EE281 - Second Order Circuits
Step Response

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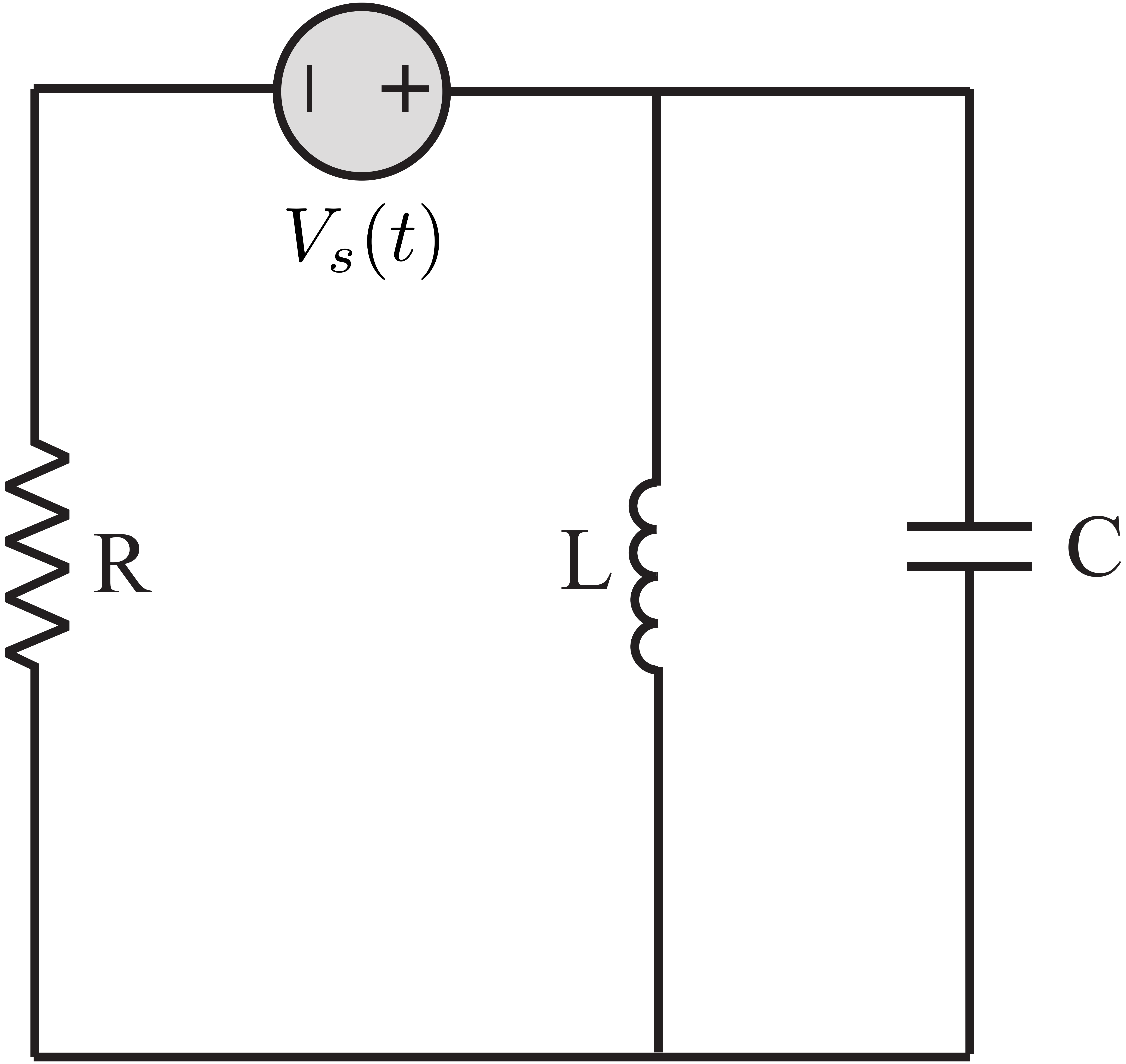
