**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.

Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is **FALSE**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

*X1* ~ *N*(μ, σ2) , *X*2 ~ *N*(μ, σ2)

Let A be random variable such that A=2X1

E(X1)= μ , E(A)= E(2X1)=2E(X1)=2 μ

Var(X1)= σ2 , Var(A)= Var(2X1)=22Var(X1)=4 σ2

**A~N(2 μ, 4 σ2)**

Let B be random variable such that B= *X*1 + *X*2

E(X1)= μ, E(X2)= μ

E(B)=E(X1+X2)=E(X1)+E(X2)=2 μ

Var(X1)= σ2 , Var(X2)= σ2

Var(B)= Var(X1+X2)=Var(X1)+Var(X2)=2 σ2

**B~N(2 μ, 2 σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Soln:

Profit1 ~ N(5, 32), Profit2 ~ N(7, 42)

Let C be random variable be overall profit from both the division.

C = Profit1+ Profit2 = N(5, 32) + N(7, 42)…….1

From Q3 we get,

*X*1 + *X*2 = N(µ1 + µ2, σ12+ σ22 )…….2 comparing 1 and 2 we get

C = N(12, 52)

1-α = 95% , Z(1-α) = 1.959964

Z = (X-µ)/σ

Substituting value of Z=1.959964, µ=12, σ= 5, we get, X =21.79982

As NormalDistribution, 50% data on either side of mean so we get following range

9.7995

Range : (9.7995,2.2005)

Range : 9.7995\*10,00,000$ - 2.2005\*10,00,000$

2.2005

µ=12

**Rs.44,09,77,500 – Rs. 9,90,22,500**

1. Specify the 5th percentile of profit (in Rupees) for the company

P(C) = 5%

Zscore of P(C)=0.05 = -1.644854

Z = (X-µ)/σ

Substituting value of Z=-1.644854, µ=12, σ= 5, we get, X =3.77573

**3.77573\*10,00,000$=16,99,07,850**

1. Which of the two divisions has a larger probability of making a loss in a given year?

The two division makes Profit .Profit1 ~ N(5, 32), Profit2 ~ N(7, 42)

So to make loss X1 be the random variable which makes loss by 1st division and X2 be for 2nd Division respectively.

For loss X1~N(5, 32) X1=0 so Z1=-5/3

P(Z1<0)= 0.04005916………1

For Loss X2~ N(7, 42) X2=0 so Z2 = -7/4

P(Z2<0)=0.04779……………..2

Comparing 1 & 2

X2 has larger probability than X1 to make a loss.

**Division2 larger probability than Division 1 to make a loss.**