Roll no :- 2150

Modern College of Engineering L_3

Shivajinagar, Pune 5.

Name :- Vaibhav Atmaram Padghan

1. If $x = u\tan v$, $y = u\sec v$, then prove that
$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x .$$

→ $x = u\tan v$.

$y = u\sec v$.

① $\left(\frac{\partial u}{\partial x}\right)_y$.

$$\frac{x}{u} = \tan v \quad \text{--①}$$

$$\frac{y}{u} = \sec v \quad \text{--②}$$

We know that :

$$1 + \tan^2 v = \sec^2 v$$

$$1 + \left(\frac{x}{u}\right)^2 = \left(\frac{y}{u}\right)^2$$

$$\therefore u^2 + x^2 = y^2$$

$$\therefore u^2 = y^2 - x^2 .$$

$$\therefore \left(\frac{\partial u}{\partial x}\right)_y = -2x$$

Scanned by CamScanner

(2). $\left(\frac{\partial v}{\partial x}\right)_y$

$$\frac{x}{y} = \frac{\tan v}{\sec v}$$

$$\therefore \frac{x}{y} = \frac{\sin v \cdot \cos v}{\cos v}$$

$$\sin v = \frac{x}{y}$$

$$\left(\frac{\partial v}{\partial x}\right)_y \cos v = \frac{1}{y}$$

$$\therefore \left(\frac{\partial v}{\partial x}\right)_y = \frac{1}{y \cdot \cos v}$$

(3). $\left(\frac{\partial u}{\partial y}\right)_x$

$$u^2 = y^2 - x^2$$

$$\therefore \left(\frac{\partial u}{\partial y}\right)_x = 2y$$

(4). $\left(\frac{\partial v}{\partial y}\right)_x$

$$\sin v = \frac{x}{y}$$

$$\cos v \left(\frac{\partial v}{\partial y}\right)_x = \frac{-x}{y^2}$$

$\therefore$ L.H.S $= \left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial v}{\partial x}\right)_y$

$= -2x \times \frac{1}{y\cos v}$.

R.H.S $= \left(\frac{\partial u}{\partial y}\right)_y \cdot \left(\frac{\partial v}{\partial y}\right)_x$

$= \frac{2\cancel{y}}{y^{\cancel{2}}} \times \frac{(-x)}{\cos v}$

$= -\frac{2x}{y\cos v}$

$\therefore$ L.H.S = R.H.S

$\therefore \left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_y \cdot \left(\frac{\partial v}{\partial y}\right)_x$.

is proved.

2. If $u = 2x+3y$, $v = 3x-2y$ prove that

(i) $\left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u = \frac{13}{4}$,

(ii) $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{4}{13}$

$\rightarrow$

① $v = 3x - 2y$.

$2y = 3x - v$

$\therefore y = \frac{1}{2}(3x - v)$

$\therefore \left(\frac{\partial y}{\partial v}\right)_x = \frac{1}{2}[-1] = \frac{-1}{2}$

②. $\left(\frac{\partial v}{\partial y}\right)_u$

$\therefore u = 2x + 3y$.

$\frac{u - 3y}{2} = x$

$v = 3x - 2y$.

$\frac{v + 2y}{3} = x$

equating both values of x.

$\frac{u - 3y}{2} = \frac{v + 2y}{3}$

$3u - 9y = 2v + 4y$.

$3u - 2v = 13y$

$3u - 13y = 2v$

$\therefore v = \frac{1}{2}[3u - 13y]$

$\therefore \left(\frac{\partial v}{\partial y}\right)_u = \frac{1}{2}[-13] = \frac{-13}{2}$

3. $\left(\frac{\partial u}{\partial x}\right)_y = 2$

4. $\left(\frac{\partial x}{\partial u}\right)_v$

$u = 2x + 3y$

$\frac{u-2x}{3} = y.$

and

$\frac{3x-v}{2} = y.$

equating both values of v.

$\frac{u-2x}{3} = \frac{3x-v}{2}$

$2u - 4x = 9x - 3v$

$2u + 3v = 13x$

$x = \frac{1}{13}[2u+3v]$

$\left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{13}[2] = \frac{2}{13}$

$\therefore$

i) $\left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial v}{\partial y}\right)_u = -\frac{1}{2} \times -\frac{13}{2} = \frac{13}{4}$

is proved.

ii) $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = 2 \times \frac{2}{13} = \frac{4}{13}$.

is proved.