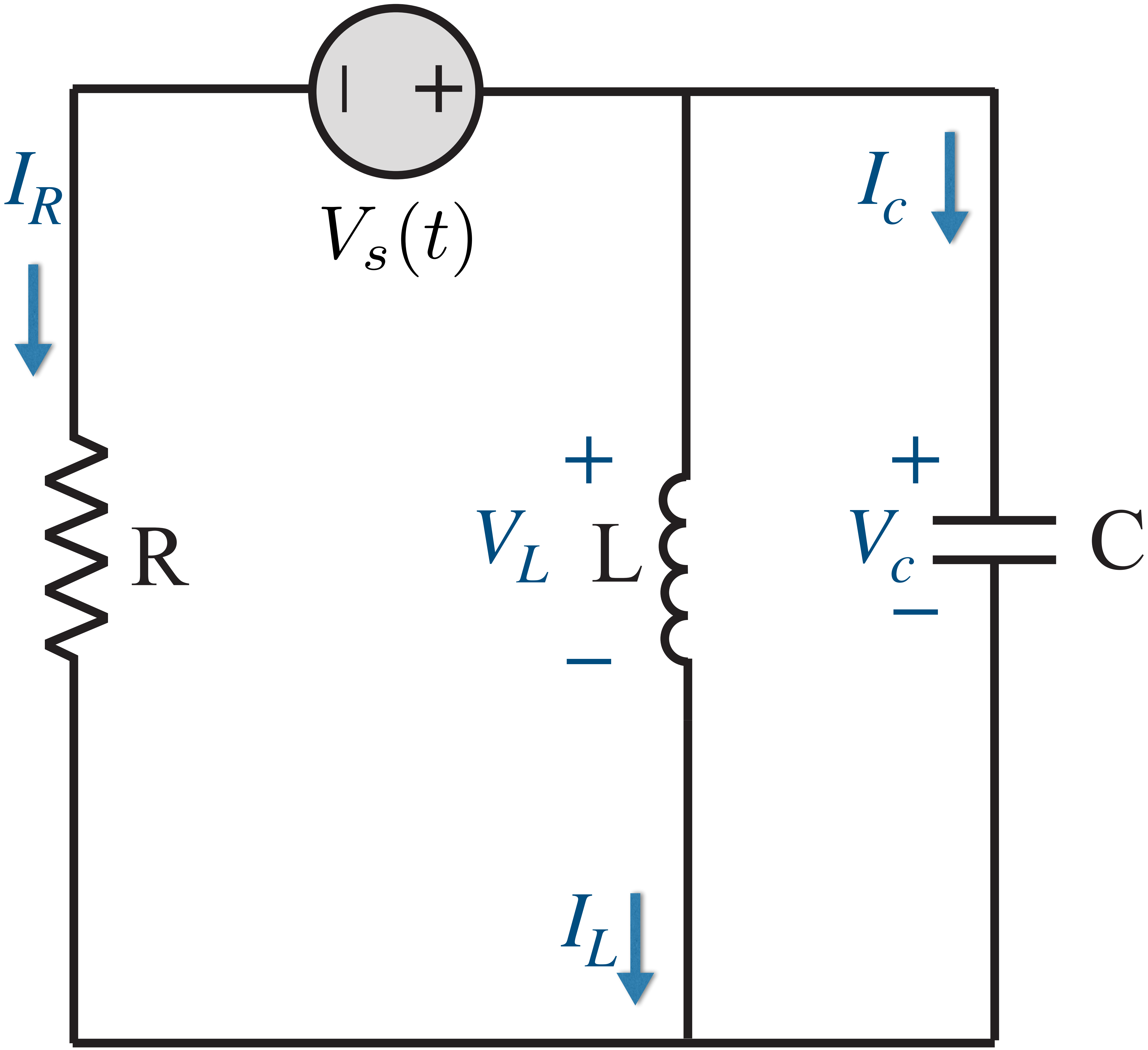
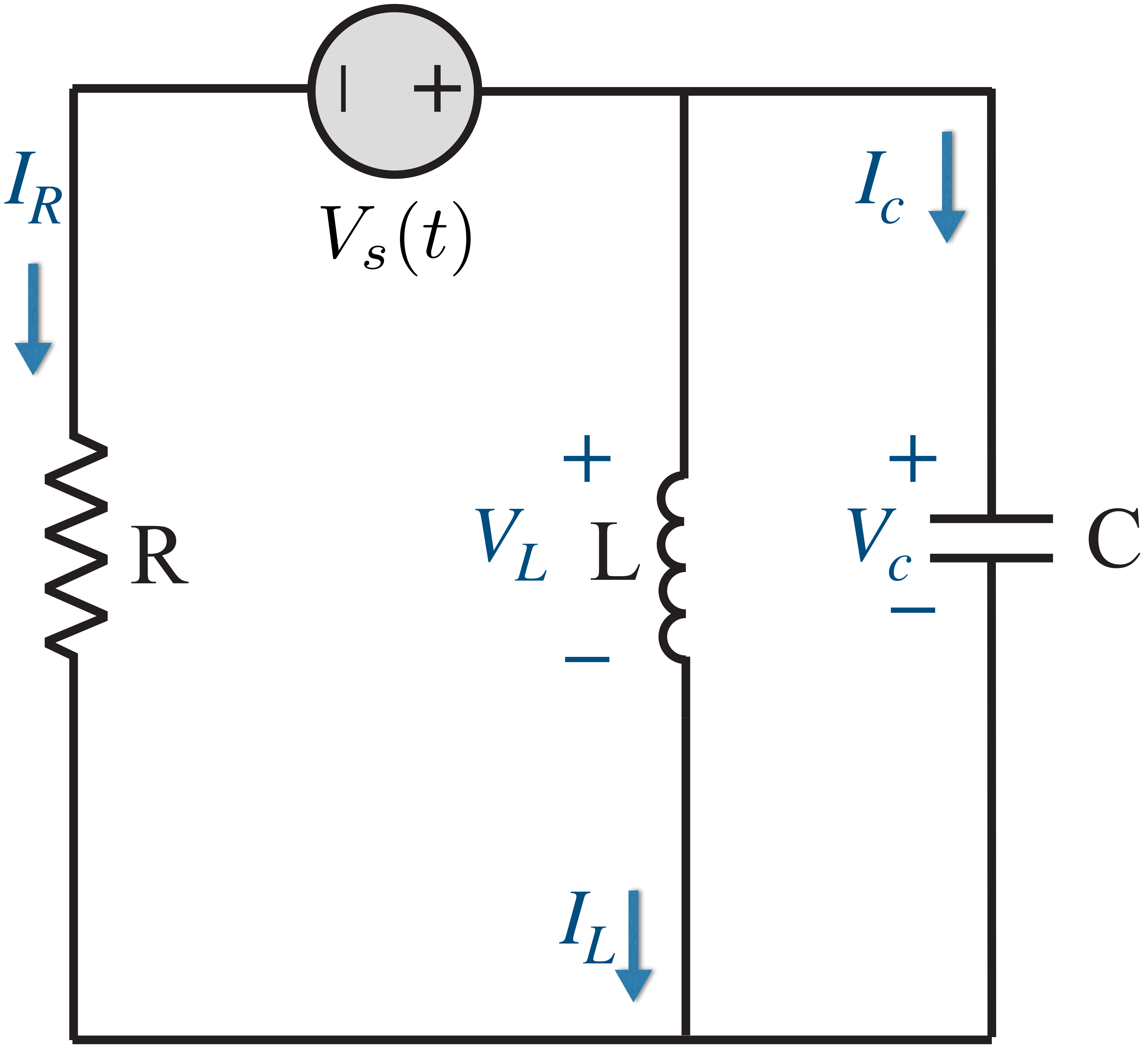
RLC - Forced





