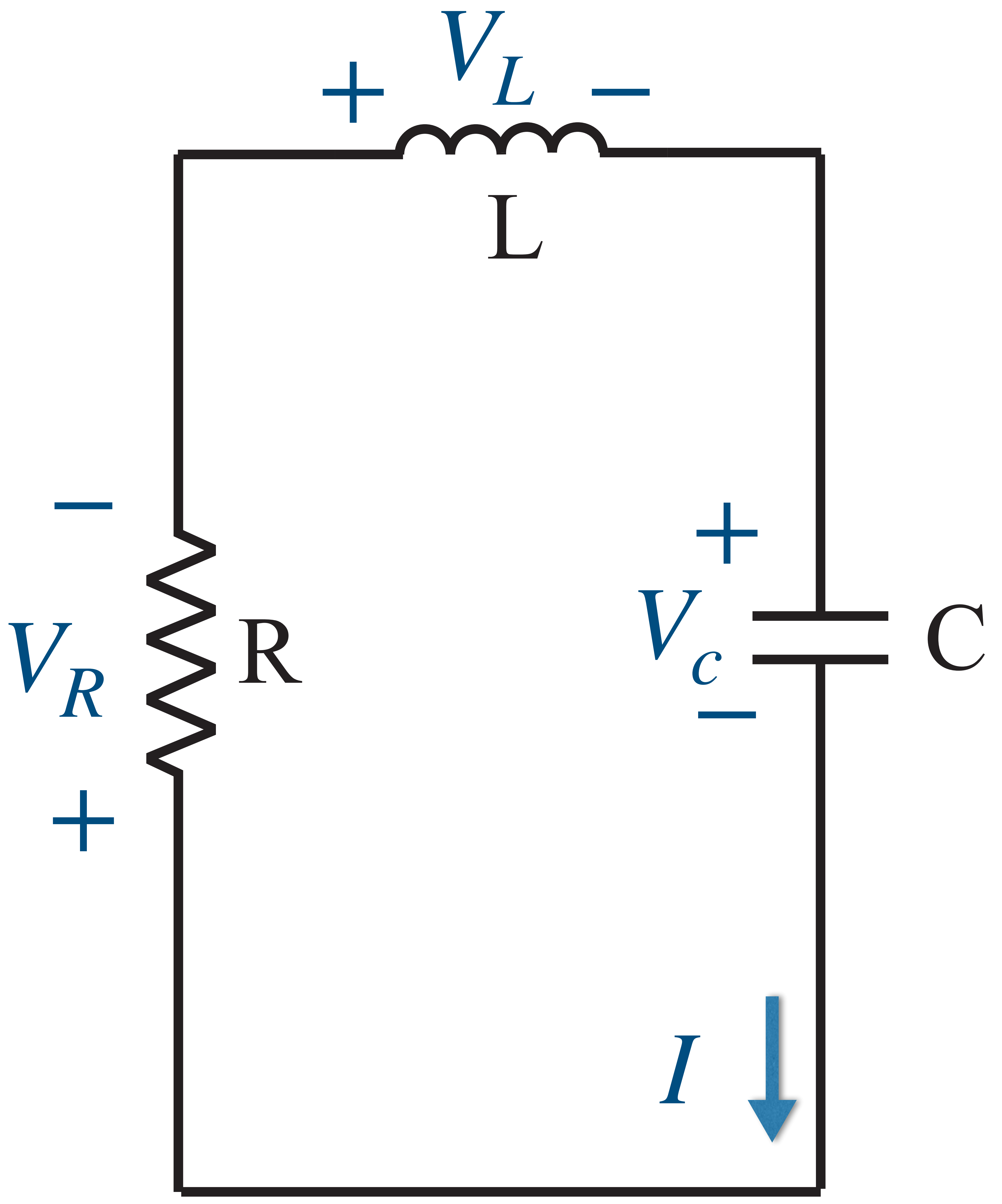
