where,

t is time taken for electron to transit through channel.

- The above formula is also applicable for MESFET. Transit time (t) should be very small so that the device can be operated at very high frequencies.

5.2.4 D.C. Characteristics

- D.C. characteristics (I_{DS} versus V_{DS} for different V_{GS}) of a typical n-channel GaAs MESFET and a typical HEMT is shown in Fig. 5.2.2.
- From D.C. characteristics it is observed that MESFET can be operated under $V_{GS} \leq 0$, where as HEMT can be operated under positive values of V_{GS}.

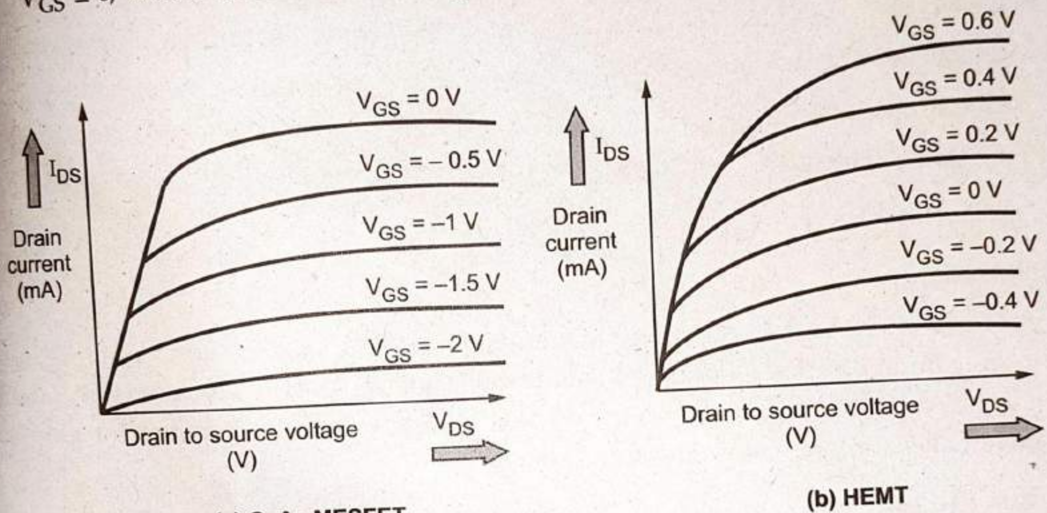


Fig. 5.2.2 D.C. characteristics

5.3 Basic Concepts of RF Design

[A] RF Characteristics

- All RF waves have characteristics that vary to define the wave. Some of these properties can be modified to modulate information onto the wave. These properties are wavelength, frequency, amplitude, and phase.

[B] RF Behaviour

- RF waves that have been modulated to contain information are called RF signals. These RF signals have behaviours that can be predicted and detected. They become stronger, and they become weaker. They react to different materials differently, and they can interfere with other signals.

5.2.1 Advantages of HEMT over MESFET

1. HEMT device offers high gain at microwave frequencies upto 70 GHz.
2. HEMT device has low noise figure.
3. The power handling capacity of HEMT device is better because of short gate length, reduced gate to source contact resistance.

5.2.2 Structure of HEMT

- Fig. 5.2.1 shows structure of HEMT.

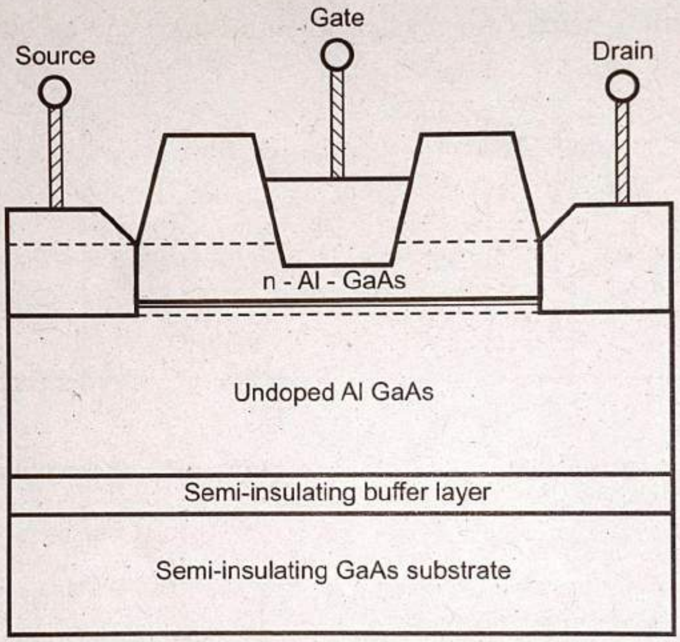


Fig. 5.2.1 Structure of HEMT

- On semi-insulating GaAs substrate a semi-insulating buffer-layer is grown followed by undoped GaAs layer. Between these layer a semi-insulating buffer layer is grown. Above this silicon-doped n-type AlGaAs layer is grown thermally.
- Between undoped AlGaAs layer and n-type AlGaAs layers a two dimensional electron gas (2-DEG) is formed.
- Ohmic contacts are fabricated by using AuGe alloy. Terminals of ohmic contacts are brought out.

5.2.3 Bandwidth of HEMT

- Bandwidth of HEMT device is given by

$$f_t = \frac{1}{2\pi t}$$

Two additional diodes L_1 and L_2 called leakage diodes are introduced to deal with collector-dependent forward current gain $(\beta_F)^*I_C$ and reverse current gains $(\beta_R)^*I_C$.

FET Model at RF

Considering non-insulated gate FET from group called MESFET or GaAsFET and HEMT, basic n-channel depletion mode MESFET model with transfer and output characteristics is shown in Fig. 5.1.3

The equations for drain current in forward mode of operation follow from analysis. Thus drain current for linear and saturations can be obtained.

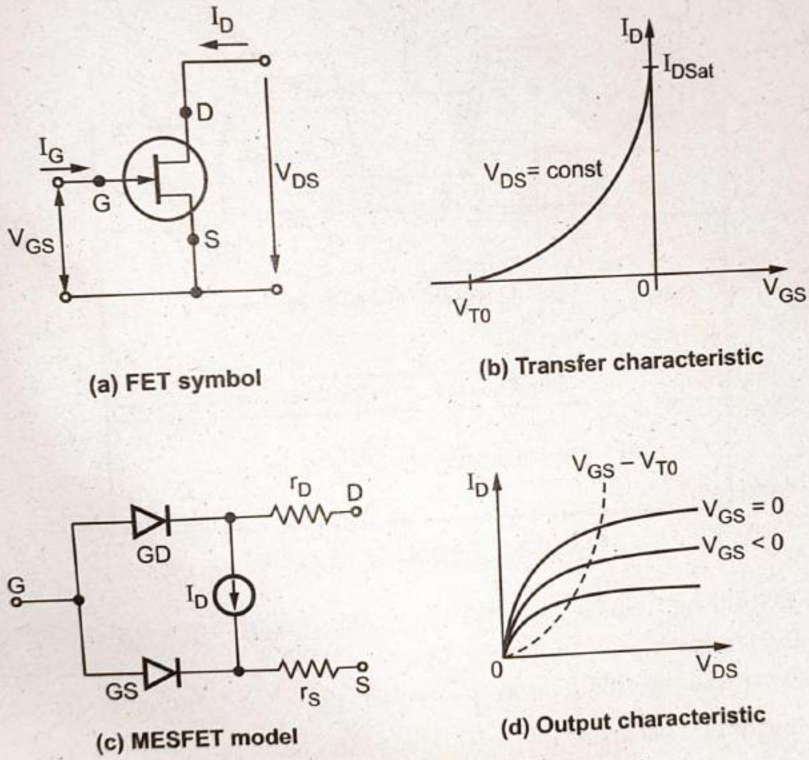


Fig. 5.1.3 n channel MESFET model

5.2 High Electron Mobility Transistors (HEMT)

- In a MESFET channel having concentration of about 10^{17} atoms/cm^3 at room temperature (300° K) has electron mobility varying from 4×10^3 cm^2/volt-sec to 5×10^3 cm^2/volt-sec.
- The electron mobility can be increased to 8×10^3 cm^2/volt-sec by using GaAs-AlGaAs modulation doped single heterojunction structure. Such transistors with high electron mobility is called **High Electron Mobility Transistors (HEMT).**

5.1 Active RF Components

5.1.1 Large Scale Diode Model

- Fig. shows large scale diode model.
- The effect of diffusion capacitance C_d and junction capacitance C_J are shown by single capacitance C.
- The injection and extraction of charges is accomplished by the electric field that constitutes a voltage drop that is shown as series resistance R_s.

