Calculate Joint Angles using Inverse Kinematics and Check with Rigid Body Tree

Here we calculate the joint angles from the left and right foot trajectories.

We use an analytic solution of inverse kinematics for the leg based on Closed-Form Inverse Kinematic Joint Solution for Humanoid Robots by Ali et. al. The function for inverse kinematics is called `invKinBody2Foot`. The input to the simulation will be the feet transform matrix that contain position and orientation information.

Finidng the analytic solution to the inverse kinematics requires deriving an expression based on the forward kinematics, which can be time consuming, difficult, and even impossible for some robot geometries and/or constraints. In that case, you may want to consider using the optimization-based `inverseKinematics` solver from Robotics System Toolbox. Although it is slower than the analytical solution, it can be applied to arbitrary kinematic chains with constraints.


```
animateOn = true;
speedupfactor = 12; % animation speed up

% load default parameters if previous script wasn't run
if ~exist('footinfos','var')
    load('defaultfootinfos.mat')
end

% NOTE: make sure parameters match in inverse kinematics function
L1 = 0.12;
L2 = 0;
L3 = 0.4;
L4 = 0.38;
L5 = 0;
```

Set up Rigid Body Tree

```
robot = rigidBodyTree;
```

The Denavit-Hartenberg (DH) parameters for the kinematic tree are defined as below. If you want to know more about this, refer to the Build Manipulator Robot Using Denavit-Hartenberg Parameters page in the documentation.

NOTE: The `setFixedTransform` function ignores rotation in `theta` because this angle depends on joint configurations. So we need to keep this in mind and offset joint angles by any nonzero `theta` values in the reference paper for those joints.

Right Leg

```
dhparams = [L1      0       -L2     0;      % Base -> hip yaw
            0       -pi/2   0       0;      % Hip yaw -> hip roll
            0       -pi/2   0       0;      % Hip roll -> hip pitch
```

```
            L3      0        0       0;      % Hip pitch -> knee pitch
            L4      0        0       0;      % Knee pitch -> ankle pitch
            0       pi/2     0       0;      % Ankle pitch -> ankle roll
            0       0        L5      0];     % Ankle roll -> end effector
(foot)

for idx = 1:size(dhparams,1)
    rightLeg(idx) = rigidBody("rightleg"+idx);
    rightJnt(idx) = rigidBodyJoint("rightjnt"+idx, 'revolute');
    setFixedTransform(rightJnt(idx),dhparams(idx,:),'dh');
    rightLeg(idx).Joint = rightJnt(idx);
    if idx==1
        addBody(robot,rightLeg(idx),"base");
    else
        addBody(robot,rightLeg(idx),"rightleg"+(idx-1));
    end

end
```

Left Leg

```
dhparams = [-L1     0        -L2     0;     % Only difference with right leg
is the
            0       -pi/2    0       0;     % first element is -L1 instead
of L1
            0       -pi/2    0       0;
            L3      0        0       0;
            L4      0        0       0;
            0       pi/2     0       0;
            0       0        L5      0];

for idx = 1:size(dhparams,1)
    leftLeg(idx) = rigidBody("leftleg"+idx);
    leftJnt(idx) = rigidBodyJoint("leftjnt"+idx, 'revolute');
    setFixedTransform(leftJnt(idx),dhparams(idx,:),'dh');
    leftLeg(idx).Joint = leftJnt(idx);
    if idx==1
        addBody(robot,leftLeg(idx),"base");
    else
        addBody(robot,leftLeg(idx),"leftleg"+(idx-1));
    end
end
```

