SHORT EXAM 3

Variation on Parametres (EXAMPLE I)

$$2y'' - y' - y = 2e^t$$

$$2y_H'' - y_H' - y_H = 0$$

$$2r^2 - r - 1 = 0$$

$$(2r-1)(r-1) = 0$$

$$r = -1/2, 1$$

$$\rightarrow y_H = C_1 e^{-1/2t} + C_2 e^t$$

$$\rightarrow y_p = u_{1(t)} e^{-1/2t} + u_{2(t)} e^t$$

$$\rightarrow y_p' = -\frac{1}{2} u_1 e^{-1/2t} + \underbrace{u_1' e^{-1/2t} + u_2' e^t}_{0} + u_2 e^t$$

$$= -\frac{1}{2} u_1 e^{-1/2t} + u_2 e^t$$

$$\rightarrow y_p'' = -\frac{1}{2} u_1' e^{-1/2t} + \frac{1}{4} u_1 e^{-1/2t} + u_2' e^t + u_2 e^t$$

$$\begin{cases} -u_1' e^{-1/2t} + 2u_2' e^t = 2e^t \\ u_1' e^{-1/2t} + u_2' e^t = 0 \end{cases}$$

$$3u_2' e^t = 2e^t$$

$$u_2' = 2/3$$

$$u_2 = \frac{2}{3} t$$

$$u_1' e^{-1/2t} + \frac{2}{3} e^t = 0$$

$$u_1' e^{-1/2t} = -\frac{2}{3} e^t$$

$$u_1' = -\frac{2}{3} e^{3/2t}$$

$$u_1 = -\frac{4}{9} e^{3/2t}$$

$$y = y_H + y_p$$

$$y = c_1 e^{-1/2t} + c_2 e^{t} - \frac{4}{9} e^{3/2t} \cdot e^{-1/2t} + \frac{2}{3} t e^{t}$$

$$= c_1 e^{-1/2t} + c_2 e^{t} - \frac{4}{9} e^{t} + \frac{2}{3} t e^{t}$$

$$\Rightarrow \boxed{c_1 e^{-1/2t} + c_2 e^{t} + \frac{2}{3} t e^{t}}$$

Variation on Parametres (EXAMPLE II)

$y'' + y = \tan t$ (Şuna tekrar dön, yaz, sadeleştir) ve eşitle

$$y_H'' + y_H = 0$$

$$r^2 + 1 = 0$$

$$r = \mp 1$$

$$y_H = c_1 \cos t + c_2 \sin t$$

$$y_p = u_1 \cos t + u_2 \sin t \quad 0$$

$$y_p' = -u_1 \sin t + u_1' \cos t + u_2' \sin t + u_2 \cos t$$

$$y_p' = -u_1 \sin t + u_2 \cos t$$

$$y_p'' = -u_1' \sin t - u_1 \cos t + u_2' \cos t - u_2 \sin t$$

$$\begin{bmatrix} -u_1' \sin t + u_2' \cos t = \tan t \\ u_1' \cos t + u_2' \sin t = 0 \end{bmatrix}$$

$$u_1' = -u_2' \frac{\sin t}{\cos t}$$

$$u_2' \frac{\sin^2 t}{\cos t} + u_2' \cos t = \tan t$$

$$u_2' \sin^2 t + u_2' \cos^2 t = \tan t \cdot \cos t$$

$$u_2' = \tan t \cdot \cos t = \sin t$$

$$u_2 = -\cos t$$

$$u_1' = -\frac{\sin^2 t}{\cos t} = -\sin t \cdot \tan t$$

$$u_1 = -\ln \left| \sec t + \tan t \right| + \sin t$$

$$y = y_H + y_p$$

$$y = c_1 \cos t + c_2 \sin t + \left[-\ln(\sec t + \tan t) + \sin t \right] \cos t - \cos t \cdot \sin t$$

$$\Rightarrow \boxed{c_1 \cos t + c_2 \sin t - \cos t \cdot \ln \left| \sec t + \tan t \right|}$$

Example 2 ⇒ Wronskin } Çok kolay ama öğrenmeyeceğim.

$y'' + y = \tan t$

High Order Linear Equations

$$y''' + y'' - 6y' + 4y = 0$$

$$r^3 + r^2 - 6r + 4 = 0$$

Check $r = -1$: $10 \neq 0$ ✗

Check $r = 1$: $0 = 0$ ✓

$(r-1)$ is a factor

$(r-1)(r^2+2r-4) = 0$

$(r^3+r^2-6r+4) - (r-1)$

$r = 1$

$(r-1)$'den ← 1

	1	1	−6	4
+		1	2	−4
	1	2	−4	0 ← remainder

$r^2 + 2r - 4 = 0$ → Quadratic E.