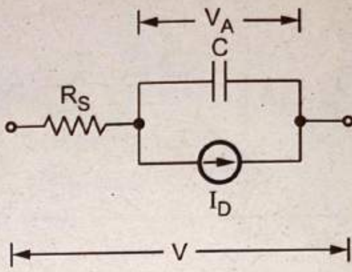


Fig. 5.1.1 Large scale diode model

- The total voltage across diode is given by two components :

$$V = R_S I_D + nV_T \ln(1+I_D/I_S)$$

- The reverse saturation current I_S is influenced by temperature, according to equation :

$$I_s(T) = I_s(T_0)\left(\frac{T}{T_0}\right)^{p/n} \exp\left[-\frac{W_9(T)}{V_T}\left(1-\frac{T}{T_0}\right)\right]$$

Where,

T_0 represents reference temperature

P_t represents reverse saturation current temperature coefficient typically 2 or 3

$W_g(T)$ represents bandgap energy

5.1.2 BJT Model at RF

- For representing transistors at radio and microwave frequencies there is need to take account for various second-order effects such as low current and high injection phenomena.
- After various refinement in the original Ebers-moll model, the new model called Gummel-Poon model is developed as shown in figure...

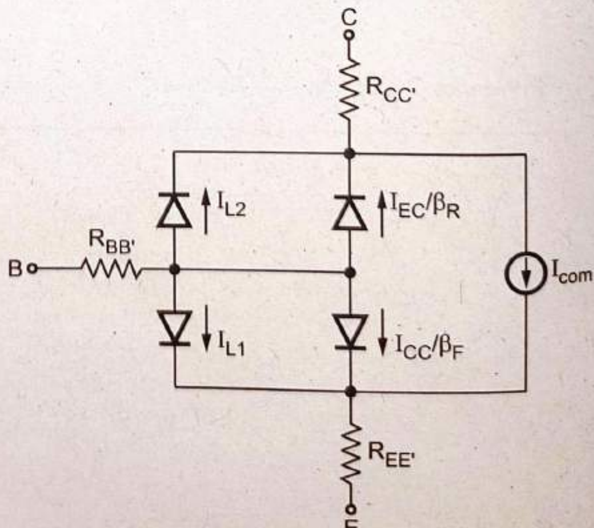


Fig. 5.1.2 Gummel-Poon model

UNIT - V

5 RF System Design Concepts

Syllabus

Active RF components: Semiconductor basics in RF, bipolar junction transistors, RF field effect transistors, High electron mobility transistors Basic concepts of RF design, Mixers, Low noise amplifiers, voltage control oscillators, Power amplifiers, transducer power gain and stability considerations.

Contents

5.6.3.2 Configuration at Output Port

- Similarly condition for stabilization of output port through series resistance and output port configuration is shown below.

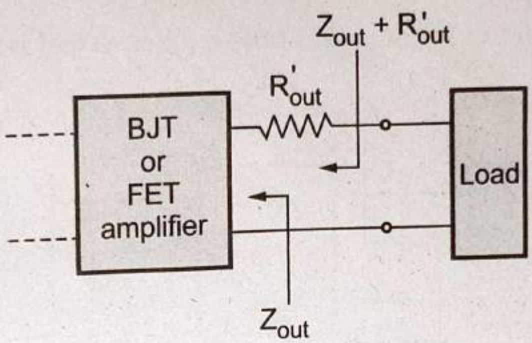


Fig. 5.6.13 Stabilization of output port through series resistance

$$\boxed{\text{Re}\{Z_{out} + R'_{out} + Z_L\} > 0}$$

- Configuration for stabilization of output port through shunt conductance is shown in Fig. 5.6.14.

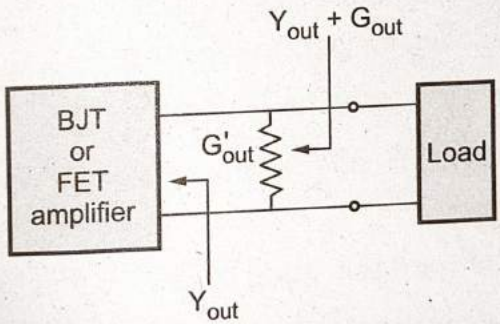


Fig. 5.6.14 Stabilization of output port through shunt conductance

Condition : $\boxed{\text{Re}\{Y_{out} + G'_{out} + Y_L\} > 0}$

5.6.4 Gain Considerations

5.6.4.1 Unilateral Design

- After ensuring stability, there is need to obtain a desired gain performance. If the transistor's feedback is neglected i.e. $S_{12} = 0$; the unilateral power gain can be expressed as

$\text{Re}\{Z_{out}\} < 0$

- A method to stabilize active device is to add a series resistance or a shunt conductance to the port.

5.6.3.1 Configuration at Input Port

- Fig. 5.6.4 shows stabilization input port through series resistance.

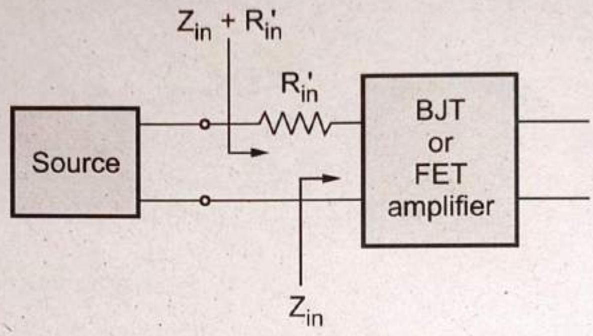


Fig. 5.6.11 Stabilization of input port through series resistance

- This loading along with $\text{Re}\{Z_s\}$ must compensate the negative contribution of $\text{Re}\{Z_{in}\}$. Therefore condition should be -

$$\boxed{\text{Re}\{Z_{in} + R'_{in} + Z_s\} > 0}$$

- The stabilization of input port through shunt conductance is shown in Fig. 5.6.12.

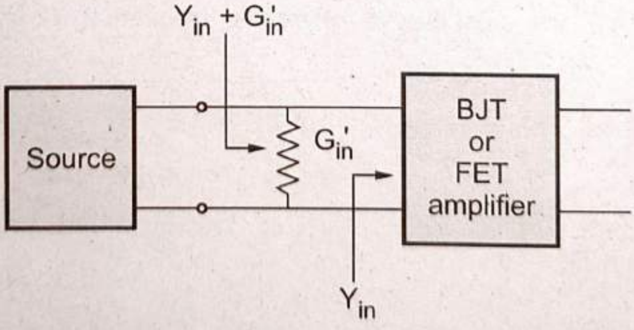


Fig. 5.6.12 Stabilization of input port through shunt conductance

- This loading along with $\text{Re}\{ Z_s \}$ must compensate the negative contribution of $\text{Re}\{ Z_{in} \}$. Therefore condition is :

$$\boxed{\text{Re}\{Y_{in} + G'_{in} + Y_s\} > 0}$$

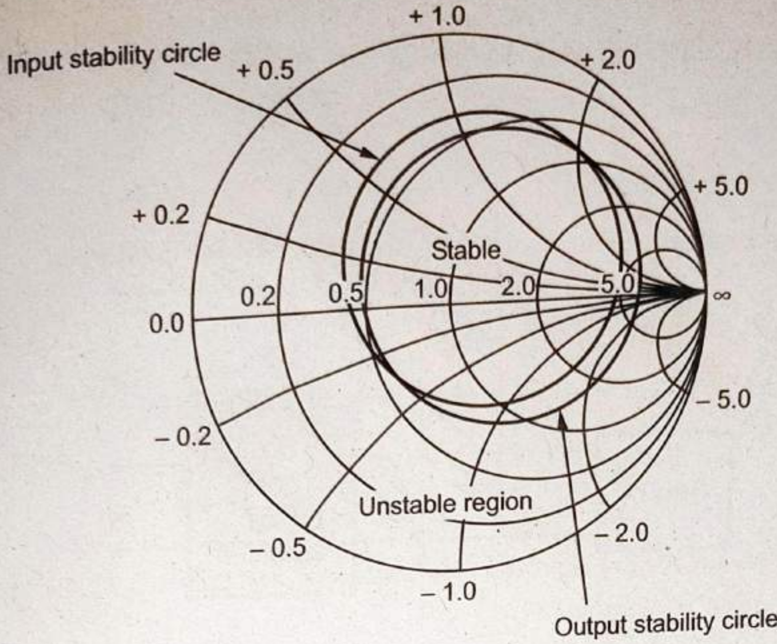


Fig. 5.6.10 Stability circles

University Questions

1. *With reference to RF transistor amplifier, discuss the considerations for stability and gain.* **AU : Dec.-14, Marks 16**
2. *Write the mathematical analysis of amplifier stability.* **AU : May-15, Marks 8**

5.6.3 Stabilization Methods

- The BJT and FET_s are unstable at operating frequency, it means $|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$

$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right| > 1 \text{ ; and}$$

$$|\Gamma_{out}| = \left|\frac{Z_{out} - Z_0}{Z_{out} + Z_0}\right| > 1.$$

Above expression indicates that -

$$\text{Re}\{Z_{in}\} < 0 \text{ and}$$

Parameters for output stability circle :

$$C_L^* = S_{22}^* - \Delta^* S_{11}$$

$$= 0.89\angle 26.5^\circ - (0.385\angle -53^\circ \times 0.316\angle -79.73^\circ)$$

$$= 0.89\angle 26.5^\circ - 0.12166\angle -132.73^\circ$$

$$= (0.7964 + j\,0.3970) - (0.0825 + j\,0.0843) = 0.7139 + j\,0.3077$$

$$\text{or } |C_L^*| = (0.517 + 0.095)^{1/2} = 0.783$$

$$\phi = \tan^{-1}\left(\frac{0.3077}{0.7139}\right) = \angle 23.31^\circ$$

$$\therefore \quad C_L = \frac{C_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{0.783\angle 23.31^\circ}{0.7921 - 0.0998} = 1.131\angle 23.31^\circ$$

$$r_L = \frac{|S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2} = 0.1755$$