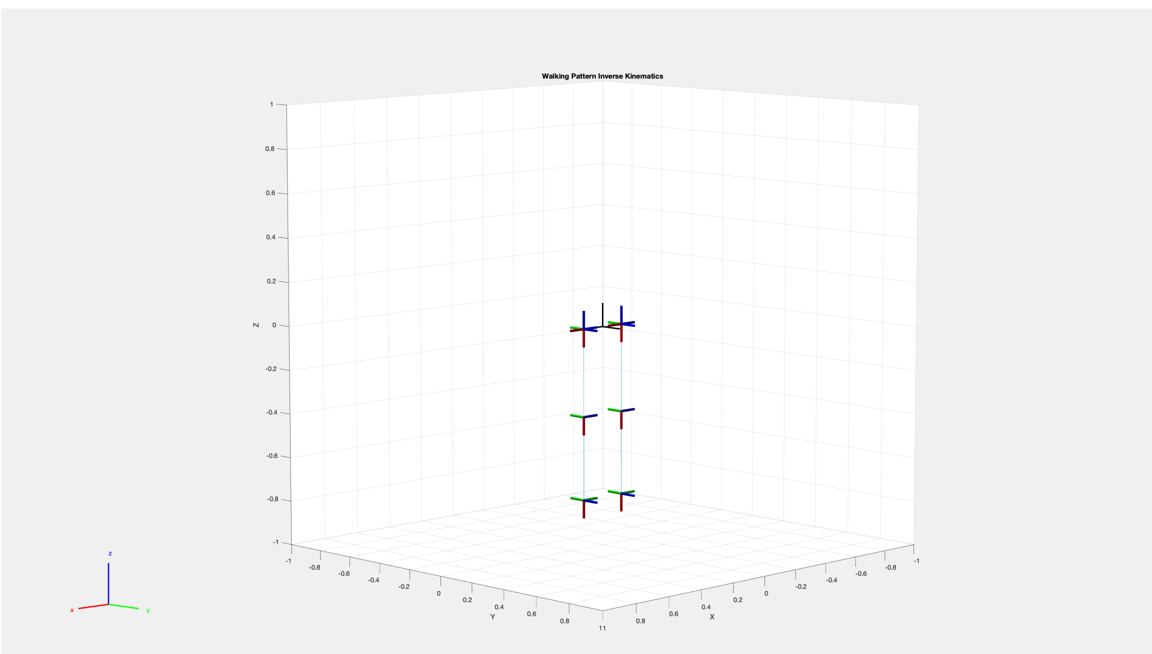
Show the Rigid Body Tree

```
showdetails(robot)
```

```
--------------------
Robot: (14 bodies)

 Idx        Body Name        Joint Name        Joint Type        Parent Name(Idx)   Children Name(s)
```

```
 ---         ---------        ----------        ----------          ----------------    ----------------
   1         rightleg1         rightjnt1          revolute                   base(0)    rightleg2(2)
   2         rightleg2         rightjnt2          revolute              rightleg1(1)    rightleg3(3)
   3         rightleg3         rightjnt3          revolute              rightleg2(2)    rightleg4(4)
   4         rightleg4         rightjnt4          revolute              rightleg3(3)    rightleg5(5)
   5         rightleg5         rightjnt5          revolute              rightleg4(4)    rightleg6(6)
   6         rightleg6         rightjnt6          revolute              rightleg5(5)    rightleg7(7)
   7         rightleg7         rightjnt7          revolute              rightleg6(6)
   8          leftleg1          leftjnt1          revolute                   base(0)    leftleg2(9)
   9          leftleg2          leftjnt2          revolute               leftleg1(8)    leftleg3(10)
  10          leftleg3          leftjnt3          revolute               leftleg2(9)    leftleg4(11)
  11          leftleg4          leftjnt4          revolute              leftleg3(10)    leftleg5(12)
  12          leftleg5          leftjnt5          revolute              leftleg4(11)    leftleg6(13)
  13          leftleg6          leftjnt6          revolute              leftleg5(12)    leftleg7(14)
  14          leftleg7          leftjnt7          revolute              leftleg6(13)
--------------------
```

```
hFig = figure;
hFig.Visible = 'on';
hFig.Units = 'Normalized';
hFig.OuterPosition = [0 0 1 1];
desconfig = robot.homeConfiguration;

qright0 = zeros(1,6);
qleft0 = zeros(1,6);
updateJoints(robot, qright0, qleft0)
```

