University School of Automation and Robotics
GURU GOBIND SINGH INDRAPRASTHA UNIVERSITY
East Delhi Campus, Surajmal Vihar
Delhi - 110092

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates
- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.
- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.
- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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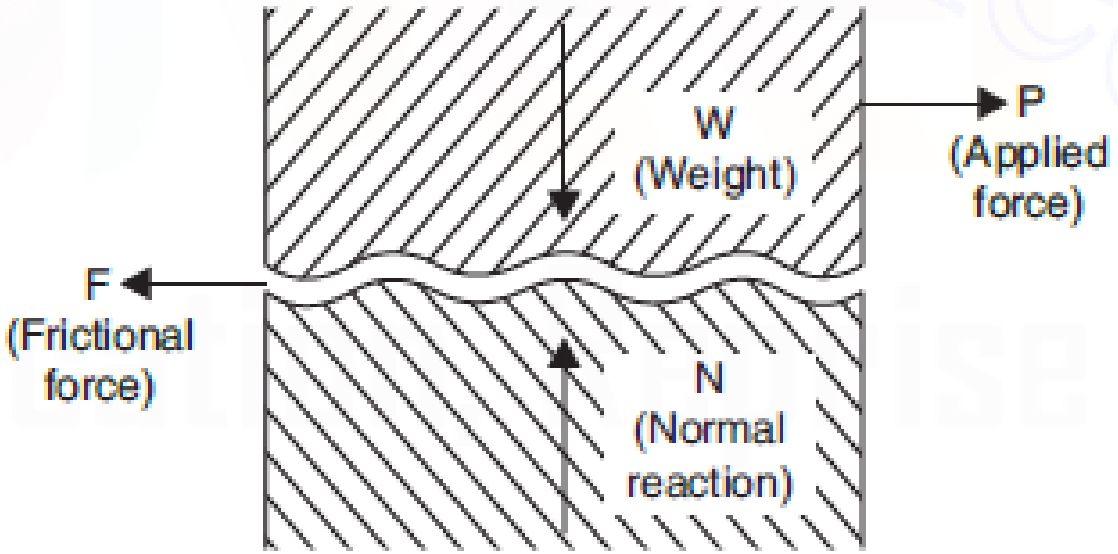
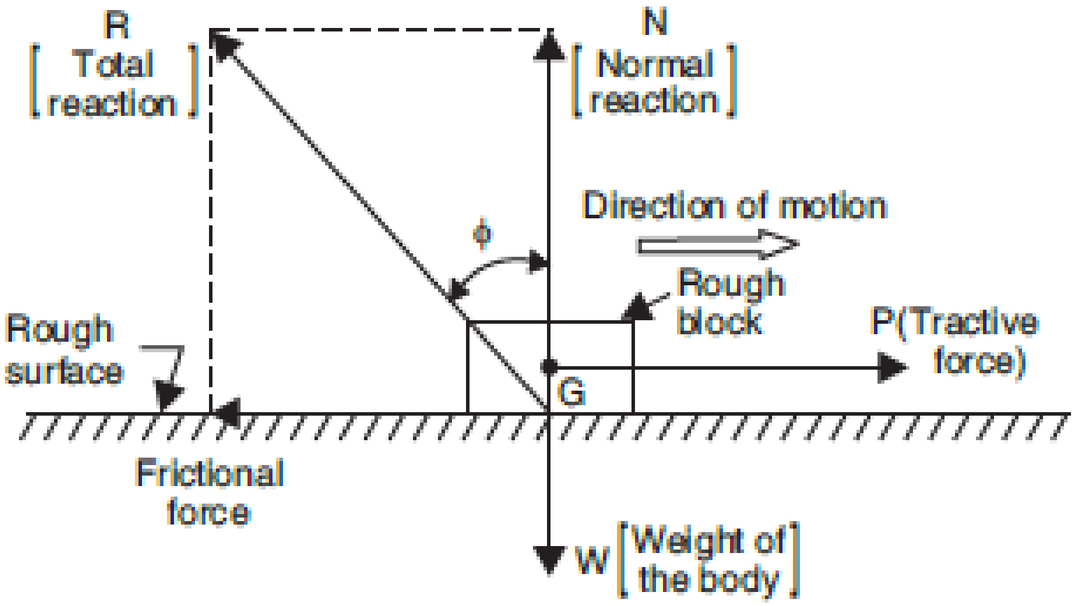
CONCEPT OF FRICTION

- It has been observed that surfaces of bodies, however smooth they may be, are not perfect and possess some irregularities and roughness.
- Therefore, if a block of one substance is placed over the level surface of another, a certain degree of interlocking of minutely projecting particles takes place.
- This interlocking properties of projecting particles oppose any tendency of the body to move.
- The resisting force acts in the direction opposite to that of the motion of the upper block and is called *friction.*



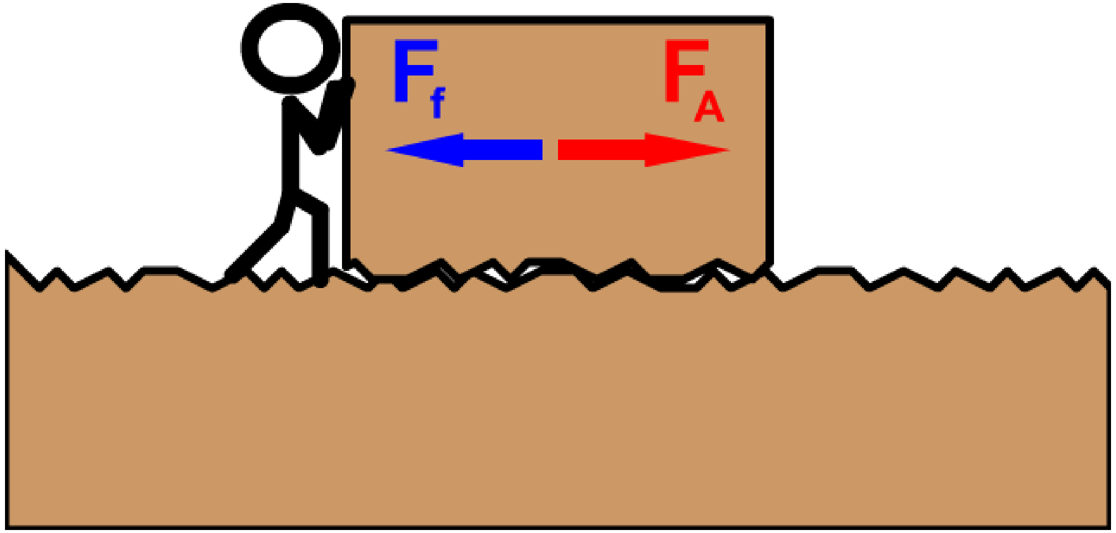
CONCEPT OF FRICTION

- Thus, *wherever there is a relative motion between two parts, a force of friction comes into play, and hence to overcome friction some energy is wasted.*
- Hence, *force of friction or frictional force* may be defined as the *opposing force which is called into play in between the surfaces of contact of two bodies, when one body moves over the* surface of another body.



CONCEPT OF FRICTION

- Friction is a force distribution at the surface of contact which acts tangential to the surface of contact and opposite to the direction of motion.
- Its magnitude depends on the roughness of surfaces.
- In engineering friction is desirable and undesirable.
- There are appliances and devices known as *friction* devices such as belts and ropes, friction clutches, jib and cotter joints, brakes, nuts and bolts, in which friction is *desirable* and efforts are made to maximise it.
- On the contrary, the friction is very *undesirable* in *moving* parts of *machines*. It causes the loss of energy which manifests itself in the forms of heat energy. Due to friction a more force is required to cause motion of the parts.
- To improve the efficiency of the machines the friction force is reduced to the minimum possible by *lubrication.*



CHARACTERISTICS OF FRICTIONAL FORCE

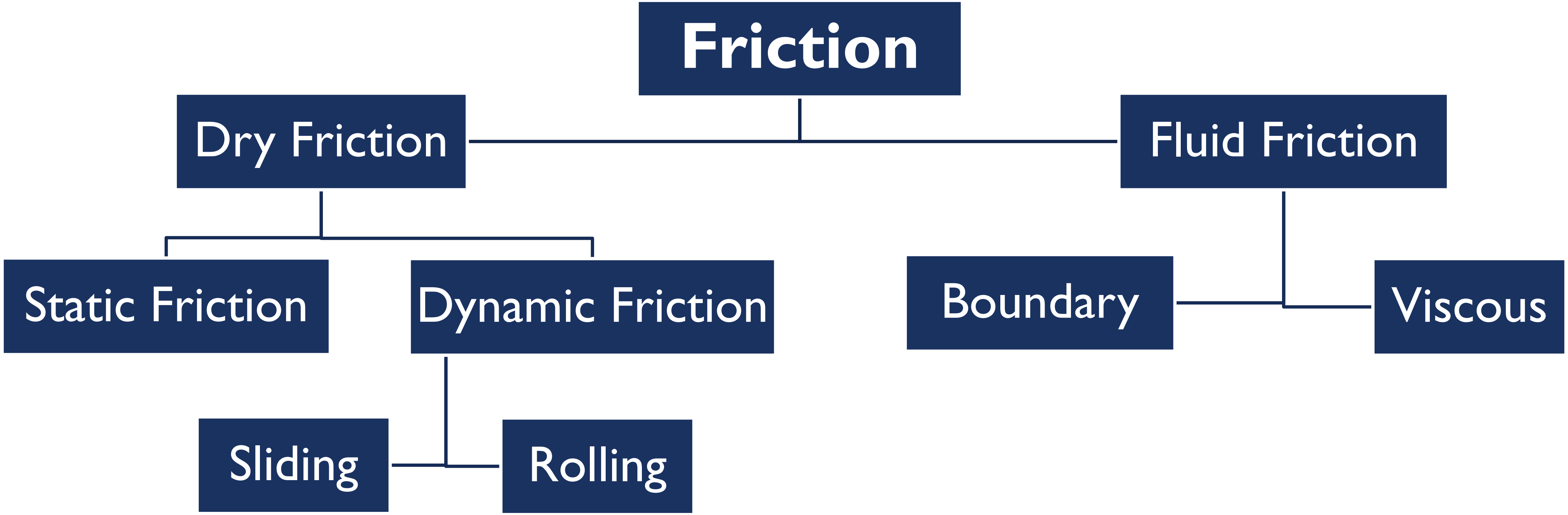
- The force of friction or frictional force entails the following characteristics :

i. It is *self-adjusting*. As tractive force P increases, the frictional force F also increases, and at any instant only as much frictional force comes into play as is necessary to prevent the motion.

ii. It always acts in a direction opposite to the motion (*i.e.*, always opposes the tractive force).

iii. It is a *passive force* (since it exists only if the tractive force P exists).

