Physics through Computational Thinking

Practice Problems: Week 1 (Submission not required)

1. Explore numerical function $N[x]$.

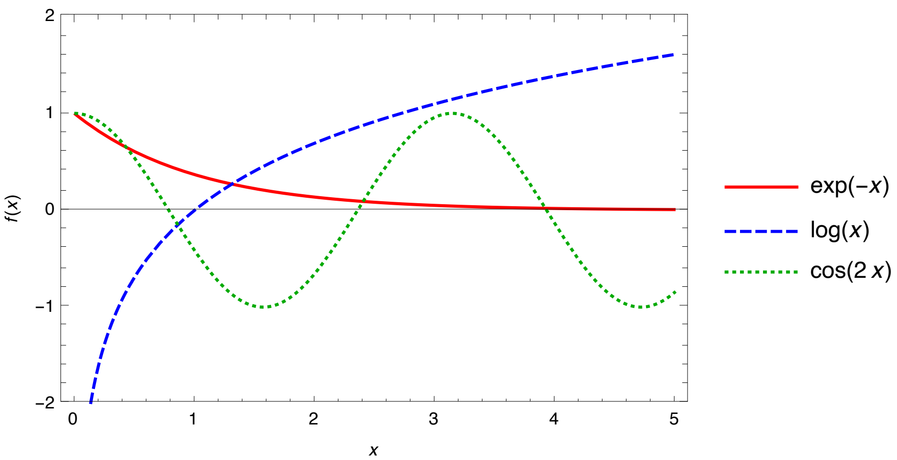
(a) N calculates numerical value of any expression. Lets find out Pi and E (the Euler number e) to 10 digits by evaluating the following commands.

```
N[Pi]
N[Pi, 10]
N[E, 10]
```

(b) Find Pi to 100 digits.

(c) Find $2^{1/2}$ and $2^{1/3}$ up to 16 digits.

2. Can you reproduce the plot below by figuring out the suitable Mathematica code (one line only). Reproduce also the plot styling that is x-range, y-range, labeling, colors, line stroke, frame etc. You may need to look up documentation of the Plot function to be able to do this. Its a good idea to start navigating into documentation and also learn how to make your figures look nicer. See if you can figure out a few styling techniques on your own to make the figure look even better that what is presented here.



3. Sketch the following functions, first on a piece of paper analyzing them for their zeros, divergences, extrema and asymptotes. Next cross - check your sketch by plotting the function on Mathematica.

Hyperbolic functions:

1. $\cosh(x)$ 2. $\sinh(x)$ 3. $\tanh(x)$ 4. $\operatorname{cosech}(x)$ 5. $\operatorname{sech}(x)$ 6. $\coth(x)$.

7. $\ln x$ 8. $\ln(\ln(x))$ 9. $\ln(x)/x$ 10. $\ln(e^x - 1)$ 11. $\ln\left(\frac{1-x}{1+x}\right)$ 12. $\frac{1}{x}\ln\left(\frac{1-x}{1+x}\right)$

13. $e^{-x}\cos(x)$ 14. $e^{-x}\sin(x)$ 15. $e^{-|x|}\cos(x)$ 16. $e^{-|x|}\sin(x)$ 17. $x\,e^{-x^2}$ 18. $x - 1 + e^{-x}$

19. x^x 20. $x^{1/x}$ 21. $x^{|x|}$ 22. $|x|^{|x|}$ 23. $\frac{|x|^{1/2}}{1+|x|^{1/2}}$ 24. $\frac{|x|^{\frac{1}{2}}}{e^x+1}$

25. $e^{\frac{1}{x}}$ 26. $e^{\frac{-1}{x^2}}$ 27. $x^{-12} - x^{-6}$ 28. $\cosh^{-1}(x)$ 29. $\coth^{-1}(x)$ 30. $\coth(x) - \frac{1}{x}$

4. For a quadratic function given by $y = a\,x^2 + b\,x + c$, where $a > 0$, find the equation of the minimum $(x_{\min}, y_{\min})$ as a function of b. Now rewrite $y_{\min}$ as a function of $x_{\min}$. What is this function? Can you make a plot to show that the minima of the parabola $y = a\,x^2 + b\,x + c$ lies on the curve $y_{\min}(x_{\min})$? Use Manipulate to vary b and demonstrate that the minima always lies on this curve.

Hint: Try the following code. Can you figure what's going on? Modify this code to manipulate a and c also.

```
Manipulate[Plot[{a x^2 + b x + c, c - a x^2, -b^2/(4 a) + c} /. {a → 2, b → b1, c → 4} // Evaluate,
  {x, -2, 2}, Frame → True, PlotRange → {-5, 10}], {b1, -6, 6}]
```

5. Find the point of intersection for curves $y = \log x$ and $y = x^{1/3}$ up to three decimal accuracy. **Hint:** You can do this by hit and trial or use a built in Mathematica function like **NSolve**.

6. Plot $\cosh(x)$ and its quadratic approximation at $x = 0$. Find the fractional deviation the quadratic approximation near $x = 0$ has with respect to $\cosh(x)$ at $x = 0.5$ and $x = 1$, that is if $y(x)$ represent quadratic approximation to $\cosh(x)$ near $x = 0$, then find $\left|\frac{y(x)-\cosh(x)}{\cosh(x)}\right|_{x=0.5}$ and $\left|\frac{y(x)-\cosh(x)}{\cosh(x)}\right|_{x=1.0}$.

7. Electric field lines of a quadrupole: Plot the electric field lines and equipotential surfaces for the quadrupolar configuration: four charges of same magnitude and alternating sign on the corners of a square of side a, that is, $+q$ at $(0, 0)$ and (a, a) while $-q$ at $(a, 0)$ and $(0, a)$. Use combination of StreamPlot and ContourPlot as shown in the lecture inside a Show function.

8. Magnetic field of a magnetic dipole: Magnetic field of a magnetic dipole is given by

$$\vec{B}(\vec{r}) = \frac{\mu_0 \, |\vec{m}|}{4 \pi r^3} \left(2 \cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right) \tag{1}$$

Plot the magnetic field lines in the x-z plane as vector plot, stream plot and stream density plot. [**Hint:** Convert the magnetic field to Cartesian coordinates first.]