Maths

Modern College of Engineering

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Assignment No - 5

Q.1] Reduce the following matrices to echelon form & find its ~~form~~ rank.

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

→ Given matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & +1 & +3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 - 2R_1,\ R_3 - 3R_1,\ R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 - 4R_2,\ R_4 - 9R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

$R_4 - 2R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 / 33$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which echelon form of matrix A.

Q.2 Examine for consistency and if consistent then solve it.

(1) $2x_1 - 3x_2 + 5x_3 = 1$

$3x_1 + x_2 - x_3 = 2$

$x_1 + 4x_2 - 6x_3 = 1$

→ Given system of eq^n,

$2x_1 - 3x_2 + 5x_3 = 1$

$3x_1 + x_2 - x_3 = 2$

$x_1 + 4x_2 - 6x_3 = 1$

It can be written as,

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -1 \\ 1 & 4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$(A|B) = \left[\begin{array}{ccc|c} 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \\ 1 & 4 & -6 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 3 & 1 & -1 & 2 \\ 2 & -3 & 5 & 1 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & -11 & 17 & -1 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A|B) = 2$ ... (cond^n for consistency)

$\therefore r = 2$; number of unknowns $= 3$.

$\therefore r < n$. Therefore system have infinitely many sol^n.

$$x_1 + 4x_2 - 6x_3 = 1$$

$$-11x_2 + 17x_3 = -1$$

Put $\boxed{x_3 = t}$

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$$-11x_2 + 17t = -1$$

$$-11x_2 = -1 - 17t$$

$$x_2 = \frac{-1-17t}{-11} = \boxed{\frac{1-17t}{11}}$$

$$x_1 + 4\left(\frac{1-17t}{11}\right) - 6t = 1$$

$$x_1 + \frac{4-68t}{11} - 6t = 1$$

$$x_1 + \frac{4}{11} - \frac{68t}{11} - \frac{66t}{11} = 1$$

$$\cancel{x_1 + \frac{4}{11} - \frac{2t}{11} = 1}$$

$$\cancel{x_1 = \frac{11}{11} - \frac{4}{11}} + \frac{8t}{11} = \cancel{\frac{7+8t}{11}}$$

$$\boxed{x_1 = \frac{134t-3}{11}}$$

(2) $x - y - z = 2$

$x + 2y + z = 2$

$4x - 7y - 5z = 2$

→

$AX = B$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$$

$R_2 - R_1, R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & -3 & -1 & -6 \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$\rho(A) = \rho(A|B) = 3$

$\therefore r = 3$; n = number of unknowns $= 3$.

$r = n$

System has unique sol^n.

$x - y - z = 2$

$3y + 2z = 0$

$\boxed{z = -6}$

$3y + 2(-6) = 0$

$\boxed{y = 4}$

$x - 4 + 6 = 2$

$\therefore \boxed{x = 0}$

Ans: $x = 0, \ y = 4, \ z = -6$.

Q.3] Investigate the values of λ & μ so that the equation

$2x+3y+5z=9$

$7x+3y-2z=8$

$2x+3y+\lambda z=\mu$

have (i) unique sol^n. (ii) No sol^n. (iii) Infinitely many sol^n.

→ The given eq^n are.

$2x+3y+5z=9$

$7x+3y-2z=8$

$2x+3y+\lambda z=\mu$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 7 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_1 - 3R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 2R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 0 & 15 & 39 & 47 \\ 0 & 15 & \lambda+34 & \mu+38 \end{array}\right]$$

$R_3 - R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 0 & 15 & 39 & 47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

This is the echélon form of matrix A.

i) No solution :- if $\rho(A) \neq \rho(A/B)$
then system have no solution.
if $\lambda = 5$ and $\mu \neq 9$

ii) Unique solⁿ :- If $\rho(A) = \rho(A/B) =$ number of unknowns.
then system have unique solⁿ i.e. $n=3$
this is possible only if $\lambda \neq 5$ & μ can be any value.

iii) Infinitely many solⁿ :-
If $\rho(A) = \rho(A/B) = r$
$n =$ number of unknowns.
Here $r < n$. i.e $r < 3$.
It is possible only if

$\lambda = 5$ & $\mu = 9$.

~~Q.5] Examine for linear~~

Q.4] Show that system

$3x + 4y + 5z = \alpha$
$4x + 5y + 6z = \beta$
$5x + 6y + 7z = \gamma$ is consistent only when α, β, γ

are in Arithematic progression.

→ Given system of eq^{ns}.