TYPES OF FRICTION



TYPES OF FRICTION

1. **Friction in unlubricated surfaces.** The *friction that exists between two unlubricated surfaces is called solid friction or dry friction.*

- The friction between dry surfaces in contact is called dry friction. It is also called coulomb friction.
- The major cause of such friction is believed to be the interlocking of microscopic protuberances (i.e., minute projections on the surfaces) which oppose the relative motion.
- Such protuberances are always present howsoever smooth the surfaces may be.
- It may be of the following two types :

(*i*) Sliding friction: The friction that exists when one surface *slides* over the other is called *sliding friction.*

(*ii*) Rolling friction: The friction that exists between two surfaces separated by *balls* or rollers, is called the *rolling friction. Rolling friction* is always *less than the sliding friction*

TYPES OF FRICTION: FRICTION IN LUBRICATED SURFACES.

- When lubricant (*i.e.* oil or grease) is applied between two surfaces in contact, then the friction may be classified into the following two types depending upon the thickness of layer of a lubricant.

1. ***Boundary friction*** (***or greasy friction or non-viscous friction***). It is the friction, experienced between the rubbing surfaces, when the surfaces have a very thin layer of lubricant.
   - The thickness of this very thin layer is of the molecular dimension.
   - In this type of friction, a thin layer of lubricant forms a bond between the two rubbing surfaces.
   - The lubricant is absorbed on the surfaces and forms a thin film.
   - This thin film of the lubricant results in less friction between them.
   - The boundary friction follows the laws of solid friction.

TYPES OF FRICTION: FRICTION IN LUBRICATED SURFACES.

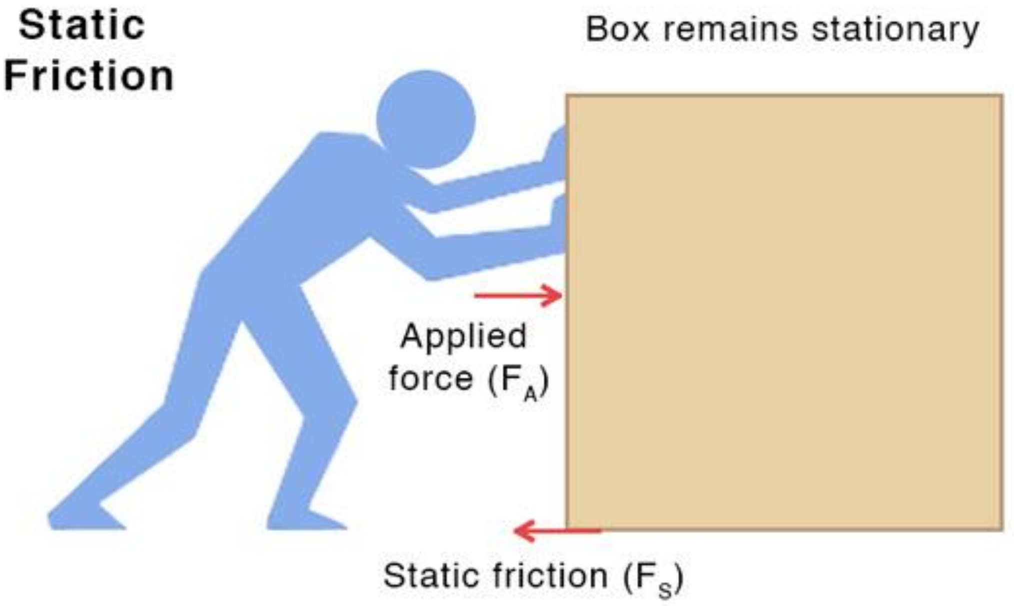
- When lubricant (*i.e.* oil or grease) is applied between two surfaces in contact, then the friction may be classified into the following two types depending upon the thickness of layer of a lubricant.

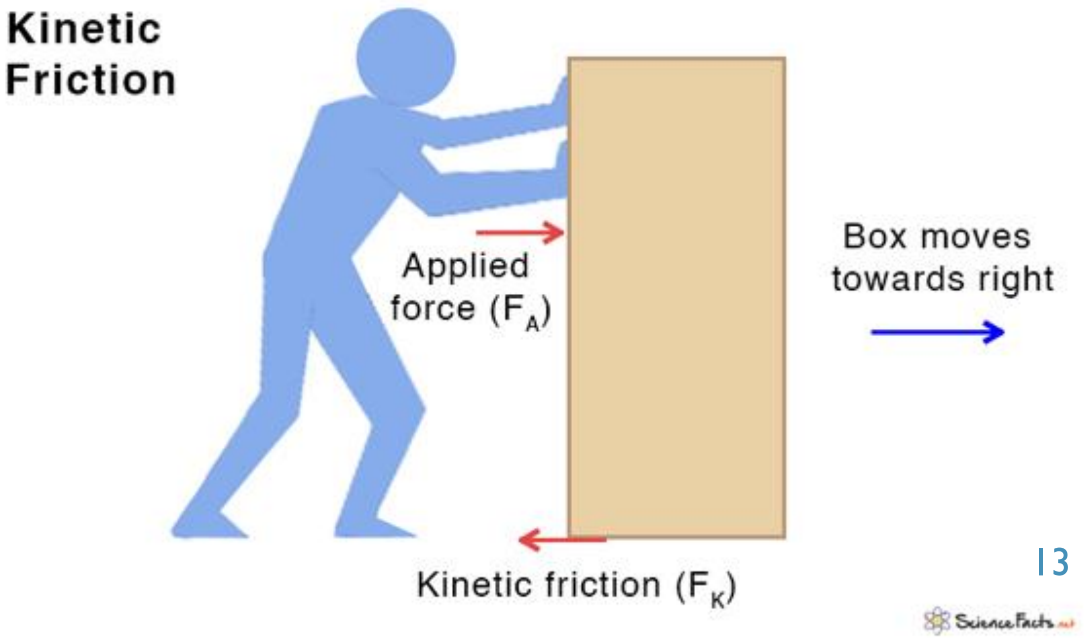
2. ***Fluid friction (or film friction or viscous friction)***. It is the friction, experienced between the rubbing surfaces, when the surfaces have a thick layer of the lubricant.
   - In this case, the actual surfaces do not come in contact and thus do not rub against each other.
   - It is thus obvious that fluid friction is not due to the surfaces in contact but it is due to the ***viscosity*** and ***oiliness*** of the lubricant.

STATIC AND DYNAMIC FRICTION

- **Static friction.** *The static friction is the friction offered by the surfaces subjected to external forces until there is no motion between them.*

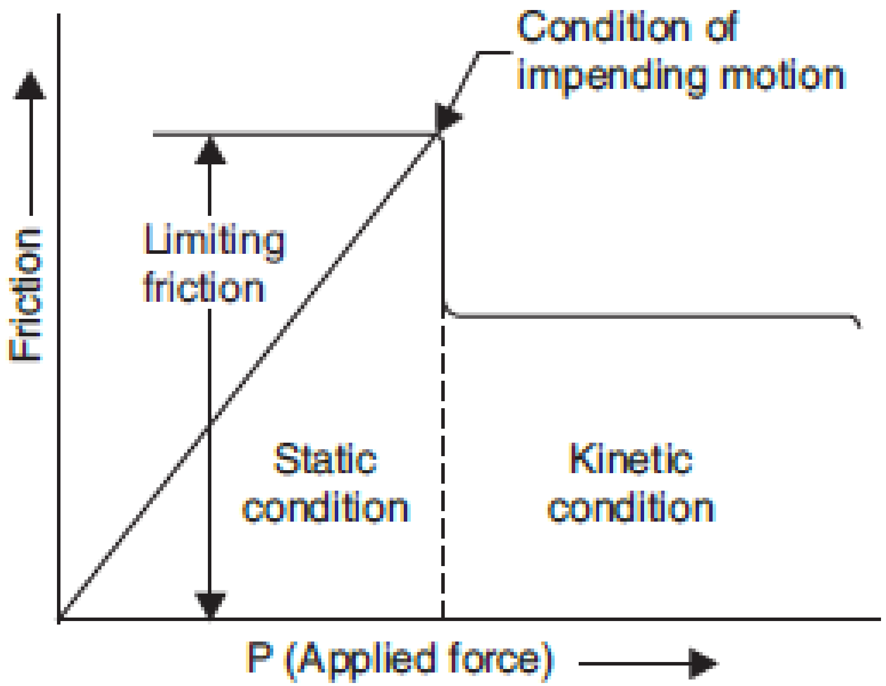
- **Dynamic friction.** It is the *friction experienced by a body* when it is in motion. It is also known *as kinetic friction and is always less than static friction* (the kinetic friction is about 40 to 75 per cent of the limiting static friction).