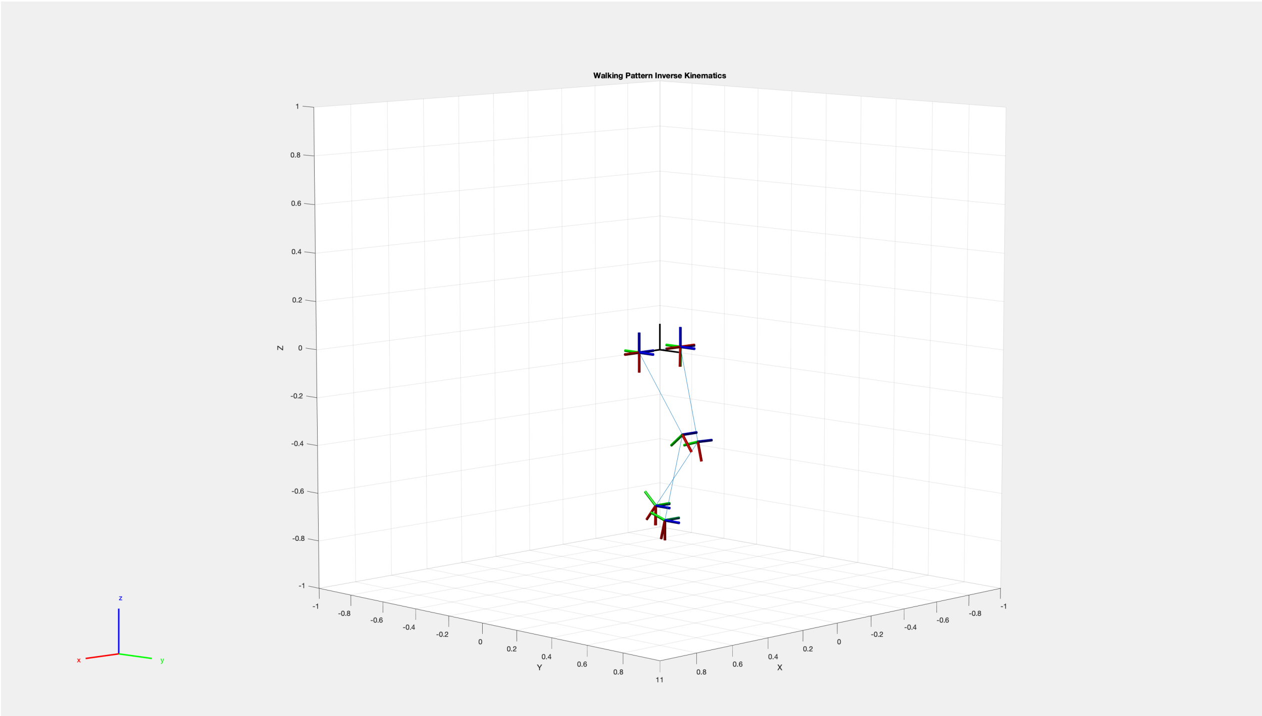


Animate Joint Trajectory

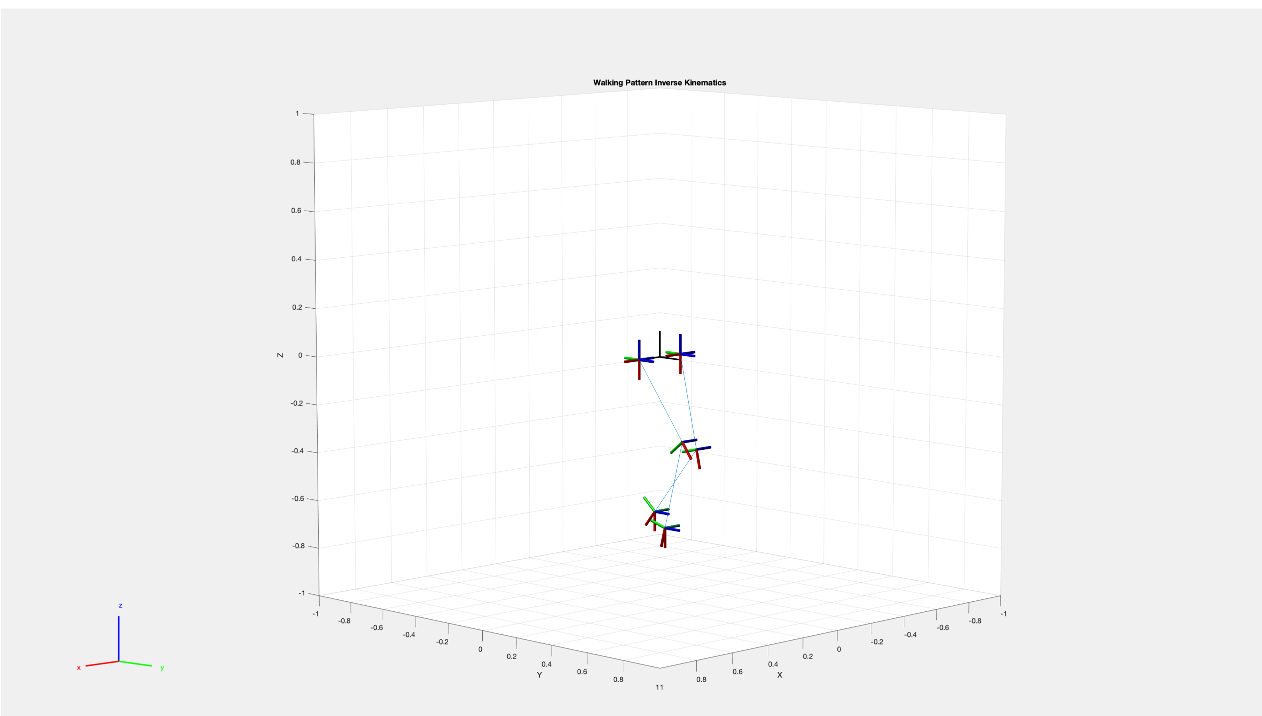
Note that feet orientation is always normal to the ground. Also note the Z vector of the end effector is pointed toward the ground.

```
n = [0;  0; -1]; % x
s = [-1; 0; 0];  % y
```

```
a = [0;  1; 0];  % z
R = [n s a];

for sIdx = 1:length(footinfos)
    % Extract X Y Z position states (indices 1, 3, and 5)
    stateL = footinfos{sIdx}.footleft([1 3 5],:);
    stateR = footinfos{sIdx}.footright([1 3 5],:);

    % Initialize matrices
    numIdx = size(stateL,2);
    jointsLeft = zeros(6,numIdx);
    jointsRight = zeros(6,numIdx);
    transMatLeft = zeros(4,4,numIdx);
    transMatRight = zeros(4,4,numIdx);

    % Skip some intermediate steps when visualizing
    for idx = 1:numIdx
        % Get Left joints
        p = stateL(:,idx);
        transmat =  [R     p;
                     [0 0 0 1]];
        isLeft = true;
        qLeft = invKinBody2Foot(transmat, isLeft); % Call IK function
        jointsLeft(:,idx) = qLeft;
        transMatLeft(:,:,idx) = transmat;

        % Get Right joints
        p = stateR(:,idx);
        transmat =  [R     p;
                     [0 0 0 1]];
        isLeft = false;
        qRight = invKinBody2Foot(transmat, isLeft);
        jointsRight(:,idx) = qRight;
        transMatRight(:,:,idx) = transmat;

        % Animate
        if animateOn
            if rem(idx,speedupfactor) == 0
                updateJoints(robot, qRight, qLeft);
            end
        end
    end
    % save joints info
    footinfos{sIdx}.jointsleft = jointsLeft;
    footinfos{sIdx}.jointsright = jointsRight;
    footinfos{sIdx}.transmatleft = transMatLeft;
    footinfos{sIdx}.transmatright = transMatRight;
end
```




```
% last update of Animation in case Animation is off
updateJoints(robot, qRight, qLeft);