I_R
$V_s(t)$
I_c
R
V_L
L
V_c
C
I_L

Let $I_L = I \qquad V_L = V_c = V$



$$I_R + I_L + I_c = 0$$

$$\frac{V - V_s}{R} + I + C\frac{dV}{dt} = 0$$

$$V = L\frac{dI}{dt}$$

$$I + \frac{L}{R}\frac{dI}{dt} + LC\frac{d^2I}{dt^2} = \frac{V_s}{R}$$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = \frac{V_s}{RLC}$$

$$I + \frac{L}{R}\frac{dI}{dt} + LC\frac{d^2 I}{dt^2} = \frac{V_s}{R}$$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = \frac{V_s}{RLC} \quad \longrightarrow \quad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = F(t)\,\omega_o^2$$

$$I \rightarrow y$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad F(t) = \frac{V_s}{R}$$

$$\alpha = \frac{1}{2RC}$$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = \frac{V_s}{RLC} \qquad \longrightarrow \qquad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = F(t)\,\omega_o^2$$

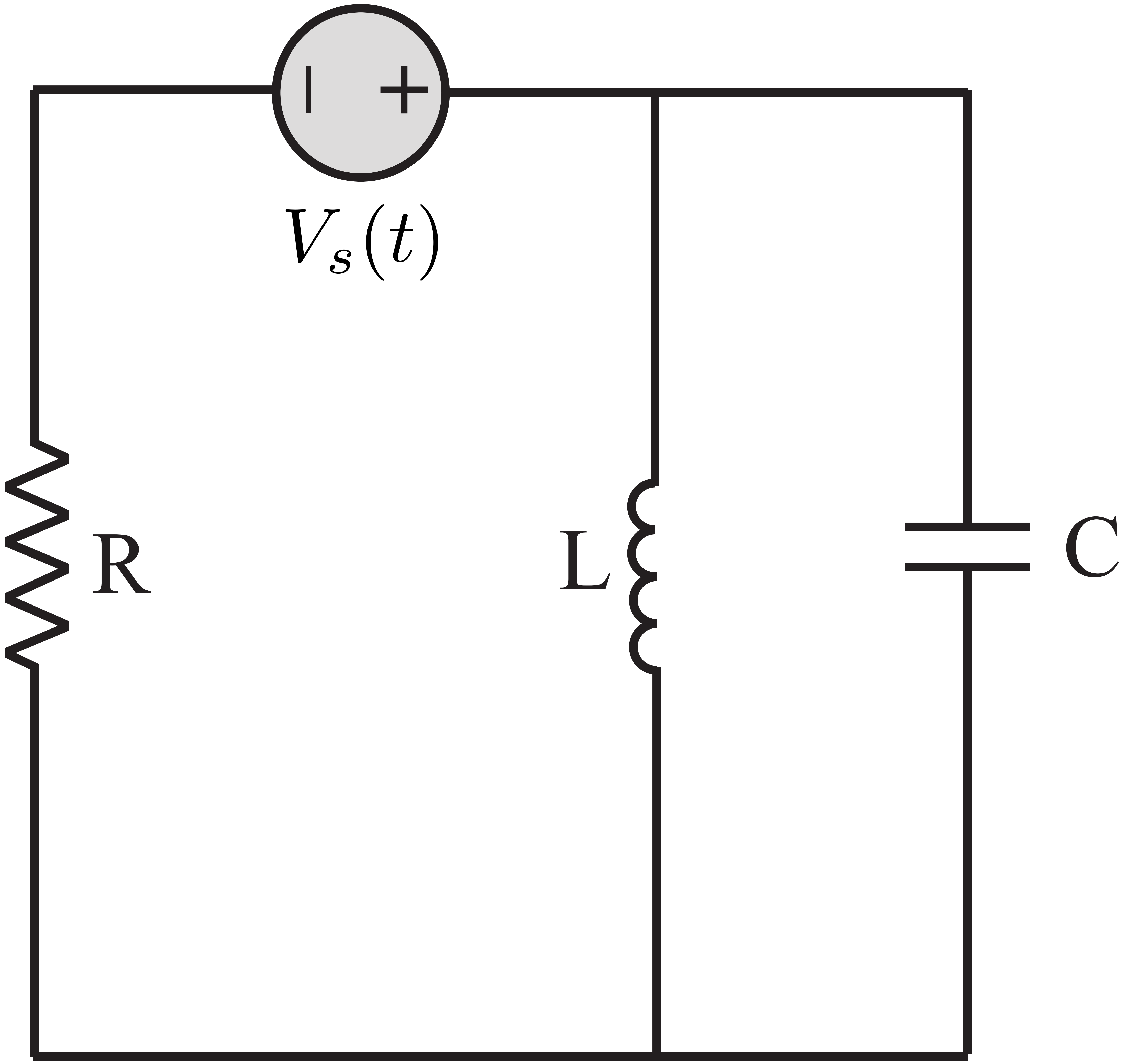
$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = \frac{V_s}{RLC} \quad \longrightarrow \quad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = \frac{F(t)}{\omega_o^2}$$