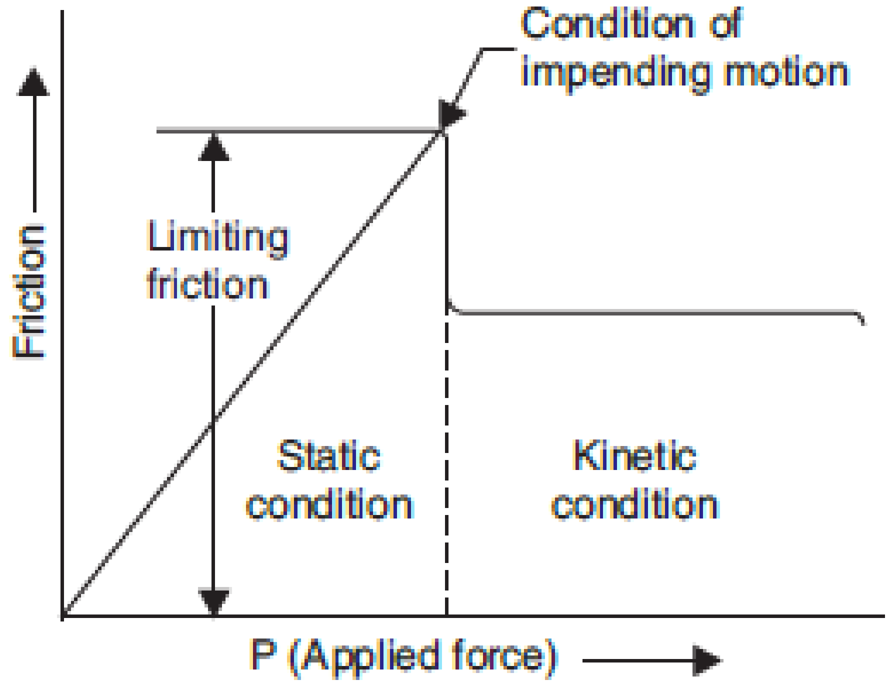
LIMITING FRICTION

- Figure shows a graph between the applied force and the friction.
- During static condition as the applied force is increased from zero value the frictional force increases in direct proportion to the applied force.
- A certain stage is reached when the applied force is just sufficient to overcome friction and motion of the body takes place.
- After this the friction *suddenly decreases* to a magnitude which remains constant throughout the motion period as shown in Figure.



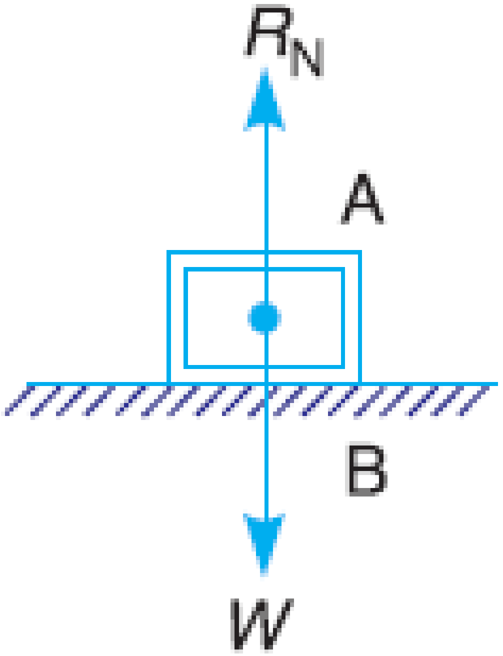
LIMITING FRICTION

- When the motion is just to commence, maximum friction is encountered. This condition is known as limiting *equilibrium. The friction acting at this stage is termed as limiting friction.*
- Hence, *limiting force of friction may be defined as the maximum value of friction force which exists when a body just begins to slide over the surface of the other body.*
- When the applied force or tractive force *P* is less than the limiting friction, the body remains at rest, and the friction is called *static friction*, which may have any value between zero and limiting friction.



LIMITING FRICTION

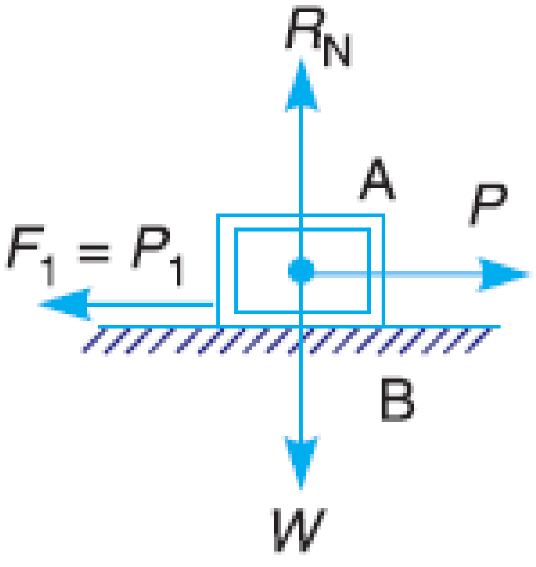
- Consider that a body *A* of weight *W* is lying on a rough horizontal body *B* as shown in Fig. (*a*).
- In this position, the body *A* is in equilibrium under the action of its own weight *W*, and the normal reaction R_N (equal to *W*) of *B* on *A*.



(*a*)

LIMITING FRICTION

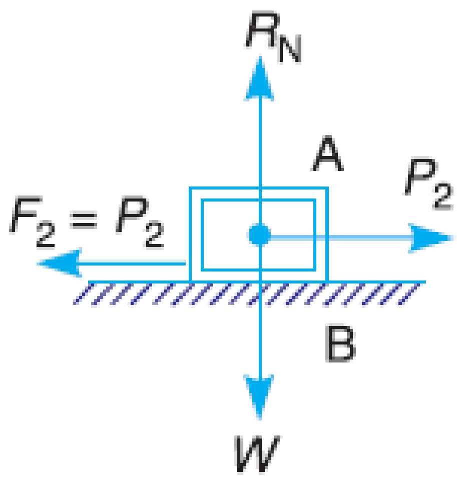
- Consider that a body *A* of weight *W* is lying on a rough horizontal body *B* as shown in Fig. (*a*).
- In this position, the body *A* is in equilibrium under the action of its own weight *W*, and the normal reaction R_N (equal to *W*) of *B* on *A*.
- Now if a small horizontal force P_1 is applied to the body *A* acting through its centre of gravity as shown in Fig. (*b*), it does not move because of the frictional force which prevents the motion.
- This shows that the applied force P_1 is exactly balanced by the force of friction F_1 acting in the opposite direction.



(*b*)

LIMITING FRICTION

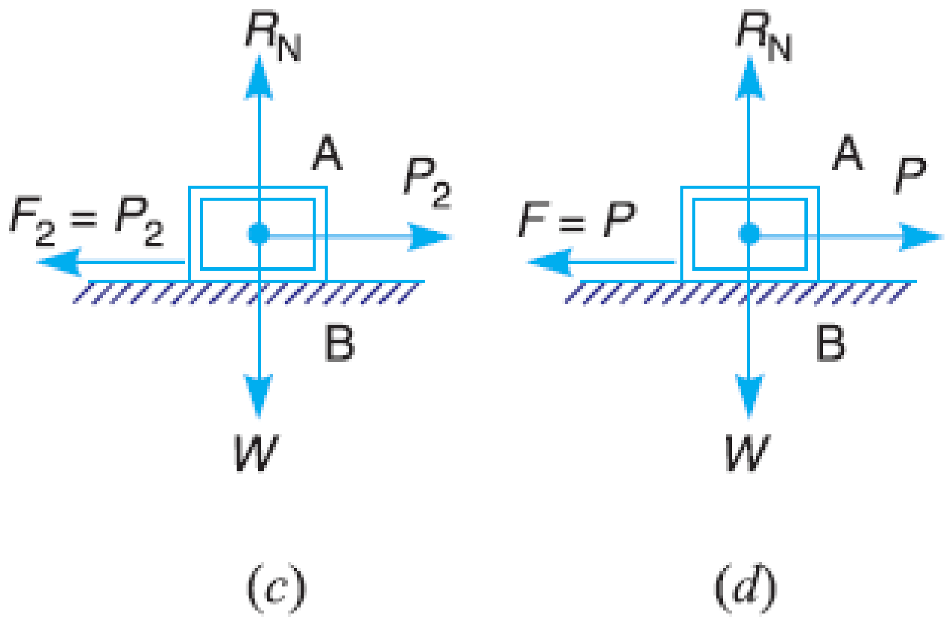
- If we now increase the applied force to P_2 as shown in Fig. (*c*), it is still found to be in equilibrium.
- This means that the force of friction has also increased to a value $F_2 = P_2$.
- Thus every time the effort is increased the force of friction also increases, so as to become exactly equal to the applied force.



(*c*)

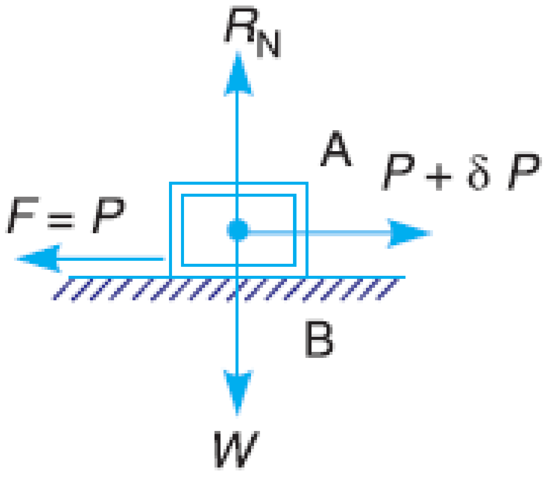
LIMITING FRICTION

- If we now increase the applied force to P_2 as shown in Fig. (*c*), it is still found to be in equilibrium.
- This means that the force of friction has also increased to a value $F_2 = P_2$.
- Thus every time the effort is increased the force of friction also increases, so as to become exactly equal to the applied force.
- There is, however, a limit beyond which the force of friction cannot increase as shown in Fig. (*d*).



