Physics through Computational Thinking
Lecture-20

Random walks

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Outline

In this module, we discuss

1. numerical simulation of random walks to verify some of the main results.

Numerical Simulation of a random walk

Special Case: The unbiased random walk

Let us numerically generate a few sample random walks and visualize them.

```
a = {0};
Do[AppendTo[a, a[[n - 1]] + RandomChoice[{1, -1}]], {n, 2, 500}]
ListPlot[a, Joined → True];
```

Such structures are commonly encountered in the study of polymers, stock price time series data, and trajectories of Brownian particles, to name only a few. We recall the key result

$$< m^2 > = N. \tag{1}$$

Can we verify this with our numerics?

```
data = Table[{n, Table[Table[RandomChoice[{1, -1}], {n}] // Total, {10 000}]^2 // N // Mean}, {n, 100, 1000, 100}];
Show[ListPlot[data], Plot[x, {x, 0, 1500}]];
```

```
nMax = 1000;
binsize = 10;
histdata = Histogram[Table[Table[RandomChoice[{1, -1}], {nMax}] // Total, {10 000}], {binsize}, "PDF"];
```

$$\texttt{Show}\left[\texttt{histdata, Plot}\left[\sqrt{\frac{1}{2\,\pi\,\texttt{nMax}}}\;e^{-x^2/(2\,\texttt{nMax})},\ \left\{x,\ -5\sqrt{\texttt{nMax}},\ 5\sqrt{\texttt{nMax}}\right\}\right]\right];$$

The biased random walk

What about the general biased random walk, for which p maybe different from $\frac{1}{2}$?

```
p = 0.55;
a = {0};
Do[AppendTo[a, a[[n - 1]] + 1 - 2 UnitStep[RandomReal[] - p]], {n, 2, 500}]
ListPlot[a, Joined → True];
```

Homework

Show that for the biased random walk in the limit of large N

$$< m^2 > = N(N-1)(2p-1)^2 \tag{2}$$

```
data = Table[{n, Table[Table[1 - 2 UnitStep[RandomReal[] - p], {n}] // Total, {10 000}]^2 // N // Mean}, {n, 1000, 10 000, 1000}];
Show[ListPlot[data], Plot[x (x - 1) (2 p - 1)^2, {x, 0, 10 000}]];
```

Homework

Work out the distribution $P_N(m)$ for the biased random walk and check that your numerical simulation result agrees with your expectation.