$$y(t) = y_h(t) + y_p(t)$$

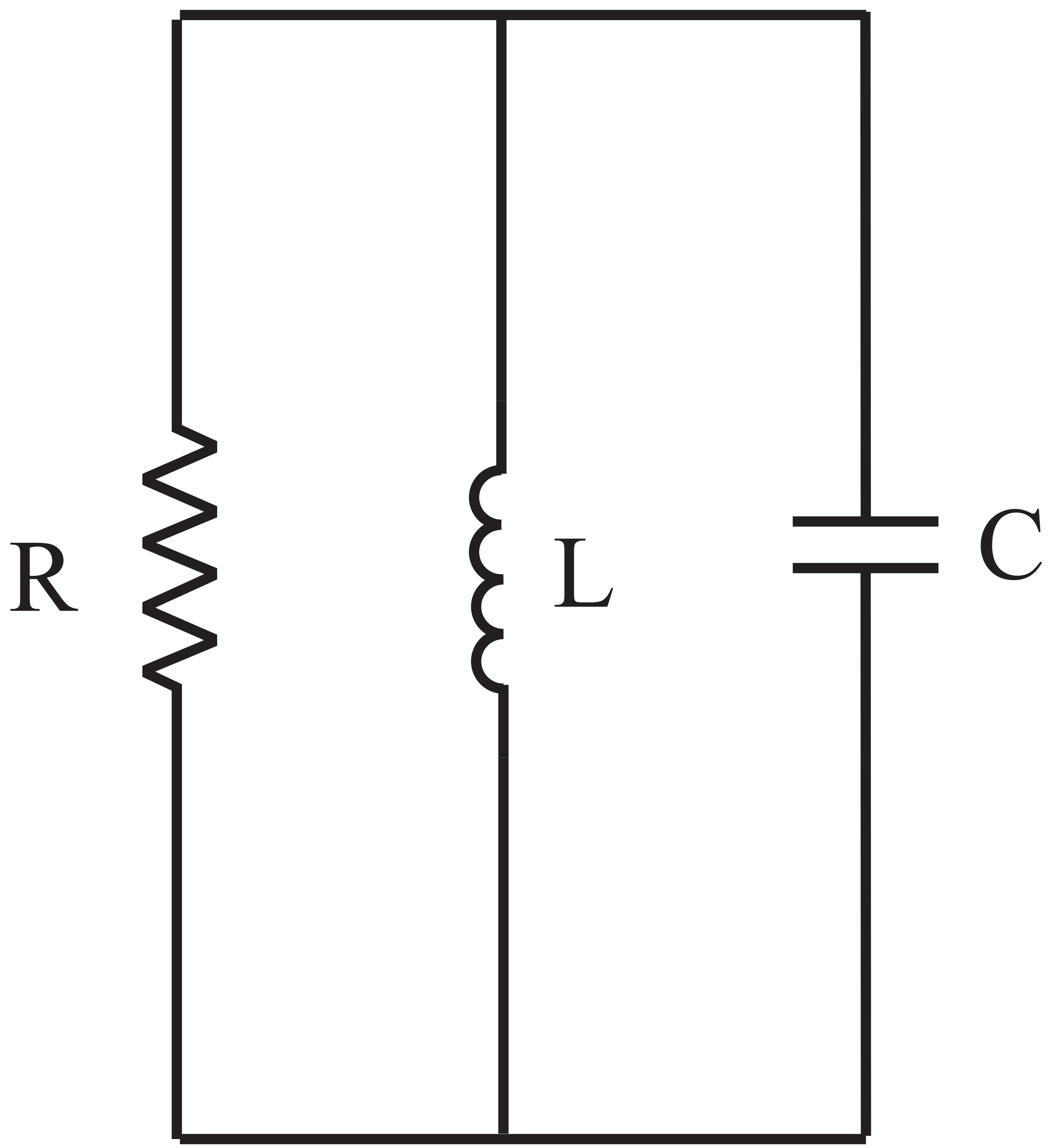
$$Let\ F(t) = A \text{ for } t \geq 0 \quad \Rightarrow \quad y_p(t) = A = y_\infty = y_{ss}$$