Example 5.6.4 *Investigate the stability regions of a transistor whose S-parameters are recorded as follows :* $S_{12} = 0.2\angle -10^\circ$; $S_{11} = 0.7\angle -70^\circ$; $S_{21} = 5.5\angle 85^\circ$ *and* $S_{22} = 0.7\angle -45^\circ$ *at 750 MHz.* **AU : May-16, Marks 16**

Solution : Given : $S_{11} = 0.7\angle -70$, $S_{22} = 0.7\angle -45$

$$S_{12} = 0.2\angle -10, \; S_{21} = 5.5\angle 85$$

K > 1 and |Δ| < 1 must simultaneously hold for unconditionally stable

- Rollet's Condition (K-Δ test) :

For unconditional stability

$$\boxed{K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}\,S_{21}|} > 1} \quad \boxed{\text{and} \quad |\Delta| = |S_{11}\,S_{22} - S_{12}\,S_{21}| < 1}$$

$$\Delta = (0.7\angle -70 \times 0.7\angle -45) - (0.2\angle -10 \times 5.5\angle 85)$$

$$= (0.49\angle -115) - (1.1\angle 75)$$

$$= 1.5848\angle -108.0775 = -0.491784 + j\,1.5066$$

$$|\Delta| \approx 1.5$$

Substituting values of S_{11}, S_{22} and $|\Delta|$

$$K \approx 1.35$$

Even though $K > 1$; transistor is potentially unstable because $|\Delta| > 1$

This results in input and output stability circles being located inside the smith chart.

Also, $|S_{11}|$ and $|S_{22}|$ are less than unity, the centre of the chart is stable point.

Fig. 5.6.10 shows stability circle sketches for $K>1$ and $|\Delta| > 1$.

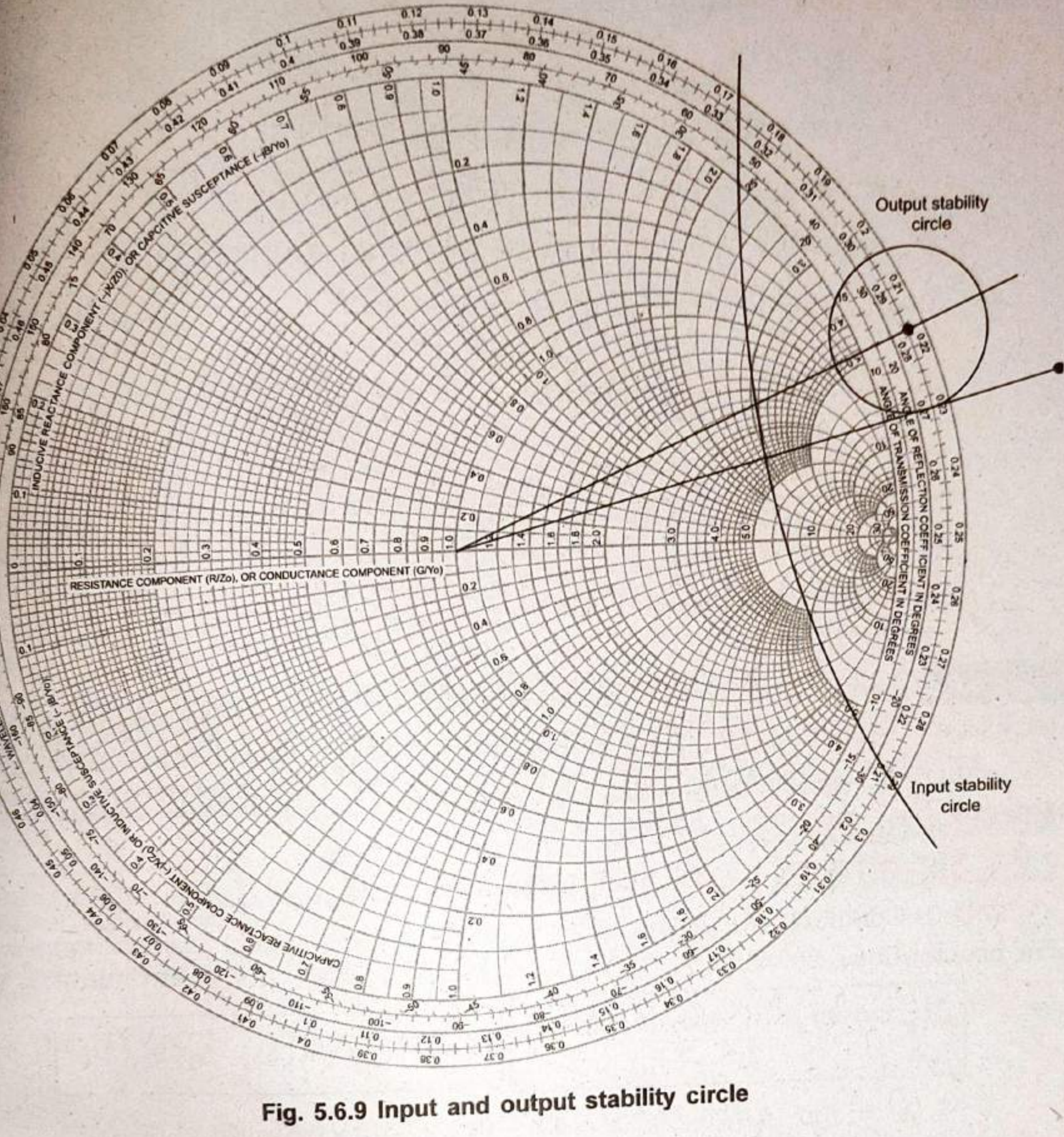


Fig. 5.6.9 Input and output stability circle

$$= 0.385 \angle 53^{\circ} - 0.20124 \angle -106.23^{\circ}$$

$$= (0.2316 + j\ 0.3073) - (0.0786 + j\ 0.2687) = 0.153 + j\ 0.0386$$

$$|C_S^*| = (0.0234 + 0.00148)^{1/2} = 0.1577$$

$$\phi = \tan^{-1}\left(\frac{0.0386}{0.153}\right) = \angle 14.15^{\circ}$$

$$\therefore \quad C_S = \frac{C_S^*}{|S_{11}|^2 - |\Delta|^2} = \frac{0.1577 \angle 14.15^{\circ}}{0.1482 - 0.0998} = 3.25 \angle 14.15^{\circ}$$

$$r_S = \frac{|S_{12} S_{21}|}{|S_{11}|^2 - |\Delta|^2} = \frac{0.1215}{0.0484} = 2.510$$

The stability of a device is tested by equation,

$$\boxed{k = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}\cdot S_{21}|}} > 1$$

Substituting values

$$k = \frac{1-|0.5\angle-60^{\circ}|^2-|0.6\angle-35^{\circ}|^2+0.42^2}{2\ |(0.02\angle 0^{\circ}\times 6.5\angle 115^{\circ})|} = \mathbf{2.17}$$

$$\boxed{|\Delta| = |S_{11}S_{22}-S_{12}S_{21}|} = 0.42$$

$\because$ $k > 1$ and $|\Delta| < 1$; the transistor is unconditionally stable.

Example 5.6.3 *The S-parameters for a transistor is given below. Determine its stability and draw the input and output stability circles (use Smith chart).*

$S_{11} = 0.385\angle{-53^{o}}$, $S_{12} = 0.045\angle{90^{o}}$, $S_{21} = 2.7\angle{78^{o}}$ and $S_{22} = 0.89\angle{-26.5^{o}}$,

AU : May-12, Marks 16

Solution :

$$\boxed{\Delta = S_{11}S_{22}-S_{21}S_{12}}$$

$$= (0.385\angle-53^{\circ}\times 0.89\angle-26.5^{\circ})-(0.045\angle 90^{\circ}\times 2.7\angle 78^{\circ})$$

$$= 0.343\angle-79.5^{\circ}-0.122\angle 168^{\circ}$$

$$= 0.6624-j\,0.3366-(0.1188-j\,0.02525)$$

$$|\Delta| = (0.0564-j0.3113)^{1/2} = 0.316$$

$$\phi = \tan^{-1}\left(\frac{0.3113}{0.0564}\right) = \angle 79.73^{\circ}$$

$$\Delta = 0.316\angle 79.73^{\circ} \text{ and } |\Delta| = 0.316$$

$$\boxed{K = \frac{1+|\Delta|^2-|S_{11}|^2-|S_{22}|^2}{2\ |S_{12}S_{21}|}}$$

$$= \frac{1+0.0998-0.1482-0.792}{0.243} = 0.6563$$

$\because$ $|\Delta| < 1$ and $K < 1$; it is the case of a potentially unstable amplifier.

Input stability circle parameters :

$$C_S^* = S_{11}-\Delta^* S_{22}$$

$$= 0.385\angle 53^{\circ}-(0.316\angle-79.73^{\circ}\times 0.89\angle-26.5^{\circ})$$

In other words, the stability circles have to reside completely outside the $|\Gamma_3| = 1$ and $\Gamma_L = 1$. In the following, we concentrate on the $|\Gamma_S| = 1$ circle shown in Fig. 5.6.8.

