# Physics through Computational Thinking

## Visual Thinking

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1. *learn various ways of visually representing data, information or functions*
2. *apply skills of visual thinking to represent and solve a few maths and physics problems*
3. *apply skills of visual thinking to interpret results from graphs*

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# What is Visual Thinking

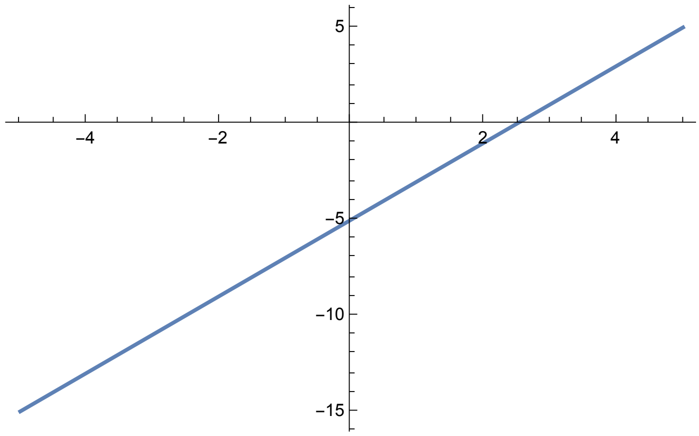
- Visual thinking is ability to represent and interpret data or any other information through a visual medium such as a data chart, a graph, a picture, mind maps or relationship maps, flow charts etc.
- Visual thinking is a way to organize your thoughts and your ability to think and communicate. An image or a graphic is thousand words.
- In this course we will learn a lot of visual thinking through computer and also some off the computer as we go along.

# Plotting a graph

- Lets start out by learning how to plot a simple function, such as a linear function.

In[∘]:=
```
Plot[2 x - 5, {x, -5, 5}, PlotStyle → Thick]
```

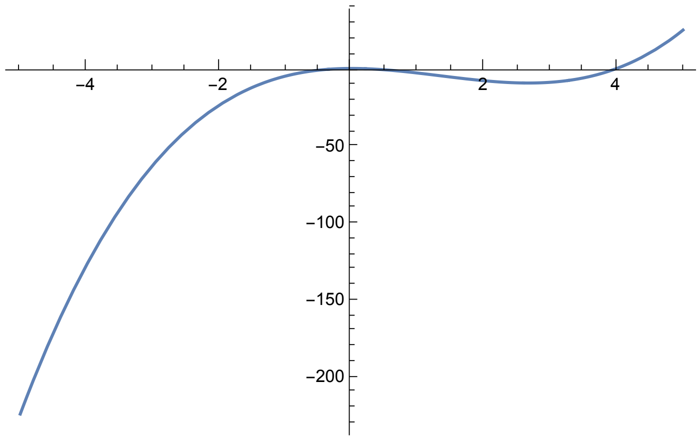
Out[∘]=




- Plot a polynomial

In[∘]:=
```
Plot[x ^ (3) - 4 x ^ 2 + 1, {x, -5, 5}]
```

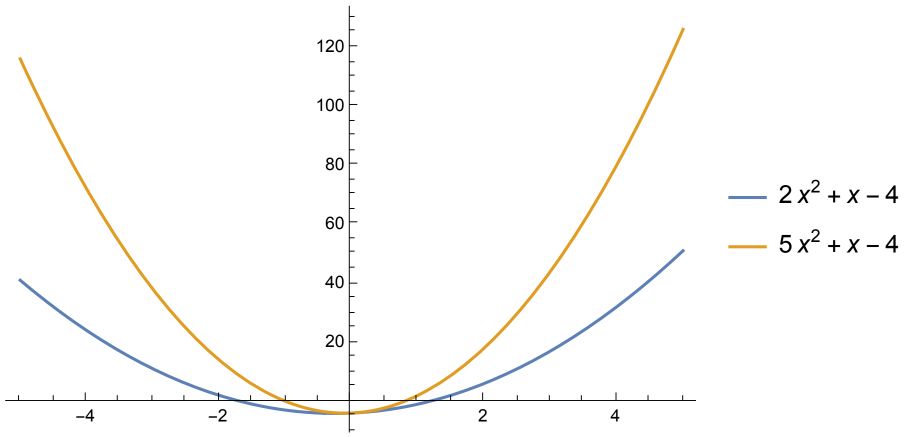
Out[∘]=




- Plot a quadratic function

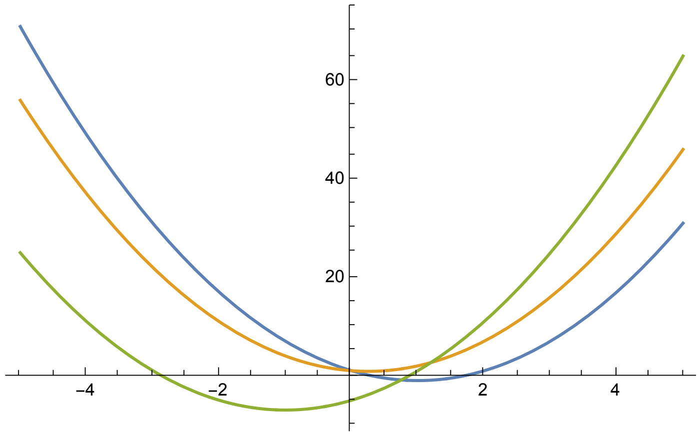
In[∘]:=
```
Plot[{2 x^2 + x - 4, 5 x^2 + x - 4}, {x, -5, 5}, PlotLegends → "Expressions"]
```