$L = 0.4H$, $C = 0.1F$, $R = 1\Omega$

$V_s = 4u(t)V$, $V_C(0) = 0V$, $I_L(0) = 0A$

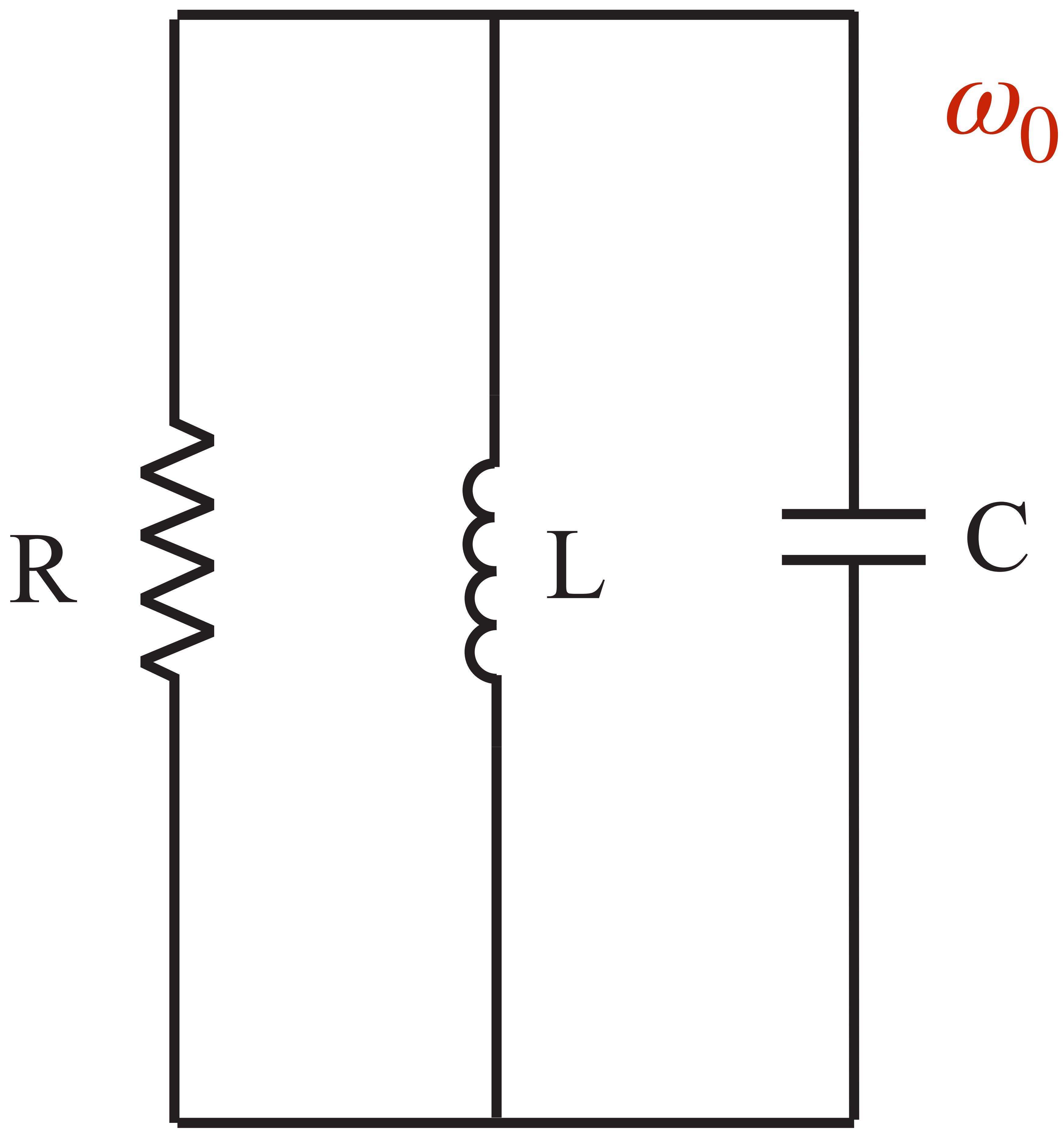


$L = 0.4H$, $C = 0.1F$, $R = 1\Omega$



$L = 0.4H \;,\; C = 0.1F \;,\; R = 1\Omega$

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = 0 \longrightarrow \quad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