3. Find the value of n for which $z = t^n e^{-\frac{r^2}{4t}}$, satisfies the partial differential equation. $\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial z}{\partial r}\right)\right] = \frac{\partial z}{\partial t}$.

soln:

$$z = t^n \cdot e^{-\frac{r^2}{4t}}$$

$$\frac{\partial z}{\partial r} = t^n \cdot e^{\frac{r^2}{4t}} \cdot \frac{-2r}{4t}.$$

$$r^2 \cdot \frac{\partial z}{\partial r} = t^n \cdot e^{-\frac{r^2}{4t}} \cdot \left(\frac{-1}{2}\right) \frac{r^3}{t}.$$

$$= -\frac{t^{n-1}}{2} \cdot r^3 \cdot e^{-\frac{r^2}{4t}}.$$

$$\frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial z}{\partial r}\right) = -\frac{t^{n-1}}{2}\left[3r^2 \cdot e^{-\frac{r^2}{4t}} + r^3 \cdot e^{-\frac{r^2}{4t}} \cdot \left(\frac{-2r}{4t}\right)\right]$$

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial z}{\partial r}\right)\right] = \frac{-t^{n-1}}{2}\left[3 \cdot e^{-\frac{r^2}{4t}} - \frac{r^2}{2t} \cdot e^{-\frac{r^2}{4t}}\right]$$

$$= \frac{-t^{n-1}}{2} \cdot e^{-\frac{r^2}{4t}}\left[3 - \frac{r^2}{2t}\right]$$

$$z = t^n . e^{\frac{-r^2}{4t}}.$$

$$\frac{\partial z}{\partial t} = t^n . e^{\frac{-r^2}{4t}} . \frac{r^2}{4t^2} + n . t^{n-1} . e^{\frac{-r^2}{4t}}.$$

$$= \frac{e^{\frac{-r^2}{4t}} . t^{n-1}}{2} \left[\frac{r^2}{2t} + n \right]$$

$$\frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 . \frac{\partial z}{\partial r} \right) \right] = \frac{\partial z}{\partial t}.$$

$$\therefore \frac{\cancel{e^{\frac{-r^2}{4t}}} . \cancel{t^{n-1}}}{\cancel{2}} \left[\frac{r^2}{2t} + n \right] = \frac{-\cancel{t^{n-1}}}{\cancel{2}} . \cancel{e^{\frac{-r^2}{4t}}} \left[3 - \frac{r^2}{2t} \right]$$

$$\cancel{\frac{r^2}{2t}} + n = \cancel{\frac{r^2}{2t}} - 3$$

$$\therefore \boxed{n = -3}$$

4. If $u = 4 . e^{-6x} \sin[pt - 6x]$ satisfies the partial differential equation, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, then find p.

Sol^n.

$\rightarrow \quad u = 4.e^{-6x}\cdot\sin[pt-6x]$

$$\frac{\partial u}{\partial t} = 4.\,e^{-6x}\cos(pt-6x)\,p.$$

$$\frac{\partial u}{\partial x} = 4\left[e^{-6x}(-6)\sin[pt-6x] + e^{-6x}\cdot\cos(pt-6x)(-6)\right]$$

$$\frac{\partial^2 u}{\partial x^2} = \cancel{4}\,{-24}\left[e^{-6x}\cancel{(-6)\sin(pt-6x)} + e^{-6x}\cos(pt-6x)(-6) + e^{-6x}(-6)\cos(pt- - \sin\cancel{(pt-6x)(-6)}\right]$$

$$\therefore \frac{\partial^2 u}{\partial x^2} = 48\times 6\times e^{-6x}\times\cos(pt-6x)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad [\text{Given}]$$

$$\therefore 48\times 6\times \cancel{e^{-6x}}\times\cos(\cancel{pt-6x}) = 4.\cancel{e^{-6x}}.\cos(\cancel{pt-6x}).p$$

$$\therefore p = \frac{48\times 6}{4}$$

$$= 12\times 6$$

$$\therefore \boxed{p = 72}$$

(5)

5. If $u = x^8 \phi\left(\frac{y}{x}\right) + \frac{1}{y^8} \phi\left(\frac{x}{y}\right)$ then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \cdot \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 64u.$

soln :-
$$u = \underbrace{x^8 f\left(\frac{y}{x}\right)}_{z} + \underbrace{y^{-8} \phi\left(\frac{x}{y}\right)}_{v}.$$

i.e. $u = z + v$.

