MID SEMESTER EXAMINATION, January - 2022
First Semester, B.Tech(IIOT-B1)
Paper Code - BS- 111: Engineering Mathematics - I

Time: 1:00 hrs. Max. Marks:30

The paper consists of 2 groups:
Group A- containing 10 objective type questions(1 mark each). Attempt all questions.
Group B- containing 6 subjective type questions(4 marks each). Attempt any 5 questions.
Mail the answer script to **maths.usar@gmail.com** as well

Group-A

1. If $f(x, y) = \dfrac{sin(yx^2 + y)}{1 + x^2}$, the value of f_{xy} at $(0, 1)$ is
   - A. 1
   - B. 0
   - C. 50
   - D. 68
2. If $u = x^y$ then the value of $\dfrac{\partial u}{\partial y}$ is equal to
   - A. 1
   - B. yx^{y-1}
   - C. xy^{x-1}
   - D. $x^y log(x)$
3. If $f(x, y) = sin(x)cos(y)$ then which of the following is a stationary point?
   - A. $(0, \frac{\pi}{2})$
   - B. $(0, 0)$
   - C. $(\frac{\pi}{4}, \frac{\pi}{4})$
   - D. $(\frac{-\pi}{4}, \frac{\pi}{4})$
4. The jacobian of p,q,r w.r.t x,y,z given p=x+y+z, q=y+z, r=z is
   - A. 2
   - B. 1
   - C. 0
   - D. -1
5. Implicit functions are those functions ______
   - A. Which can be eliminated to give zero
   - B. Which are rational in nature
   - C. Which can not be solved for a single variable
   - D. Which can be solved for a single variable
6. Which of the following correctly defines Leibnitz rule of a function given by $I(\alpha) = \int_a^b f(x, \alpha)dx$ where a and b are functions of α?
   - A. $\dfrac{dI}{d\alpha} = \int_a^b \dfrac{\partial}{\partial \alpha} f(x, \alpha)dx$

B. $\dfrac{dI}{d\alpha} = \dfrac{d}{d\alpha}\displaystyle\int_a^b f(x,\alpha)dx$

C. $\dfrac{dI}{d\alpha} = \displaystyle\int_a^b \dfrac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\dfrac{da}{d\alpha} - f(a,\alpha)\dfrac{db}{d\alpha}$

D. $\dfrac{dI}{d\alpha} = \displaystyle\int_a^b \dfrac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\dfrac{db}{d\alpha} - f(a,\alpha)\dfrac{da}{d\alpha}$

7. What is the value of $\dfrac{\partial^2 z}{\partial x \partial y}$ for the function $z = 4x^3y + 10y$?

    A. $3xy$

    B. $6x^2$

    C. $12x^2$

    D. $12x + 10$

8. Let $w = f(x,y)$, where x and y are functions of t. Then according to the chain rule $\dfrac{dw}{dt}$ is equal to

    A. $\dfrac{dw}{dx}\dfrac{dx}{dt} + \dfrac{dw}{dy}\dfrac{dt}{dt}$

    B. $\dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt}$

    C. $\dfrac{dw}{dx}\dfrac{\partial x}{\partial t} + \dfrac{dw}{dy}\dfrac{\partial y}{\partial t}$

    D. $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}$

9. The Laplacian of a function $f = f(x,y)$ is defined to be $f_{xx} + f_{yy}$. Which of the following functions has Laplacian equal to zero?

    A. $f = x^2 - 2y^2 + 2xy$

    B. $f = 3x^2 + 3y^2 + 6xy$

    C. $f = x^2 - 2y^2 + 3xy$

    D. $f = 3x^2 - 3y^2 - 6xy$

10. If $f(x,y) = x^2 - 2y^2$, $x = u - v$ and $y = u + v$ then $\dfrac{\partial f(x,y)}{\partial v}$ is equal to

    A. $6u + 2v$

    B. $-(2u + 6v)$

    C. $-(6u + 2v)$

    D. $2(u^2 + 6uv + v2)$

Group-B

1. If $u = f(r,s,t)$ and $r = x/y$, $s = y/z$, $t = z/x$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.

2. If $\dfrac{x^2}{a^2+u} + \dfrac{y^2}{b^2+u} + \dfrac{z^2}{c^2+u} = 1$, prove that $\left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial y}\right)^2 + \left(\dfrac{\partial u}{\partial z}\right)^2 = 2\left(x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}\right)$

3. If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$ where $x = rcos\theta$,$y = rsin\theta$, find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$.

4. The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

5. If $f(x,y)=0$, show that $\left(\frac{\partial f}{\partial y}\right)^3 \frac{d^2y}{dx^2} = 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial^2 f}{\partial x \partial y}\right) - \left(\frac{\partial f}{\partial y}\right)^2\left(\frac{\partial^2 f}{\partial x^2}\right) - \left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial^2 f}{\partial y^2}\right)$

6. Prove that $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx = log(1+a)$, $(a > -1)$

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