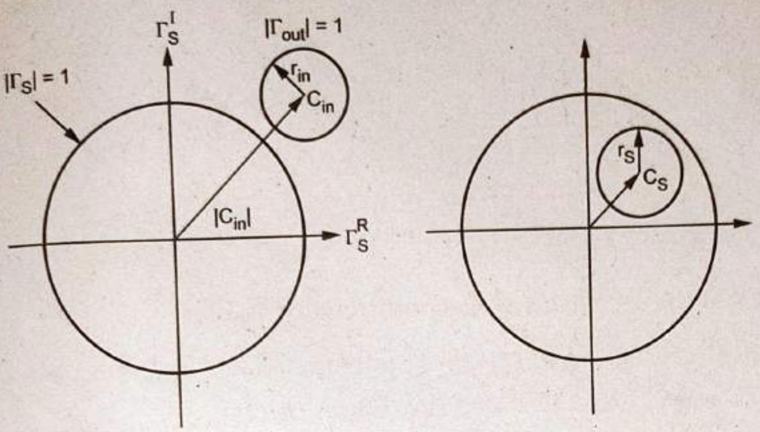


Fig. 5.6.8 : Unconditional stability in the Γ_S and Γ_{out} planes for $|S_{11}| < 1$

- The condition for stability is expressed in terms of stability factor k as -

$$k = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}||S_{21}|} > 1$$

- The stability factor k is also called as **Rollet factor**. It applies for both input and output ports.

Inequality $|\Delta| = |S_{11}\,S_{22} - S_{12}\,S_{21}| \leq |S_{21}\,S_{22}| + |S_{12}\,S_{21}|$

- For an **unconditional stable design both conditions are to be fulfilled** i.e.

  I] $|\Delta| < 1$ II] $k > 1$

Solved Examples

Example 5.6.2 *A MESFET operated at 5.7 GHz has the following S-parameters :*

$S_{11} = 0.5\angle -60^\circ$, $S_{12} = 0.02\angle 0^\circ$

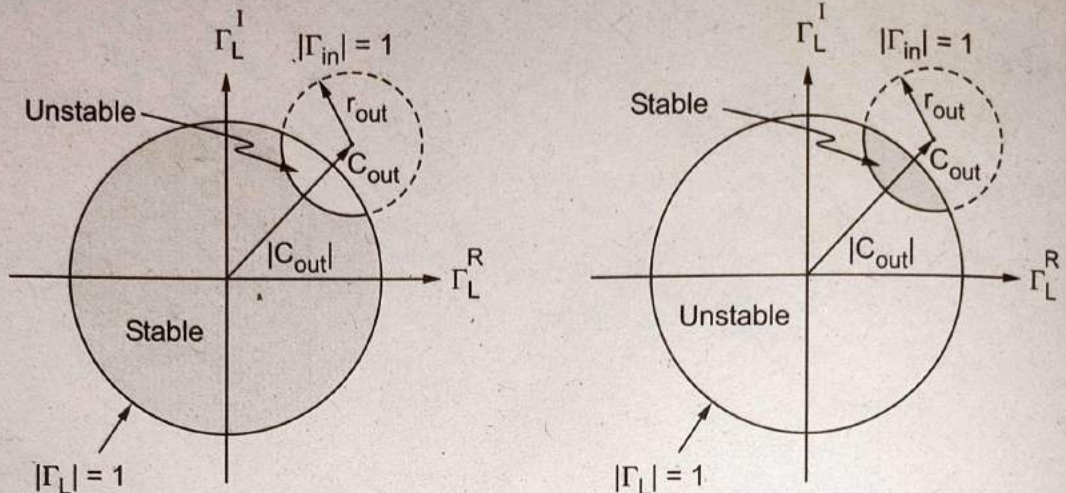
$S_{21} = 6.5\angle 115^\circ$, $S_{22} = 0.6\angle -35^\circ$

Verify the circuit, whether it is unconditionally stable or not ?

AU : Dec.-13, Marks 6

Solution : $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}|$

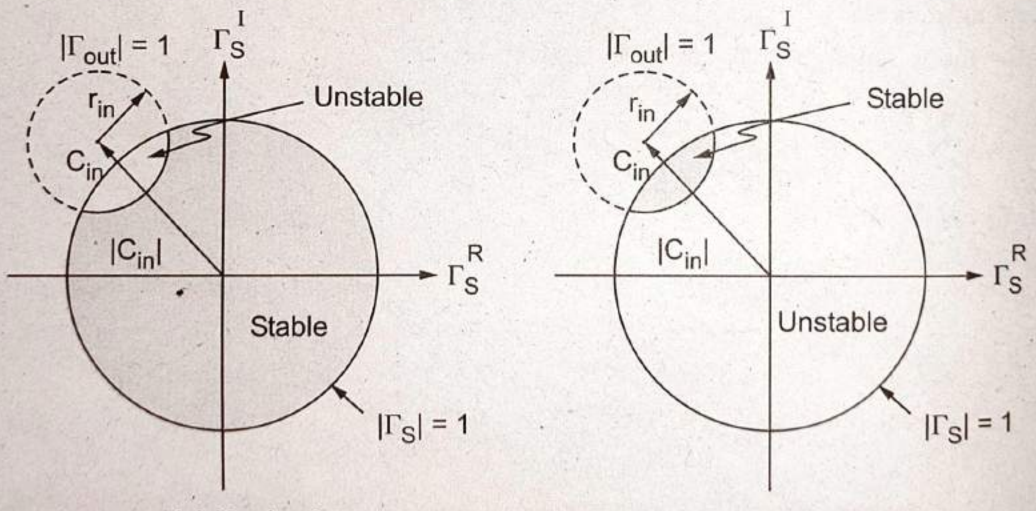
$$= |(0.5\angle -60^\circ \times 0.6\angle -35^\circ) - (0.02\angle 0^\circ \times 6.5\angle 115^\circ)| = 0.42$$



(a) Shaded region is stable since $|S_{11}| < 1$

(b) Stable region excludes the origin, $\Gamma_L = 0$, since $|S_{11}| > 1$

Fig. 5.6.6 Output stability circles denoting stable and unstable regions



(a) $|S_{22}| < 1$

(b) $|S_{22}| > 1$

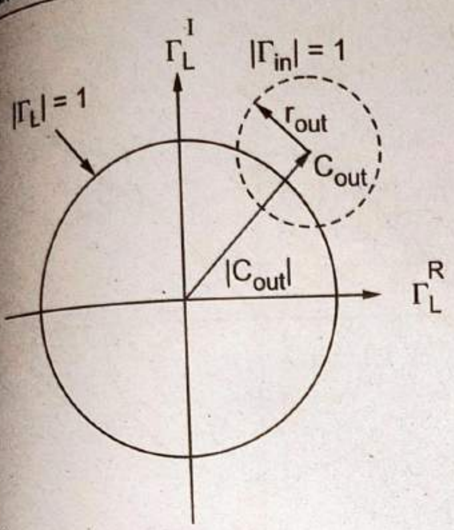
Fig. 5.6.7 Input stability circles denoting stable and unstable regions

5.6.2.2 Unconditional Stability

- An amplifier when remains stable throughout the entire domain of the Smith chart at the selected frequency and bias conditions. Such a situation is referred as **Unconditional stability.**

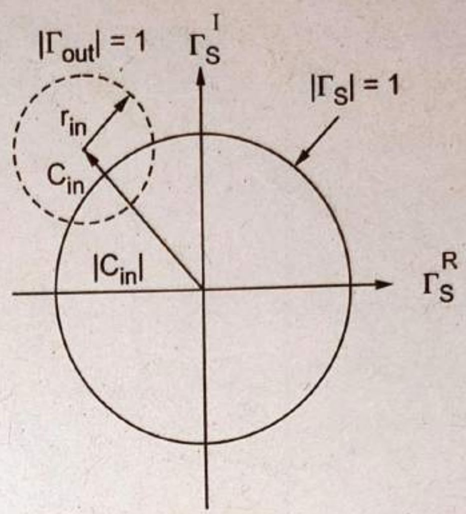
$$\left| |C_{in}| - r_{in} \right| > 1$$

$$\left| |C_{out}| - r_{out} \right| > 1$$



Output stability circle

Fig. 5.6.4 Stability circle $|\Gamma_{in}| = 1$ in the complex Γ_L plane



Input stability circle

Fig. 5.6.5 Stability circle $|\Gamma_{out}| = 1$ in the complex Γ_s plane

Input stability circle

- The input stability circle equation is given by

$$\boxed{\left(\Gamma_s^R - C_{in}{}^R\right)^2 + \left(\Gamma_s^I - C_{in}^I\right)^2 = r_{in}^2}$$

The circle radius is given as

$$r_{in} = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{11}|^2 - |\Delta|^2 \right|}$$

$$C_{in} = C_{in}^R + jC_{in}^I = \frac{(S_{11} - S_{22}^* \Delta)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \ldots(5.6.1)$$

- If $\Gamma_L = 0$, then $|\Gamma_{in}| = |S_{11}|$ and two cases have to be differentiated depending on $|S_{11}| < 1$ or $|S_{11}| > 1$. In this case, the only stable region is shaded domain between output stability circle $|\Gamma_{in}| = 1$ and $|\Gamma_L| = 1$ circle, represented in Fig. 5.6.6.
- In Fig. 5.6.7 shows the two stability domain for input stability circle. The rule is that if $|S_{22}| < 1$, center ($\Gamma_3 = 0$) must be stable ; otherwise the center becomes unstable for $|S_{22}| > 1$.