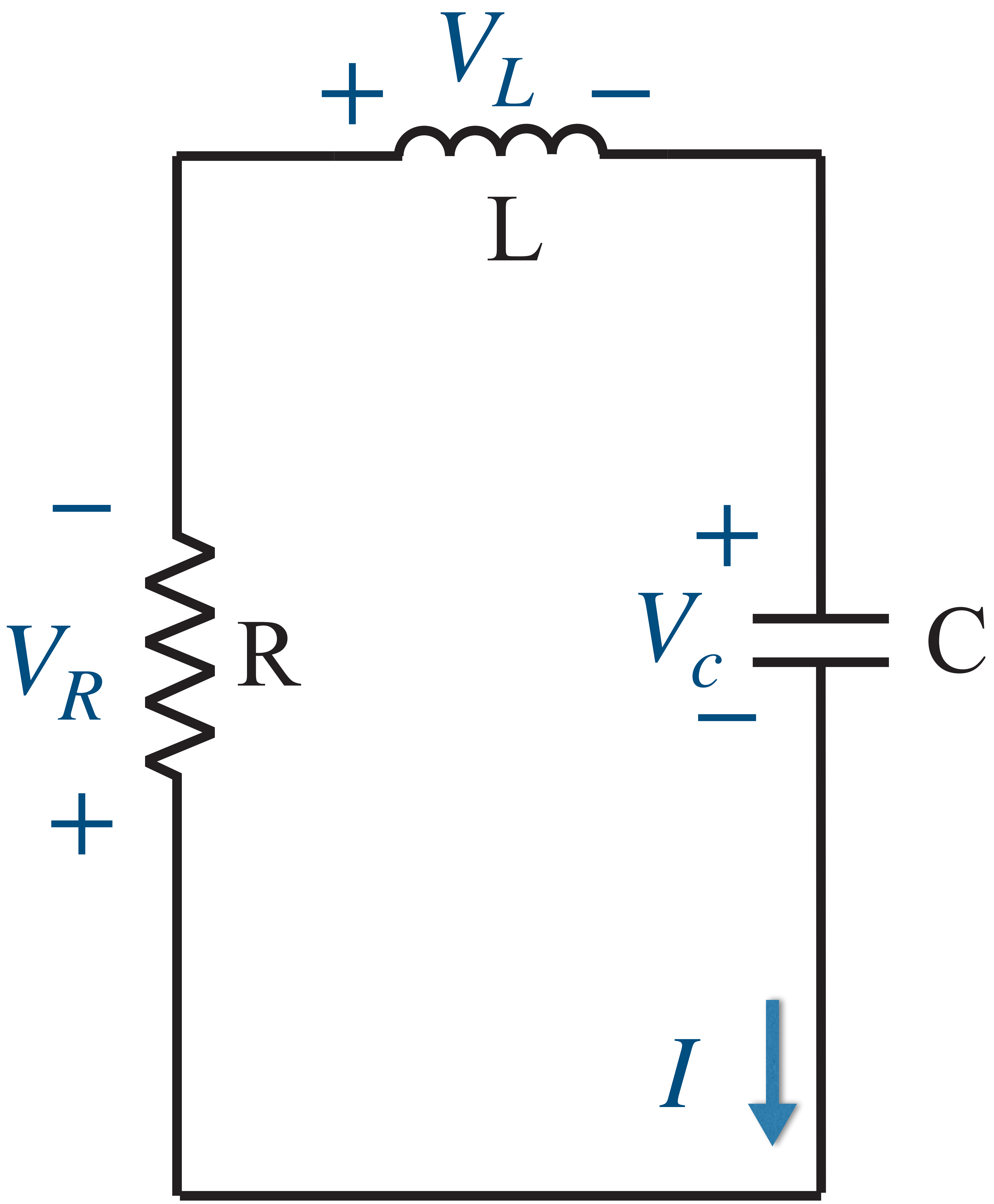
Series RLC