$\because$ $z = x^8 f\left(\frac{y}{x}\right)$ is a homogenous function in x and y of degree $n=8$.

Also, $v = y^{-8} \phi\left(\frac{x}{y}\right)$ is a homogenous function in x and y of degree $n=-8$.

$\therefore$ By Euler's theorem on homogenous function.

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \cdot \frac{\partial^2 z}{\partial x \partial y} + y^2 \cdot \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$
$$= 8 \times 7 z$$
$$= 56 z \quad \text{-- (1)}$$

and $x^2 \frac{\partial^2 v}{\partial x^2} + 2xy.\frac{\partial^2 v}{\partial x.\partial y} + y^2 \frac{\partial^2 v}{\partial y^2} = n(n-1)v$

$= -8(-9)v$

$= 72v.$ — (2)

Adding Equation (1) and (2), we get,

$x^2 \frac{\partial^2}{\partial x^2}(z+v) + 2xy.\frac{\partial^2}{\partial x.\partial y}(z+v) + y^2 \frac{\partial^2}{\partial y^2}(z+v)$

$= 56z + 72v.$

$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy.\frac{\partial^2 u}{\partial x.\partial y} + y^2.\frac{\partial^2 u}{\partial y^2} = 56z + 72v$ — (A)

~~Now,~~

~~R.H.S = 64.~~

Again by Euler's theorem

$x.\frac{\partial z}{\partial x} + y.\frac{\partial z}{\partial y} = 8z$ -- (3)

and

$x\frac{\partial v}{\partial x} + y.\frac{\partial v}{\partial y} = -8v$ -- (4)

adding (3) and (4)

$x\frac{\partial}{\partial x}(z+v) + y\frac{\partial}{\partial y}(z+v) = 8z - 8v$

$$x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 8z - 8v \quad \cdots \text{(5)}$$

adding (A) and (5)

$$\therefore x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \cdot \frac{\partial^2 u}{\partial x \cdot \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} = 56z + 72v + 8z - 8v$$

$$= 64(z+v)$$

$$= 64u$$

is proved.

6. If $u = \cos^{-1}\left(\frac{x^3 y^2 + 4y^3 x^2}{\sqrt{x^4 + 6y^4}}\right)$. Find the value of (i) $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y}$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \cdot \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$,

Solⁿ.

Let $y = tx$.

$$\therefore u = \cos^{-1}\left(\frac{t^2 x^5 + 4t^3 x^5}{x^2 \sqrt{1+6t^4}}\right)$$

$$u = \cos^{-1}\left(x^3 \frac{(t^2+t^3)}{\sqrt{1+6t^4}}\right)$$

$$\therefore \cos u = x^3 \frac{(t^2+t^3)}{\sqrt{1+6t^4}}$$

Let $z = \cos u$.

$$\therefore z = x^3 \frac{(t^2+t^3)}{\sqrt{1+6t^4}}.$$

$\therefore$ z is homogenous function of deg $n=3$.

and $z = \cancel{\tan u}$, $\cos u$.

using Euler's theorem

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \cdot \frac{\partial^2 u}{\partial x \cdot \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2}$$

$$= g(u)\,[g'(u) - 1]$$

where, $g(u) = \dfrac{n \cdot f(u)}{f'(u)}$

$$= 3\,\frac{\cos u}{-\sin u}$$

$$= -3\cot u.$$

$$g'(u) = -\cancel{\cot u}\cdot \operatorname{cosec}^2 u$$

$$\therefore\ g(u)\cdot[g'(u)-1] = -3\cot u\,[-\cancel{\cot u}\cdot\operatorname{cosec}^2 u - 1]$$

$$= -3\,\frac{\cos u}{\sin u}\left[-\frac{\operatorname{cosec}^2 u}{\cancel{\cos u}\ \cancel{\sin^2 u}} - 1\right]$$

$$\therefore\ \text{(ii)}\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\cdot\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

$$= +3\cot u\,[\cancel{\cot u}\operatorname{cosec}^2 u - 1]$$

$$= 3\cot^3 u,$$

Again using Euler's theorem.

$$x\cdot\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y} = n\,\frac{f(u)}{f'(u)}$$

$$= -3\cot u.$$

$$\therefore\ \text{(i)}\ x\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y} = -3\cot u$$

and

$$\text{(ii)}\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\cdot\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 3\cot u[\cot u\cdot\operatorname{cosec} u - 1]$$

7. If $u = \tan^{-1}\left[\frac{x^3+y^3}{x+y}\right]$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u\left[1-4\sin^2 u\right]$$

