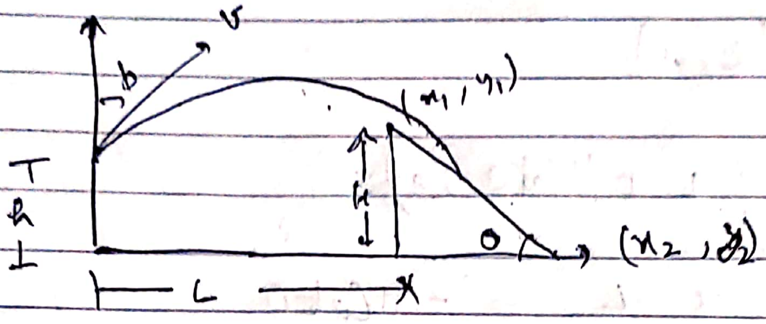
1)



a) x - comp. of velocity $\rightarrow$ $v\sin\phi$

y - comp. of velocity $\rightarrow$ $v\cos\phi$

Along x-axis, distance transverse by ball is

$$x = v\sin\phi\, t \Rightarrow t = \frac{v\sin\phi}{x}$$

Along y-axis

$$y - h = t\, v\cos\phi - \frac{1}{2} g t^2$$

$$\Rightarrow \boxed{y = x\cot\phi - \frac{g x^2}{2v^2\sin^2\phi} + h} \quad -(1)$$

b) $(x_1, y_1) \equiv (L, H)$

$(x_2, y_2) \equiv (L + H\cot\theta, 0)$

$\therefore$ eqⁿ $\rightarrow$ $\dfrac{x - L}{y - H} = -H\cot\theta$

$\Rightarrow$ $x - L = -(y - H)\cot\theta$

$\Rightarrow$ $\boxed{y = H - (x - L)\tan\theta}$

c) Non-dim. eqⁿ of trajectory of ball

$$\frac{y}{h} = \frac{x}{h}\cot\phi - \frac{g x^2}{2U^2 h \sin^2\phi} + 1$$

Take $\dfrac{y}{h} = Y$ & $\dfrac{x}{h} = X$

$$\Rightarrow \quad Y = X\cot\phi - \frac{g\ell}{2v^2\sin^2\phi}X^2 + 1$$

let $\quad w = \dfrac{gh}{2v^2}$

$$\therefore \quad Y = X\cot\phi - \frac{wX^2}{\sin^2\phi} + 1$$

$\therefore$ 2 free parameters in eq^n of trajectory ~~trajectory~~ of ball.

Again, Eq^n of slope of hill,

~~Again~~,

$$y = H - (x - L)\tan\theta$$

$$\frac{y}{h} = \frac{H}{h} - \left(\frac{x}{h} - \frac{L}{h}\right)\tan\theta$$

$$\Rightarrow \quad Y = \frac{H}{h} - \left(X - \frac{L}{h}\right)\tan\theta$$

$$Y = w_1 - (X - w_2)\tan\theta$$

$w_1 = \dfrac{H}{h}$; $w_2 = \dfrac{L}{h}$ $\quad \Rightarrow$ 3 free parameters.

(Q)

$$x(t) = A\cos(\omega t) + B\sin(\omega t) \quad \text{—— (1)}$$

$$x(t) = Ce^{i\omega t} + De^{-i\omega t} \quad \text{—— (2)}$$

$$x(t) = F\cos(\omega t + \phi) \quad \text{—— (3)}$$

$$x(t) = G\sin(\omega t + \phi) \quad \text{—— (4)}$$

From (1) & (2)

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$= (C + D)\cos(\omega t) + i(C - D)\sin(\omega t)$$

Comparing both sides

$$A = C + D$$

$$B = i(C - D) \Rightarrow C - D = -iB$$

$$\Rightarrow \boxed{C = \frac{A - iB}{2}}$$

$$\boxed{D = \frac{A + iB}{2}}$$

Again from (2) & (3)

$$(C+D)\cos\omega t + i(C-D)\sin\omega t$$

$$= F\cos\omega t\cos\phi - F\sin\omega t\sin\phi$$

$$\therefore \quad F\cos\phi = C+D = A$$
$$-F\sin\phi = i(C-D) = B$$

$$\therefore \quad \tan\phi = -\frac{B}{A}$$

$$\boxed{\phi = -\tan^{-1}\frac{B}{A}}$$

Also, $F^2 = A^2 + B^2$

$$\Rightarrow \boxed{F = \pm\sqrt{A^2+B^2}}$$

From (2) & (4)

