Physics through Computational Thinking

Random walks: First passage

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Outline

In this module we will cover

1. The problem of first passage.

The First Passage Problem

```
Clear["Global`*"]
```

For an investor, often times it is interesting to understand if and when a certain stock price will hit a certain value. Let us model the movement of stock prices as a random walk and formulate this question as a problem of *first passage* (to be defined shortly). Consider a random walk starting at the origin and moving to the right or left with equal probability. Let each random walk run for a maximum of N_{steps} number of time steps. The event of the walker returning to the origin for the first time, is called the first passage.

We are interested in two questions: (i) does first passage happen for a given N_{steps} (ii) the time taken for the first passage when it does happen. In order to examine these questions, numerically generate a large number of random walks N_{walks} starting from the origin.

(a) Suppose you fix $N_{\text{walks}} = 10\,000$. Find the fraction of walks f that have completed first passage for a given N_{steps}. Make a plot of f as a function of N_{steps} in which you allow N_{steps} to vary from 100 to 10000 in steps of 100.

```
data = Table[num = Table[a = {0};
      fp = 0;
      Do[AppendTo[a, a[[n - 1]] + RandomChoice[{1, -1}]];
       If[a[[n]] == 0, fp = 1;
        Break[]], {n, 2, nsteps}];
      fp, {10 000}];
    {nsteps, Total[num] / Length[num] // N}, {nsteps, {10, 50, 100, 200, 500, 1000, 2000, 5000, 10 000}}];

ListPlot[data, Joined → True, PlotMarkers → Automatic];
```

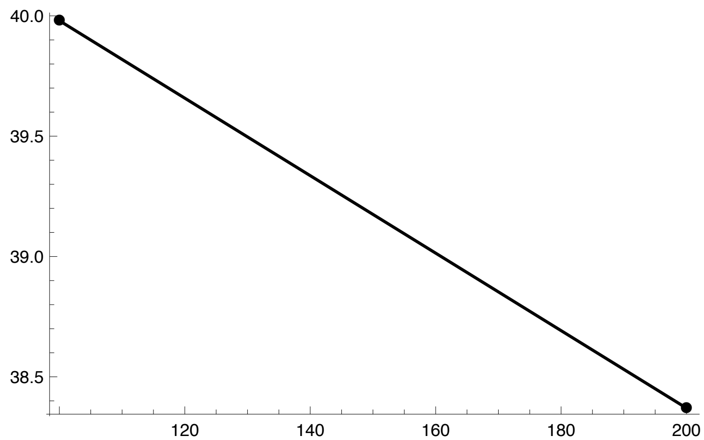
(b) Generate $N_{\text{walks}} = 10\,000$ random walks, and for each of these walks, count the number of time steps t for the random walk to return to the origin for the first time. Continue your random walk until the first passage takes place. Make a list of these times $\{t_1, t_2, \ldots, t_{N_{\text{walks}}}\}$, and make a histogram of this list. Is this a sharply peaked distribution or is it a wide distribution? Find mean value $\langle t \rangle$.

```
data = Table[a = {0};
   AppendTo[a, a[[1]] + RandomChoice[{1, -1}]];
   n = 2;
   While[a[[n]] ≠ 0, AppendTo[a, a[[n]] + RandomChoice[{1, -1}]]; n++];
   tfp = n - 1;
   tfp, {100}];

TableForm[data];

Tally[data // Sort] // TableForm;

Histogram[data];

Mean[data] // N;
```

(c) Repeat **(b)** by varying N_{walks}. For each value of N_{walks} obtain $\langle t \rangle$. Plot ⟨t⟩ as a function of N_{walks}, where N_{walks}goes from 100 to 2000 in steps of 100. Is this a smooth curve? What do you conclude from this plot?

```
means = Table[data = Table[a = {0};
      AppendTo[a, a[[1]] + RandomChoice[{1, -1}]];
      n = 2;
      While[a[[n]] ≠ 0, AppendTo[a, a[[n]] + RandomChoice[{1, -1}]]; n++];
      tfp = n - 1;
      tfp, {nWalks}];
   {nWalks, Mean[data] // N}, {nWalks, 100, 200, 10}];
```

```
ListPlot[means, Joined → True, PlotMarkers → Automatic]
```



(d) What can we say about the mean of the first-passage times? Would it converge if the average is taken over a larger and larger number of random walks?

```
data = Table[a = {0};
   AppendTo[a, a⟦1⟧ + RandomChoice[{1, -1}]];
   n = 2;
   While[a[[n]] ≠ 0, AppendTo[a, a⟦n⟧ + RandomChoice[{1, -1}]]; n++];
   tfp = n - 1;
   tfp, {100}];

means = Table[Mean[Take[data, n]] // N, {n, 1, 100}];

ListPlot[means, Joined → True, PlotMarkers → Automatic];
```