$3x + 4y + 5z = \alpha$

$4x + 5y + 6z = \beta$

$5x + 6y + 7z = \gamma$

Matrix form $AX = B$

$$\begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 4 & 5 & \alpha \\ 4 & 5 & 6 & \beta \\ 5 & 6 & 7 & \gamma \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 4 & 5 & 6 & \beta \\ 3 & 4 & 5 & \alpha \\ 5 & 6 & 7 & \gamma \end{array}\right]$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \beta - \alpha \\ 3 & 4 & 5 & \alpha \\ 5 & 6 & 7 & \gamma \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \beta - \alpha. \\ 0 & 1 & 2 & 4\alpha - 3\beta \\ 0 & 1 & 2 & 5\alpha - 5\beta + \gamma \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \beta - \alpha \\ 0 & 1 & 2 & 4\alpha - 3\beta \\ 0 & 0 & 0 & \alpha - 2\beta + \gamma \end{array}\right]$$

Rank of matrix = Total number of rows − Number of rows containing all zero

System is consistent only if $\rho(A) = \rho(A|B)$

For above matrix this is possible only if.

$$\alpha - 2\beta + \gamma = 0$$
$$\beta = \frac{\alpha + \gamma}{2}$$

This shows that α, β, γ are in A.P.

Q.5] Examine the linear dependence or independence for the following system of eq^n vector. If dependent Find the relation bet^n them.

→

① $x_1 = (1, 2, -1, 0)$, $x_2 = (1, 3, 1, 2)$, $x_3 = (4, 2, 1, 0)$, $x_4 = (6, 1, 0, 1)$

→ Consider

$$c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 = 0$$

$$c_1(1, 2, -1, 0) + c_2(1, 3, 1, 2) + c_3(4, 2, 1, 0) + c_4(6, 1, 0, 1) = 0$$

$$c_1 + c_2 + 4c_3 + 6c_4 = 0$$
$$2c_1 + 3c_2 + 2c_3 + c_4 = 0$$
$$-c_1 + c_2 + c_3 + 0 = 0$$
$$0 + 2c_2 + 0 + c_4 = 0$$

In matrix form,

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$[A|z] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 2 & 3 & 2 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

By $R_2 - 2R_1$, $R_3 + R_1$

$$(A|Bz) = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 1 & -6 & -11 & 0 \\ 0 & 2 & 5 & 6 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

By, $R_3 - 2R_2$, $R_4 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 1 & -6 & -11 & 0 \\ 0 & 0 & 17 & 28 & 0 \\ 0 & 0 & 12 & 23 & 0 \end{array}\right]$$

By, $R_4 - 12/17 \, R_3$

$$[A|z] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 1 & -6 & -11 & 0 \\ 0 & 0 & 17 & 28 & 0 \\ 0 & 0 & 0 & 55/17 & 0 \end{array}\right]$$

$\rho(A) = \rho(A|z) = 4 - 0 = 4$ = number of unknown

System have unique sol^n. i.e Trival sol^n

$\therefore c_1 = c_2 = c_3 = c_4 = 0$

$\therefore x_1, x_2, x_3, x_4$ are linearly independent.

2) $x_1 = (2,-1,3,2), x_2 = (1,3,4,2), x_3 = (3,-5,2,2)$

$\rightarrow$ $c_1x_1 + c_2x_2 + c_3x_3 = 0$

$c_1(2,-1,3,2) + c_2(1,3,4,2) + c_3(3,-5,2,2) = 0$

$2c_1 + c_2 + 3c_3 = 0$

$-c_1 + 3c_2 - 5c_3 = 0$

$3c_1 + 4c_2 + 2c_3 = 0$

$2c_1 + 2c_2 + 2c_3 = 0$

In matrix form,

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & -5 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A|z) = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ -1 & 3 & -5 & 0 \\ 3 & 4 & 2 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right]$$

By, $R_1 + R_2$, $R_3 + 3R_2$, $R_4 + 2R_2$.

$$[A|z] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 0 \\ -1 & 3 & -5 & 0 \\ 0 & 13 & -13 & 0 \\ 0 & 8 & -8 & 0 \end{array}\right]$$

$R_2 + R_1, \frac{1}{13} R_3, \frac{1}{8} R_4$

$$[A|Z] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_2 - 7R_3, R_4 - R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$[A|Z] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(A) = \rho(A|Z) = 4 - 2 = 2 < 3$$

System have non trival sol^n

By, R_2,

$C_2 - C_3 = 0$

$C_1 + 6C_2 - 2C_3 = 0$ Let $C_3 = t$

$C_2 = C_3 \quad \therefore C_2 = t$

$C_1 + 4t - 2t = 0$

$C_1 = -2t$

X_1, X_2, X_3 are linearly dependent,

$c_1 X_1 + c_2 X_2 + c_3 X_3 = 0$

$-2t X_1 + t X_2 + t X_3 = 0$

$X_2 + X_3 = 2 X_1$

This is the relation betn vectors.

