Computer Graded Assignment: Week 1

1. The 100^{th} digit in $\frac{\pi^2}{6}$ is

☑ 5

☐ 0

☐ 4

☐ 8

Solution: Use $N[\pi^2/6, 101]$ to to see 101 significant digits. The second last digit is your answer

In[∘]:=
```
N[π^2/6, 101]
```

Out[∘]=
```
1.6449340668482264364724151666460251892189499012067984377355582293700074704032008738336289006197587053
```

2. The curves $y = x^4$ and $y = e^{x/4}$

☐ never intersect with each other

☐ intersect exactly once with each other

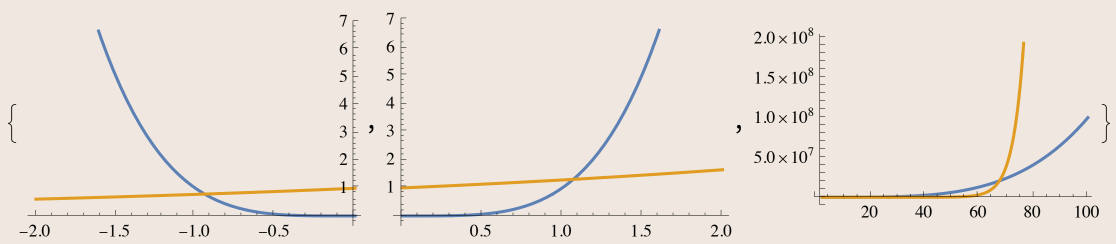
☐ intersect exactly twice with each other

☑ intersect exactly thrice with each other

Solution: On the positive-x side, $e^{x/4}$ will dominate x^p for any p. We can easily find the intersections by plotting the function and keeping the large x-behaviour in mind. The best approach is to do an analysis region-wise. You can also use NSolve as shown below:

In[∘]:=
```
{Plot[{x^4, e^(x/4)}, {x, -2, 0}], Plot[{x^4, e^(x/4)}, {x, 0, 2}], Plot[{x^4, e^(x/4)}, {x, 2, 100}]}
```

Out[∘]=



In[∘]:=
```
NSolve[x^4 == e^(x/4), x, Reals]
```

Out[∘]=
```
{{x → -0.942779}, {x → 1.0691}, {x → 67.3611}}
```

3. The smallest value of x for the point of intersection of the curves $y = x^{1/3}$ and $y = \log(x)$ is

☐ 6.5074

☑ 6.4057

☐ 93.355

☐ 95.353

Solution:

In[∘]:=
```
NSolve[x^(1/3) == Log[x], x, Reals]
```

Out[∘]=
```
{{x → 6.40567}, {x → 93.3545}}
```

4. Asymptotes for $\coth^{-1}(x)$ are

☐ $y = 1, y = -1$ and $x = 0$

☑ $x = 1, x = -1$ and $y = 0$

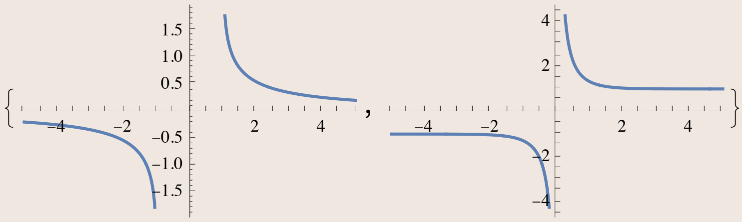
☐ $x = 1$ and $y = 1$

☐ $x = 0$ and $y = 0$

Solution: It is evident from plotting. Also plotting the inverse of this function Coth[x] helps in identifying its asymptotes and confirming the results.