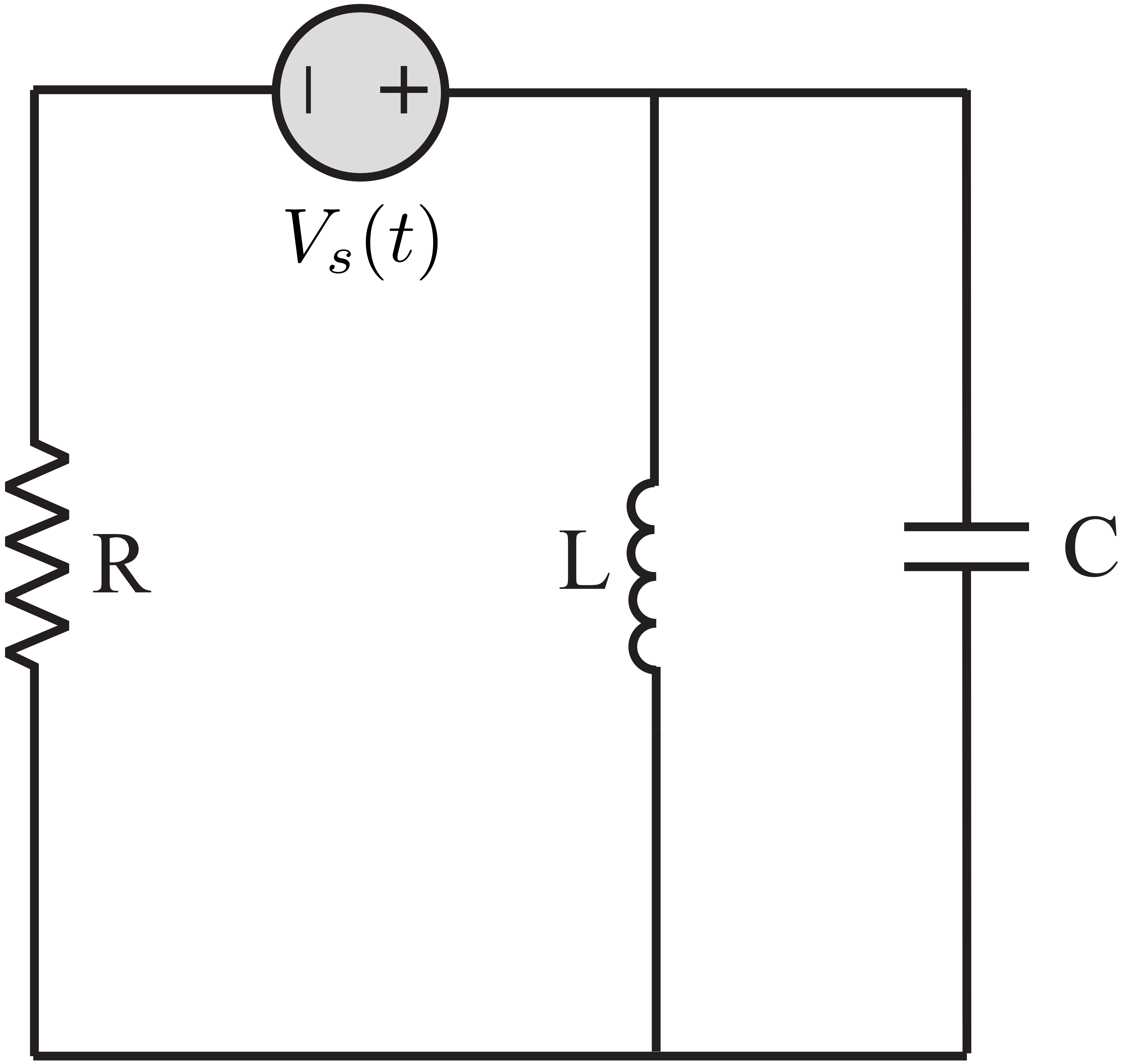


$$\omega_0 = \sqrt{\frac{1}{LC}} = 5 \text{ rad/s} \qquad \alpha = \frac{1}{2RC} = 5 \text{ rad/s}$$

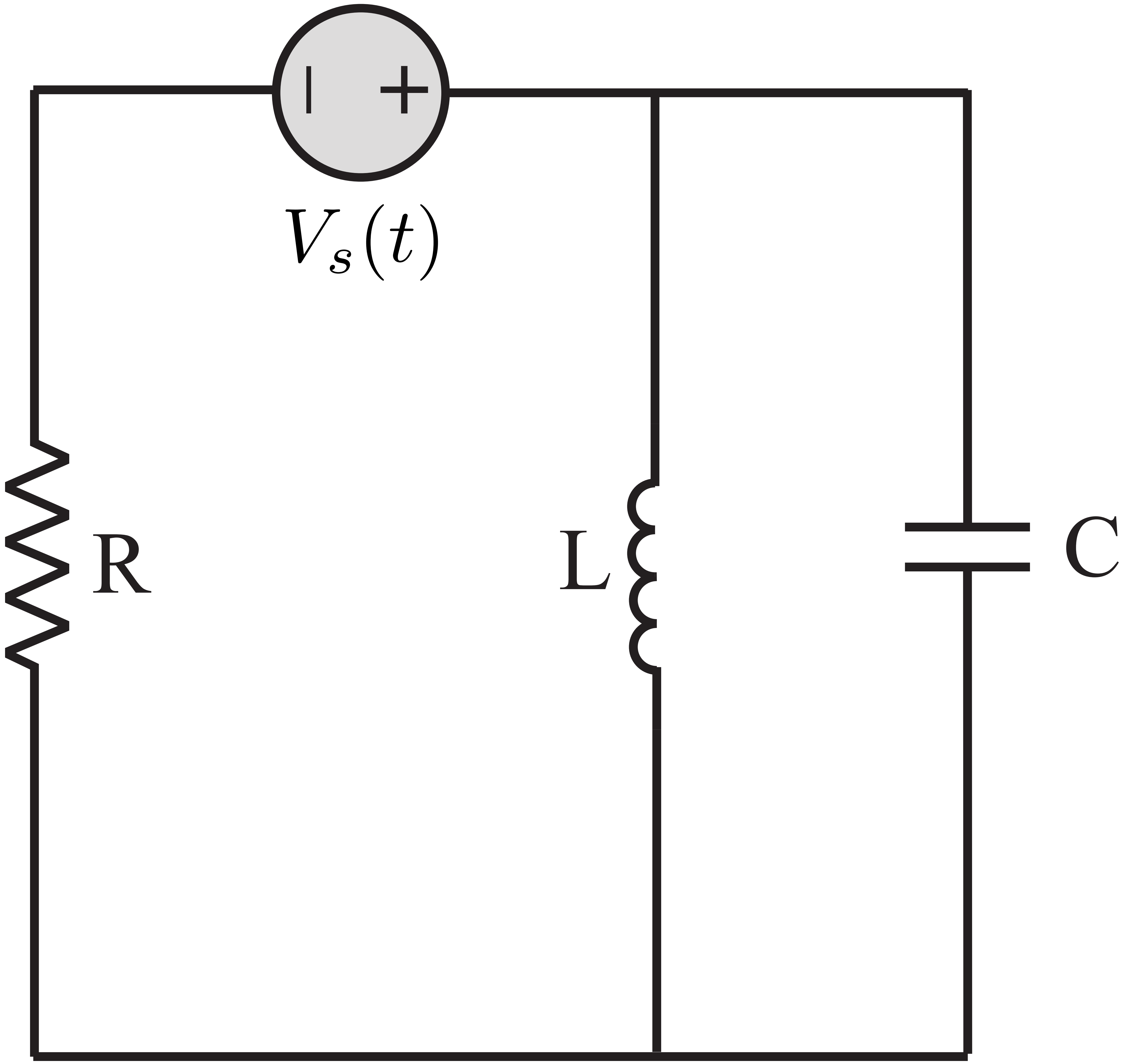
$\alpha = \omega_0 \;\rightarrow$ **Critically-damped**