5.6.2.1 Stability Circles

- An amplifier must be stable for a specified range of frequencies. Usually the amplifier tend to oscillate depending on operating frequency and load (termination).
  1. If $|\Gamma_0|>1$; then return voltage increases in magnitude called positive feed back, which causes instability.
  2. If $|\Gamma_0|<1$; causes diminished return voltage wave (negative feed back)
- For a two-port network, stability implies that the magnitudes of reflection coefficients are less than unity.

$$|\Gamma_L| < 1 \quad \text{and}$$

$$|\Gamma_s| < 1$$

$$|\Gamma_{in}| = \left|\frac{S_{11}-\Gamma_L\Delta}{1-S_{22}\Gamma_L}\right| < 1$$

$$|\Gamma_{out}| = \left|\frac{S_{22}-\Gamma_3\Delta}{1-S_{11}\Gamma_3}\right| < 1$$

$$\Delta = S_{11}S_{22}-S_{12}S_{21}$$

$$S_{11} = S_{11}^R+jS_{11}^I \; ; \; S_{22} = S_{22}^R+jS_{22}^I$$

$$\Delta = \Delta^R+j\Delta^I$$

$$\Gamma_L = \Gamma_L^R+j\Gamma_L^I$$

Output stability circle

- The output stability circle equation is given by -

$$\boxed{\left(\Gamma_L{}^R-C_{out}{}^R\right)^2+\left(\Gamma_L{}^I-C_{out}{}^I\right)^2 = r^2{}_{out}}$$

The circle radius is given by

$$r_{out} = \frac{|S_{12}\cdot S_{21}|}{\left||S_{22}|^2-|\Delta|^2\right|}$$

Center of this circle is located at

$$C_{out} = C_{out}^R+jC_{out}^I = \frac{(S_{22}-S_{11}^*\Delta)^*}{|S_{22}|^2-|\Delta|^2}$$

$$G_{TU} = 12.569$$

3) Available Power Gain :

$$G_A = \frac{|S_{21}|^2 \cdot (1-|\Gamma_s|^2)}{(1-|\Gamma_{out}|^2 \cdot |1-S_{11} \cdot \Gamma_s|^2)} = \mathbf{14.739}$$

4) Operating Gain :

$$G = \frac{(1-|\Gamma_L|^2) \cdot |S_{21}|^2}{(1-|\Gamma_{in}|^2) \cdot |1-S_{22}\,\Gamma_L|^2} = \mathbf{13.742}$$

University Questions

1. *A microwave amplifier is characterized by its S-parameters. Derive equations for power gain, available gain and transducer gain.* **AU : Dec.-12, Marks 16**
2. *Derive the transducer power gain for a transistor amplifier.* **AU : May-13, Marks 8**
3. *Write brief note on : i) Operating power gain ii) Available power gain* **AU : Dec.-13, Marks 3 + 3**
4. *Design a microwave amplifier for maximum transducer power gain.* **AU : May-15, Marks 8**
5. *Derive the equation for power gain, available power gain and transducer power gain.* **AU : Dec.-16, Marks 16**

5.6.2 Stability Considerations

- The stability of an amplifier is a very important considerations in microwave circuit design. There are two types of stability.

1) Conditional Stability

- A network is conditionally stable if the real part of the input impedance Z_{in} and output impedance Z_{out} is greater than zero for some positive real source and load impedances at a specific frequency.

2) Unconditional Stability

- A network is unconditionally stable if the real part of input impedance Z_{in} and the output impedance Z_{out} is greater than zero for all positive real source and load impedances at a specific frequency.

$$G = \frac{P_i}{P_{in}} = \frac{P_L}{P_A} \cdot \frac{P_A}{P_{in}} = G_T \cdot \frac{P_A}{P_{in}}$$

$$\boxed{G = \frac{\left(1-|\Gamma_L|^2\right) \cdot |S_{21}|^2}{\left(1-|\Gamma_{in}|^2\right) \cdot |1-S_{22}\Gamma_L|^2}}$$

Example 5.6.1 *An RF amplifier has the following S - parameters. $S_{11} = 0.3 \angle -70°$, $S_{21} = 3.5 \angle 85°$, $S_{12} = 0.2 \angle -10°$ and $S_{22} = 0.4 \angle -45°$. Furthermore, the input side of the amplifier is connected to a voltage source with $Vs = 5V \angle 0°$ and source impedance $Z_S = 40\ \Omega$. The output is utilized to drive an antenna which has an amplifier of $Z_L = 73\ \Omega$. Assuming that S - parameters of the amplifier are transducer gain G_T, unilateral transducer gain G_{TU}, available gain G_A, Operating gain G.*

AU : Dec.-17, Marks 16

Solution :

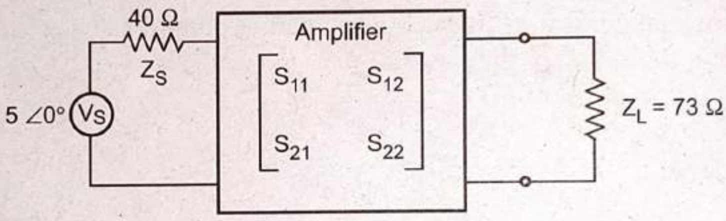


Fig. 5.6.3

Given : $Z_S = 40\ \Omega$, $Z_L = 73\ \Omega$, $Z_O = 50\ \Omega$.

$$\Gamma_s = \frac{Z_S - Z_O}{Z_S + Z_O} = \frac{40-50}{40+50} = -0.111$$

$$\Gamma_L = \frac{Z_L - Z_O}{Z_L + Z_O} = \frac{73-50}{73+50} = 0.186$$

1) Transducer Power Gain :

$$G_T = \frac{1-|\Gamma_L|^2 \cdot |S_{21}|^2 \cdot (1-|\Gamma_s|^2)}{|1-|\Gamma_L|\Gamma_{out}|^2 \cdot |1-S_{11}|\Gamma_s|^2}$$

$$G_T = \mathbf{12.562}$$

2) Unilateral Power Gain :

Effect of negative feedback is neglacted i.e.

$$S_{12} = 0$$

$$G_{TU} = \frac{(1-|\Gamma_L|^2) \cdot |S_{21}|^2 \cdot (1-|\Gamma_s|^2)}{|1-|\Gamma_L S_{22}|^2 \cdot |1-S_{11}|\overline{\Gamma_s}|^2}$$

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right) \cdot |S_{21}|^2 \cdot \left(1-|\Gamma_s|^2\right)}{|1-\Gamma_L S_{22}|^2 \cdot |1-S_{11}\Gamma_s|^2}$$

- The unilateral transducer power gain G_{TU} is the forward power gain in a feedback amplifier having its reverse power gain set to zero ($|S_{12}|^2 = 0$) by adjusting lossless reciprocal feedback network connected around the microwave amplifier.

2) Matched Transducer Power Gain ($\Gamma_s = \Gamma_e = 0$)

- When both the input and output networks are perfectly matched to the source impedance and the load impedance, respectively, the transducer power gain is given by -

$$G_{TM} = |S_{21}|^2$$

3) Maximum Unilateral Transducer Power Gain

- The maximum unilateral transducer power gain is obtained when

$$\Gamma_s = S_{11}^* \qquad \text{and} \qquad \Gamma_e = S_{22}^*$$

$$G_{TU \cdot max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

5.6.1.3 Additional Power Relations

Available Power Gain (G_A) :

- The available power gain for load side matching $\left(\Gamma_L = \Gamma_{out}^*\right)$ is givens as -

$$G_A = \frac{\text{Power available from the amplifier}}{\text{Power available from the source}} = G_T\big|_{\Gamma_L = \Gamma_{out}^*}$$

$$G_A = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_s|^2\right)}{\left(1-|\Gamma_{out}|^2\right) \cdot |1-S_{11} \cdot \Gamma_s|^2}$$

Operating Power Gain (G)

- The **operating power gain** or **power gain** is defined as the ratio of power delivered to load to the power supplied to the amplifier.

$$G = \frac{\text{Power delivered to load}}{\text{Power supplied to amplifier}}$$