LIMITING FRICTION

- After this, any increase in the applied effort will not lead to any further increase in the force of friction, as shown in Fig. (e), thus the body *A* begins to move in the direction of the applied force.
- This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as ***limiting force of friction*** or simply ***limiting friction***.
- It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction into play is called **static friction** which may have any value between zero and limiting friction.



(e)

LAWS OF STATIC FRICTION

- Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = constant$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

LAWS OF KINETIC OR DYNAMIC FRICTION

- Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

LAWS OF SOLID FRICTION

- Following are the laws of solid friction :

1. The force of friction is directly proportional to the normal load between the surfaces.
2. The force of friction is independent of the area of the contact surface for a given normal load.
3. The force of friction depends upon the material of which the contact surfaces are made.
4. The force of friction is independent of the velocity of sliding of one body relative to the other body.

LAWS OF FLUID FRICTION

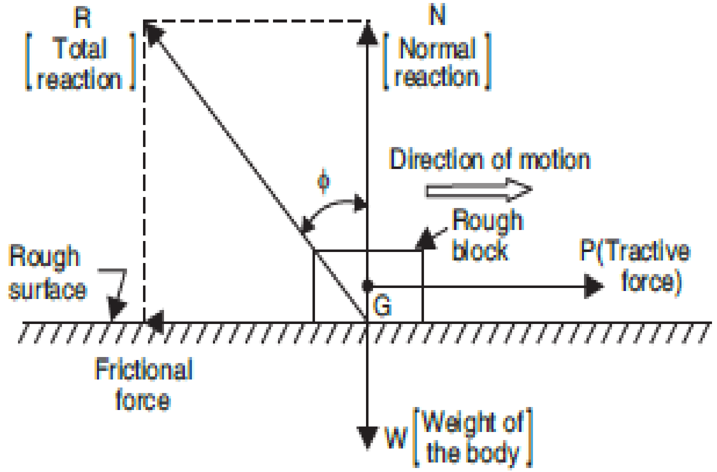
- Following are the laws of fluid friction :

1. The force of friction is almost independent of the load.
2. The force of friction reduces with the increase of the temperature of the lubricant.
3. The force of friction is independent of the substances of the bearing surfaces.
4. The force of friction is different for different lubricants.

ANGLE OF FRICTION

- Consider a body of weight *W* resting on an inclined plane
- Body is in equilibrium under the action of the following forces :

1. Weight (*W*) of the body, acting vertically downwards,
2. Friction force (*F*) acting upwards along the plane, and
3. Normal reaction (*R*) acting at right angles to the plane.



- Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane.
- This angle of inclined plane, at which a body just begins to slide down the plane, the angle which the normal reaction makes with resultant force, is called the angle of friction or limiting angle of friction.
- This is also equal to the angle, which the normal reaction makes with the vertical.

CO-EFFICIENT OF FRICTION

- It is *defined as the ratio of limiting force of friction to the normal reaction between the two bodies.*
- It is denoted by μ.
- The ratio remains constant.
- Coefficient of friction μ = tan ϕ = $\frac{limiting\ friction\ or\ static\ frictional\ force}{Normal\ Reaction}$
- Coefficient of friction μ = $\frac{F}{N}$

$$\therefore F = \mu N$$

- where ϕ = Angle of friction, *F* = Limiting friction, and *R* = Normal reaction

ANGLE OF REPOSE

- Consider a body of weight *W* acting on a rough horizontal plane inclined at angle α.
- If the angle of inclination is slowly increased, a stage will come when the block of mass *m* will tend to slide down.
- This angle of the plane with horizontal plane is known as ***angle of repose.***
- The body is in equilibrium under the action of the following forces :

(*i*) Weight, *W* (which may be resolved into two components $W\sin\theta$ and $W\cos\theta$)

(*ii*) Normal reaction, *N* and

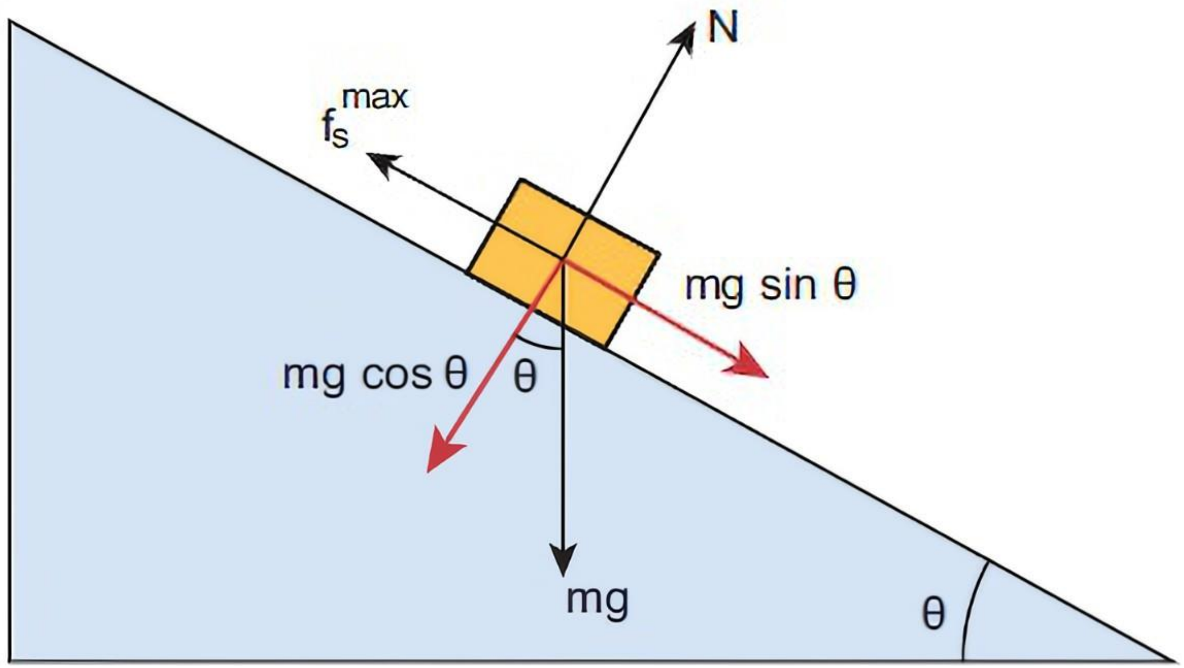
(*iii*) Frictional force, *F* (= μ*N*).

ANGLE OF REPOSE

- In the limiting condition when the block is about to slide down the inclined plane, the frictional force must act up the plane and for equilibrium; considering the forces along and perpendicular to the plane.

$$F = W \sin\theta$$

$$N = W \cos\theta$$



- Getting value of F and N

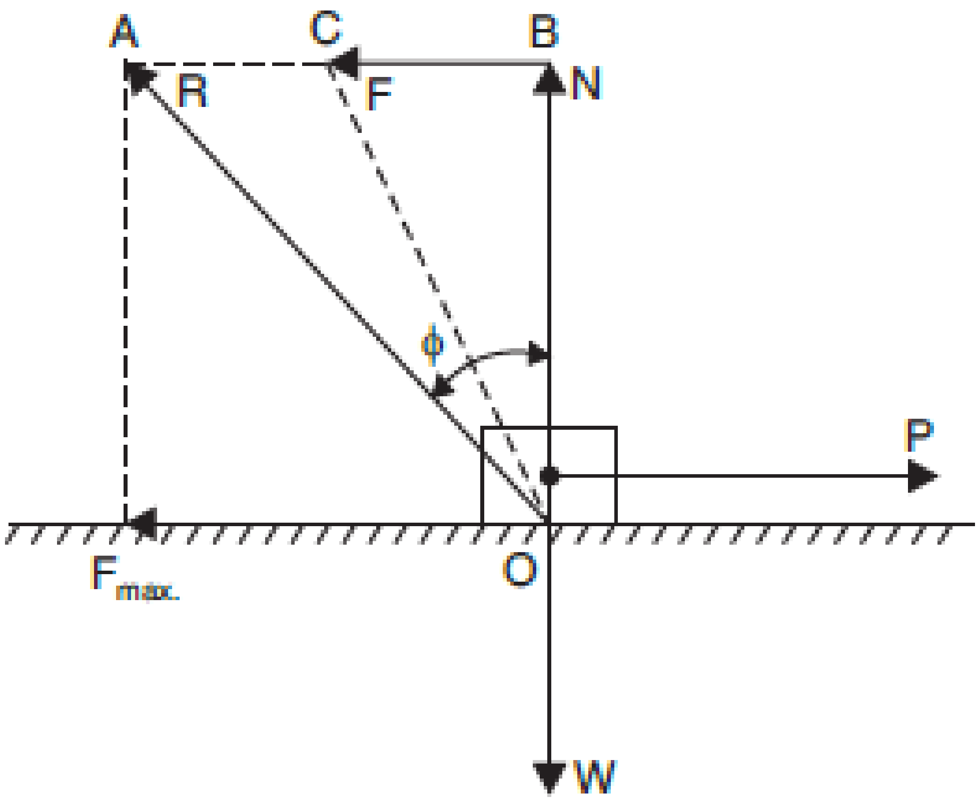
$$\frac{F}{N} = \frac{W \sin\theta}{W \cos\theta} = tan\,\theta$$

$$\frac{F}{N} = \mu = tan\,\phi$$

- where ϕ is the angle of friction.
- The angle θ is called *angle of repose* and is *equal to the angle of friction when the body is in the condition of limiting equilibrium on an inclined plane.*

CONE OF FRICTION

- If the line *OA* of Figure making the maximum angle of friction ϕ with the normal is revolved about *OB* as an axis, the cone generated is called the cone of *friction*.
- If the resultant *R* of the normal reaction and the friction falls within the cone of friction, the forces acting on the body are not great enough to cause motion.
- This principle is used in *self-locking* mechanisms and also in *taper pins*.



