Step 2) $4s + 3 - 3 = 5 - 5$
Step 3) $4s = 0$
Step 4) $\frac{4s}{4} = \frac{0}{4}$
Step 5) $s = 0$

(c)

Step 1) $5x + 3 = 9 - 2x$
Step 2) $5x + 3 + 2x - 3 = 9 - 2x + 2x - 3$
Step 3) $7x = 6$
Step 4) $\frac{7x}{7} = \frac{6}{7}$
Step 5) $x = \frac{6}{7}$

6. Suppose $x = 0$. If you divide both sides of an equation by x, you may incorrectly obtain $1 = 0$, which is a contradiction. Although the division property of equality is commonly used, explain why it cannot be applied in this situation. *Hint: consider the meaning of the expression* $\frac{0}{0}$.