Roll No.

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Assignment - 1.

WAVE EQUATION.

Q.1) What is wave function? Explain the physical significance of ψ and $|\psi|^2$.

Ans. 1] Wave function :- The wave variable associated with the matter waves being a function of space and time is called the wave function ψ and it represents mathematically the motion of particle.

2] Physical significance of ψ and $|\psi|^2$.

i) According to Max Born, $|\psi|^2$ represents the ~~prop~~ probability density that is it represents the probability per unit volume of finding a particle described by the wave function ψ at a particular time, t, at a particular point (x, y, z) contained in the volume.

ii) The wave function ψ describing the particle is considered as being spread out in space, but it does not mean that the particle is also spread out. ψ represents a particle which is actually in existence

iii) A large value of $|\Psi|^2$ means a strong probability of finding the particle there, and a small value of $|\Psi|^2$ means a little probability of its existence there. $|\Psi|^2 = 0$ means that the particle is absent at that point at time t.

Q.2. Derive schroondinger Time independent wave equation.

Ans. Consider a particle of mass m moving with velocity v. Let (x, y, z) be the co-ordinates representing the position of particle at time t.

According to De Broglie's hypothesis,

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

Step 1 :- Let Ψ be the wave variable associated with the matter waves. Ψ is a function of x, y, z and t.

In analogy with the equation

$$\frac{d^2y}{dt^2} = v^2 \cdot \frac{d^2y}{dx^2}.$$

We can write differential equation for matter waves with wave velocity u as

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right]$$

or $\frac{\partial^2 \psi}{\partial t^2} = u^2 . \bar{\nabla}^2 \psi$ -- (1)

where, $\bar{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called Laplacian operator.

step. 2 :-

The solution of equation (1) will give us ψ as a function of space and time $\psi(x,y,z,t) = \psi_0 (x,y,z) . e^{-i\omega t}$ --- (2)

where $\psi_0 (x,y,z)$ is amplitude of wave at point (x,y,z) and is a function of x,y,z. Equation (2) can be written as, $\psi(\bar{r},t) = \psi_0(\bar{r}) . e^{-i\omega t}$ -- (3)

where $\bar{r} = \bar{i}x + \bar{j}y + \bar{k}z$ is known as position vector of point (x,y,z).

Step 3 :- Differentiating eq^n ③ w.r.t t we have,

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 \cdot e^{-i\omega t}$$

$$\therefore \frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0 e^{-i\omega t}$$

or

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \text{--- ④}$$

(since $i^2 = -1$ and $\psi_0 \cdot e^{-i\omega t} = \psi$).

$\therefore$ From equations ① and ④ we have,

$$u \cdot \nabla^2 \psi = -\omega^2 \psi.$$

or $$\nabla^2 \psi + \frac{\omega^2}{u^2} \cdot \psi = 0 \quad \text{--- ⑤}$$

Step 4 :-

Now $\omega = 2\pi\nu$ and $u = \nu \cdot \lambda$.

where, ν = frequency of waves.

$$\frac{\omega}{u} = \frac{2\pi\nu}{\nu\lambda} = \frac{2\pi}{\lambda} \quad \text{--- ⑥}$$

Hence, eq^n ⑤ becomes.

$$\nabla \psi + \frac{4\pi^2}{\lambda^2} \cdot \psi = 0 \quad \text{--- ⑦}$$

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The De-Broglie wavelength of particle is given by.

$$\lambda = \frac{h}{p}.$$

Hence eqn 7 becomes.

$$\nabla^2 \Psi + \frac{4\pi^2 \cdot p^2}{h^2} \cdot \Psi = 0 \quad \text{--- (8)}$$

step-5 :-

the total energy E of the particle is the sum of kinetic and potential energies.

