



Name :- Vaibhav Atmanam Padgikan

Assignment - 5  
Quantum Mechanics.

Q.17 State De-Broglie's Hypothesis of matter waves. Derive the equation for de Broglie's wavelength in terms of momentum of particle.

Ans. i) De-Broglie's Hypothesis of matter waves :-

According to De-Broglie's concept, a moving particle always has a wave associated with it and the motion of the particle is guided by that wave in a similar manner as photon is controlled by a wave.

ii) A light wave of frequency  $\nu$  is associated with photon of energy  $E$ , given by plank's relation as :-

$$E = h\nu \quad \text{--- (1)}$$

According to Einstein's theory of relativity, a particle of mass  $m$  is equivalent to

$$E = mc^2 \quad \text{--- (2)}$$

Comparing (1) and (2),

$$h\nu = mc^2 \quad \text{--- (3)}$$

If  $p$  is momentum of photon, then

$$p = mc$$

From eq<sup>n</sup> ③ we get,

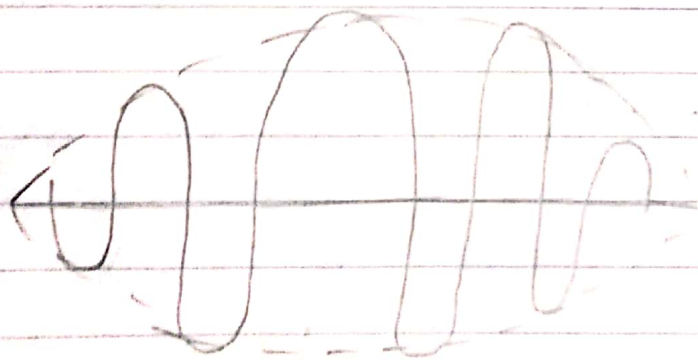
$$h\nu = pc$$

$$\text{or } h\nu = p v \lambda \quad (c = v \lambda)$$

$$\therefore \left[ \lambda = \frac{h}{p} = \frac{h}{mv} \right]$$

Q.2) Define wave group and group velocity. Explain why the concept of phase velocity is meaningless and that for group velocity is significant?

Ans. 1) Wave Group :- The hump or envelope of waves is called the wave group.



2) Group velocity :- The velocity with which a wave packet moves forward in the medium is called group velocity.

3) Group velocity is defined only for the superimposed waves and group





velocity is velocity of wave with lower frequency.

4) Phase velocity is velocity of wave with higher frequency. The average velocity of the advancement of individual monochromatic wave in the medium with which a wave packet is constructed is called as phase velocity.

5) Phase velocity is always greater than the velocity of light.

6) Group velocity is equal to the velocity of particle which is always less than  $c$ .

7) Therefore, phase velocity becomes meaningless and group velocity is significant.

Q.3]. State Heisenberg's Uncertainty Principle. Explain it using the concept of narrow and broad wave packet.

Ans. i) Heisenberg's Uncertainty Principle :-  
In classical physics, the dynamical variables of like momentum and coordinates of a particle can be measured accurately at any instant. But such measurements cannot be



- performed beyond a certain limit in quantum mechanics.
- ii) According to the principle's of wave mechanics a moving material particle is associated with a wave which describes all about the particle.
  - iii) A particles is always localised in space and hence a wave packet or wave group represents the moving particle. The particle lies somewhere in the wave packet and the probability of finding the particle at a given point is proportional to the wave amplitude at that point.
  - iv) Thus although the particle is somewhere within the wave packet moving with the group velocity, it is impossible to determine the exact position and exact velocity at any given moment. There is always a certain uncertainty in the experimental measurement of position and momentum of a particle of very small size, like an electron, in motion.
  - v) This uncertainty is not due to the imperfection of the measuring instrument.





## Modern College of Engineering

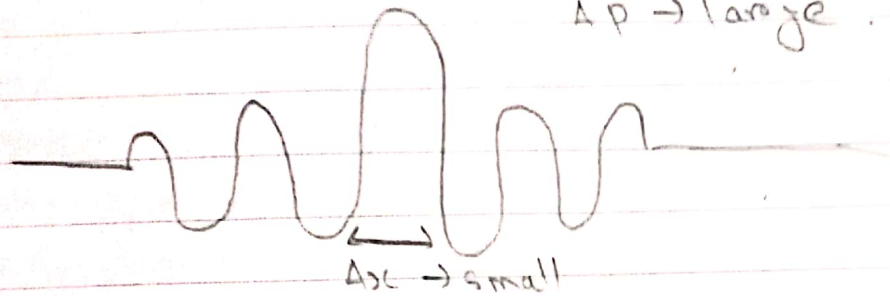
Shivajinagar, Pune 5.

(3)

ent but is something inherent in the wave nature of the moving particle.

vi) Narrow wave packet :-

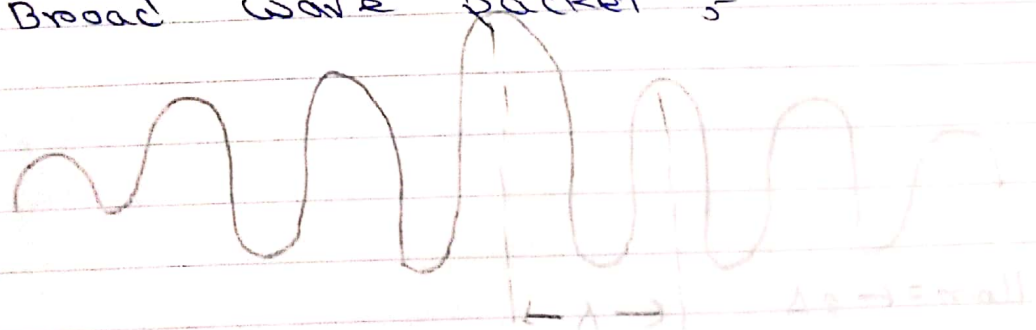
$\Delta p \rightarrow \text{large}$



a) In case of narrow wave packet, the amplitude is large over a very small region of the space and is negligible elsewhere as shown in figure

b) The small region of space can be associated with the position of particle can be fixed with minimum errors or uncertainty. But at the same time wavelength  $\lambda$  and momentum ( $p = \frac{h}{\lambda}$ ) cannot be measured accurately and hence is uncertain.

7) Broad wave packet :-



momentum of photon, then



- i) If wave packet is sufficiently wide as shown in figure, then the wavelength  $\lambda$  and hence the momentum  $P$  can be determined with more accuracy.
- ii) Thus consideration of infinitely small and infinitely large wave packets show that certainty about momentum involves complete uncertainty about the position and vice-versa. Hence, it is impossible to determine simultaneously both the momentum and position of moving particle.
- iii) Experiments never tell us that the particle occupies a particular position and has particular velocity at that instant. They only give us the probability.

- II -