- For example, if the angle of taper pin is less than the angle of friction, no force at right angles in the axis of the pin could cause it to move in the direction of its axis.

FRICTIONAL LOCK MECHANISM AND DEVICES

- Locking device is designed to prevent mated shafts and components from loosening out of place when they are subjected to movement, varying temperatures, vibrations, stresses, and other conditions.
- They are critical components, as they often ensure the safety of the system.
- They appear frequently in systems that require coupling various components together.
- *Frictional locking devices are devices that perform operations using the coefficient of friction between the two contacting surfaces.*
- *A primary example occurs when inserting the locking device between the shaft and the hub of a system.*
- *The locking device then expands to fill the gap, holding the components in place by friction.*
- *These usually take the form of metallic or non-metallic hollow cylinders, often with a slit on one side.*
- *Another familiar friction locking device is the nut. These ubiquitous pieces of assembly and mating components work with a combination of friction on the threads of the shaft, slight tension on the bolt and compression of the parts held together.*

FRICTIONAL LOCK MECHANISM AND DEVICES

- Frictional locking devices have the advantage that they do not require keying.
- That is, no need to properly align keys and key-ways, and no need to worry if these will be compatible when designing systems.
- Indeed, because the locking is completely performed by friction between the locking device and the shaft, the system can even deal with oversized and undersized shafts.
- No keys also means no worry over loose keyed components at reduced torque ratings; loose keys can cause vibrations and injuries, and damage equipment.
- All that engineers need out of the system is the ability to insert the shaft into the locking device, the frictional locking device then exerts radial pressure, locking the components in place.

FRICTIONAL LOCK MECHANISM AND DEVICES

- When compared to keyed connections, they can be backlash free with proper fit tolerances, they allow the ability to make adjustments to the axial position and angular timing in a system, and no impact between key and key-way occurs when reversing the system because no keys are present.
- Avoid employing them in situations with high external centrifugal forces. These situations can cause a drop in the pressure between the components and lead to slipping.
- In such applications, use slit-less friction locking devices, which have stricter machining and application tolerances, or use another type of locking device.
- Frictional locking devices come in varying configurations, usually anywhere from one to three pieces, Smaller sizes are usually reserved for lower torque, less demanding operations.

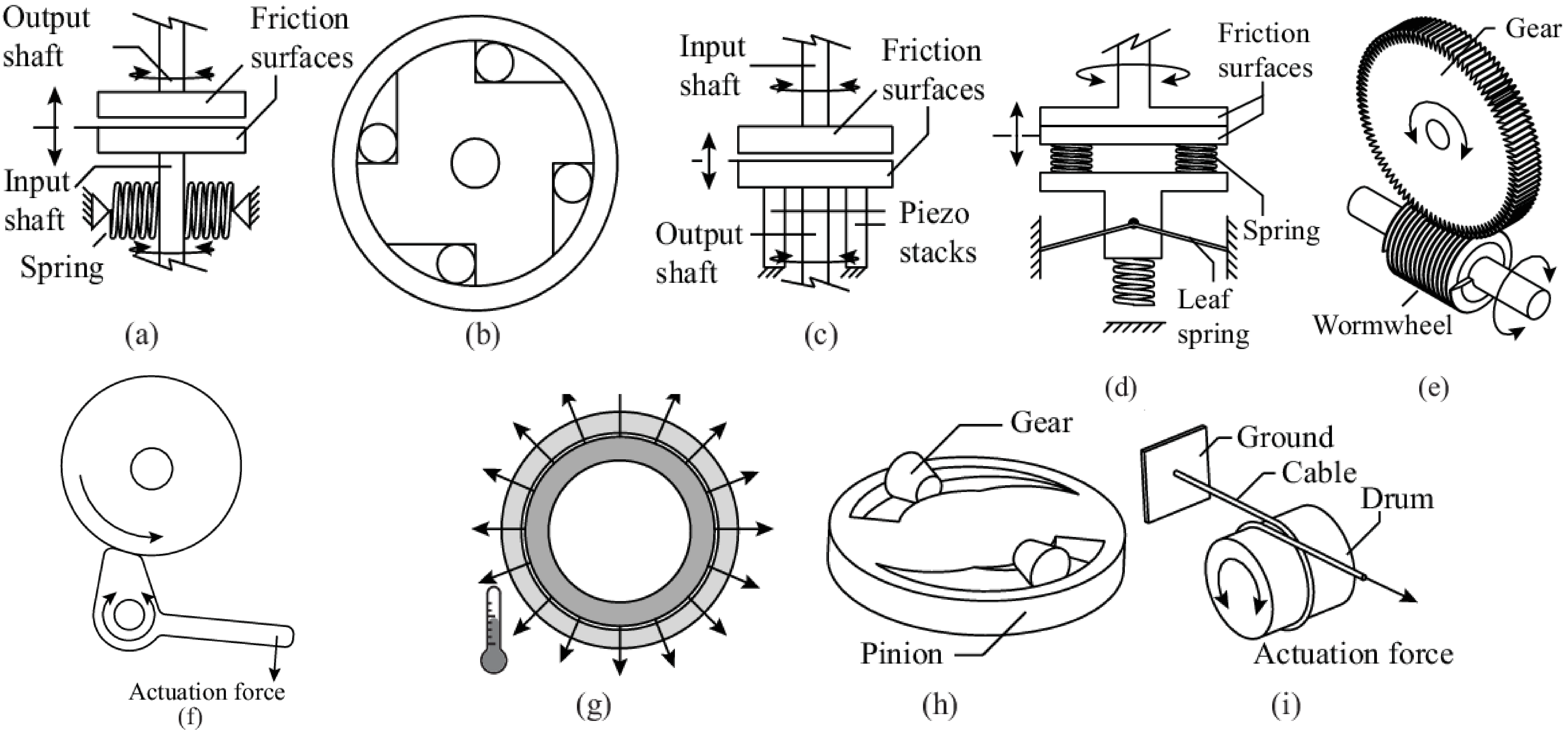
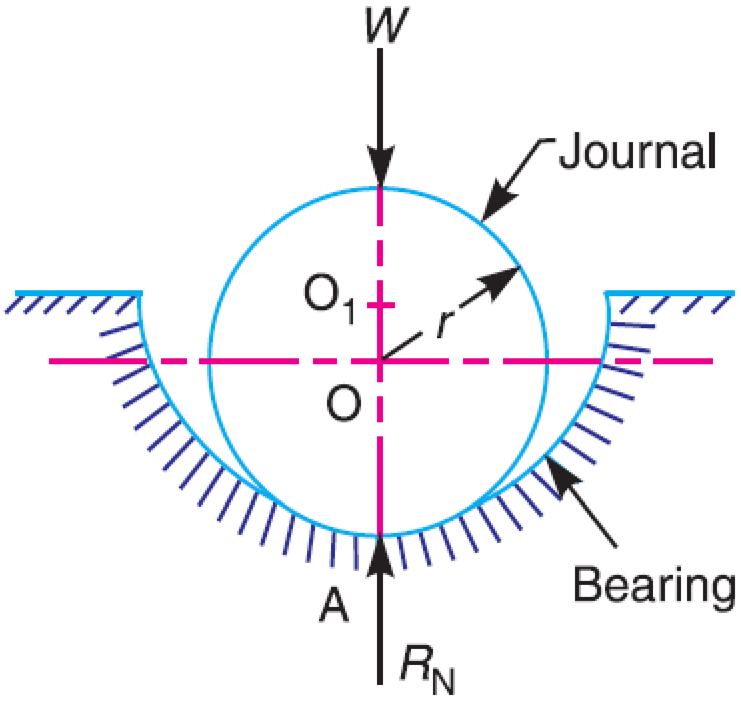


Figure shows types of friction based locking devices with: (a) a bi-stable brake, (b) an overrunning clutch, (c) a piezo actuated brake, (d) a statically balanced brake, (e) a wormwheel, (f) a self-engaging brake, (g) a thermic lock, (h) a self-engaging pinion-gear mechanism and (i) a capstan.

FRICTION IN JOURNAL BEARING

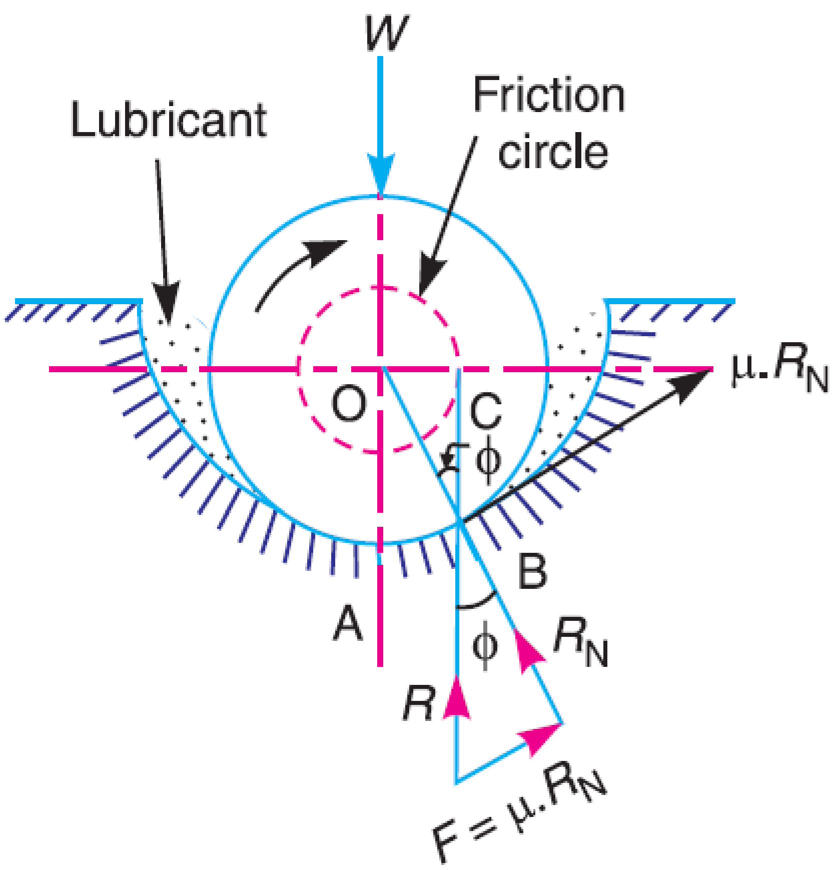
- A journal bearing forms a turning pair as shown in Fig(*a*).
- The fixed outer element of a turning pair is called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.***
- The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.
- When bearing is not lubricated (or journal is stationary), then there is a line contact between two elements as shown in Fig. (*a*).
- The load *W* on the journal and normal reaction *RN* (equal to *W*) of the bearing acts through the centre.
- The reaction *RN* acts vertically upwards at point *A*. This point *A* is known as ***seat*** or ***point of pressure.***



(*a*)

FRICTION IN JOURNAL BEARING

- Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig. (b).
- The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction.
- Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B.
- This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point A to point B.



