Physics through Computational Thinking

Solutions for Computer Graded Assignment: Week 3

1. It turns out that *Mathematica*'s function called StandardDeviation in fact computes what is called a bias-corrected sample standard deviation. Generate the sequence of squares {1,4,9,16,...,10000}. Directly find the standard deviation of this set of numbers by computing the difference between the mean of the squares and the square of the means, and then taking its square root. Next find the standard deviation of these numbers using the in-built function StandardDeviation. What is the difference between the answers for the standard deviation?

☑ 15.1598

☐ 1.00000

☐ 0.03452

☐ 0.04610

Solution

```
data1 = Table[i^2, {i, 100}];
(* Table of sqaure of number from 1,2,....,100*)
error1 = Sqrt[Mean[data1^2] - (Mean[data1])^2] // N;
(*Standard deviation from normal formula i.e. σ=√(<Xi^2> -<Xi>^2) *)
error11 = N[StandardDeviation[data1]];
(*Standard deviation from direct formula in Mathematica*)
Diff1 = error11 - error1
  (*Differnce between two Standard deviations*)
```

Out[12]=
```
15.1598
```

2. An experiment is conducted 100 times, yielding the values 11(10 times), 11.5(15 times), 12 (50 times), 12.5(15 times), and 13(10 times). Which of the following is a good estimate of the error-bar for the observable?

☑ 0.052705

☐ 0.527046

☐ 0.524404

☐ 0.030541

Solution

In[1]:=
```
A = Join[Table[11, {10}], Table[11.5, {15}],
  Table[12, {50}], Table[12.5, {15}], Table[13, {10}]]
```

Out[1]=
```
{11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11.5, 11.5, 11.5, 11.5, 11.5, 11.5, 11.5, 11.5, 11.5,
 11.5, 11.5, 11.5, 11.5, 11.5, 11.5, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12,
 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12,
 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12.5, 12.5, 12.5, 12.5, 12.5, 12.5, 12.5,
 12.5, 12.5, 12.5, 12.5, 12.5, 12.5, 12.5, 12.5, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13}
```

In[4]:=

```
Sqrt[(Mean[A^2] - Mean[A]^2)/(Length[A] - 1)]
```

$$\text{Sqrt}\left[\frac{\text{Mean}\left[\text{A}^2\right] - \text{Mean}[\text{A}]^2}{\text{Length}[\text{A}] - 1}\right]$$

Out[4]=

```
0.0527046
```

3. In an experiment measuring the constant *g* the following numbers were obtained: 9.4,9.5,9.6,9.7,9.8,9.9,10.0,10.1,10.2,10.3. Which of the following is a good estimate (with error-bar) for *g*, based on the available experimental data?

☐ 9.85±0.009

☐ 9.9±0.287

☐ 9.71±0.01

☑ 9.85±0.096

■ Solution

In[13]:=

```
B = {9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 10, 10.1, 10.2, 10.3}
```

Out[13]=

```
{9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 10, 10.1, 10.2, 10.3}
```

In[14]:=

```
Mean[B]
```

Out[14]=

```
9.85
```

In[15]:=

$$\text{Sqrt}\left[\frac{\text{Mean}\left[\text{B}^2\right] - \text{Mean}[\text{B}]^2}{9}\right]$$

Out[15]=

```
0.0957427
```

4. We wish to create a list of values of the e^{-n^2}, where n goes from -10 to 10 in steps of unity. We think of doing it in two different ways:

- a) Table[Exp[$-n^2$], {n, -10, 10, 1}]
- b) Exp[Table[$-n^2$, {n, -10, 10, 1}]].

Try out both on *Mathematica*, and answer which of the following is correct

☐ Only a is correct.

☐ Only b is correct.

☐ Neither a nor b is correct.

☑ Both a and b are correct.