Series RLC

$$I_c = I_R = I_L = I$$



$$V_R + V_L + V_C = 0$$

$$R \cdot I + L \cdot \dot{I} + V = 0$$

$$LC\ddot{V} + RC\,\dot{V} + V = 0$$

$$\ddot{V} + \frac{R}{L}\dot{V} + \frac{1}{LC}V = 0$$

$$\ddot{V} + \frac{R}{L}\dot{V} + \frac{1}{LC}V = 0 \qquad \longrightarrow \qquad \ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

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$$V \rightarrow y$$

Natural (Resonant) Frequency: $\omega_0 = \sqrt{\dfrac{1}{LC}}$

Attenuation (Neper Frequency): $\alpha = \dfrac{R}{2L}$

$L = 0.1H$, $C = 0.1F$, $R = 2/5\Omega$

$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

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$$\alpha = \frac{R}{2L}$$

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$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$$

$$\omega_0 = 10\,\text{rad/s}$$

$$\alpha < \omega_0 \quad \rightarrow$$ **Under-damped**

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$\alpha = 2\,\text{rad/s}$$

$$\omega_d = \sqrt{10^2 - 2^2} \approx 9.8\,\text{rad/s}$$

$$\alpha = \frac{R}{2L}$$

$$V(t) = C_1\, e^{-2t} \cos(9.8\, t) + C_2\, e^{-2t} \cos(9.8\, t)$$

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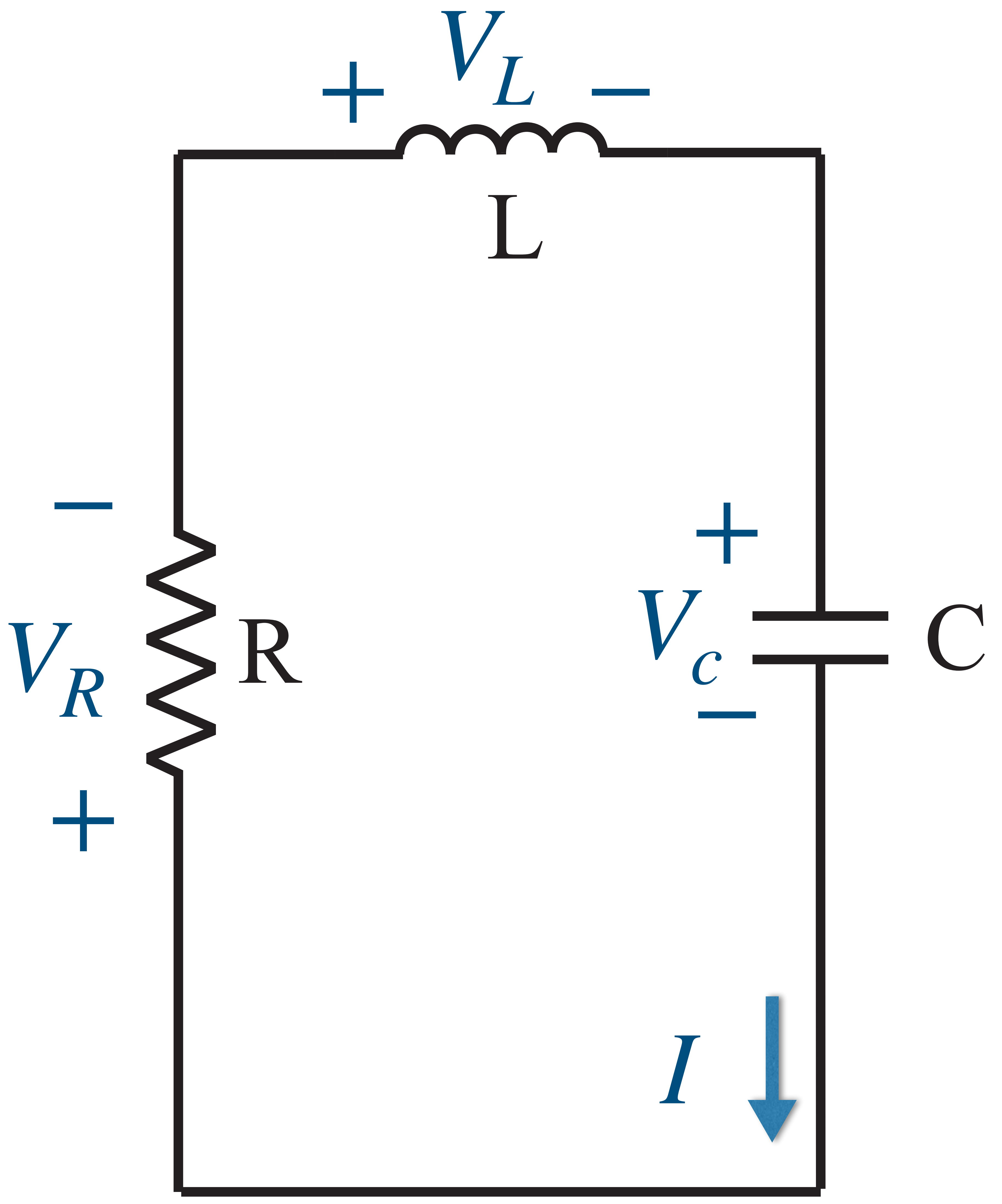
We need $V(0)$ and $\dot{V}(0)$ to compute $V(t)$

$Let\ V_c(0) = 10V\ \&\ I_L(0) = 0A$



$$V(0) = 10 \;\&\; \dot{V}(0) = 0$$

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$$V(0) = C_1 = 10$$

$$\dot{V}(0) = -2C_1 + 9.8C_2 = 0 \;\rightarrow C_2 \approx 2$$

$$V(t) = 10\,e^{-2t} \cos(9.8\,t) + 2\,e^{-2t} \sin(9.8\,t)\;\; V$$