(b)

FRICTION IN JOURNAL BEARING

- In order that the rotation may be maintained, there must be a couple rotating the shaft.
- Let ϕ = Angle between R (resultant of F and R_N) and R_N,

  μ = Coefficient of friction between the journal and bearing,

  T = Frictional torque in N-m, and r = Radius of the shaft in metres.
- For uniform motion, resultant force and resultant turning moment on the shaft must be zero

$$\boldsymbol{R = W} \quad \text{and} \quad \boldsymbol{T = W \times OC = W \times OB \sin\phi = W.r \sin\phi}$$

- Since ϕ is very small, therefore substituting $\sin\phi = \tan\phi$

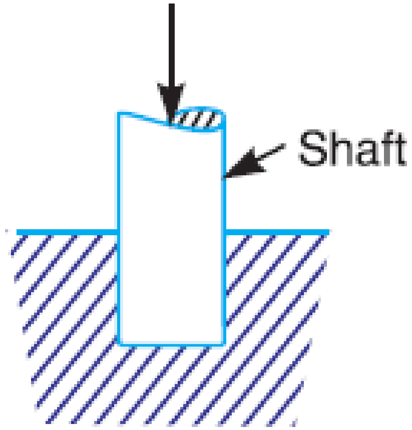
$$\boldsymbol{T = W.r \tan\phi = \mu.W.r} \ \ldots \qquad (\because \mu = \tan\phi)$$

- If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

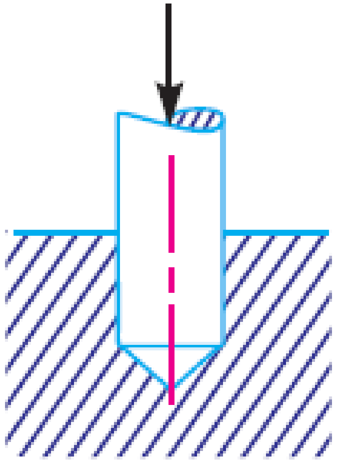
$$\boldsymbol{P = T.\omega = T \times 2\pi N/60 \text{ watts}} \qquad (N = \text{Speed of shaft in r.p.m.})$$

FRICTION OF PIVOT & COLLAR BEARING

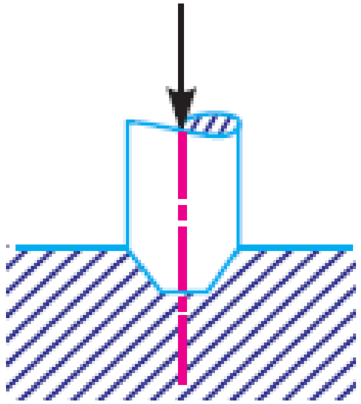
- Rotating shafts are frequently subjected to axial thrust.
- The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.
- Propeller shafts of ships, shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.
- Bearing surfaces placed at the end of a shaft to take the axial thrust are known as ***pivots.***
- The pivot may have a flat surface or conical surface as shown in Fig. (*a*) and (*b*) respectively.
- When cone is truncated, it is then known as truncated or trapezoidal pivot as shown in (*c*).



(*a*) Flat pivot.



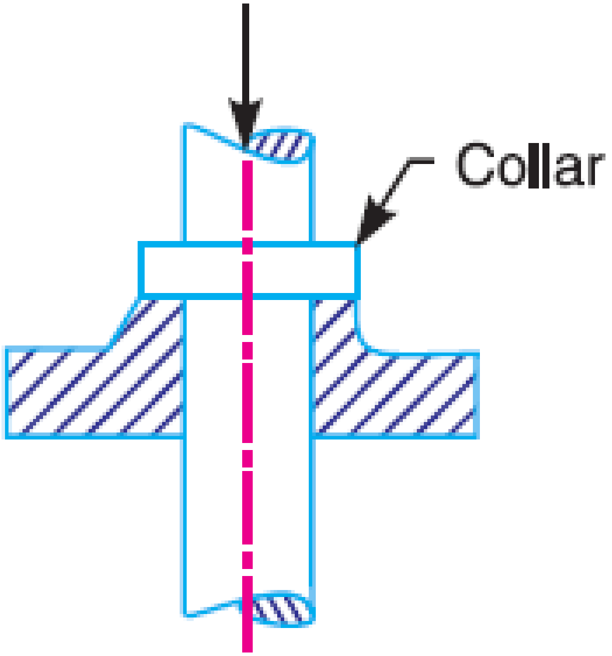
(*b*) Conical pivot.



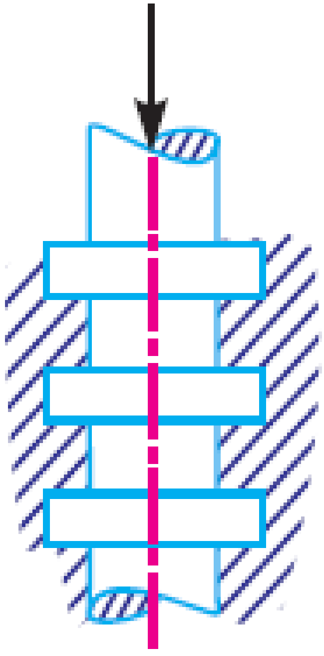
(*c*) Truncated pivot.

FRICTION OF PIVOT & COLLAR BEARING

- The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used.
- There may be a single collar, as shown in Fig. (*d*) or several collars along the length of a shaft, as shown in Fig. (e) in order to reduce the intensity of pressure.



(*d*) Single flat collar.



(*e*) Multiple flat collar.

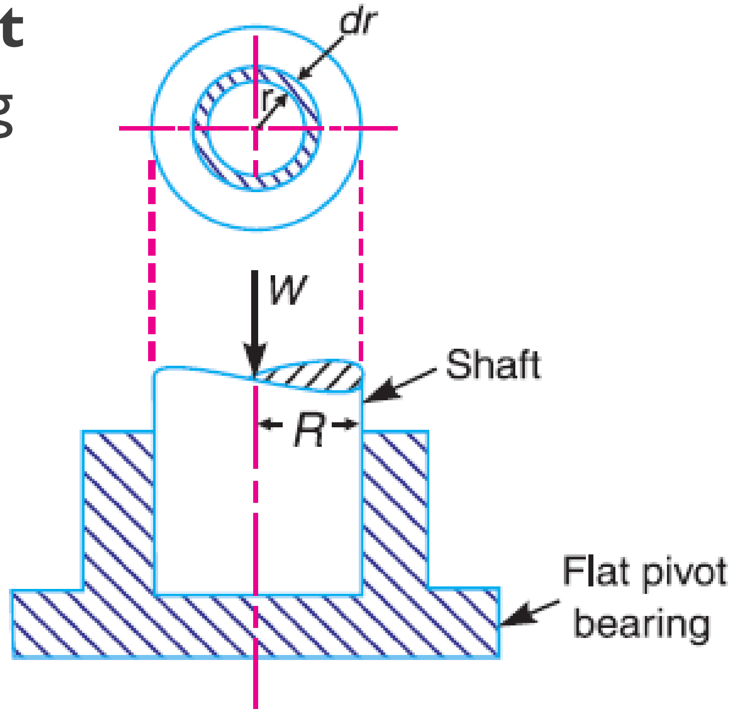
FRICTION OF PIVOT & COLLAR BEARING

- In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.
- In a new bearing, the contact between the shaft and bearing may be good over the whole surface and pressure over the rubbing surfaces is uniformly distributed.
- But when the bearing becomes old, all parts of rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing.
- This means that wear may be different at different radii and which alters distribution of pressure.
- Hence, in the study of friction of bearings, it is assumed that

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

FLAT PIVOT BEARING

- When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Figure, the sliding friction will be along the surface of contact between the shaft and the bearing.



- Let W =Load transmitted over the bearing surface,

  R =Radius of bearing surface,

  p =Intensity of pressure per unit area of bearing surface

  between rubbing surfaces, and

  μ =Coefficient of friction.

- **We will consider the following two cases :**

  1. When there is a uniform pressure and **2.** When there is a uniform wear.

FLAT PIVOT BEARING: *CONSIDERING UNIFORM PRESSURE*

- When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

- Consider a ring of radius *r* and thickness *dr* of the bearing area.