Out[∘]=



In[∘]:= `Plot[{2 x^2 - 4 x + 1, 2 x^2 - x + 1, 2 x^2 + 4 x - 5}, {x, -5, 5}]`

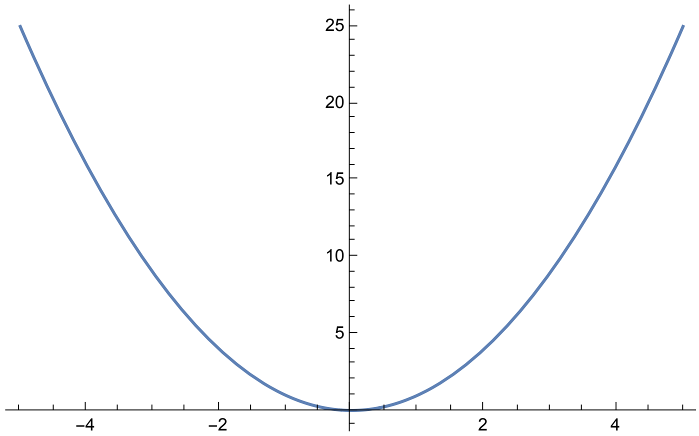
Out[∘]=




- Plot the most simple quadratic function

In[∘]:= `Plot[x^2, {x, -5, 5}]`

Out[∘]=



# Understanding Quadratic Function

- Lets vary the parameters of a general quadratic function using **Manipulate** construct and see what is the effect of each parameter on the function.

$$a x^2 + b x + c = 0$$

In[•]:=
```
Manipulate[Plot[a x^2 + b x + c, {x, -5, 5}, PlotRange → {{-5, 5}, {-10, 10}},
  Frame → True], {{a, 1}, -5, 5}, {{b, 0}, -5, 5}, {{c, 0}, -5, 5}]
```

# Quadratic function & Quadratic-like behaviour

- Quadratic functions are of great importance in physics as ***quadratic potentials*** are associated with ***simple harmonic oscillator*** which has a simple sinusoidal oscillatory motion.
- Thus it is often of great importance to identify regions or domains where other functions behave like a harmonic oscillator potential or quadratic potential.
- Lets consider the following functions

$$f_1(x) = -\cos(x)$$

$$f_2(x) = -\frac{\sin(x)}{x}$$

$$f_3(x) = \frac{-1}{x} + \frac{1}{x^2} \qquad \text{for } x > 0$$

**Exercise:** Plot these functions and identify if these functions have quadratic behaviour at their minima and maxima?

# Function behaviour near $x = 0$

**Exercise:** Can you plot $\sqrt{x}$, $x$, $x^2$, $x^3$, $x^4$ etc. for $x > 0$ on the same plot/graph on paper. Keep in mind how they differ near $x = 0$.

# Radicals and Logarithms

**Exercise 1:** Plot $x$, $\sqrt{x}$ and $\log(x)$. Do they intersect at any point? Plot them and find out how they behave at small $x$ and large $x$?

**Exercise 2:** Plot $x^{1/3}$ and $\log(x)$. Do they intersect at any point?

**Exercise 3:** Visually find the solution of the transcendental equation $x^{1/n} = \log(x)$. For what values of $n$ there are solutions to this equation. When do you have exactly one solution?

```
Plot[{∛x, Log[x]}, {x, 0, 10}, Frame → True, PlotLegends → "Expressions"]
```

```
Manipulate[Plot[{x^(1/n), Log[x]}, {x, 0, 100},
  Frame → True, PlotLegends → {x^(1/"n"), Log[x]}], {{n, 2}, 1, 5}]
```

**Solution 3:** We can also solve this analytically. Let's say that exactly one solution happens for $n = n_0$ and (say) at $x = x_0$. Then, for $n = n_0$ and $x = x_0$ we have both the functions evaluate to the same value and their derivatives also evaluate to the same value, thus

$$x_0{}^{1/n_0} = \log(x_0)$$

$$\text{and,} \qquad \left.\frac{d\, x^{1/n_0}}{d\, x}\right|_{x=x_0} = \left.\frac{d\, \log(x)}{d\, x}\right|_{x=x_0}$$

last equation simplifies to

$$\frac{1}{n_0}\frac{x_0^{1/n_0}}{x_0} = \frac{1}{x_0} \qquad \Rightarrow \qquad x_0^{1/n_0} = n_0 \qquad \Rightarrow \qquad \log(x_0) = n_0 \log(n_0)$$

Substituting in the first equation we get

$$n_0 = n_0 \log(n_0) \qquad \Rightarrow \qquad \log(n_0) = 1 \qquad \Rightarrow \qquad n_0 = e$$

Solving for $x_0$ we get

$$\log\,(x_0) = n_0 \log\,(n_0) \qquad \Rightarrow \qquad \log(x_0) = e \qquad \Rightarrow \qquad x_0 = e^e$$

Numerically

```
In[•]:= N[E]
Out[•]= 2.71828

In[•]:= N[E^E]
Out[•]= 15.1543
```

# Exercises

**1.** Explore numerical function *N*[*x*].

(a) N calculates numerical value of any expression. Lets find out Pi and E (the Euler number *e*) to 10 digits by evaluating the following commands.

```
N[Pi]
```

```
N[Pi, 10]
```

```
N[E, 10]
```

(b) Find Pi to 100 digits.

(c) Find $2^{1/2}$ and $2^{1/3}$ up to 16 digits.

**2.** Can you reproduce the plot below by figuring out the suitable Mathematica code (one line only). Reproduce also the plot styling that is x-range, y-range, labeling, colors, line stroke, frame etc. You may need to look up documentation of the Plot function to be able to do this. Its a good idea to start navigating into documentation and also learn how to make your figures look nicer. See if you can figure out a few styling techniques on your own to make the figure look even better that what is presented here.