Soln:-

Let $u = \tan^{-1}\left[\frac{x^3+y^3}{x+y}\right]$

put, $y = tx$.

$$\therefore u = \tan^{-1}\left[\frac{x^3+t^3x^3}{x-tx}\right]$$

$$u = \tan^{-1}\left[x^2\left(\frac{1+t^3}{1-t}\right)\right]$$

$$\therefore \tan u = x^2\left(\frac{1+t^3}{1-t}\right).$$

$\therefore$ z is homogenous function of degree $n=2$.

and $z = \tan u$

Using Euler's theorem

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

$$= g(u)\left[g'(u)-1\right].$$

where , $g(u) = \frac{n.f(u)}{f'(u)}$.

$$= 2\left[\frac{\tan u}{\sec^2 u}\right]$$

$$= 2\left[\frac{\sin u}{\cos u} \times \cos u . \cos u\right]$$

$$= \sin(2u).$$

$$g'(u) = 2\cos(2u).$$

$$L.H.S = g(u)\left[g'(u) - 1\right]$$

$$= \sin(2u)\left[2\cos(2u) - 1\right]$$

$$= \sin(2u)\left[2(1 - 2\sin^2 u) - 1\right]$$

$$= \sin(2u)\left[2 - 4\sin^2 u - 1\right]$$

$$= \sin(2u)\left[1 - 4\sin^2 u\right]$$

$= R.H.S$

$\therefore L.H.S = R.H.S$

$\therefore$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x . \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin(2u)\left[1 - 4\sin^2 u\right]$$

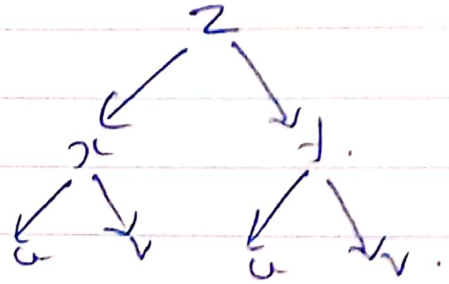
is proved.

Scanned by CamScanner

8. If $z = F(x, y)$, where $x = e^u \cos v$, $y = e^u \sin v$, then prove that

$$y \cdot \frac{\partial z}{\partial u} + x \cdot \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}.$$

Solⁿ :-



① $$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$$

$$= \frac{\partial z}{\partial x} \times e^u \cos v + \frac{\partial z}{\partial y} \times e^u \sin v$$

② $$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial v}.$$

$$= \frac{\partial z}{\partial x} \times e^u (-\sin v) + \frac{\partial z}{\partial y} \times e^u \cos v$$

$$\text{L.H.S} = y \cdot \frac{\partial z}{\partial u} + x \cdot \frac{\partial z}{\partial v}$$

$$= y \cdot \left[\frac{\partial z}{\partial x} \times e^u \cos v + \frac{\partial z}{\partial y} e^u \cancel{\cos v} \sin v \right]$$

$$+ x \left[\frac{\partial z}{\partial x} \times e^u (-\sin v) + \frac{\partial z}{\partial y} \times e^u \cos v \right]$$

$$= e^{u}\sin v\left[\frac{\partial z}{\partial x} \times e^{u}\cos v + \frac{\partial z}{\partial y} \times e^{u} \times \sin v\right]$$

$$+ e^{u}\cos v\left[\frac{\partial z}{\partial x} \times e^{u}(-\sin v) + \frac{\partial z}{\partial y} \times e^{u}\cos v\right]$$

$$= \frac{\partial z}{\partial y}\, e^{2u}\sin^2 v + \frac{\partial z}{\partial y} \times e^{2u} \times \cos^2 v$$

$$= e^{2u} \cdot \frac{\partial z}{\partial y}.$$

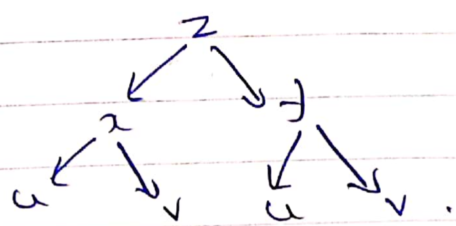
= R.H.S

∴ L.H.S = R.H.S.

∴ $y \cdot \frac{\partial z}{\partial u} + x \cdot \frac{\partial z}{\partial v} = e^{2u} \cdot \frac{\partial z}{\partial y}$ is proved.

