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Neural Networks

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675086474

1) a) Given :- For some j, $w_{ji} = 1$ & $b_j = 0$

To prove :- z_j is equivalent to logical OR of x_i

Inference :-

$$z_j = \text{step1}\left(b_j + \sum_{i=1}^{9} w_{ji} x_i\right) \quad \text{step1}(t) = \begin{cases} 0, & t<1 \\ 1, & t\geq 1 \end{cases}$$

Since $b_j = 0$ & $w_{ij} = ~~0~~ 1$

$$\boxed{z_j = \text{step1}\left(\sum_{i=1}^{9} x_i\right)}$$

According to the step fn if there is atleast ~~one~~ one $x_i = 1$ then $z_j = 1$, or if no $x_i = 1$, then $z_j = 0$.

Proof :-

i) Let us consider $x_4 = 1$ and rest as zero.

Thus $\sum_{i=1}^{9} x_i = 0+0+0+1+0+0+0+0+0$
$= 1$

$z_j = \text{step1}(1) = 1$

$$\boxed{z_j = 1}$$

ii) Consider all x_i as zeros,

Thus $\sum_{i=1}^{9} x_i = 0+0+0+0+0+0+0+0+0$
$= 0$

$z_j = \text{step1}(0) = 0$

$$\boxed{z_j = 0}$$

Thus with given condition it is logical OR.

b) Given :- If $w_{ji} = -1$ and $b_j = N$ (no. of such weights)

To prove :- z_j is inverted in these cases.

Inference :-

Consider $b_j = N$, where N is the no. of such inverted weights.

So, $z_j = \text{Step1}\left(N + \sum_{i=1}^{9} w_{ji} x_i\right)$

Case 1 :- For $x_i = 1$, $w_{ji} = 1$, $b_j = 0$

$$z_j = \text{Step1}\left(0 + \sum_{i=1}^{9} x_i\right) = \text{Step}(1)$$

$$= 1$$

$\boxed{z_j = 1}$ For nrml weights. works as logical OR.

Case 2 :-

For $x_i = 0$, $w_{ij} = 1$, $b_j = 0$

$$z_j = \text{Step1}\left(0 + \sum_{i=1}^{9} x_i\right)$$

$$= \text{Step}(0)$$

$$= 0$$

$\boxed{z_j = 0}$ works as logical OR

Case 3 :- here $w_{ij} = -1$, $b_j = N$

$$z_j = \text{Step1}\left(N + \sum_{i=1}^{9} w_{ij} x_i\right)$$

Say, consider $x_2 = 1$, $w_{2j} = -1$

then $b_j = 1$

$$z_j = \text{Step1}\left(1 + \sum_{i=1}^{9} w_{ij} x_i\right)$$

for, $\sum_{i=1}^{9} w_{ij} x_i = 0 + (-1)(1) + 0 + 0 + 0 + 0 + 0 + 0 + 0$

$= -1$

$= \text{Step1}(1 - 1) = \text{Step1}(0)$

$z_j = 0$.

so if atleast one $x_i = 1$ and its weight is negative, it acts as a Inverted logical OR

Case 4:- here $w_{ij} = -1$, $b_j = N$

Say, consider $x_2 = 0$, $w_{ij} = -1$

then $b_j = 1$

$$z_j = \text{step1}\left(1 + \sum_{i=1}^{9} w_{ij} x_i\right)$$

for, $\sum_{i=1}^{9} w_{ij} x_i = 0 + (-1)(0) + 0 + 0 + 0 + 0 + 0 + 0 + 0$

$= 0$

$= \text{Step1}(1 + 0) = \text{step}(1)$

$z_j = 1$

So, if the weight is negative and all $x_i = 0$ then, it acts as a Inverted logical OR

Thus when weight is negative it acts as a Inverted logical OR

c) Case 1: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$

$$z_1 = \text{not}\left(x_1 \cdot x_2 \cdot \overline{x_3} \cdot x_4 \cdot x_5 \cdot \overline{x_6} \cdot \overline{x_7} \cdot \overline{x_8} \cdot \overline{x_9}\right)$$

using DeMorgan's law

$$= (\overline{x_1} + \overline{x_2} + x_3 + \overline{x_4} + \overline{x_5} + x_6 + x_7 + x_8 + x_9)$$

