EE281 - Second Order Circuits

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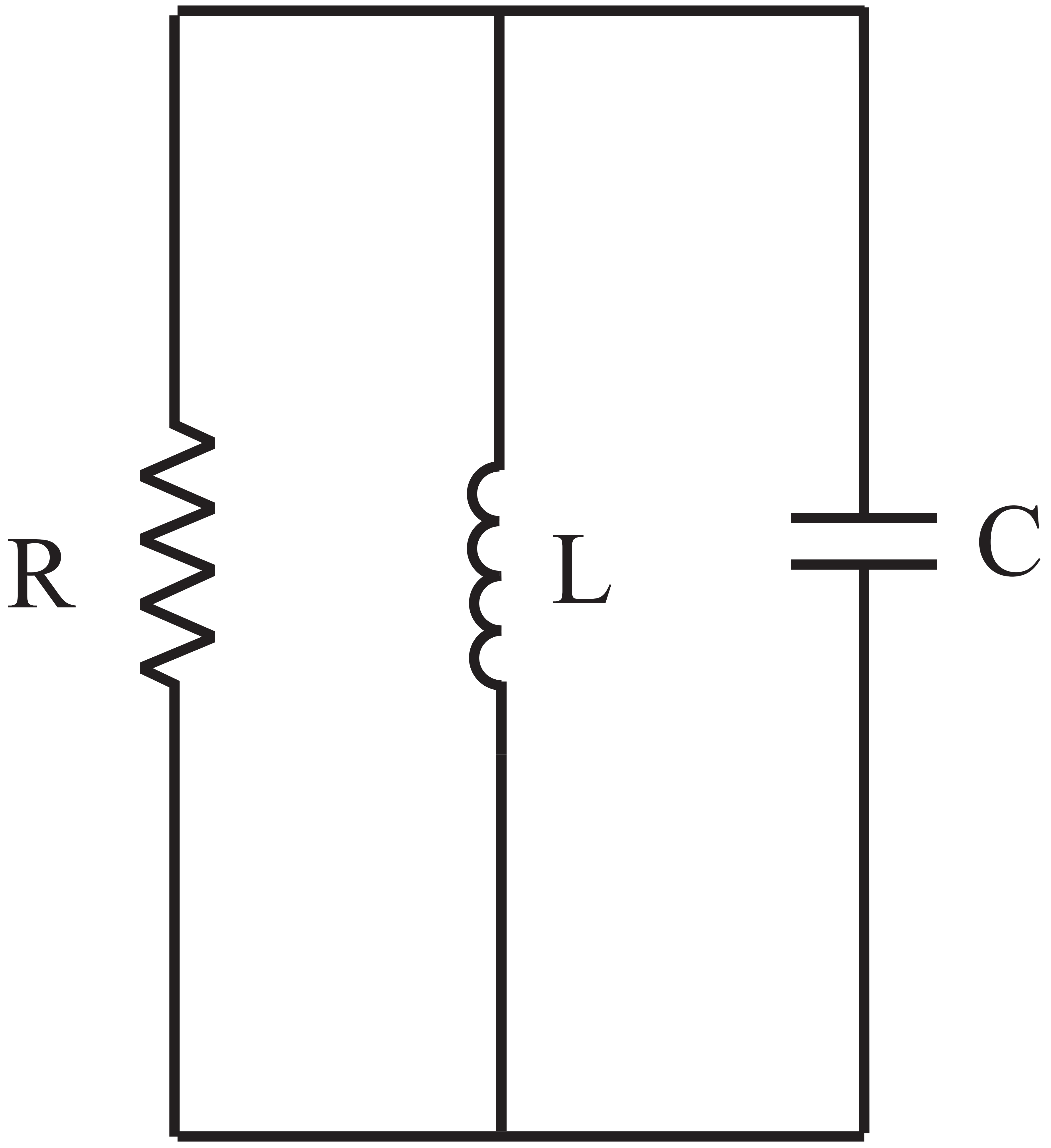
RLC



