Euclidian Division In $\mathbb{Z}$

$Definition:$ ––––––––––––––––––––––––––––––––

$for\ a,b \in \mathbb{Z}^{+} \times \mathbb{Z}_{*}\ we\ say\ that\ b\ divid\ a:$

$\exists q \in \mathbb{Z}\ ,\ \ a = q{*}b\ and\ we\ donated\ by\ b|a$

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1) $if\ a|1\ if\ and\ only\ if\ a = \pm 1$

$Proof:$

$a|1 \Leftrightarrow \exists q \in \mathbb{Z}\ ,\ \ 1 = q \times a\ this\ it\ true\ only\ if\ q = a = 1\ or\ q = a = -1$

$\Leftrightarrow a = \pm 1$

2) $\forall a \in \mathbb{Z}^{*},\ \ a|0$

$Proof:$

$\forall a \in \mathbb{Z}^{*}\ a|0 \Leftrightarrow \exists q \in \mathbb{Z}\ ,\ \ 0 = q \times a\ this\ is\ always\ true\ for\ q = 0$

3) $a|b\ and\ b|a\ if\ and\ only\ if\ a = \pm b$

$Proof:$

$a|b \Leftrightarrow \exists q \in \mathbb{Z}\ ,\ \ b = q \times a$

$b|a \Leftrightarrow \exists q' \in \mathbb{Z}\ ,\ \ a = q' \times b$

$\Leftrightarrow b = q \times q' \times b\ this\ it\ true\ only\ if\ q = q' = 1\ or\ q = q' = -1$

$\Leftrightarrow a = \pm b$

4) $if\ a|b\ and\ b|c\ then\ a|c$

$Proof:$

$a|b \Leftrightarrow \exists q \in \mathbb{Z}\ ,\ \ b = q \times a$

$b|c \Leftrightarrow \exists q' \in \mathbb{Z}\ ,\ \ c = q' \times b$

$\Leftrightarrow q'' = q \times q' \in \mathbb{Z}\ ,\ c = q''a$

$\Leftrightarrow a|c$

5) $if\ b|a\ and\ b|c\ then\ \forall k,l \in \mathbb{Z}\ \ \ b|ka + lc$

$Proof:$

$b|a \Leftrightarrow \exists q \in \mathbb{Z}\ ,\ \ a = q \times b$

$b|c \Leftrightarrow \exists q' \in \mathbb{Z}\ ,\ \ c = q' \times b$

$\Leftrightarrow \forall k,l \in \mathbb{Z}\ \ \ ka + lc = (kq + lq') \times b\ and\ q'' = kq + lq' \in \mathbb{Z}$

$\Leftrightarrow b|ka + lc$