Euclidian Division In $\mathbb{Z}$

$Definition:$ ——————————————————————————————————————

$$for\ a,b \in \mathbb{Z}^+ \times \mathbb{Z}_+^*\ we\ say\ that\ b\ devid\ a\ in\ the\ euclidian\ sense:$$

$$\exists q,r \in \mathbb{Z}^+\ ,\ \ a = q*b + r \quad with\ q = \left\lfloor \frac{a}{b} \right\rfloor\ and\ 0 \leqslant r < b$$

$$q:\ is\ the\ quotion$$
$$r: is\ the\ reminder$$

$$\left\lfloor \frac{a}{b} \right\rfloor\ is\ the\ floor\ function\ meaning\ the\ largest\ integer\ smaller\ then\ \frac{a}{b}$$

$$Proof:$$

$$we\ know\ that\ \left\lfloor \frac{a}{b} \right\rfloor \leqslant \frac{a}{b} < \left\lfloor \frac{a}{b} \right\rfloor + 1 \implies b*\left\lfloor \frac{a}{b} \right\rfloor \leqslant b*\frac{a}{b} < b*\left(\left\lfloor \frac{a}{b} \right\rfloor + 1\right)$$

$$\implies b*q \leqslant a < b*q + b \implies 0 \leqslant a - b*q < b \implies 0 \leqslant r < b$$

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