∴ Area of bearing surface, $A = 2\pi r . dr$

- Load transmitted to the ring, $\delta W = p \times A = p \times 2\,\pi\, r . dr$ ...**(*i*)**
- Frictional resistance to sliding on the ring acting tangentially at radius *r*,

$$F_r = \mu . \delta W = \mu\, p \times 2\pi\, r . dr = 2\pi\, \mu . p . r . dr$$

- ∴ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi\, \mu\, p\, r . dr \times r = 2\,\pi\,\mu\, p\, r^2\, dr \;\ldots(\boldsymbol{ii})$$

- Integrate this equation within the limits from 0 to *R* for total frictional torque on pivot bearing.

FLAT PIVOT BEARING: *CONSIDERING UNIFORM PRESSURE*

- ∴ Total frictional torque,

$$T = \int_0^R 2\pi\mu\, p r^2 dr = 2\pi\mu\, p \int_0^R r^2 dr$$

$$T = 2\pi\mu\, p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p * \frac{R^3}{3} = \frac{2}{3}\pi\mu\, p * R^3$$

$$T = \frac{2}{3}\pi\mu * \frac{W}{\pi R^2} * R^3 = \frac{2}{3}\mu W R \qquad \ldots\ldots \left(p = \frac{W}{\pi R^2}\right)$$

- When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi\, N/60 \;\ldots (\,\omega = 2\pi N\,/\,60)$$

- where N = Speed of shaft in r.p.m.

FLAT PIVOT BEARING: *CONSIDERING UNIFORM WEAR*

- Rate of wear depends upon intensity of pressure (*p*) and velocity of rubbing surfaces (*v*).
- Assume rate of wear is proportional to product of intensity of pressure and velocity of rubbing surfaces (*i.e. p.v.*.).
- Since velocity of rubbing surfaces increases with distance (*i.e.* radius *r*) from axis of bearing, therefore for uniform wear

$$p.r = C\ (a\ constant)\ or\ p = C\ /\ r$$

- and the load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi r.dr$$

$$\delta W = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

- ∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi C.dr = 2\pi\ C|r|_o^R = 2\pi\ CR$$

$$\mathrm{C} = \frac{W}{2\pi r}$$

FLAT PIVOT BEARING: *CONSIDERING UNIFORM WEAR*

- We know that frictional torque acting on the ring

$$T_r = 2\pi\mu\, pr^2.dr = 2\pi\mu * \frac{C}{r} * r^2.dr \qquad \ldots\left(\because\ p = \frac{C}{r}\right)$$

$$T_r = 2\pi\mu\, Cr.dr$$

- ∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\mu C.rdr = 2\pi\mu\, C\left|\frac{r^2}{2}\right|_o^R = 2\pi\mu C * \frac{R^2}{2}$$

$$\mathrm{T} = \pi\mu C R^2$$

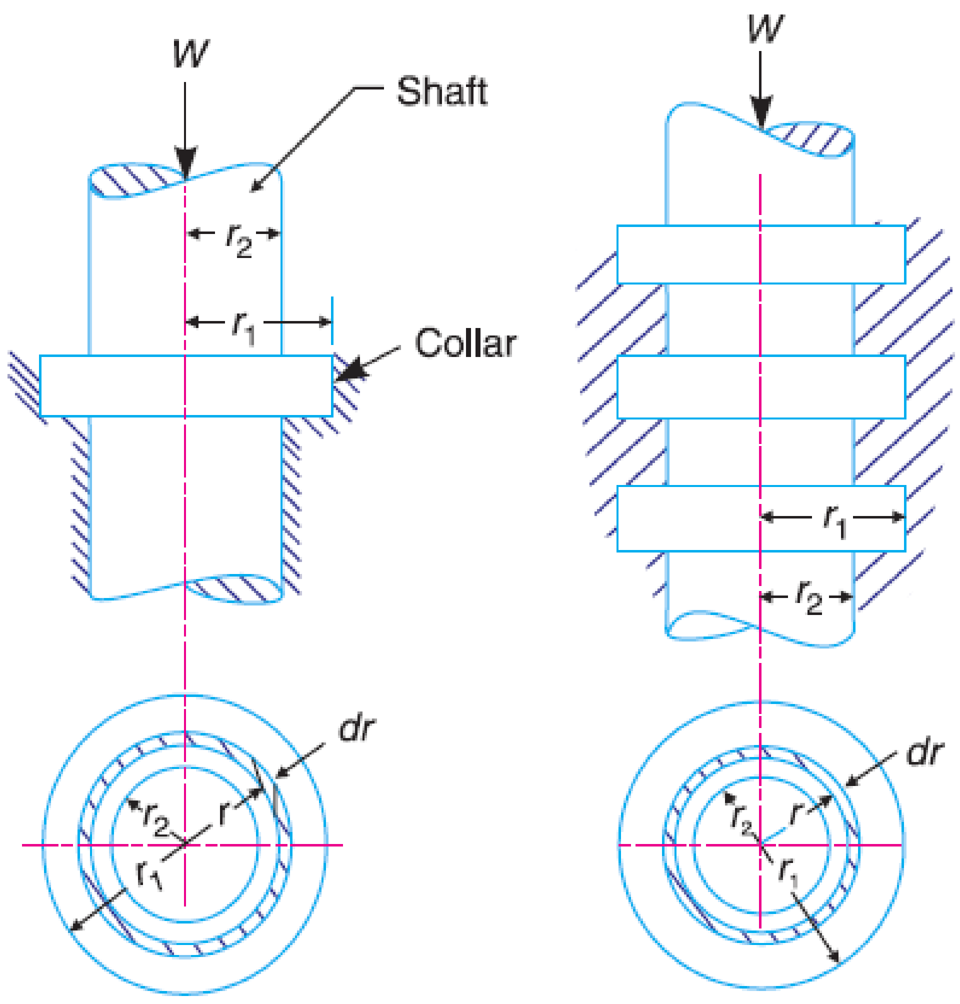
$$\mathrm{T} = \pi\mu \frac{W}{2\pi R} * R^2 = \frac{1}{2}\mu W.R \qquad \ldots\left(\because\ C = \frac{W}{2\pi R}\right)$$

FLAT COLLAR BEARING

- Collar bearings are used to take axial thrust of the rotating shafts.
- There may be a single collar or multiple collar bearings as shown in Fig. (*a*) and (*b*) respectively.
- Collar bearings are also known as ***thrust bearings.***
- Consider a single flat collar bearing supporting a shaft as shown in Fig. (*a*).
- Let r_1 = External radius of the collar, and
- r_2 = Internal radius of the collar.

∴ Area of the bearing surface,

$$A = \pi\,[(r_1)^2 - (r_2)^2]$$



(*a*) Single collar bearing (*b*) Multiple collar bearing.

FLAT COLLAR BEARING: *CONSIDERING UNIFORM PRESSURE*

- When the pressure is uniformly distributed over the bearing area, then intensity of pressure

$$p = \frac{W}{A} = \frac{W}{\pi(r_1)^2 - (r_2)^2}$$

- Consider a ring of radius *r* and thickness *dr* of the bearing area.

∴ Area of bearing surface, $A = 2\pi r.dr$

- Load transmitted to the ring, $\delta W = p \times A = p \times 2\pi r.dr$ ...**(*i*)**
- Frictional resistance to sliding on the ring acting tangentially at radius *r*,

$$F_r = \mu.\delta W = \mu p \times 2\pi r.dr = 2\pi \mu.p.r.dr$$

- ∴ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi \mu p r.dr \times r = 2\pi \mu p r^2 dr \quad \ldots(\boldsymbol{ii})$$

- Integrate this equation within the limits from r_2 to r_I for total frictional torque on collar bearing.

FLAT COLLAR BEARING: *CONSIDERING UNIFORM PRESSURE*

- ∴ Total frictional torque,

$$T = \int_{r_1}^{r_2} 2\pi\mu\, p r^2 dr = 2\pi\mu\, p \int_{r_1}^{r_2} r^2 dr$$

$$T = 2\pi\mu\, p \left[\frac{r^3}{3}\right]_{r_1}^{r_2} = 2\pi\mu\, p * \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

- Substituting the value of *p* from equation i

$$T = 2\pi\mu * \frac{W}{\pi(r_1)^2 - (r_2)^2} * \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3}\mu W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

FLAT COLLAR BEARING: *CONSIDERING UNIFORM WEAR*

- Rate of wear depends upon intensity of pressure (*p*) and velocity of rubbing surfaces (*v*).
- Assume rate of wear is proportional to product of intensity of pressure and velocity of rubbing surfaces (*i.e. p.v.*.).
- Since velocity of rubbing surfaces increases with distance (*i.e.* radius *r*) from axis of bearing, therefore for uniform wear

$$p.r = C\ (a\ constant)\ or\ p = C\,/\,r$$

- and the load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi r.dr$$

$$\delta W = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

- ∴ Total load transmitted to the collar bearing

$$W = \int_{r_1}^{r_2} 2\pi C.dr = 2\pi\, C|r|_{r_1}^{r_2} = 2\pi\, C(r_1 - r_2)$$

$$\mathrm{C} = \frac{W}{2\pi(r_1 - r_2)}$$

FLAT COLLAR BEARING: *CONSIDERING UNIFORM WEAR*

- We know that frictional torque acting on the ring

$$T_r = 2\pi\mu\, pr^2.dr = 2\pi\mu * \frac{C}{r} * r^2.dr \qquad \ldots\left(\because\ p = \frac{C}{r}\right)$$

$$T_r = 2\pi\mu\, Cr.dr$$

- ∴ Total frictional torque on the bearing,

$$T = \int_{r_1}^{r_2} 2\pi\mu C.rdr = 2\pi\mu\, C \left|\frac{r^2}{2}\right|_{r_1}^{r_2} = 2\pi\mu C * \frac{(r_1)^2 - (r_2)^2}{2}$$

