①P

NN - HW2

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1) a) Given: To design a feed fwd NN for the below logical statement.

$$(x_1 \wedge x_2 \wedge \neg x_3) \vee (\neg x_2 \wedge x_3)$$

+1 ⇒ True -1 ⇒ False

$$\varphi(v) = sign(v) = \begin{cases} +1 & ; \ v > 0 \\ 0 & ; \ v = 0 \\ -1 & ; \ v < 0 \end{cases}$$

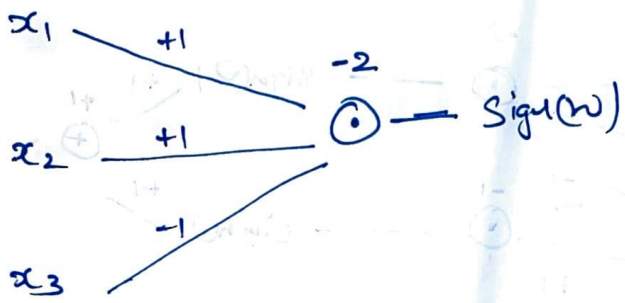
Inference:

$$(x_1 \wedge x_2 \wedge \neg x_3) \vee (\neg x_2 \wedge x_3)$$

↓ ① ↓ ②

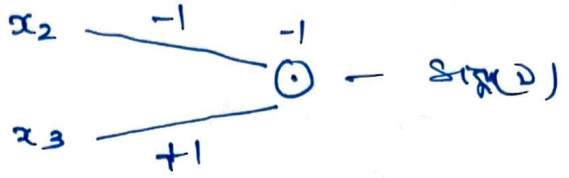
① ⇒ $(x_1 \wedge x_2 \wedge \neg x_3)$

We can choose the $w = [1, 1, -1]$ since x_3 is a not operation and x_1, x_2 is a And operation. Let the bias $b_1 = -2$, so that it activates only when it satisfies the logical statement.

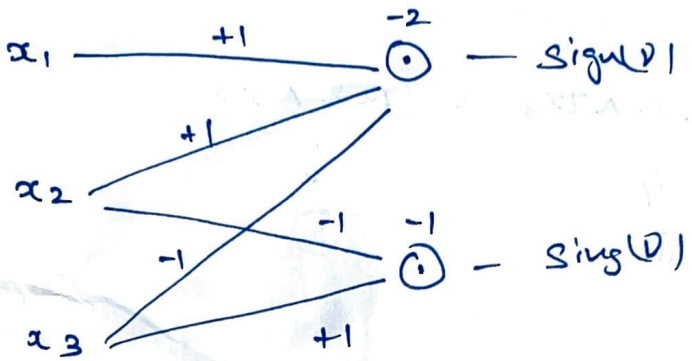


(2) $\Rightarrow (\neg x_2 \wedge x_3)$

Similarly we assign the weights as $[-1, 1]$ and bias $b_2 = -1$ so that it activates satisfying the logical statement.



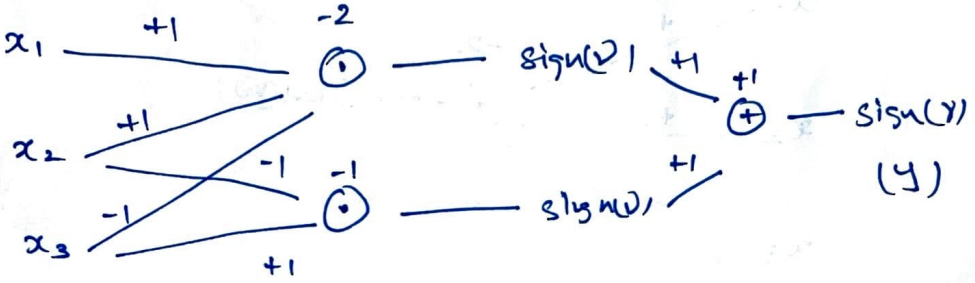
Combining (1) & (2)



Second layer:- Since its an OR operation, we need to make sure that it satisfies the condition for the given bias & weight. Thus we choose

$$\text{weight} = [1, 1]$$

$$\text{bias} = +1$$



(3) P

Thus we have 3 — input

2 — hidden layer

1 — output

3-2-1 NN.

b) First layer (hidden)

$$z_j = \text{sign}\left(\sum_{i=1}^{3} w_{ji} x_i + b_j\right) - (1)$$

Second layer (output)

$$y = \text{sign}\left(\sum_{i=1}^{2} v_{ji} z_j + c_j\right) - (2)$$

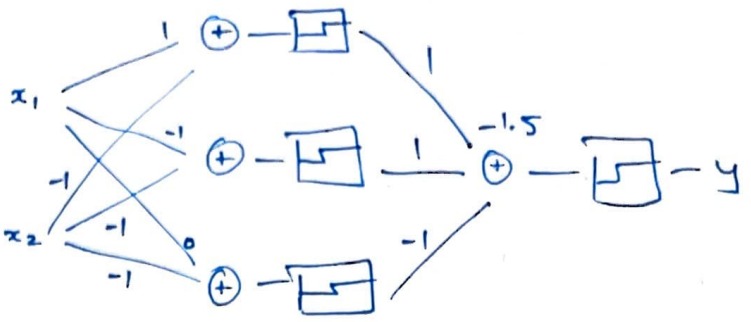
(2) in (1)

$$y = \text{sign}\left(\sum_{i=1}^{2} v_{ji} \left(\text{sign}\left(\sum_{i=1}^{3} w_{ji} x_i + b_j\right)\right) + c_j\right)$$

c)

x1	x2	x3	Logical	NN
1	1	1	False	-1
1	1	-1	True	1
1	-1	1	True	1
1	-1	-1	False	-1
-1	1	1	False	-1
-1	1	-1	False	-1
-1	-1	1	True	1
-1	-1	-1	False	-1

2) a) Given: (4p)



$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times1} \quad \text{input}$$

$$w = \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ 0 & -1 \end{bmatrix}_{3\times2} \quad \text{weight}$$

$$b = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}_{3\times1} \quad \text{bias}$$

$$v = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}_{1\times3} \qquad c = \begin{bmatrix} -1.5 \end{bmatrix}_{1\times1}$$

b) $$z_j = \text{step}\left(\sum_{i=1}^{2} w_{ji} x_i + b_j \right) \quad -(1)$$

$$y = \text{step}\left(\sum_{i=1}^{1} v_{ji} z_j + c_i \right) \quad -(2)$$

(2) in (1)

$$y = \text{step}\left(\sum_{i=1}^{1} v_{ji} \left(\text{step}\left(\sum_{i=1}^{2} w_{ji} x_i + b_j \right) \right) + c_j \right)$$

$$y = \text{step}\left(\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \left(\text{step}\left(\begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right) \right) + \begin{bmatrix} -1.5 \end{bmatrix} \right).$$

c) code submitted

d)