```



Create Simulation Input (End Effector)

The two variables used inside the simulation are end effector position and orientation information for the left leg and the right leg. This end effector position and orientation information is in the form of 4-by-4 transformation matrices.

```
timeVec = footinfos{1}.timevec;
transMatLeft = footinfos{1}.transmatleft;
transMatRight = footinfos{1}.transmatright;

siminL.time = timeVec;
siminL.signals.values = transMatLeft;
siminL.signals.dimensions = [4,4];

siminR.time = timeVec;
siminR.signals.values = transMatRight;
siminR.signals.dimensions = [4,4];

for idx = 2:length(footinfos)
    timeVec = footinfos{idx}.timevec;

    transMatLeft = footinfos{idx}.transmatleft;
    transMatRight = footinfos{idx}.transmatright;

    siminL.time = [siminL.time timeVec];
    siminL.signals.values = cat(3,siminL.signals.values, transMatLeft);

    siminR.time = [siminR.time timeVec];
    siminR.signals.values = cat(3,siminR.signals.values, transMatRight);
end
```

Helper Functions

```
function updateJoints(robot, anglesright, anglesleft)

    desconfig = robot.homeConfiguration;

    desconfig(1).JointPosition = pi; % angle offset
    for idx = 1:length(anglesright)
        desconfig(idx+1).JointPosition = anglesright(idx);
    end
    desconfig(2).JointPosition = desconfig(2).JointPosition - pi;
    desconfig(3).JointPosition = desconfig(3).JointPosition + pi/2;

    desconfig(8).JointPosition = pi;
    for idx = 1:length(anglesleft)
        desconfig(idx+8).JointPosition = anglesleft(idx);
    end
    desconfig(9).JointPosition = desconfig(9).JointPosition - pi;
    desconfig(10).JointPosition = desconfig(10).JointPosition + pi/2;

    % update graphics
    show(robot, desconfig, 'PreservePlot', false);
    title('Walking Pattern Inverse Kinematics')
    pause(0.001)
```

```
end
```