In[∘]:=
```
{Plot[ArcCoth[x], {x, -5, 5}], Plot[Coth[x], {x, -5, 5}]}
```

Out[∘]=



5. For the function $e^{-x/4}\cos(x)$, the distance between two consecutive minima is

☐ π

☐ $2\tan^{-1}\left(\frac{-1}{4}\right)$

☑ 2π

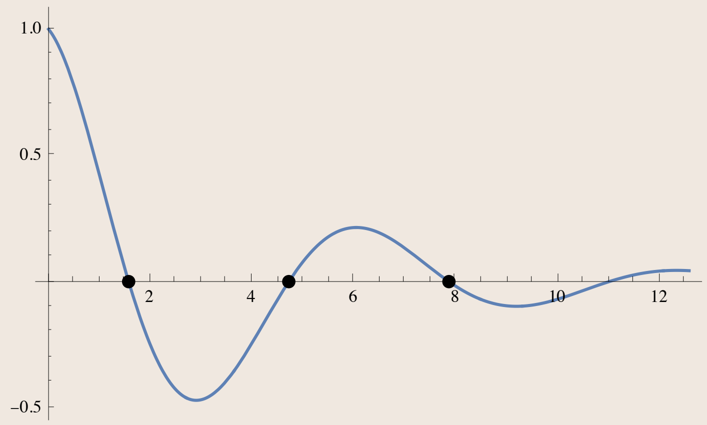
☐ $\cos^{-1}\left(\frac{4}{\sqrt{17}}\right)$

Solution: $e^{-x/4}$ provides an envelope to $\cos(x)$ which has period 2π. $e^{-x/4}$ is monotonically decreasing thus minima and maxima positions are determined by cosine function. This is evident from plotting

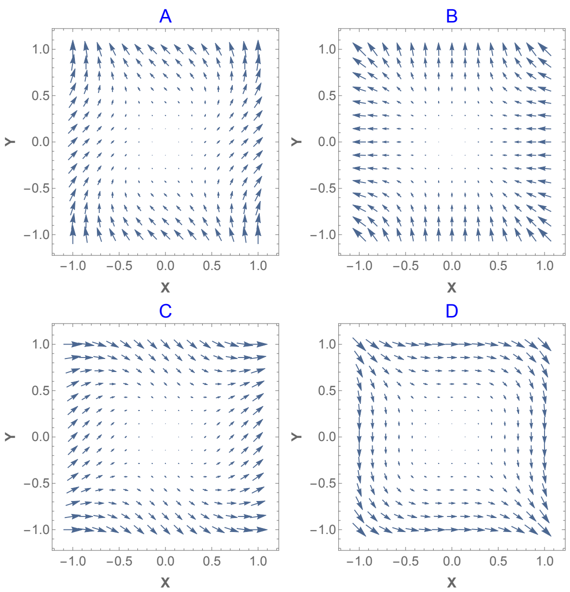
In[∘]:=

```
Plot[e^(-x/4) Cos[x], {x, 0, 4π},
 Epilog → {PointSize[0.02], Point[{π/2, 0}], Point[{3π/2, 0}], Point[{5π/2, 0}]}]
```

Out[∘]=



6. Which of the following images represent plot of the vector field $\vec{v}(\vec{r}) = \left(x^2 + y^2\right)\hat{\imath} + \left(x^2 - y^2\right)\hat{\jmath}$?



☐ A

☐ B

☑ C

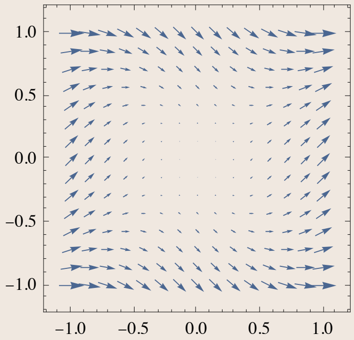
☐ D

Solution: This is evident from plotting the vector field

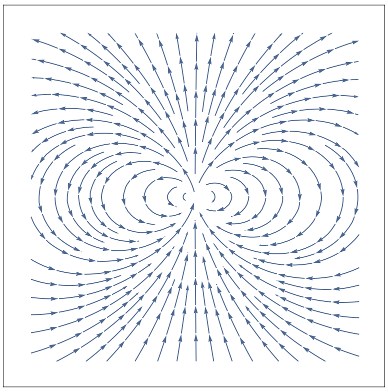
In[∘]:=

```
VectorPlot[{x^2 + y^2, x^2 - y^2}, {x, -1, 1}, {y, -1, 1}, ImageSize → Small]
```