9. If $z = f(x,y)$ where, $x = u+v$, $y = uv$, then proove that, $u \cdot \frac{\partial z}{\partial u} + v \cdot \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} + 2y \cdot \frac{\partial z}{\partial y}$.

Soln:



$x = u+v$

$y = uv$.

Scanned by CamScanner

① $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$.

$= \frac{\partial z}{\partial x} \times 1 + \frac{\partial z}{\partial y} \times v$.

② $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial v}$.

$= \frac{\partial z}{\partial x} \times 1 + \frac{\partial z}{\partial y} \times u$.

③ $\frac{\partial z}{\partial x} =$

$L.H.S = u \cdot \frac{\partial z}{\partial u} + v \cdot \frac{\partial z}{\partial v}$

$= u\left[\frac{\partial z}{\partial x} + v\frac{\partial z}{\partial y}\right] + v\left[\frac{\partial z}{\partial x} + u \cdot \frac{\partial z}{\partial y}\right]$

$= u \cdot \frac{\partial z}{\partial x} + uv \cdot \frac{\partial z}{\partial y} + v\frac{\partial z}{\partial x} + uv \cdot \frac{\partial z}{\partial y}$.

$= \frac{\partial z}{\partial x}(u+v) + 2uv\frac{\partial z}{\partial y}$.

$= \frac{\partial z}{\partial x}x + 2y\frac{\partial z}{\partial y}$.

$= R.H.S$.

Modern College of Engineering
Shivajinagar, Pune 5.

L.H.S = R.H.S.

$\therefore$

$$\therefore\ u\cdot\frac{\partial z}{\partial u}+v\cdot\frac{\partial z}{\partial v}=x\cdot\frac{\partial z}{\partial x}+2y\cdot\frac{\partial z}{\partial y}.$$

10. If $u=f(r)$, where $r=\sqrt{x^2+y^2}$ then prove that

$$u_{xx}+u_{yy}=f''(r)+\frac{1}{r}f'(r).$$

Solⁿ:- $u=f(r)$, $r=\sqrt{x^2+y^2}$.

$$\frac{\partial r}{\partial x}=\frac{1}{2}\times\frac{2x}{\sqrt{x^2+y^2}}=\frac{x}{r}. \quad \text{--- (1)}$$

similarly.

$$\frac{\partial r}{\partial y}=\frac{y}{r}.$$

$$\frac{\partial u}{\partial x}=\frac{d\,f(r)}{dr}\cdot\frac{\partial r}{\partial x}=f'(r)\cdot\frac{x}{r}.$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial x}\left[f'(r)\cdot\frac{x}{r}\right]=$$

$$=f'(r)\cdot\frac{\partial}{\partial x}\left(\frac{x}{r}\right)+\frac{x}{r}\cdot\frac{\partial}{\partial x}\left(f'(r)\right)$$

$$=f'(r)\left[\frac{r(1)-x\cdot\frac{\partial r}{\partial x}}{r^2}\right]+\frac{x}{r}f''(r)\cdot\frac{\partial r}{\partial x}$$

$$= \frac{f'(r)}{r} - f'(r)\cdot\frac{x}{r^2}\cdot\frac{\partial r}{\partial x} + \frac{x}{r} f''(r)\cdot\frac{\partial r}{\partial x}.$$

$$= \frac{f'(r)}{r} - f'(r)\frac{x}{r^2} \times \frac{x}{r} + \frac{x^2}{r^2} f''(r)$$

[from ①]

i)

$$\frac{\partial^2 u}{\partial x^2} = \frac{f'(r)}{r} - f'(r)\frac{x^2}{r^3} + \frac{x^2}{r^2} f''(r)$$

similarly,
we get

$$\frac{\partial^2 u}{\partial y^2} = \frac{f'(r)}{r} - f'(r)\frac{y^2}{r^3} + \frac{y^2}{r^2} f''(r).$$

ii)

$\therefore$ L.H.S $= u_{xx} + u_{yy}$

$$= \frac{2f'(r)}{r} - \frac{f'(r)}{r^3}(x^2+y^2) + \frac{f''(r)}{r^2}(x^2+y^2)$$

$$= \frac{2f'(r)}{r} - \frac{f'(r)}{\cancel{r^3}\, r}\cancel{r^2} + \frac{f''(r)}{\cancel{r^2}}\cancel{r^2}$$

$$= \frac{f'(r)}{r} + f''(r)$$

$= R.H.S.$

$\therefore L.H.S = R.H.S.$

$\therefore u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r)$

is prooved

— || —