Available power

- Maximum power transfer condition exists when input impedance is complex conjugate matched with source impedance i.e. $Z_{in} = Z_S^*$.
- The maximum power transfer condition can also be expressed in terms of reflection coefficient as $\Gamma_{in} = \Gamma_s^*$
- The available power is gives as -

$$P_A = P_{in}\big|_{\Gamma_{in}=\Gamma_s^*}$$

$$P_A = \frac{1}{2} \cdot \frac{|b_s|^2}{|1-\Gamma_{in}\,\Gamma_s|^2}\Bigg|_{\Gamma_{in}=\Gamma_s^*} \cdot \left(1-|\Gamma_{in}|^2\right)$$

$$\boxed{P_A = \frac{1}{2} \cdot \frac{|b_s|^2}{1-|\Gamma_s|^2}}$$

- From the expression it is concluded that available power is dependent on $\Gamma_s \cdot I_f$ $\Gamma_{in} = 0$ and $\Gamma_s \neq 0$ then

$$P_{inc} = \frac{|b_s|^2}{2}$$

5.6.1.2 Transducer Power Gain

- The gain of amplifier when placed between source and load is called the **transducer power gain (G_T)**. It is computed as

$$G_T = \frac{\text{Power delivered to load}(P_L)}{\text{Available power from source}(P_A)}$$

- The power delivered to the load is the resultant of the power incident at the load minus power reflected from the load.
- The transducer power gain is expressed as -

$$\boxed{G_T = \frac{\left(1-|\Gamma_L|^2\right)\cdot|S_{21}|^2\cdot\left(1-|\Gamma_s|^2\right)}{|1-\Gamma_L\Gamma_{out}|^2\cdot|1-S_{11}\Gamma_s|^2}}$$

1) Unilateral Power Gain (G_{TU}) ($|S_{12}|^2 = 0$)

- The amplifier power gain when feedback effect of amplifier is neglected i.e. $S_{12} = 0$, it is called **unilateral power gain.**

5.6.1 Amplifier Power Relations

5.6.1.1 RF Source

- A general single-stage amplifier with two matching networks at input and output is shown in Fig. 5.6.1

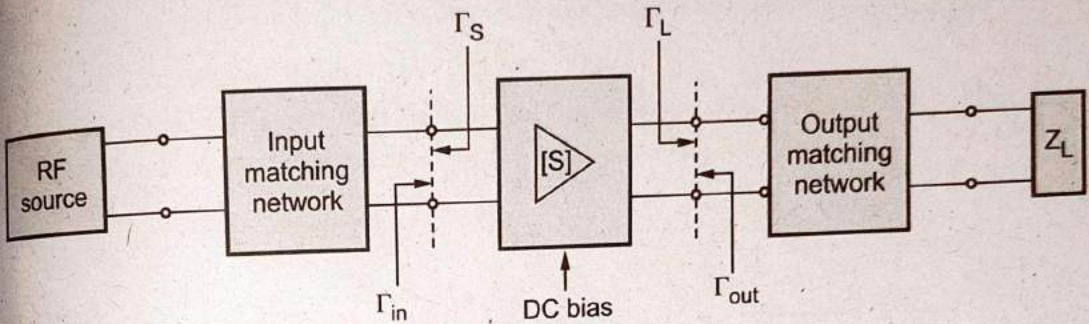


Fig. 5.6.1 General amplifier system

- Assuming in terms of power flow relations, the two matching networks are included in the source and load impedances. The amplifier circuit is than reduced and shown in Fig. 5.6.2.

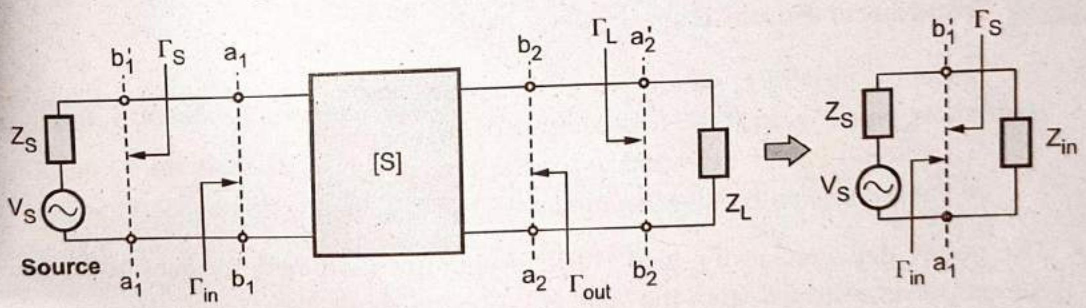


Fig. 5.6.2 Simplified diagram of single stage amplifier

Incident power

- The incident power associated with b_1' is given by

$$\boxed{P_{inc} = \frac{1}{2} \cdot \frac{Z_0}{(Z_S + Z_0)} \cdot \frac{|V_S|^2}{|1 - \Gamma_{in}\,\Gamma_s|}}$$

- The actual input power P_{in} is comprises of incident and reflected power waves.

$$P_{in} = P_{inc}\left(1 - |\Gamma_{in}|^2\right)$$

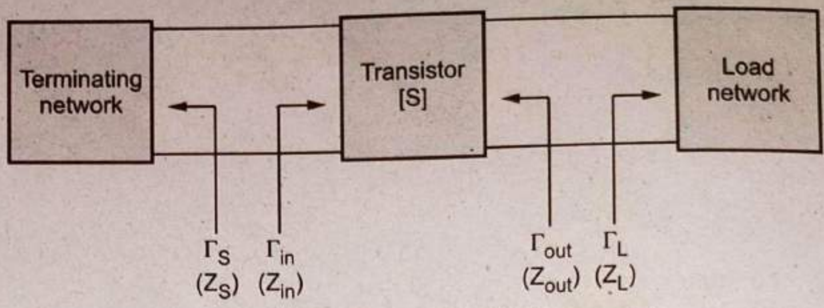


Fig. 5.5.2 Transistor oscillator model

- For steady-state oscillation at the input port, $\Gamma_S = \Gamma_{in} = 1$.

$$\frac{1}{\Gamma_{in}} = \Gamma_{in} = S_{11} \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = \frac{S_{11} - \Delta \Gamma_L}{1 - S_{22} \Gamma_L}$$

5.6 Power Amplifiers

- The amplifier is characterised by its s-matrix at a specified DC bias point.
- Some important parameters of amplifiers are :

1. Gain and gain flatness (expressed in dB)
2. Operating frequency (expressed in Hz)
3. Operating bandwidth (expressed in Hz)
4. Output power (expressed in dBm)
5. Power supply requirements (expressed in V and A)
6. Input and Output reflection coefficients (VSWR)
7. Noise figure (expressed in dB)
8. Intermodular distortion (IMD) products.
9. Harmonics
10. Feedback
11. Heating effects

- Amplifier performance is greatly affected by all above parameters.
- To analyze amplifier performance number of definitions are established stating various power relations.
- Other important analysis tools are stability, gain, noise and VSWR performance. All these can be expressed as circle equations and represented in Smith chart.

Mixers used in Microwaves

1. Single ended diode mixer
2. Single ended FET mixer
3. Balanced mixer
4. Image reject mixer
5. Differential FET mixer

5.5 RF Oscillators

- RF and microwave oscillators are used in all modern wireless communications, radar, and remote sensing systems to provide signal sources for frequency conversion and carrier generation.
- At microwave frequencies negative resistance diodes or transistors and used to generate oscillations.
- The RF circuit for a one-port negative resistance oscillator is shown in Fig. 5.5.1

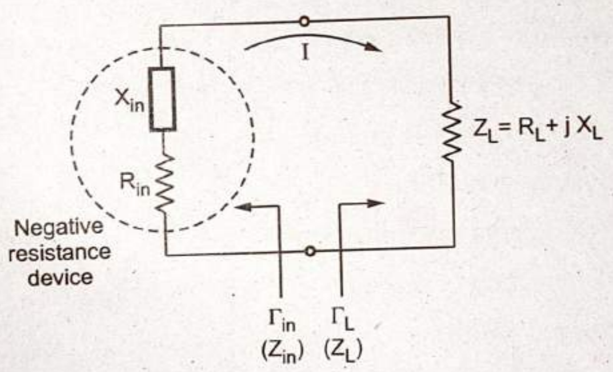


Fig. 5.5.1 One port negative resistance oscillator

Here, $Z_{in} = R_{in} + j$ Xin is the input impedance of the active device.

- Condition for steady-state oscillation is : $Z_L = -Z_{in}$. It implies the relation between reflection coefficients Γ_L and Γ_{in} :

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-Z_{in} - Z_0}{-Z_{in} + Z_0} = \frac{Z_{in} + Z_0}{Z_{in} - Z_0} = \frac{1}{\Gamma_{in}}$$

RF Transistor Oscillators

- In a transistor oscillator, a negative resistance one-port network is effectively created by terminating a potentially unstable transistor with impedance designed to drive the device in an unstable region.
- Circuit for a two-port transistor oscillator is shown in Fig. 5.5.2

- Major RF signal behaviours-

1. Gain
2. Loss
3. Reflection
4. Refraction
5. Diffraction
6. Scattering
7. Absorption
8. VSWR
9. Return Loss
10. Amplification and Attenuation
11. Wave Propagation
12. Free Space Path Loss
13. Delay Spread

[C] Link Budget

- This is a term used to describe the cumulative effects of gains and nonideal losses in a communications system. Historically, link budgets were used in satellite channel calculations but they have become more common in their use for gain and loss analysis in any communication system.