Solution

In[18]:=

```
Table[Exp[-n^2], {n, -10, 10, 1}]
Exp[Table[-n^2, {n, -10, 10, 1}]]
```

Out[18]=

$$\left\{\frac{1}{e^{100}}, \frac{1}{e^{81}}, \frac{1}{e^{64}}, \frac{1}{e^{49}}, \frac{1}{e^{36}}, \frac{1}{e^{25}}, \frac{1}{e^{16}}, \frac{1}{e^{9}}, \frac{1}{e^{4}}, \frac{1}{e}, 1, \frac{1}{e}, \frac{1}{e^{4}}, \frac{1}{e^{9}}, \frac{1}{e^{16}}, \frac{1}{e^{25}}, \frac{1}{e^{36}}, \frac{1}{e^{49}}, \frac{1}{e^{64}}, \frac{1}{e^{81}}, \frac{1}{e^{100}}\right\}$$

Out[19]=

$$\left\{\frac{1}{e^{100}}, \frac{1}{e^{81}}, \frac{1}{e^{64}}, \frac{1}{e^{49}}, \frac{1}{e^{36}}, \frac{1}{e^{25}}, \frac{1}{e^{16}}, \frac{1}{e^{9}}, \frac{1}{e^{4}}, \frac{1}{e}, 1, \frac{1}{e}, \frac{1}{e^{4}}, \frac{1}{e^{9}}, \frac{1}{e^{16}}, \frac{1}{e^{25}}, \frac{1}{e^{36}}, \frac{1}{e^{49}}, \frac{1}{e^{64}}, \frac{1}{e^{81}}, \frac{1}{e^{100}}\right\}$$

5. Use the Table command to create a list of values of the function e^{-n^2}, where n goes from -10 to 10 in steps of unity. Numerically evaluate the mean and standard deviation of this list. *Note:* Do not blindly use the *Mathematica* function for standard deviation, because it gives what is called a bias-corrected sample standard deviation, which is not what we want.

☐ Mean is 0.0348761 and standard deviation is 0.789645.

☑ Mean is 0.0844113 and standard deviation is 0.231116.

☐ Mean is 0.0189643 and standard deviation is 0.132456.

☐ Mean is 0.0102304 and standard deviation is 0.214852.

Solution

In[20]:=

```
data5 =
  Table[Exp[-n^2], {n, -10, 10, 1}];
(*Data for question no. 5*)
Mean[data5]  //
 N
(*The required MEAN of the data5*)
error5 = Sqrt[Mean[data5^2] - (Mean[data5])^2] // N
(*Standard Deviation from normal formula =√(<Xi^2> -<Xi>^2) *)
```

Out[21]=

```
0.0844113
```

Out[22]=

```
0.231116
```

6. Let us superpose the waves $x_1(t) = A_1 \cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2 \cos(\omega_2 t + \phi_2)$. Play with Lissajous figures obtained from each of the following parameters. Which of the following yields a straight line?

☐ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = \pi,\ \phi_2 = 2$

☐ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = 2,\ \phi_2 =$ RandomReal[] π

☑ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = \pi,\ \phi_2 = 2\pi$

☐ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = 1,\ \phi_2 = 2\pi$

■ Solution

```
A1 = 1; ω1 = π; ϕ1 = 0;
A2 = 1; ω2 = π; ϕ2 = 2 π;
f[t_] = A1 Cos[ω1 * t + ϕ1];
g[t_] = A2 Cos[ω2 * t + ϕ2];
h[t_] = f[t] + g[t];
Plot[{h[t]}, {t, 0, 10 π}, PlotStyle → Thick];
ParametricPlot[{f[t], g[t]}, {t, 0, 100}, AxesLabel → {"f", "g"}];
(*Parameteric plot of two function *)
```

7. If we superpose the waves $x_1(t) = A_1 \cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2 \cos(\omega_2 t + \phi_2)$, which of the following yields a circle?