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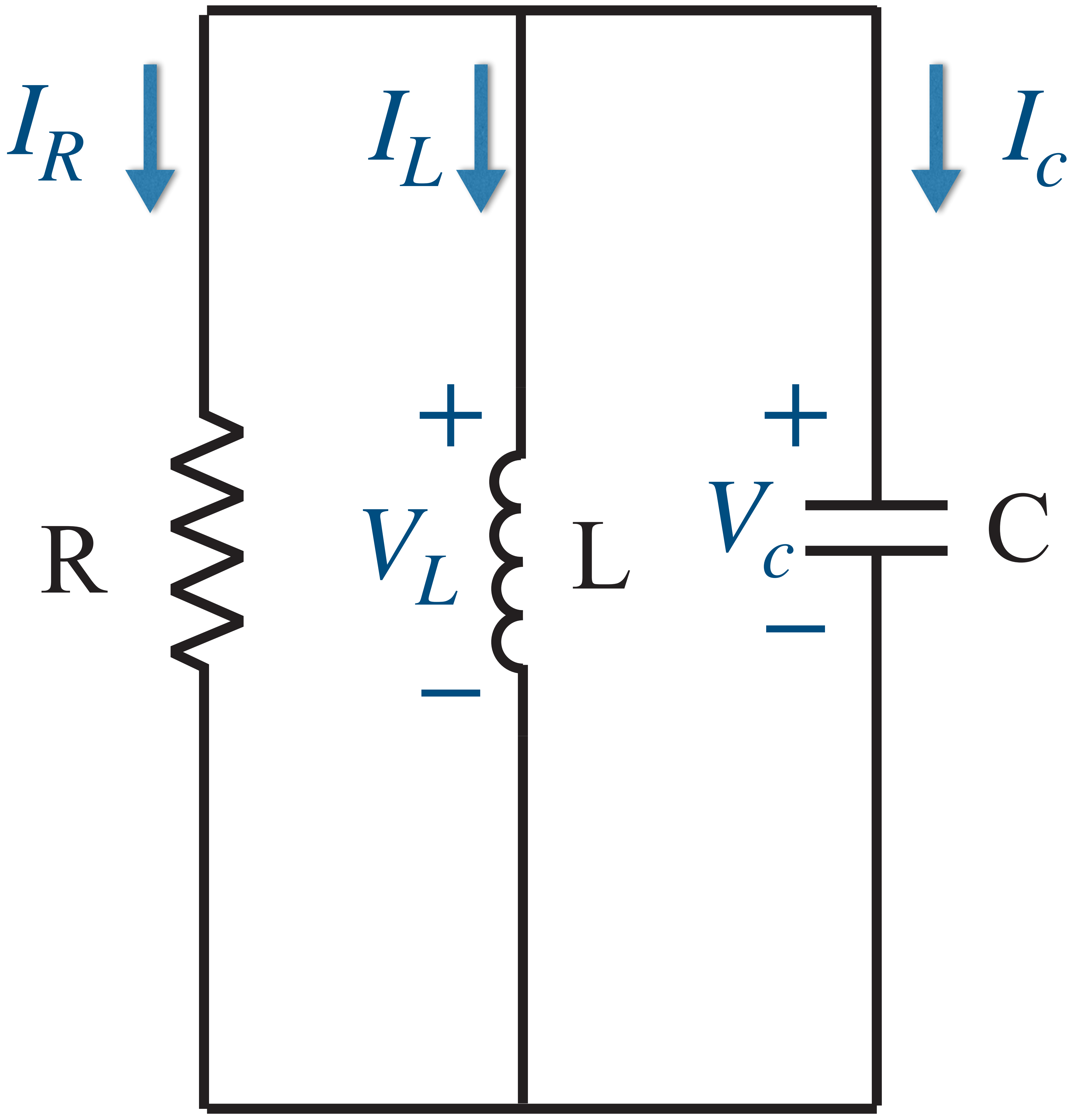
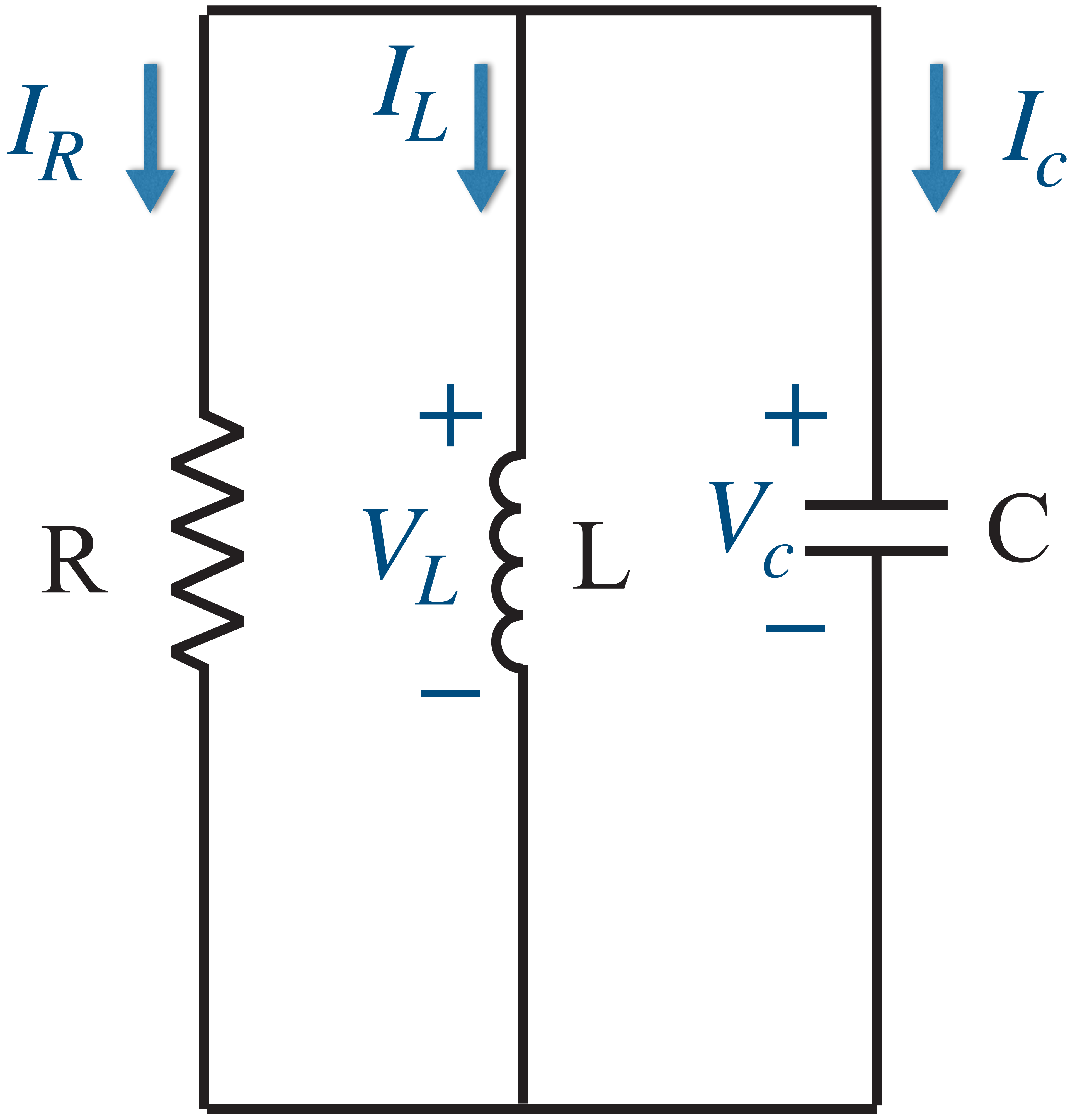


