Practice Problems: Week 5 (Submission not required)

1. Solve the following differential equations using Euler's method, Improved Euler's method and 4th-order Runge-Kutta method for the given initial condition and range with number of steps $n_{\max} = 100$. Compare your numerical result with corresponding analytical result on a plot:

(a)	$\dot{x}(t) = -x\,t;$	$x(0) = 1$	$t \in [0, 5]$
(b)	$\dot{x}(t) = 1 - t;$	$x(0) = 0$	$t \in [0, 1]$
(c)	$\dot{x}(t) = \cos(t)$	$x(\pi) = 1$	$t \in [\pi, 3\pi]$
(d)	$\dot{x}(t) = \operatorname{sech}^2(t)$	$x(1) = 1$	$t \in [-1, 1]$

Find the Global Mean error for each of these cases in each of the methods. An examples is shown below:

Solution (a) using RK4: Using RK4 method and the Global Mean Error function we have

In[∘]:=

```
rk4[F_, X0_, tf_, nMax_] := Module[{h, datalist, prev, rate1, rate2, rate3, rate4, next},
  h = (tf - X0[[1]]) / nMax // N;
  For[datalist = {X0},
   Length[datalist] ≤ nMax,
   AppendTo[datalist, next],
   prev = Last[datalist];
   rate1 = F@prev;
   rate2 = F@(prev + h/2 rate1);
   rate3 = F@(prev + h/2 rate2);
   rate4 = F@(prev + h rate3);
   next = prev + h/6 (rate1 + 2 rate2 + 2 rate3 + rate4);
  ];
  Return[datalist];
 ]
```

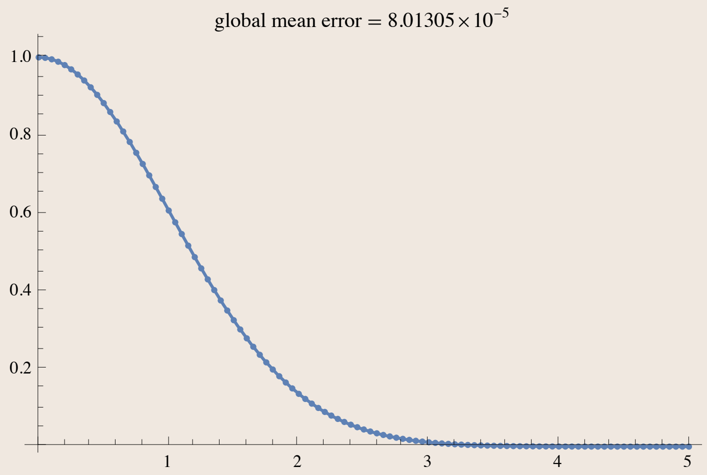
In[∘]:=

```
err[dataset_, func_] := Module[{tlist, xlist, Fxlist},
  tlist = dataset[[ ;; , 1]];       (*Extract each time value*)
  xlist = dataset[[ ;; , 2]];       (*Extract each x value*)
  Fxlist = func /@ tlist;           (*Apply func to each time value to get list of func[ti]*)
  Return[xlist - Fxlist // Abs // Mean];
 ]
```

In[∘]:=

```
solx[t_] := e^(-t^2/2);
rateFunc[{t_, x_}] = {1, -x t};
initial = {0, 1};
tf = 5;
nMax = 100;
data = rk2[rateFunc, initial, tf, nMax];
Show[ListPlot[data, PlotRange → Full], Plot[solx[t], {t, 0, 5}],
 PlotLabel → "global mean error" == ScientificForm[err[data, solx]]]
```

Out[∘]=



2. Solve the following differential equation

$$\dot{x}(t) = x^2 - t^2$$

using Improved Euler and RK4 method for $t \in [0, 1]$ for the following initial conditions

(a) $x(0) = 1$

(b) $x(0) = 1/2$

(c) $x(0) = 0$

(d) $x(0) = -1$

Compare the solutions you got with Euler Method and NDSolve.