5.4 RF Mixer

- Modern microwave systems typically use several mixers and filters to perform the functions of frequency up-conversion and down-conversion between baseband signal frequencies and RF carrier frequencies.
- A mixer is a three-port device that uses a nonlinear or time-varying element to achieve frequency conversion.
- An ideal mixer produces an output consisting of the sum and difference frequencies of its two input signals.
- The performance of practical RF and microwave mixers is based on the nonlinearity provided by diode or a transistor.
- A nonlinear component generates a wide variety of harmonics and other products of input frequencies, so filtering must be used to select the desired frequency components.

- **Noise Figure** (NF) is the noise factor, expressed in decibels :

$$NF_{dB} = 10*\log(F)$$

Q.13 Define unconditional stability with regard to microwave transistor amplifier.

AU : Dec.-17

Ans. : Unconditional Stability : A network is unconditionally stable if the real part of input impedance Z_{in} and the output impedance Z_{out} is greater than zero for all positive real source and load impedances at a specific frequency.

❑❑❑

Q.10 Draw the contour of nodal quality factor Q = 3. AU : May-15

Ans. : During L-type matching network analysis at each node the impedance can be expressed in terms of equivalent series impedance $Z_s = P_s + j \times s$ or admittance $Y_P = G_P + jB_P$.

$$\text{Nodal quality factor } Q_n = \frac{|X_s|}{P_s} = \frac{|B_P|}{G_P}$$

BW of matching network can be estimated by nodal quality factor.

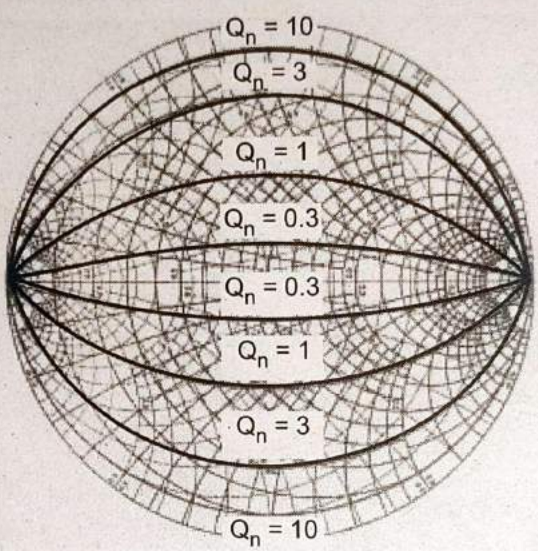


Fig. 3 Plot of various nodal quality factor

Q.11 Define maximum available gain. AU : Dec.-15

Ans. : Maximum unilateral transducer power gain is obtained when

$$\Gamma_s = S_{11}^* \quad \text{and} \quad \Gamma_l = S_{22}^*$$

$$G_{TU\,max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

Q.12 Define Noise Figure. AU : Dec.-16

Ans. : Noise Figure :

- Noise factor (F) is a measure of how the signal to noise ratio is degraded by a device :

$$F = (S_{in} / N_{in}) / (S_{out} / N_{out})$$

Where, S_{in} is the signal level at the input

N_{in} is the noise level at the input,

S_{out} is the signal level at the output and

N_{out} is the noise level at the output.

.5 Define transducer power gain. **AU : Dec.-13**

...ns. : Transducer power gain :

The gain of amplifier when placed between source and load is called transducer ...wer gain (G_T).

$$G_T = \frac{\text{Power delivered to load}}{\text{Available power from source}}$$

.6 Give expression that relates nodal quality factor (Q_n) with loaded quality factor (Q_L). **AU : Dec.-13**

...ns. : Relation between Q_L and Q_n

$$\boxed{Q_L = \frac{Q_n}{2}}$$

.7 Define stability. **AU : May-14**

...ns. : Stability :

Stability in a network means when the magnitudes of reflection co-efficients are less ...an unity.

$$|\Gamma_L| < 1 \quad \text{and} \quad |\Gamma_S| < 1$$

Q.8 Define unilateral power gain. **AU : Dec.-14**

Ans. : Unilateral power gain :

The amplifier power gain when feedback effect of amplifier is neglected i.e. $S_{12} = 0$; ...t is called **unilateral power gain.**

Q.9 Draw the VSWR circle for reflection coefficient 1. **AU : May-15**

Ans. : Reflection coefficient (Γ) for an impedance Z_L attached to a transmission live with characteristic impedance Z_0 is given by :

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

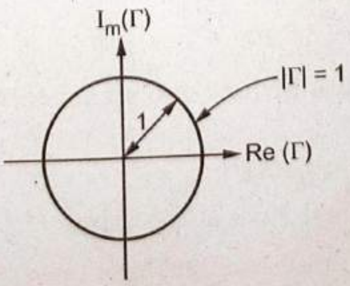


Fig. 2

Ans. : A microwave amplifier connected to a source at the input (port 1) and a load impedance at the output (port 2).

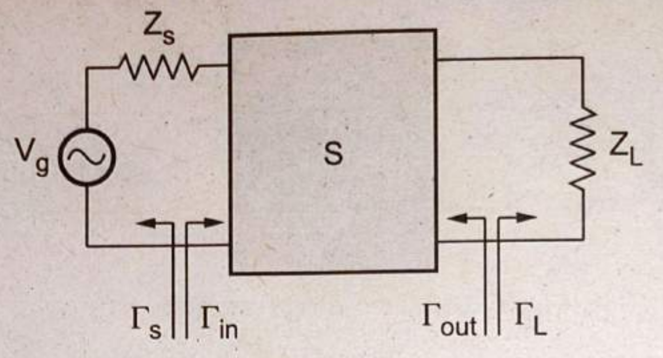


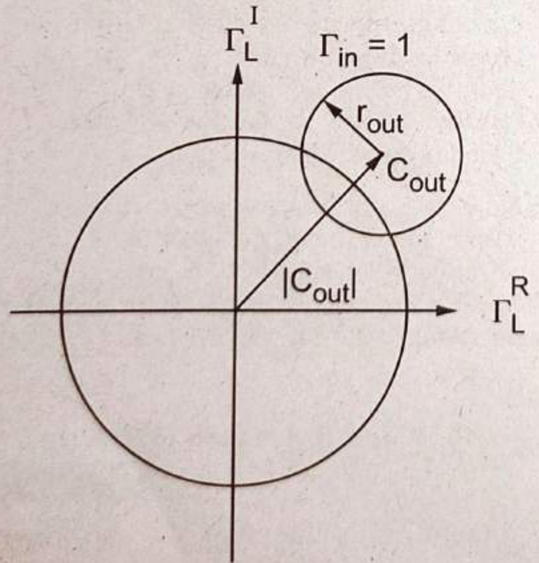
Fig. 1

$$G_P = \frac{P_L}{P_{in}} = |S_{21}|^2 \frac{(1-|\Gamma_L|^2)\,|1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2\,|1-\Gamma_s\Gamma_{in}|^2} \frac{|1-\Gamma_s\Gamma_{in}|^2}{|1-\Gamma_s|^2\,(1-|\Gamma_{in}|^2)}$$

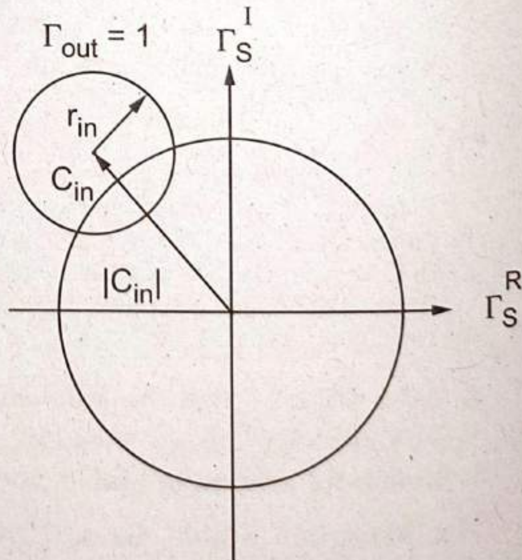
$$= |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2\,(1-|\Gamma_{in}|^2)}$$

Q.4 Draw typical output stability circle and input stability circle. **AU : May-13**

Ans. :



Output stability circle



Input stability circle

- Input and output matching networks are needed to couple source and load respectively. Also, there is need of interstage matching network for matching the output of stage-1 with input of stage-2 as shown in Fig. 5.6.18.

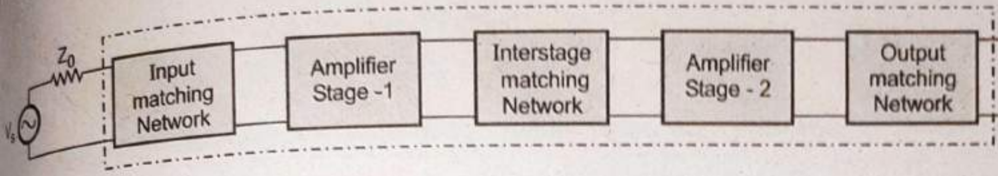


Fig. 5.6.18 Two stage amplifier

- Total power gain G_{tot} is the product of gain of individual stages.

$$G_{tot}(dB) = G_1(dB) + G_2(dB)$$

Two Marks Questions with Answers

Q.1 Distinguish between conditional and unconditional stabilities of amplifier. **AU : May-12**

Ans. :

Sr. No.	Conditional stability	Unconditional stability
1.	**Conditional stability** refers to a network that is stable when its input and output "see" the intended characteristic impedance Z_0 (usually 50 ohms, sometimes 75 ohms).	**Unconditional stability** refers to a network that can "see" any possible impedance on the Smith chart from the center to the perimeter (up to gamma=1.0) at any phase angle. Gamma < 1 means that the real part of the impedance is positive.
2.	If there is a mismatch, there is a region of either source or load impedances that will definitely cause it to oscillate. The term "potentially unstable" refers to the same condition.	Note that any network can oscillate if it sees a real impedance that is negative, so if your system goes outside the normal Smith chart all bets on stability are off.

