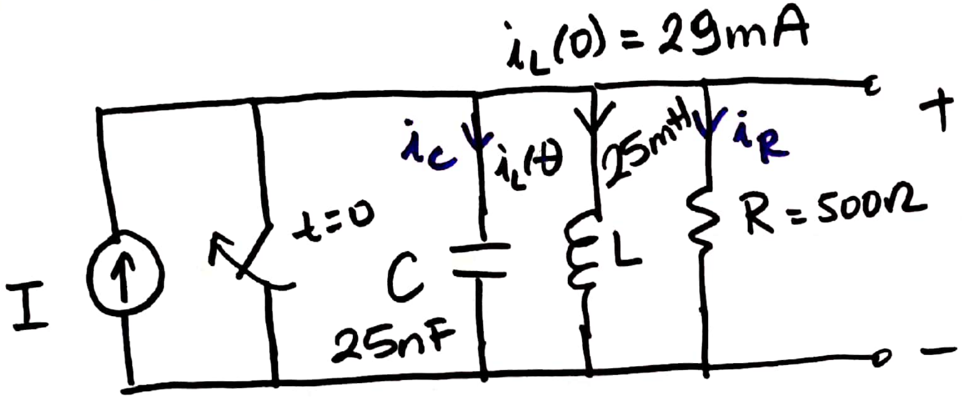
Parallel RLC forced response with nonzero initial conditions



$v_C(0) = 50V$

$I = 24mA$

$$I = i_C + i_L + i_R$$

$$C\frac{dv_C}{dt} + i_L + \frac{v_C}{R} = I$$

$$v_C = v_L = L\frac{di_L}{dt} \qquad \frac{dv_C}{dt} = L\frac{d^2 i_L}{dt^2}$$

$$CL\frac{d^2 i_L}{dt^2} + i_L + \frac{L}{R}\frac{di_L}{dt} = I$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}$$

$$i_L(t) = I_f + I_n$$

Forced (I_f): Steady-state constant value

natural response (I_n): Depending on α, ω_0 one of the 3 solutions

There are two coefficients to be determined by using these initial conditions.

$$\left.\begin{array}{l} i_L(0) \\ \frac{di_L}{dt}(0) = \frac{v_C(0)}{L} \end{array}\right\}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 500 \times 25 \times 10^{-9}} = \frac{10^6}{25} = 4 \times 10^4 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25 \times 10^{-3} \times 25 \times 10^{-9}}} = \frac{1}{25 \times 10^{-6}} = 4 \times 10^4 \text{ rad/s}$$

$\alpha = \omega_0 \Rightarrow$ critically damped

$$i_L(t) = 24\times10^{-3} + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

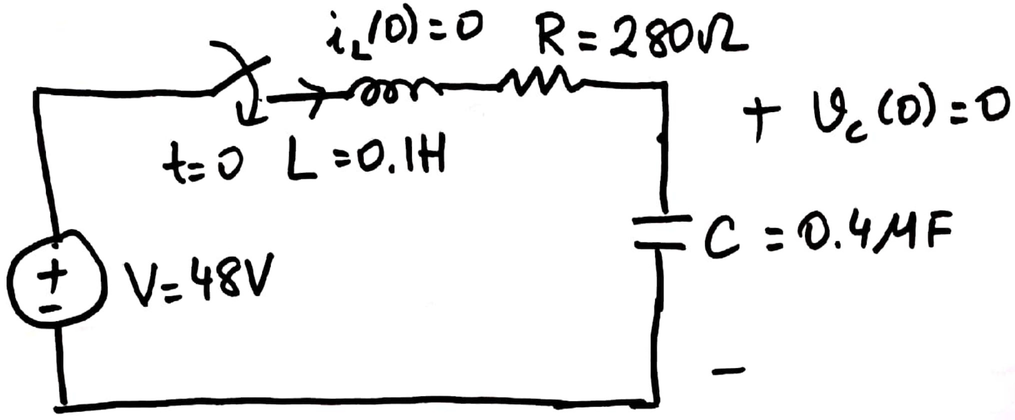
$$i_L(0) = 24\times10^{-3} + D_2 = 29\,mA = 29\times10^{-3}\,A \Rightarrow D_2 = 5\times10^{-3}\,A$$

$$\left.\frac{di_L}{dt}\right|_{t=0} = D_1\left(\underbrace{e^{-\alpha t}}_{1} + \cancel{t(-\alpha)\underbrace{e^{-\alpha t}}_{0}}\right) + D_2(-\alpha)\underbrace{e^{-\alpha t}}_{1} = D_1 - \alpha D_2$$

$$D_1 - 4\times10^{4}\times5\times10^{-3} = \frac{v_c(0)}{L} = \frac{50}{25\times10^{-3}} \Rightarrow D_1 = 2200\,A/s$$

$$i_L(t) = 24 + 2.2\,t e^{-4000t} + 5e^{-4000t}\ mA$$

Series RLC forced response



$$V = v_L + v_R + v_C$$

$$V = L\frac{di_L}{dt} + i_L R + v_c$$

$$i_L = i_C = C\frac{dv_c}{dt}$$

$$V = LC\frac{d^2 v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{V}{LC}$$