☐ $A_1 = 1,\ \omega_1 = 1,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = 1,\ \phi_2 = 2$

☑ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = \pi,\ \phi_2 = \frac{\pi}{2}$

☐ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = 0,\ A_2 = 1,\ \omega_2 = 2\pi,\ \phi_2 = \frac{\pi}{2}$

☐ $A_1 = 1,\ \omega_1 = \pi,\ \phi_1 = \pi,\ A_2 = 1,\ \omega_2 = \text{RandomReal}[]\,\pi,\ \phi_2 = \text{RandomReal}[]\,\pi$

■ Solution

```
A1 = 1; ω1 = π; ϕ1 = 0;
A2 = 1; ω2 = π; ϕ2 = 2 π;
f[t_] = A1 Cos[ω1 * t + ϕ1];
g[t_] = A2 Cos[ω2 * t + ϕ2];
h[t_] = f[t] + g[t];
Plot[{h[t]}, {t, 0, 10 π}, PlotStyle → Thick];
ParametricPlot[{f[t], g[t]}, {t, 0, 100}, AxesLabel → {"f", "g"}];
(*Parameteric plot of two function *)
```

8. A ball is dropped from atop a tower. Its position is measured after 1s, and in units of 1s thereafter. The values obtained are as follows:

```
5.31118
19.9166
44.6343
79.1902
123.028
176.601
240.228
313.908
397.164
490.82
```

Use the Fit function to fit with a quadratic function of the form: $a + b\,t + \frac{1}{2}\,g\,t^2$. What is the value of g that you extract from this fitting procedure? You can cross-check using the FindFit function.

□ 9.8000

□ 9.3248

☑ 9.8154

□ 9.7963

■ Solution

In[44]:=

```
data8 = {5.31118, 19.9166, 44.6343, 79.1902, 123.028, 176.601, 240.228,
   313.908, 397.164, 490.82};                (*data set for question 8*)
Fitt8[x_] = FindFit[data8, {a + b x + 0.5 c x^2}, {a, b, c},
  x]
(*Fitting for data8 set with curve f(t)=a+bt+1/2gt^2 *)
```

Out[45]=

```
{a → 0.604207, b → -0.0856191, c → 9.81542}
```

9. Generate a table with a hundred random numbers using the RandomReal[] function. Find its sum. Next find the sum of two hundred random numbers. Suppose we do this all the way up to 1000, and make a table of the sums. One way to do this would be with the code:

```
totals=Table[Total[Table[RandomReal[],{n}]],{n,100,1000,100}]
```

Now fit the resulting list to the form: $f(n) = a\,n^b$. The exponent b is closest to:

□ 2

□ $\frac{1}{2}$

☑ 1

□ $\frac{3}{2}$

■ Solution

In[46]:=

```
data9 = Table[Total[Table[RandomReal[], {n}]],
   {n, 100, 1000, 100}];                              (*data set for question 9*)
fitt9 = FindFit[data9, {a x^b}, {a, b},
  x]
(*Fitting for data8 set with the curve f(n)=an^b*)
```

Out[47]=

```
{a → 47.778, b → 1.02352}
```

10. Generate a table with a hundred random numbers using the RandomReal[] function. Find the square of its sum. Next find the square of the sum of two hundred random numbers. Suppose we do this all the way up to 1000, and make a table of the squares of the sums of n random numbers. Let n run from 100 to 1000 in steps of 100.

Fit the resulting list to the form: $f(n) = a\,n^b$. The exponent b is closest to:

☑ 2

☐ $\frac{1}{2}$

☐ 1

☐ $\frac{3}{2}$

Solution

In[48]:=

```
data10 = Table[(Total[Table[RandomReal[], {n}]])^2,
   {n, 100, 1000, 100}];                        (*data set for question 10*)
fitt10 = FindFit[data10, {a x^b}, {a, b},
  x]
(*Fitting for data8 set with the curve f(n)=an^b *)
(***ANSWER=b→2***)
```

Out[49]=

```
{a → 2688.06, b → 1.96195}
```