

#### Math 213

1st Midterm Exam, March 26th, 2022, 11:00-12:00

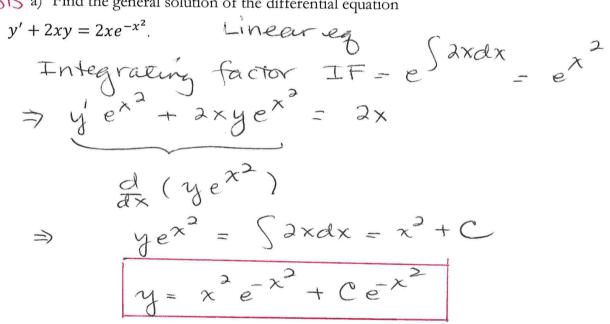
#### **IMPORTANT**

1. Write down your name and surname on top of each page of your work. 2. Show all your work. Correct answers without justification will not get credit. 3. Unless otherwise stated, you may use any method from classwork to solve the problems. 4. Calculators are not allowed. 5. All electronic devices are to be kept shut and out of sight. 6. All cellphones are to be left on the instructor's desk before the exam begins.

Q1	Q2	Q3	Q4	TOT
25 pts	25 pts	25 pts	25 pts	100 pts

Q1.

15 pts a) Find the general solution of the differential equation



1 (O ) [ b) Find a constant-coefficient homogeneous linear differential equation for which

$$y(x) = 6 + 3xe^{x} - 5\cos 3x$$
is a solution.
$$D (D-1)^{2} (D^{2} + 9) y = 0$$

Q2. Find the general solution of the differential equation

$$(D^3 + 7D^2 + 19D + 13)y = 0$$

where D is the differential operator  $\frac{d}{dx}$ .

$$m^{3} + 7m^{2} + 19m + 13 \qquad | \frac{m+1}{m^{2} + 6m + 13} |$$

$$-m^{3} + m^{2} \qquad | \frac{m}{m^{2} + 6m + 13} |$$

$$-6m^{3} + 6m \qquad | \frac{13m + 13}{m^{2} + 6m + 13} |$$

$$.: The auxiliary eq is \quad (m+1)(m^{2} + 6m + 13) = 0$$

$$(m+1)(m^{2} + 6m + 13) = 0$$

$$= -6 \pm \sqrt{36 - 4 \cdot 13} = -6 \pm \sqrt{36 - 52}$$

$$= -6 \pm \sqrt{-16} = -3 \pm 2i$$

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Froots 
$$m = \frac{-6 \pm \sqrt{36 - 4.13}}{2} = \frac{-6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{-6 \pm \sqrt{-16}}{2} = -3 \pm 2.i$$

5 ph 
$$y = C_1 e^{-X} + C_2 e^{-3x} \cos 2x + C_3 e^{-3x} \sin 2x$$

Q3. Find a particular solution of the differential equation

When a particular solution of the differential equation
$$y'' + 4y = \sin 2x. \qquad \text{Annihilator method:}$$

$$y = C_1 \sin 2x + C_2 \cos 3x + C_3 \times \sin 2x + C_4 \times \cos 3x$$

$$\text{he cause these functions cure in the solution apace of } (b^2 + 4) y = 0$$

$$\text{if } y_p = A \times \sin 2x + B \times \cos 3x$$

$$\text{App'} = A \times \sin 2x + B \times \cos 3x + B \cos 2x - 2 \cos 2x$$

$$\text{App'} = A \times \cos 2x - A \times \sin 2x - B \cos 2x - A \times \cos 2x$$

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# Q4. Given the initial value problem

$$y' - 2xy = 0$$
,  $y(0) = 1$ 

- (a) Find the recursion relation for a power series expansion about x = 0 for the solution of this problem.
- (b) Solve the recursion relation and find the power series solution to the problem.

(a) 
$$y = \sum_{n=0}^{\infty} \alpha_{n} x^{n} = \alpha_{0} + \alpha_{1} x + \dots \Rightarrow y(0) = \alpha_{0} = 1$$
 $y' = \sum_{n=1}^{\infty} n \alpha_{n} x^{n-1}$ 
 $y' = \sum_{n=1}^{\infty} n \alpha_{n} x^{n-1} - 2 \sum_{n=1}^{\infty} \alpha_{n} x^{n+1} = 0$ 
 $y' = \sum_{n=1}^{\infty} n \alpha_{n} x^{n-1} - 2 \sum_{n=1}^{\infty} \alpha_{n} x^{n} = 0$ 
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$$Q_{2m} = \frac{1}{m!}$$

$$\Rightarrow y = 1 + \sum_{i=1}^{\infty} \frac{x^{2m}}{m!}$$
or
$$y = \sum_{i=1}^{\infty} \frac{x^{2m}}{m!}$$

or 
$$y = \frac{\infty}{2} \frac{\chi^2 m}{m!}$$