$$\mathrm{T} = \pi\mu C(r_1)^2 - (r_2)^2$$

Substituting the value of *C* from equation ii

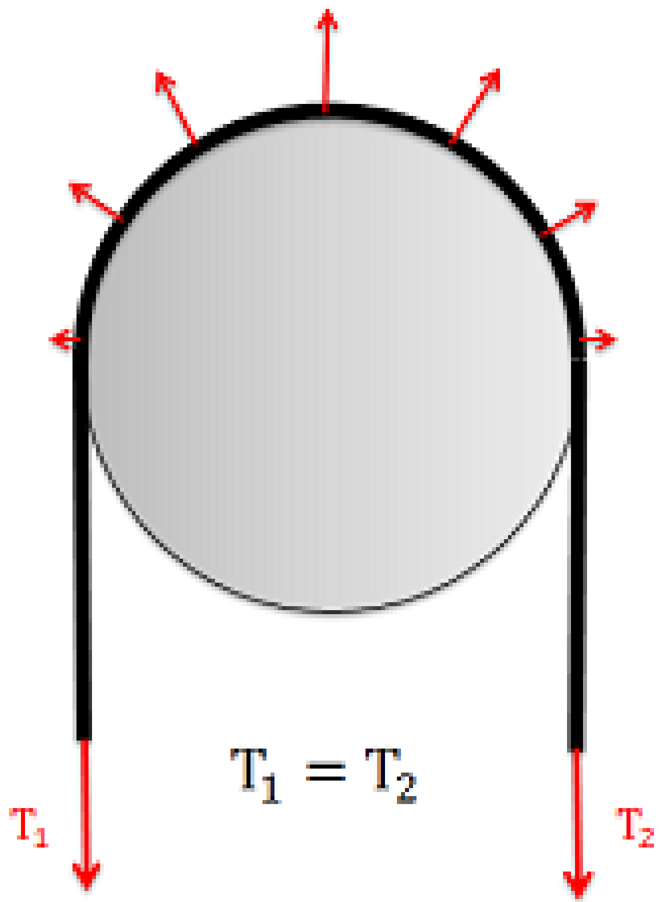
$$\mathrm{T} = \pi\mu \frac{W}{2\pi(r_1 - r_2)} * (r_1)^2 - (r_2)^2 = \frac{1}{2}\mu W.(r_1 + r_2)$$

FRICTION IN FLAT BELTS

- In any system where a belt or a cable is wrapped around a pulley or some other cylindrical surface, we have the potential for friction between the belt or cable and the surface it is in contact with.
- In some cases, such as a rope over a tree branch being used to lift an object, the friction forces represent a loss.
- In other cases such as a belt-driven system, these friction forces are put to use transferring power from one pulley to another pulley
- In many belt-driven systems, the belt friction keeps the pulley from slipping relative to the belt. This allows us to use belts to transfer forces from one pulley to another pulley

FRICTION IN FLAT BELTS

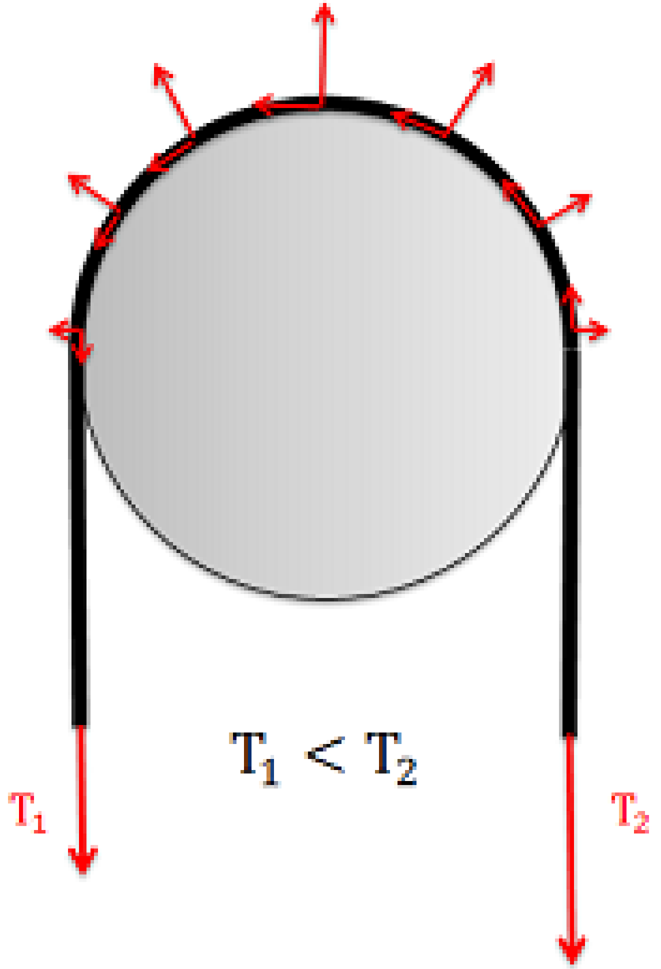
- If we were to pass a rope over a tree branch to help lift an object, the rope would experience belt friction resisting the sliding of the rope relative to the surface of the tree branch.
- For analysis, we will start a flat, massless belt passing over a cylindrical surface. If we have an equal tension in each belt, the belt will experience a non-uniform normal force from the cylinder that is supporting it.



With equal tensions on each side of the belt, only a non-uniform normal force exists between the belt and the surface

FRICTION IN FLAT BELTS

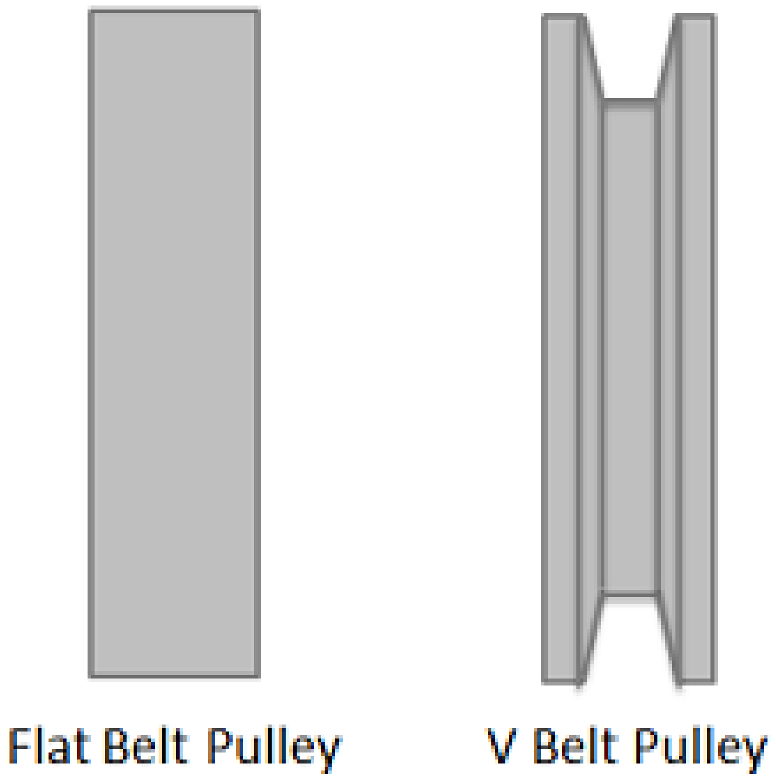
- In a frictionless scenario, if we were to increase the tension on one side of the rope it would begin to slide across the cylinder. If friction exists between the rope and the surface though, the friction force will oppose with sliding motion, and prevent it up to a point.



With unequal tensions, a friction force will also be present opposing the relative sliding of the belt to the surface

FRICTION IN FLAT BELTS

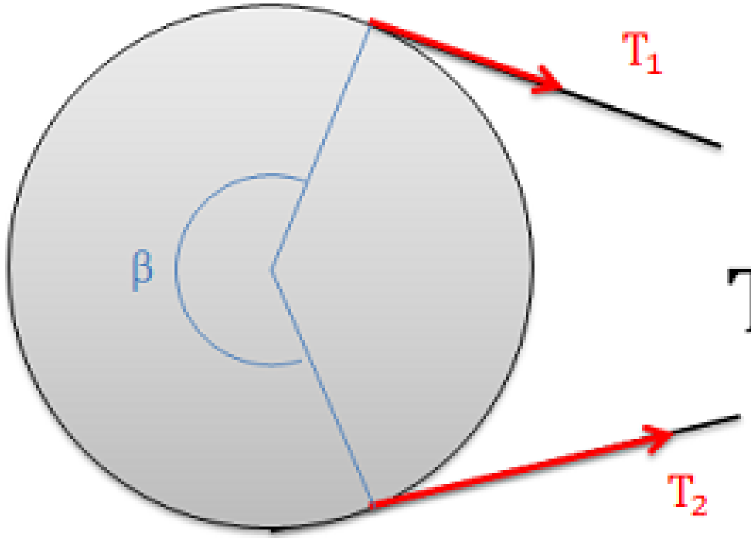
- A flat belt is any system where the pulley or surface only interacts with the bottom surface of the belt or cable.
- If belt/cable fits into a groove, then it is considered a V belt.
- When analyzing systems with belts, we are usually interested in the range of values for the tension forces where the belt will not slip relative to the surface.
- Starting with the smaller tension force on one side (T_1), we can increase the second tension force (T_2)to some maximum value before slipping.
- For a flat belt, the maximum value for T_2 will depend on the value of T_1, the static coefficient of friction between the belt and the surface, and the contact angle between the belt and the surface (β) given in radians, as described.



For a flat belt, the belt or cable will interact with the bottom surface. For a V belt, the belt or cable will interact with the sides of a groove

$$T_{2\max} = T_1 e^{\beta\mu_s}$$

FRICTION IN FLAT BELTS



Coefficient of friction between surface and belt

Euler's constant ≈2.718

Angle of contact in radians

$$T_{2max} = T_1 e^{\mu_s \beta}$$

Always the larger tension force

Always the smaller tension force

The method for determining the maximum value of T_2 before the belt starts slipping.