(using inverted logical OR)

$$= (w_{11}\overline{x_1} + w_{12}\overline{x_2} + w_{13}x_3 + w_{14}\overline{x_4} + w_{15}\overline{x_5} + w_{16}x_6 + w_{17}x_7 + w_{18}x_8 + w_{19}x_9)$$

where $w_{ij} = [-1, -1, 1, -1, -1, 1, 1, 1, 1]$

and $b = 4$

According to f^n

$$z_1 = 4 + \sum_{i=1}^{9} x_j w_{ij}$$

$$= 4 + (-1 - 1 + 0 - 1 - 1 + 0 + 0 + 0 + 0)$$

$$= 4 + (-4)$$

$$\boxed{z_1 = 0}$$

Case 2:

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$$

$$z_2 = \text{not}\left(\bar{x}_1 \cdot x_2 \cdot x_3 \cdot \bar{x}_4 \cdot x_5 \cdot x_6 \cdot \bar{x}_7 \cdot \bar{x}_8 \cdot \bar{x}_9\right)$$

using De Morgan law

$$= x_1 + \bar{x}_2 + \bar{x}_3 + x_4 + \bar{x}_5 + \bar{x}_6 + x_7 + x_8 + x_9$$

using inverted logical or

$$= \left(w_{11}x_1 + w_{12}\bar{x}_2 + w_{13}\bar{x}_3 + w_{14}x_4 + w_{15}\bar{x}_5 + w_{16}\bar{x}_6 + w_{17}x_7 + w_{18}x_8 + w_{19}x_9\right)$$

when $= w_{ij} = [1, -1, -1, 1, -1, -1, 1, 1, 1]$

and $b = 4$

According to f_4

$$z_2 = 4 + \sum_{i=1}^{9} x_i w_i$$

$$= 4 + (0 - 1 - 1 + 0 - 1 - 1 + 0 + 0 + 0)$$

$$= 4 + (-4)$$

$\cancel{z_2 = 0}$ $\boxed{z_2 = 0}$

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Case 3:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{3\times 3}$$

~~$z_3 = \text{not}(\bar{X}_1 \cdot X_2 \cdot \bar{X}_3 \cdot \bar{X}_4 \cdot \bar{X}_4 \cdot X_5 \cdot X_6$~~

$z_3 = \text{not}(\bar{X}_1 \cdot \bar{X}_2 \cdot \bar{X}_3 \cdot \bar{X}_4 \cdot X_5 \cdot X_6 \cdot \bar{X}_7 \cdot$

$X_8 \cdot X_9)$

using Demorgan law

$= (x_1 + x_2 + x_3 + x_4 + \bar{X}_5 + \bar{X}_6 + X_7 +$

$\bar{X}_8 + \bar{X}_9)$

using inverted logic dense)

$= w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 +$

$w_{15}\bar{x}_5 + w_{16}\bar{x}_6 + w_{17}x_7 + w_{18}\bar{x}_8 +$

$w_{19}\bar{x}_9)$

when $w_{ij} = [1, 1, 1, 1, -1, -1, 1, -1, -1]$

According to fn

$z_3 = u + \sum_{i=1}^{9} x_i w_{ij}$

$= 4 + (\cancel{4}) - 4$

$\boxed{\cancel{z_3 = 4}}$ $\boxed{z_3 = 0}$

Case 4:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3}$$

$$z_4 = \text{not}\left(\overline{x_1}\cdot\overline{x_2}\cdot\overline{x_3}\cdot x_4\cdot x_5\cdot\overline{x_6}\cdot x_7\cdot x_8\cdot\overline{x_9}\right)$$ (using demorgan laws)

$$= (x_1 + x_2 + x_3 + \overline{x_4} + \overline{x_5} + x_6 + \overline{x_7} + \overline{x_8} + x_9)$$