$$Let\ I_L = I$$



$$V_R = V_L = V_c \ \&\ I_R + I_L + I_c = 0$$

$$\frac{V_L}{R} + I + C\frac{dV_L}{dt} = 0 \ \&\ V_L = L\frac{dI}{dt}$$

$$I + \frac{L}{R}\frac{dI}{dt} + LC\frac{d^2I}{dt^2} = 0$$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = 0$$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = 0 \quad \longrightarrow \quad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

$$I \to y$$

Natural (Resonant) Frequency: $$\omega_0 = \sqrt{\frac{1}{LC}}$$

Attenuation (Neper Frequency): $$\alpha = \frac{1}{2RC}$$

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Characteristic equation: $$s^2 + 2\alpha s + \omega_0^2 = 0$$

Roots/Poles: $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

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Two distinct real roots (over-damped)

$$\alpha > \omega_0 \;\Rightarrow\; s_{1,2} \in \mathbb{R} \;\&\; s_1 \neq s_2$$

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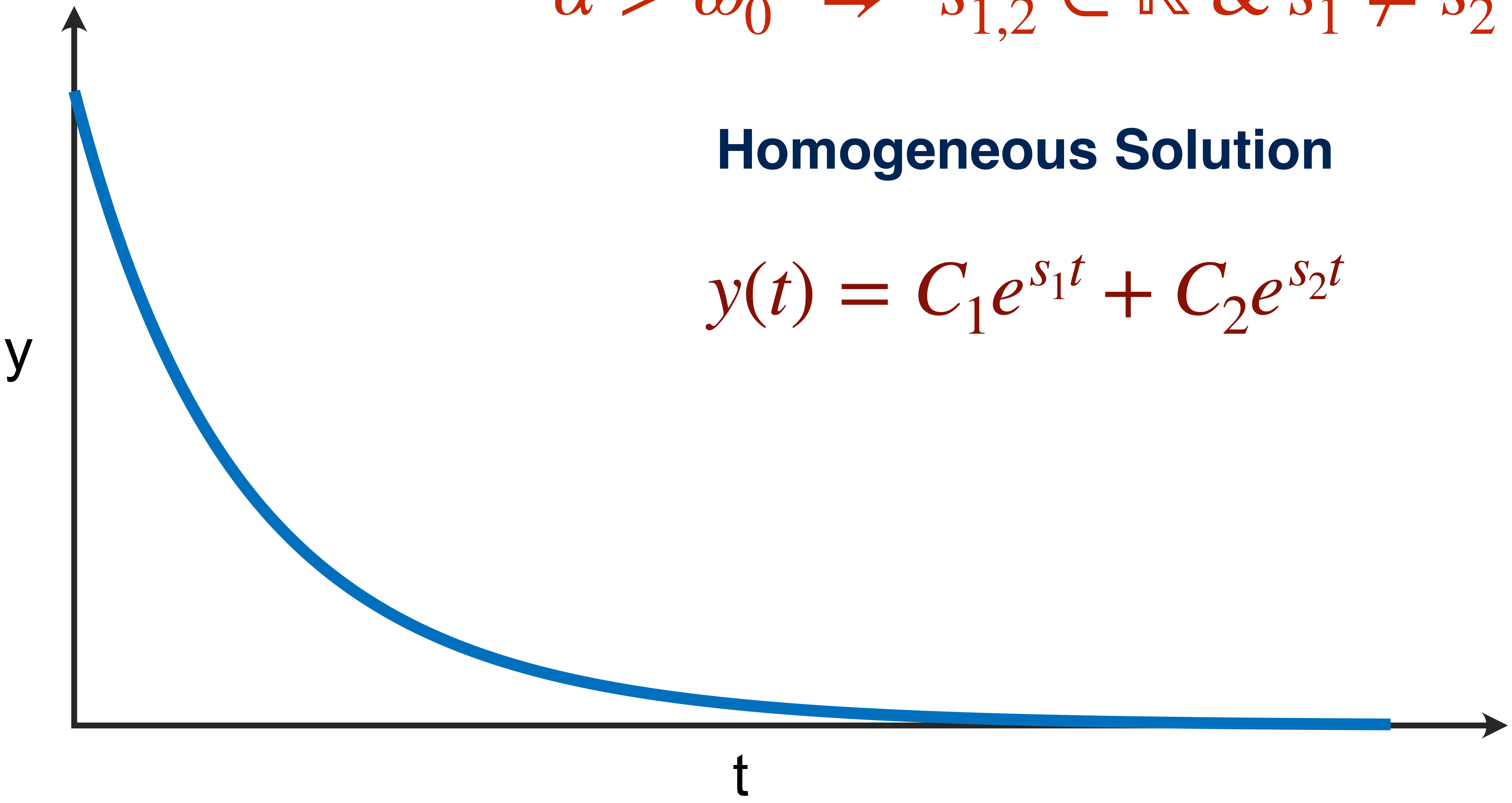
Homogeneous Solution

$$y(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$



$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

Real double root (critically-damped)

$$\alpha = \omega_0 \quad \Rightarrow \quad s_{1,2} \in \mathbb{R} \;\&\; s_1 = s_2 = s$$