$$(C+D)\cos\omega t + i(C-D)\sin\omega t$$

$$= G\sin\omega t\cos\psi + G\cos\omega t\sin\psi$$

$\therefore G\cos\psi = i(C-D) = B$

$G\sin\psi = A$

$\Rightarrow \tan\psi = \frac{A}{B}$

$$\boxed{\psi = \tan^{-1}\frac{A}{B}}$$

Also,

$$\boxed{G = \pm\sqrt{A^2+B^2}}$$

$C = \frac{A-iB}{2}$

$D = \frac{A+iB}{2}$

$\phi = -\tan^{-1}\frac{B}{A}$

$F = \pm\sqrt{A^2+B^2}$

$\psi = \tan^{-1}\frac{A}{B}$

$G = \pm\sqrt{A^2+B^2}$

3) Exact diff. form of pendulum eq^n is,

$$\ddot{\theta} = -\frac{g}{l} \sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{l} \sin\theta$$

Multiply both sides by $2\frac{d\theta}{dt}$

$$\Rightarrow \frac{2d\theta}{dt} \frac{d^2\theta}{dt^2} = -\frac{2g}{l} \sin\theta \frac{d\theta}{dt}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{d\theta}{dt}\right)^2 = -\frac{2g}{l} \sin\theta \frac{d\theta}{dt}$$

Integrating both sides

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{l}\cos\theta + A$$

$$\frac{d\theta}{dt} = \sqrt{A + \frac{2g}{l}\cos\theta}$$

$$\Rightarrow \boxed{t = \frac{d\theta}{\sqrt{A + \frac{2g}{l}\cos\theta}}}$$

3.(b) Small angle approximation :

When θ is very small then $\sin\theta \approx \theta$

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Let the Sol^n be $\theta = Ae^{i\omega t}$

$$\therefore \frac{d\theta}{dt} = Ai\omega e^{i\omega t}$$

$$\frac{d^2\theta}{dt^2} = A(-\omega^2) e^{i\omega t}$$

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

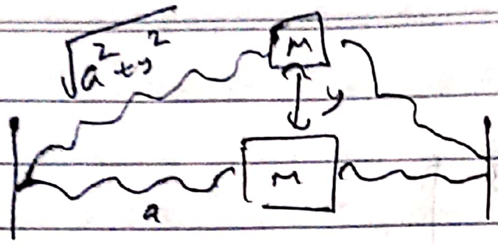
$$\therefore \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

$$\Rightarrow -\omega^2\theta = -\frac{g}{l}\theta$$

$$\Rightarrow \omega = \pm\sqrt{\frac{g}{l}}$$

$\therefore$ General Sol^n: $\boxed{\theta = Ae^{i\omega t} + Be^{-i\omega t}}$

4) (a)



Change in length $\Delta x = \sqrt{a^2+y^2} - a_0$

$$\therefore \quad V(y) = 2 \times \frac{1}{2} k \, \Delta x^2$$

$$= k\left(\sqrt{a^2+y^2} - a_0\right)^2$$

$$= k a_0^2 \left(\sqrt{\frac{a^2}{a_0^2}+\frac{y^2}{a_0^2}} - 1\right)^2$$

(c) Eq^n of motion: can be obtained,

$$M\frac{d^2y}{dt^2} = -\frac{dV(y)}{dy}$$

d) For small ~~osc~~ oscillation. i.e $y \ll a$

$$V(y) \approx k\left(a + \frac{y^2}{2a} - a_0\right)^2$$

$$\left[\sqrt{a^2+y^2} \approx a\sqrt{1+\frac{y^2}{a^2}} \approx a\left(1+\frac{y^2}{2a^2}\right)\right]$$

Again eq of motion!

$$M\frac{d^2y}{dt^2} = 2k\left(a + \frac{y^2}{2a} - a_0\right)\left(\frac{y}{a}\right)$$

$$= 2k\left(\frac{a-a_0}{a} + \frac{y^2}{2a^2}\right)y$$

$$M\frac{d^2y}{dt^2} \approx 2ky\left(\frac{a-a_0}{a}\right)$$

$$\frac{d^2y}{dt^2} = \frac{2k}{M}\left(1-\frac{a_0}{a}\right)y$$

$$\Rightarrow \boxed{\omega_y = \sqrt{\frac{2k}{m}\left(1-\frac{1}{\gamma}\right)}}$$

e) $$\frac{\omega_y}{\omega_x} = \frac{\sqrt{\frac{2k}{m}\left(1-\frac{1}{\gamma}\right)}}{\sqrt{\frac{2k}{m}}}$$

$$= \sqrt{1-\frac{1}{\gamma}}$$