Out[∘]=



7. Stream Plot shown below represents field lines for



☐ a magnetic monopole

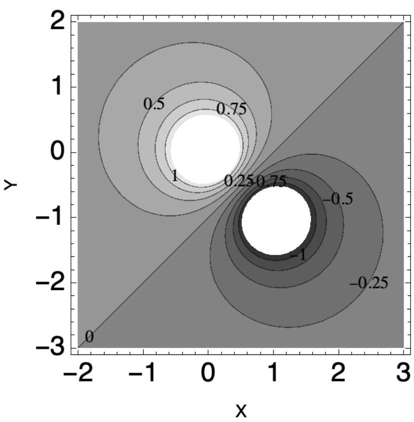
☑ a magnetic dipole

☐ an electric quadrupole

☐ a pair of positive electric charges

Solution: See solution for Week 1 Practice problem 8.

8. In the equipotential lines plot shown below for electric dipole in the *x*–*y* plane, the negative charge is placed at



☐ (0 , 0)

☐ (0 , 1)

☐ $(-1\,,\ 1)$

☑ $(1\,,\ -1)$

Solution: The potential near negative charge is negative and equipotential contours are centered about the charges.

9. If the potential $u(r) = -\frac{1}{r^2} + \frac{2}{r^4}$ for positive r, near its minimum r_0 is approximated by the quadratic potential $u(r) \approx u_0 + \alpha(r - r_0)^2$, then the value of α is

☐ $\frac{-1}{8}$

☐ 2

☑ $\frac{1}{8}$

☐ -2

Solution: You need to do a Taylor expansion about the minima at $r = 2$. You can also use Series function as shown below

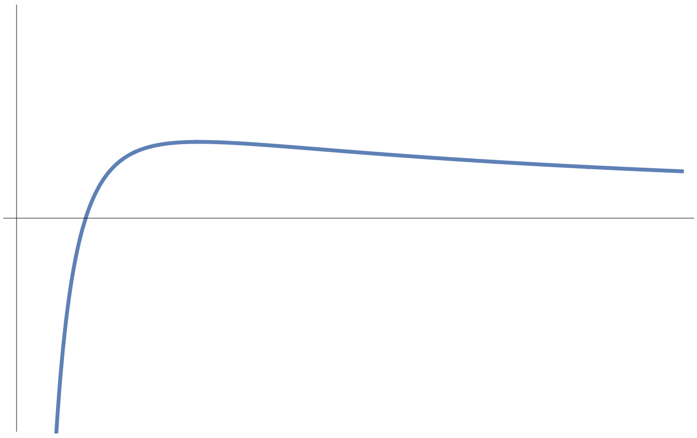
In[∘]:=

$$\text{Series}\left[\frac{-1}{r^2} + \frac{2}{r^4},\ \{r,\ 2,\ 2\}\right]$$

Out[∘]=

$$-\frac{1}{8} + \frac{1}{8}\,(r-2)^2 + O[r-2]^3$$

10. Identify the function whose plot is given by the image below:



☐ $x \log(x)$

☐ $|x \log(x)|$

☑ $\frac{\log(x)}{x}$

☐ $\frac{x}{\log(x)}$

Solution: Plotting all the cases reveals the answer. However, you should be able to do this by considering small x and large x behaviour. Absolute values are immediately ruled out because plot shows a negative value.

In[∘]:=

```
Plot[{x Log[x], Abs[x Log[x]], Log[x]/x, x/Log[x]},
  {x, 0, 5}, PlotRange → {-2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=