$v_c(t) = V_f + V_n \rightarrow$ One of 3 solutions

V_f: steady-state final value

$$\alpha = \frac{R}{2L} = \frac{280}{2\times0.1} = 1400 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\times0.4\times10^{-6}}} = 5000 \text{ rad/s}$$

$v_c(0)$

$$\frac{dv_c(0)}{dt} = \frac{i_L(0)}{C}$$

$\omega_0 > \alpha \Rightarrow$ underdamped

$$v_c(t) = 48V + B_1 e^{-\alpha t} \cos\omega_d t + B_2 e^{-\alpha t} \sin\omega_d t$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4800$$

$$v_c(0) = 48 + B_1 = 0 \Rightarrow B_1 = -48V$$

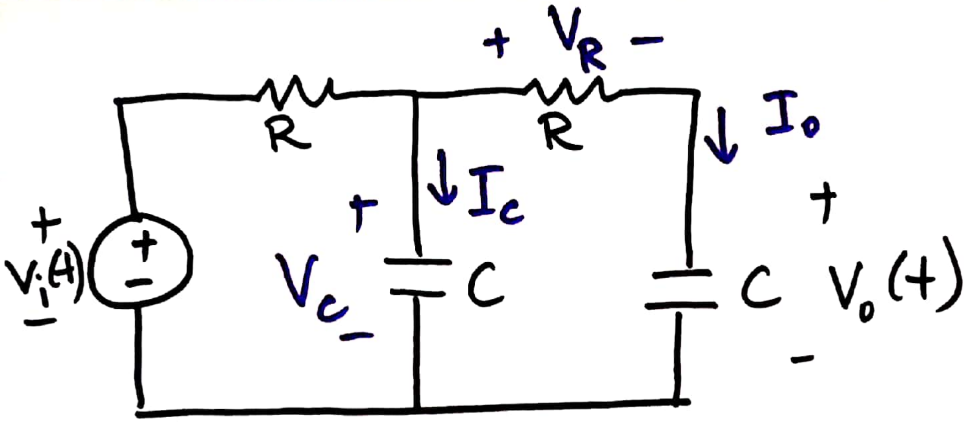
$$\frac{dv_c(0)}{dt} = \frac{i_L(0)}{C} = 0$$

$$\left.\frac{dv_c(t)}{dt}\right|_{t=0} = B_1\left(\overbrace{-\alpha e^{-\alpha t} \cos\omega_d t}^{1} + \cancel{e^{-\alpha t}\omega_d(-\sin\omega_d t)}\right) + B_2\left(\cancel{e^{-\alpha t}(-\alpha)\sin\omega_d t} + e^{-\alpha t}\omega_d \cos\omega_d t\right)^{1}$$

$$= -\alpha B_1 + \omega_d B_2 = 0$$

$$B_2 = \frac{1400(-48)}{4800} = -14V$$

$$v_c(t) = 48 + (-48)e^{-1400t} \cos 4800t + (-14)e^{-1400t} \sin 4800t$$



Find DE in terms of $V_o(t)$

$$-V_c + V_R + V_o = 0$$

$$V_c = RI_o + V_o = RC\frac{dV_o}{dt} + V_o$$

$$I_c = C\frac{dV_c}{dt} = RC^2\frac{d^2V_o}{dt^2} + C\frac{dV_o}{dt}$$

$$-V_i + R(I_c + I_o) + V_c = 0$$

$$R\left(RC^2\frac{d^2V_o}{dt^2} + C\frac{dV_o}{dt} + C\frac{dV_o}{dt}\right) + RC\frac{dV_o}{dt} + V_o = V_i$$

$$R^2C^2\frac{d^2V_o}{dt^2} + 3RC\frac{dV_o}{dt} + V_o = V_i$$

$$\frac{d^2V_o}{dt^2} + \frac{3}{RC}\frac{dV_o}{dt} + \frac{V_o}{R^2C^2} = \frac{V_i}{R^2C^2}$$