$$\therefore E = \frac{1}{2} m v^2 + V \qquad (V = E.P)$$

$$= \frac{1}{2m} m^2 v^2 + V = \frac{p^2}{2m} + V.$$

$$\therefore p^2 = 2m(E - V) \quad \text{--- (9)}$$

step-6 :-

Putting this value of in eqn 8 we have,

$$\nabla \Psi + \frac{4\pi^2}{h^2} \cdot 2m (E - V) \Psi = 0$$

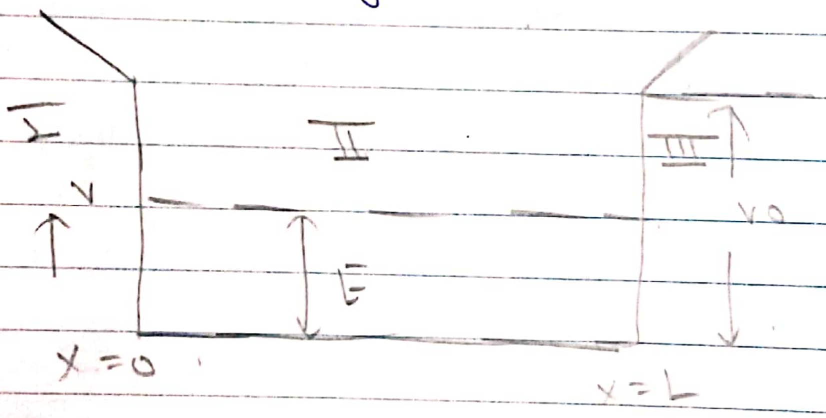
putting $\frac{h}{2\pi} = \hbar$ we have

$$\nabla^2 \psi + \frac{2m}{\hbar^2} \cdot (E - V) \psi = 0 \quad \text{--- (10)}$$

This is schrodinger's time independent equation.

Q.3. Discuss the problem of particle trapped in finite potential well. Explain tunneling effect.

Ans. I]



Finite potential well.

1) Consider a particle of mass m moving with velocity v along the x- direction between x=0 and x=L. The walls of the box are not rigid. Hence it is represented by a potential well of finite depth.

2) The problems of particle trapped in finite potential well are :-

i) The eigen functions are similar in

appearance to those of infinite well except that they extend a little outside the box.

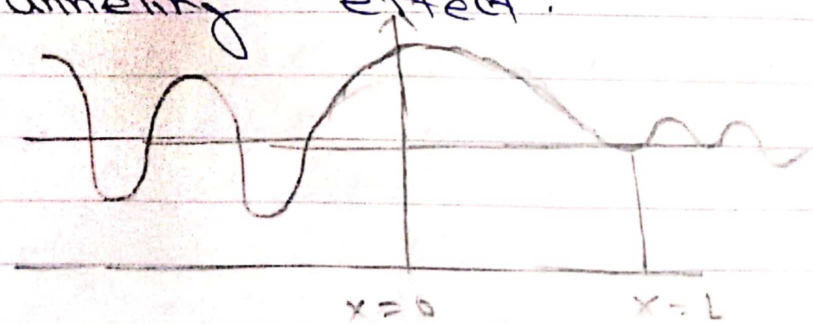
ii) Even though the particle energy E is less than the P.E V_0 there is a definite probability that the particle is found outside the box

iii) The particle energy is not enough to break through the walls of the box but it can penetrate the walls and leak out. This shows penetration of particle into the classically forbidden region.

iv) The energy levels of the particle are still discrete but there are a finite numbers of them. Such a limit exists because, soon the particle energy becomes equal to V_0.

v) For energies higher than ~~e~~ this the particle energy is not quantised but may have any value above V_0.

II] Tunneling effect.



Barrier tunneling and width L.

i) Classically we know that if $E > V_0$ the particle can enter region II from region I, but its energy will be slowed down to value $(E - V_0)$.

ii) The particle is thus transmitted into region III.

iii) But if $E < V_0$, the particle cannot enter region II and is reflected back into region I without the loss of speed.

iv) It represents the damped oscillating sine wave as shown in figure. Although it is damped, but it gives a non-zero wave function in region III, indicating the penetration of particle into classically forbidden region.

v) Thus in wave mechanics, there is a finite chance that the particle (having $E < V_0$) can penetrate the barrier and continues its motion to the right that is the particle can tunnel through classically forbidden region and come out to the outer side of it. This is called, the 'Barrier Penetration' or 'Tunnelling effect'.