$$r = \frac{-(2) \pm \sqrt{2^2 - 4.1.(-4)}}{2.1}$$

$$= \frac{-2 \pm \sqrt{20}}{2} \rightarrow \boxed{r = -1+\sqrt{5},\ -1-\sqrt{5}}$$

$$y(t) = C_1 e^{t} + C_2 e^{(-1+\sqrt{5})t} + C_3 e^{(-1-\sqrt{5})t}$$

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High Order Example 2

$$2y''' - 6y'' - 5y' + 15y = 0$$

$$2m^3 - 6m^2 - 5m + 15 = 0$$

$$2m^2(m-3) - 5(m-3) = 0$$

$$(2m^2 - 5)(m-3) = 0$$

$2m^2 - 5 = 0$ $\qquad$ $m - 3 = 0$

$m = \pm \frac{\sqrt{10}}{2}$ $\qquad$ $m = 3$

$$y = c_1 e^{3x} + c_2 e^{\sqrt{10}x/2} + c_3 e^{-\sqrt{10}x/2}$$

High Order Example 3

$$y''' + 8y'' + 12y' = 0$$

$$m^3 + 8m^2 + 12m = 0$$

$$m(m^2 + 8m + 12) = 0$$

$$m(m+6)(m+2) = 0$$

$m = 0, -6, -2 \Rightarrow y = c_1 e^{0x} + c_2 e^{-6x} + c_3 e^{-2x}$

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High Order Example 4

$$y^4 + y^3 - 7y^2 - y' + 6y = 0$$

$$m^4 + m^3 - 7m^2 - m + 6 = 0 \rightarrow$$ Bu denklemin kökleri bul

Wolpramdom

$$\rightarrow 1, -1, 2, -3$$

$$y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{-3x}$$

High Order Example 5

$$y^4 - 3y^2 - 28y = 0$$

$$m^4 - 3m^2 - 28 = 0 \rightarrow m = \pm\sqrt{7} \quad m = \mp 2i$$

$$\alpha = 0 \quad \beta = 2$$

$$y = c_1 e^{\sqrt{7}x} + c_2 e^{-\sqrt{7}x} + c_3 \cos 2x + c_4 \cdot \sin 2x$$

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Constant Coefficients

$ay'' + by' + cy = 0$ Suggest $\Rightarrow$ $y = e^{rt}$

$y' = r.e^{rt}$

$y'' = r^2.e^{rt}$

$ar^2e^{rt} + bre^{rt} + c.e^{rt} = 0$

$e^{rt}(ar^2 + br + c) = 0$

e^{rt} ↓ never zero

$(ar^2 + br + c)$ must have 0

EXAMPLE 1

$4y'' - 5y' - 6y = 0$

↳ $4r^2 - 5r - 6 = 0$

↳ Roots $r = -3/4$, $r = 2$ $\Rightarrow$ $e^{-3/4t}$, e^{2t}

$y_2 = e^{2t}$

$y_2' = 2e^{2t}$

$y_2'' = 4e^{2t}$

Denklemde yaz

$4.4e^{2t} - 5.2e^{2t} - 6.e^{2t} = 0$

$0 = 0$ Check ✓

$y = C_1e^{-3/4t} + C_2e^{2t}$

Example 2

$$\frac{d^4y}{dx^4} - 81y = 0$$

$D^4 - 81 = 0 \Rightarrow (D^2-9)(D^2+9)$

$D \pm 3 \quad D \pm 3i$

$y = C_1e^{3x} + C_2e^{-3x} + C_3\cos 3x + C_4 \sin 3x$

Eğer karmaşık sayı çıkarsa

Şöyle yaz

→ Repeated real roots

$y = C_1e^{rx} + C_2xe^{rx} + C_3x^2e^{rx} + \dots + C_nx^{n-1}e^{rx}$

↳ $y = C_1 + C_2x + C_3x^2 + C_4x^3 + C_5e^{5x} + C_6xe^{5x}$

→ $y'' - 4y' + 13y = 0$

$D^2 - 4D + 13 = 0$ → Kökleri $2 \pm 3i$ çıkıyor

↳ $y = e^{2x}[C_1\cos 3x + C_2 \sin 3x]$

→ Repeated Complex Roots $(a+bi)^2$

$y = e^{ax}[C_1\cos(bx) + C_2\sin(bx) + xe^{ax}[C_3\cos(bx) + C_4\sin(bx)]$

Undetermined Coefficient

$$y'' - 3y' + 2y = e^t \sin t$$

$$r^2 - 3r + 2 = 0$$

$$r = 1 \;,\; r = 2$$

$$y_H = c_1 e^t + c_2 e^{2t} \qquad y_p = Ae^t \sin t + Be^t \cos t$$

$$y_p' = Ae^t \cos t + Ae^t \sin t - Be^t \sin t + Be^t \cos t$$

$$y_p' = (A-B)e^t \sin t + (A+B)e^t \cos t$$

$$y_p'' = (A-B)e^t \cos t + (A-B)e^t \sin t - (A+B)e^t \sin t + (A+B)\cdot e^t \cdot \cos t$$

$$y_p'' = e^t \sin t\,(-2B) + 2A \cdot e^t \cos t \quad \rightarrow \text{Başa dön yaz}$$

$$-2Be^t \sin t + 2Ae^t \cos t - 3\left[(A-B)e^t \sin t + (A+B)e^t \cos t\right] + 2\cdot(Ae^t \sin t + Be^t \cos t) = e^t \sin t$$

$$(-A+B)e^t \sin t + (-A-B)e^t \cos t = 1e^t \sin t$$

$$\begin{bmatrix} -A+B = 1 \\ -A-B = 0 \end{bmatrix}$$

$$A = -1/2 \qquad B = 1/2$$

$$y_p = -\frac{1}{2}e^t \sin t + \frac{1}{2}e^t \cos t$$

$$y = y_H + y_p \Rightarrow \underbrace{c_1 e^t + c_2 e^t - \frac{1}{2}e^t \sin t + \frac{1}{2}e^t \cos t}_{\text{Answer}}$$