$$I_h(t) = C_1\, e^{-5t} + C_2\, t\, e^{-5t}$$

$V_s = 4u(t)V$

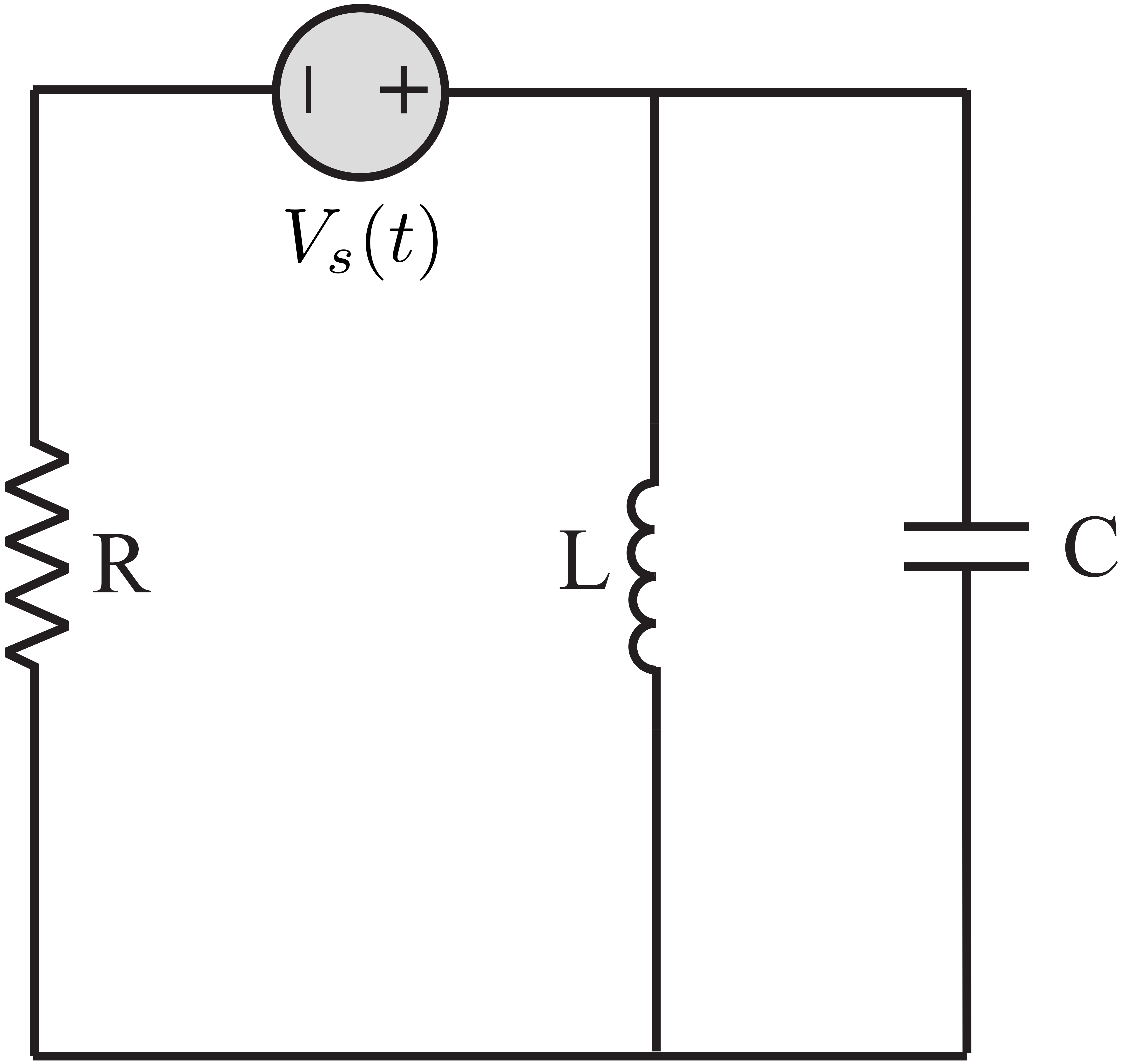


$V_s = 4u(t)V$

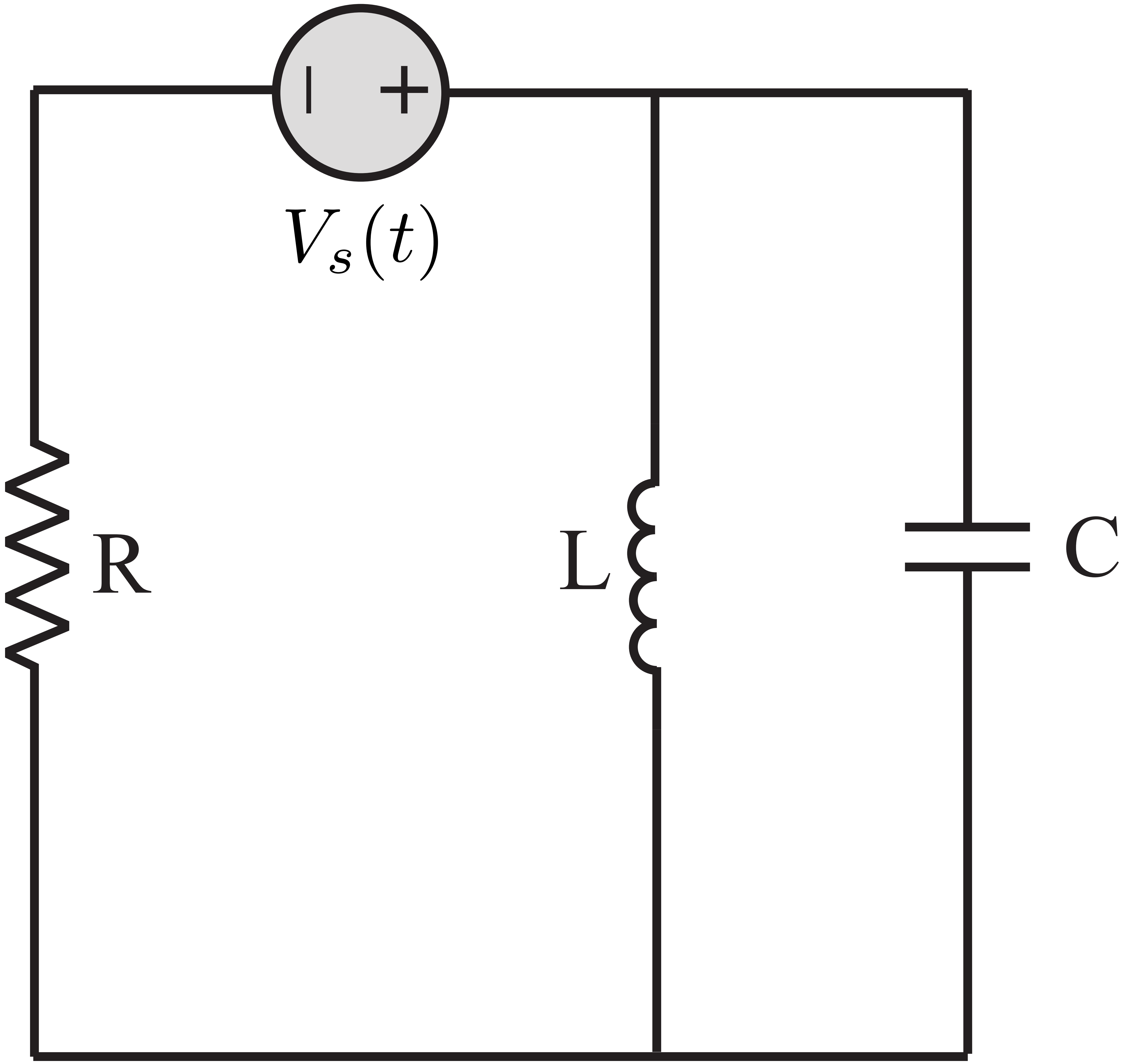


$I_p(t) = I_\infty = 4A$

$V_s = 4u(t)V$, $V_C(0) = 0V$, $I_L(0) = 0A$



$V_s = 4u(t)V$, $V_C(0) = 0V$, $I_L(0) = 0A$



$$I(0) = I_L(0) = 0A$$

$$L\dot{I} = V_C \;\rightarrow\; \dot{I}(0) = 0A$$

$$I(0) = 0A$$

$$\dot{I}(0) = 0A$$

$$I(t) = C_1 e^{-5t} + C_2 t e^{-5t} + 4\ A$$

$$I(0) = 0A$$

$$\dot{I}(0) = 0A$$

$$I(t) = C_1 e^{-5t} + C_2 t e^{-5t} + 4\ A$$

$$I(0) = C_1 + 4\ A = 0 \ \rightarrow \ C_1 = -4$$

$$\dot{I}(0) = -5C_1 + C_2 = 0 \ \rightarrow \ C_2 = -20$$

$$I(t) = \left(4 - 4e^{-5t} - 20\,t\,e^{-5t}\right)\ A$$