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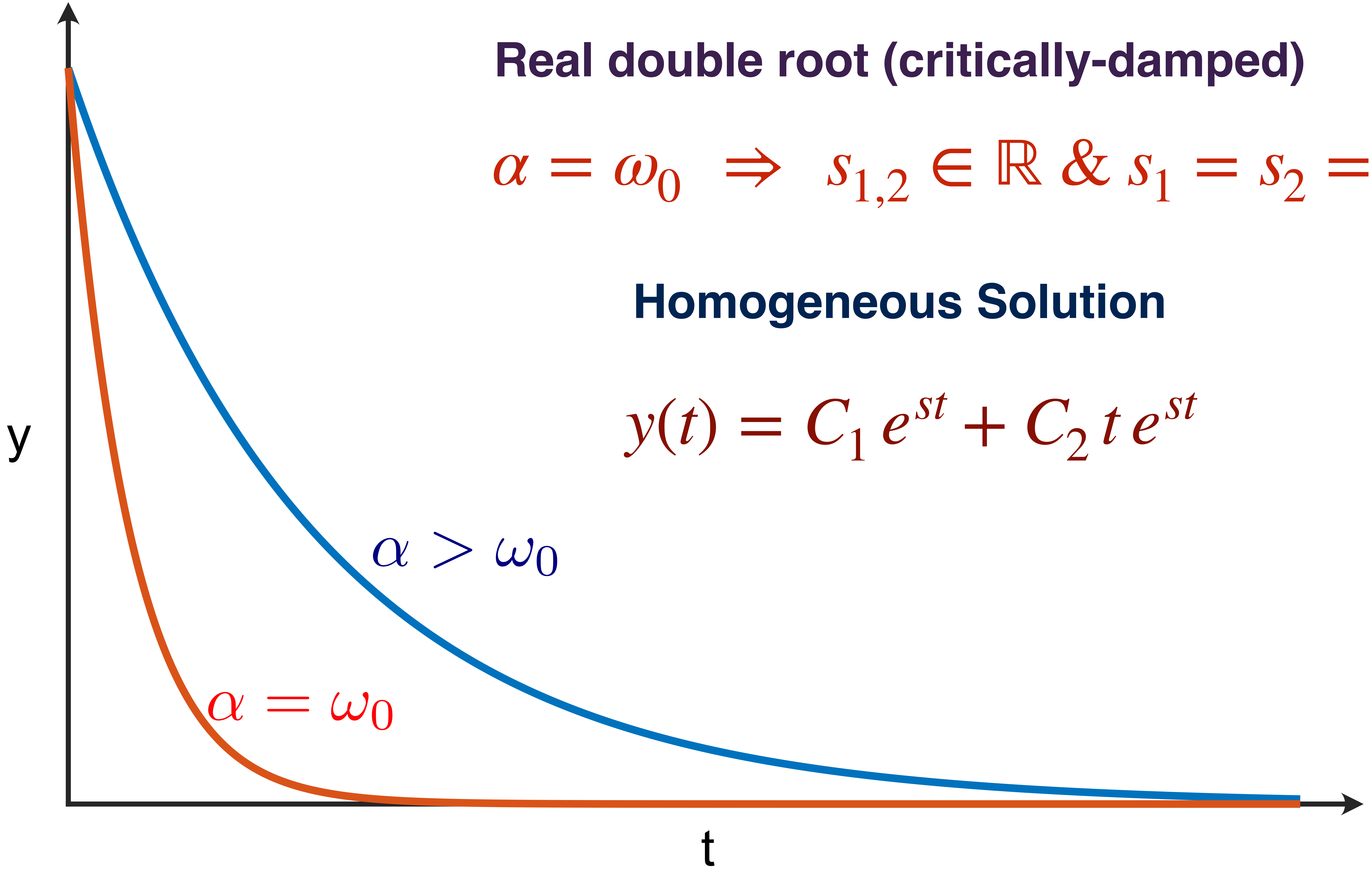
$$\alpha = \omega_0 \;\Rightarrow\; s_{1,2} \in \mathbb{R} \;\&\; s_1 = s_2 = s$$

