MID SEMESTER EXAMINATION, December - 2022

Third Semester, B.Tech

Paper Code - ABS- 201: Engineering Mathematics - III

Time: 1:30 hrs. Max. Marks:40

All the questions are compulsory. Each question is of 5 marks.

1. Let $\mathbf{R}$ be the set of all real numbers and $\mathbf{R}^2 = \{(x_1, x_2) : x_1 \in R, x_2 \in R\}$. Then which one of the following is a subspace of $\mathbf{R}^2$ over $\mathbf{R}$?

   (a) $\{(x_1, x_2) : x_1 > 0, x_2 > 0\}$
   (b) $\{(x_1, x_2) : x_1 \in \mathbf{R}, x_2 > 0\}$
   (c) $\{(x_1, x_2) : x_1 < 0, x_2 < 0\}$
   (d) $\{(x_1, 0) : x_1 \in \mathbf{R}\}$

2. Find the dimension of the subspace of $\mathbf{R}^3$ spanned by $(-3, 0, 1)$, $(1, 2, 1)$ and $(3, 0, -1)$.

3. Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear transformation defined by

$$T(x, y, z) = (x + y, x - z)$$

   Then find the dimension of the null space of T.

4. If $T : \mathbf{R}^2 \to \mathbf{R}^3$ is a linear transformation such that $T(0, 1) = (2, 3, 1)$ and $T(1, 1) = (3, 0, 2)$. Then find T(x,y)?

5. Complete the following composition table for the set $S = \{a, b, c\}$, so that $(S, *)$ may be a group:

*	a	b	c
a	a	b	c
b	b		
c	c		

6. Let $S = \{x | x \in \mathbf{R}, x \neq 0, x \neq -1\}$ and consider the following functions

$$f_i : S \to S, i = 1, 2, \ldots\ldots, 6, as$$

$f_1(x) = x$, $f_2(x) = 1 - x$, $f_3(x) = \frac{1}{x}$, $f_4(x) = \frac{1}{1-x}$, $f_5(x) = 1 - \frac{1}{x}$, $f_6(x) = \frac{x}{x-1}$.

   Show that $G = \{f_1, f_2, \ldots\ldots, f_6\}$ is a group under the operation of composition. Give the composition table.

7. Is $(\mathbf{Z}^+, .)$ a subgroup of $(\mathbf{Q}^+, .)$?

8. Given the permutation $x = (12)(34)$ and $y = (13)(56)$, find a permutation 'a' such that

$$a^{-1}xa = y$$

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