⇒ using inverted logical or)

$$= w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}\overline{x_4} + w_{15}\overline{x_5} + w_{16}x_6 + w_{17}\overline{x_7} + w_{18}\overline{x_8} + w_{19}x_9$$

where $w_{ij} = [1, 1, 1, -1, -1, 1, -1, -1, 1]$

According to f_4

$$z_4 = 4 + \sum_{i=1}^{9} x_i w_i$$

$$= 4(-4)$$

$$\boxed{z_4 = 0}$$

$$w_{ji} = \begin{bmatrix} -1, & -1, & 1, & -1, & -1, & 1, & 1, & 1, & 1 \\ 1, & -1, & -1, & 1, & -1, & -1, & 1, & 1, & 1 \\ 1, & 1, & 1, & 1, & -1, & -1, & 1, & -1, & -1 \\ 1, & 1, & +1, & -1, & -1, & +1, & -1, & -1, & 1 \end{bmatrix}$$

$$\cancel{z = [0, 0, 0, 0]}$$

$$b = [4, 4, 4, 4]$$

$= z_1 \cdot z_2 \cdot z_3 \cdot z_4$

apply demorgan law

$= \overline{z_1} + \overline{z_2} + \overline{z_3} + \overline{z_4}$

so here

bias $= 4$

$v_j =$ weight $= [-1, -1, -1, -1]$

$\boxed{c = 4}$

2) a)

$$g(x) = x^3$$

$$f(x, w) = wx$$

given composite fn

$$l(f(x,w), g(x)) = (f(x,w) - g(x))^2$$

chain rule :-

$$\boxed{\frac{\partial l}{\partial w} = \frac{\partial l}{\partial f} \cdot \frac{\partial f}{\partial w}}$$

b) In chain rule,

$$\frac{\partial l}{\partial w} = \frac{\partial l}{\partial f} \cdot \frac{\partial f}{\partial w}$$

where

$$\frac{\partial l}{\partial f} = 2\left(f(x,w) - g(x)\right) = 2(wx - x^3)$$

$$\frac{\partial f}{\partial w} = x$$

Substitud in chain rule

$$\cancel{\frac{\partial l}{\partial w} = 2(f(x,w)}$$

$$\frac{\partial l}{\partial w} = 2(wx - x^3) \cdot x$$

$$= 2x\,(\omega x - x^3)$$
$$= 2(\omega x^2 - x^4)$$

So now integrat to $\frac{\partial L}{\partial \omega}$

$$\frac{\partial L}{d\omega} = \int_{-1}^{1} 2(\omega x^2 - x^4)\,dx$$

$$\cancel{= \int_{-1}^{1} 2x^2(\omega - x^2)\,dx}$$

$$= \int_{-1}^{1} (2\omega x^2 - 2x^4)\,dx$$

$$= \int_{-1}^{1} 2\omega x^2 . dx - \int_{-1}^{1} 2x^4 . dx$$

$$= 2\omega \int_{-1}^{1} x^2\,dx - 2\int_{-1}^{1} x^4\,dx$$

Cause 2. 2 & ω is constant

$$= 2\omega\left[\frac{x^3}{3}\right]_{-1}^{1} - 2\left[\frac{x^5}{5}\right]_{-1}^{1}$$

$$= 2w\left[\frac{1}{3}\right] - 2w\left[\frac{-1}{3}\right] - \left[2\left[\frac{1}{5}\right] - 2\left[\frac{-1}{5}\right]\right]$$

$$= \frac{2w}{3} + \frac{2w}{3} - \left[\frac{2}{5} + \frac{2}{5}\right]$$

$$\frac{dL}{dw} = \frac{4w}{3} - \frac{4}{5}$$

To find optimal of w

$$\frac{dL}{dw} = 0$$

$$\frac{4w}{3} - \frac{4}{5} = 0$$

$$\frac{20w - 12}{15} = 0$$

$$20w - 12 = 0$$

$$20w = 12$$

$$w = \frac{12}{20} = \frac{3}{5}$$

$$\underline{\underline{w = 3/5}}$$

2) c) graph plotted.