Q.6] Determine values of a, b, c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.

→ Given matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.

$\therefore AA^T = I$

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ -2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$4b^2+c^2 = 1$; $2b^2-c^2 = 0$

$a^2+b^2+c^2 = 1$; $a^2-b^2-c^2 = 0$

Consider, $4b^2+c^2 = 1$, $a^2+b^2+c^2 = 1$

$2b^2 - c^2 = 0$ $\quad a^2-b^2-c^2 = 0$

gives, $6b^2 = 1$ $\quad 2a^2 = 1$

$\therefore b^2 = \frac{1}{6}$, $a^2 = \frac{1}{2}$ $\quad \therefore b = \pm\sqrt{\frac{1}{6}}$

$a = \pm \sqrt{\frac{1}{2}}$ Also $c^2 = 2b^2 \Rightarrow c^2 = \frac{1}{3}$

$\therefore c = \pm \sqrt{\frac{1}{\sqrt{3}}}$; $a = \pm \sqrt{\frac{1}{6}}$;

$c = \pm \sqrt{\frac{1}{3}}$

Q.7) Co-ordinates of a point P are (50,50,50). Origin is shifted to the point (5, -2, 3) Rotation is about z axis through 45°. Find the co-ordinates of P in new co-ordinate system.

→

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -u \\ -\sin\theta & \cos\theta & 0 & -v \\ 0 & 0 & 1 & -w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Substituting, $x = 50, y = 50, z = 50$

$u = 5, v = -2, w = 3$

$\theta = 45°$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & -5 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \\ 50 \\ 1 \end{bmatrix}$$

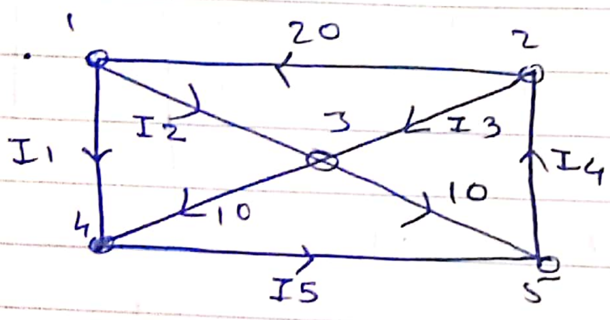
$$X = \frac{50}{\sqrt{2}} + \frac{50}{\sqrt{2}} + 0 - 5$$

$$Y = \frac{-50}{\sqrt{2}} + \frac{50}{\sqrt{2}} + 0 + 2$$

$$Z = 47$$

$$X = 65.71, \; Y = 2, \; Z = 47$$

Q-8] Find the current in various branches of the following network.



→

Applying KCL for various nodes,

At node 1	$I_1 + I_2 = 20$	$I_1 + I_2 = 20$
At node 2	$I_3 + 20 = I_4$	$I_3 - I_4 = -20$
At node 3	$I_2 + I_3 = 20$	$I_2 + I_3 = 20$
At node 4	$I_1 + 10 = I_5$	$I_1 - I_5 = -10$
At node 5	$I_5 + 10 = I_4$	$I_4 - I_5 = 10$

Writing matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 20 \\ -20 \\ 20 \\ -10 \\ 10 \end{bmatrix}$$

$AI = B$

$$[A|B] \quad \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

$R_4 - R_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 0 & -1 & -30 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$ & $R_3 \longleftrightarrow R_4$

$$(A|B) = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 0 & -1 & -30 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

$R_3 + R_2$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

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$R_4 - R_3$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

For left loop $6I_1 + 2I_2 = 0$

For Right bop $2I_2 = 3I_3 = 18$

Thus we get 3 eq^n,

$$I_1 - I_2 + I_3 = 0$$
$$6I_1 + 2I_2 = 0$$
$$2I_2 + 3I_3 = 18$$

writing matrix form.

$$\begin{bmatrix} 1 & -1 & -1 \\ 6 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - 6R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 8 & -6 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 18 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 3 \\ 0 & 8 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$

$R_3 - 4R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ -72 \end{bmatrix}$$

$\therefore I_1 - I_2 + I_3 = 0$

$2I_2 + 3I_3 = 18$

$-18 I_3 = -72$

$\boxed{I_3 = 4}$

$2I_2 + 3(4) = 18$

$2I_2 = 6$

$\boxed{I_2 = 3}$

$\therefore I_1 - 3 + 4 = 0$

$\boxed{I_1 = -1}$

$I_1 = -1$ amp , $I_2 = 3$ amp & $I_3 = 4$ amp.