Homogeneous Solution

$$y(t) = C_1 e^{st} + C_2 t\, e^{st}$$



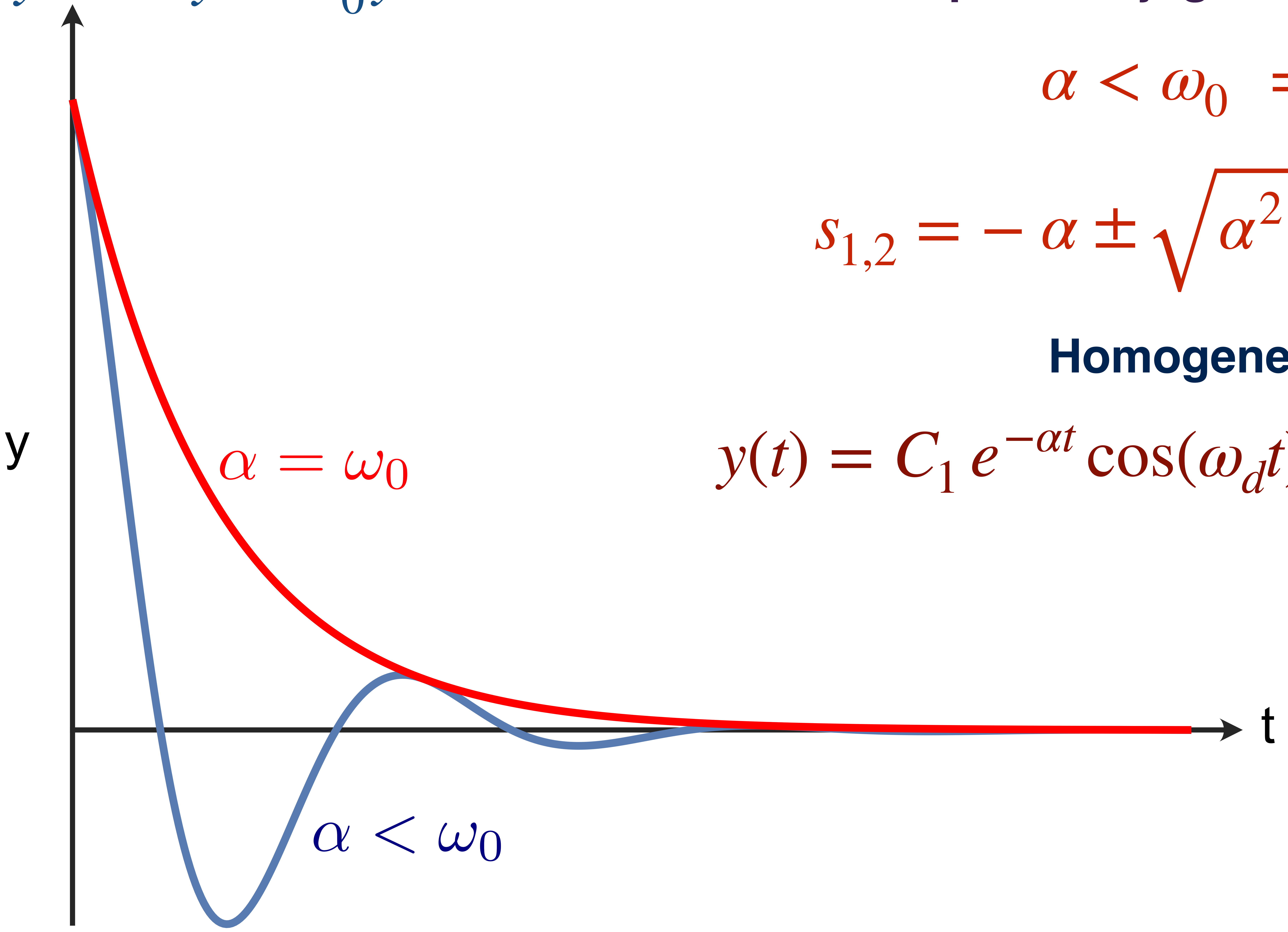
$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

Complex conjugate roots (under-damped)

$$\alpha < \omega_0 \;\Rightarrow\; s_{1,2} \in \mathbb{C}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm j\omega_d$$

$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$



Complex conjugate roots (under-damped)

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$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm j\omega_d$$

Homogeneous Solution

$$y(t) = C_1 e^{-\alpha t} \cos(\omega_d t) + C_2 e^{-\alpha t} \sin(\omega_d t)$$