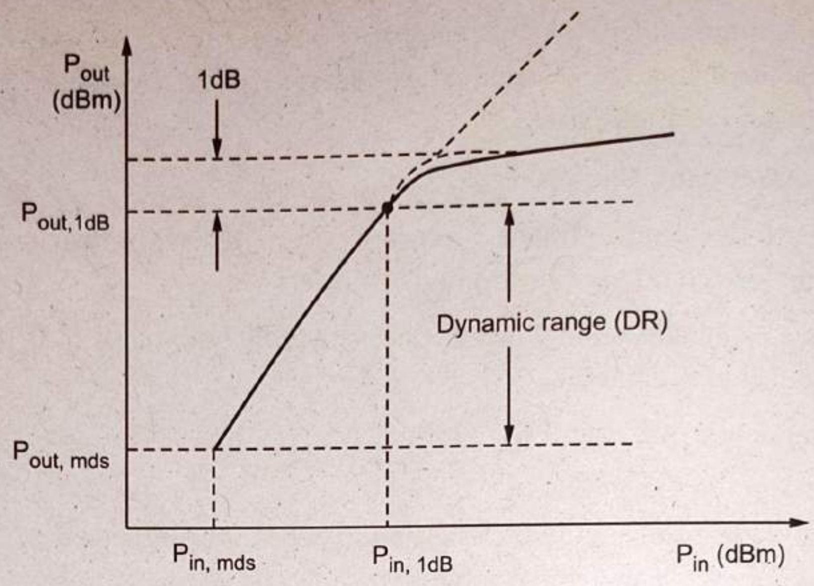
Q.2 A GaAs MESFET has the following parameters :
$S_{11} = 0.65\underline{|-154^\circ}$, $S_{12} = 0.02\underline{|40^\circ}$, $S_{21} = 2.04\underline{|185^\circ}$ and $S_{22} = 0.55\underline{|30^\circ}$
Calculate its maximum stable power gain. **AU : May-12**

Ans. : The Maximum Stable Gain (MSG) is the maximum value GMAG can have, which is achievable when K = 1 :

Maximum stable gain $G_{MSG} = [|S_{21}| / |S_{12}|] = [\,2.04/0.55] = 3.7$

Q.3 Define power gain of amplifier in terms of S-parameters and reflection coefficients. **AU : Dec.-12**

- Fig. 5.6.17 shows output power of the amplifier as a function of input power.



$P_{in,mds}$ = Minimum detectable signal input power
$P_{out,mds}$ = Minimum detectable signal output power

Fig. 5.6.17 Power-gain compression point

Dynamic Range (DR)

- The linear region is called as dynamic range. This range represents the power levels between the minimum detectable signal output power P_o, mds and P_{1dB}.
- The low power level is limited by noise power level. A minimum detectable input signal $P_{in,\ mds}$ can be detectable only if its output power level $P_{out,\ mds}$ is above the noise power level. Then dynamic range,

$$DR = P_{1dB} - P_{out,mds}$$

1-dB Compression Point

- The point where the gain of amplifier deviates from the linear, or small signal gain by 1 dB is called the 1 dB compression point.
- The 1-dB compression point used to characterise the power handling capabilities of the amplifier.

5.6.7.2 Multistage Amplifiers

- Multistage amplifier is used when power gain requirement by single-stage amplifier can not be fulfilled.

5.6.7 Broadband, High-Power and Multistage Amplifiers

- A major limitation of broadband amplifier design at RF is the limitation by gain-BW product of active device. At certain transition frequency f_T, transistor stops functioning as an amplifier.

Complications in Broadband Design

1. Parameter $|S_{21}|$ does not remains constant over the wide frequency band of operation therefore compensation needed.
2. Reverse gain $|S_{12}|$ increases, which reduces overall gain and it is possible that device may fall into oscillation.
3. Change in frequency of S_{11} and S_{22}
4. Noise figure degradation at HF.

- To overcome these problems two design approaches are suggested :

1. Frequency compensated matching networks
2. Negative feed back

5.6.7.1 High Power Amplifiers

- High-power amplifiers are typically used transmitters. For design purposes, a set of large-signal s-parameters is usually needed to characterise the device for power applications.
- However, the measurement of large signal s-parameters are not perfectly defined. Therefore alternative method is to obtain the source and load reflection coefficients in terms of output power and gain at its 1-dB gain compression point.

Gain Compression Point

- The 1-dB compression point is defined as the power gain at the non-linear region of the microwave devices reduces 1-dB power gain over the small-signal linear power gain.

$$G_{1dB} = G_{sl}(dB) - 1(dB)$$

Where $G_{sl}(dB)$ is the small signal linear power gain.

- The small signal power gain is given by :

$$G_P = \frac{P_{out}}{P_{in}} = P_{out}(dBm) - P_{in}(dBm)$$

$$\Rightarrow \quad P_{out}(dBm) = P_{in}(dBm) + G_P(dB)$$

At 1-dB gain compression point, the output power can be expressed as

$$P_{1dB}(dBm) = P_{in}(dBm) + G_P(dB) - 1(dB)$$

- Matching networks are used to maintain VSWR
- The input matching network (IMN) determines the input VSWR and the output matching network (OMN) determines the output VSWR.

For IMN :

$$\text{VSWR}_{\text{IMN}} = \frac{1+|\Gamma_{\text{IMN}}|}{1-|\Gamma_{\text{IMN}}|}$$

For OMN :

$$\text{VSWR}_{\text{OMN}} = \frac{1+|\Gamma_{\text{OMN}}|}{1-|\Gamma_{\text{OMN}}|}$$

$$P_{in} = P_A(1-|\Gamma_{\text{IMN}}|^2) \quad \text{... (5.6.2)}$$

Assuming matching network is lossless i.e. same power is available to active device

$$P_{in} = P_A \frac{(1-|\Gamma_s|^2)(1-|\Gamma_{in}|^2)}{|1-\Gamma_s\Gamma_{in}|} \quad \text{... (5.6.3)}$$

Equating both equations and solving for $|\Gamma_{\text{IMN}}|$

$$|\Gamma_{\text{IMN}}| = \frac{\sqrt{1-(1-|\Gamma_s|^2)(1-|\Gamma_{in}|^2)}}{|1-\Gamma_s\,\Gamma_{in}|}$$

$$|\Gamma_{\text{IMN}}| = \left|\frac{\Gamma_{in}-\Gamma_s^*}{1-\Gamma_s\,\Gamma_{in}}\right|$$

$$|\Gamma_{\text{IMN}}| = \left|\frac{\Gamma_{in}^*-\Gamma_s}{1-\Gamma_s\,\Gamma_{in}}\right|$$

Similarly output reflection coefficient is

$$|\Gamma_{\text{OMN}}| = \sqrt{1-\frac{|\Gamma_L|^2(1-|\Gamma_{out}|^2)}{|1-\Gamma_L\Gamma_{out}|}}$$

$$|\Gamma_{\text{OMN}}| = \frac{\Gamma_{out}-\Gamma_L^*}{1-\Gamma_L\Gamma_{out}}$$

$$|\Gamma_{\text{OMN}}| = \frac{\Gamma_L^*-\Gamma_L}{1-\Gamma_L T_{out}}$$

Total noise figure :

$$F_{tot} = \frac{P_{n3}}{P_{n1}G_{A1}G_{A2}}$$

$$F_{tot} = 1+\frac{P_{ni1}}{P_{n1}G_{A1}}+\frac{P_{ni2}}{P_{n1}G_{A1}G_{A2}}$$

Noise figure at first stage :

$$F_1 = 1+\frac{P_{ni1}}{P_{n1}G_{A1}}$$

Noise figure at second stage :

$$F_2 = 1+\frac{P_{ni2}}{P_{n1}G_{A2}}$$

Noise figure at third stage :

$$F_3 = 1+\frac{P_{ni3}}{P_{n2}G_{A3}}$$

Total noise figure for two stages :

$$F_{tot} = F_1+\frac{F_2-1}{G_{A1}}$$

Total noise figure for three stages :

$$\boxed{F_{tot} = F_1+\frac{F_2-1}{G_{A1}}+\frac{F_3-1}{G_{A1}G_{A2}}}$$

University Questions

1. *Describe the process of stabilizing the noise performance of a transistor by plotting noise circles on the* $\overline{|S}$ *plane.* **AU : May-12, Marks 8**
2. *Write brief on noise figure.* **AU : Dec.-13, Marks 4**
3. *Show that the noise figure of a three stage amplifier is* $F = F_1+\frac{F_2-1}{GA_1}+\frac{F_3-1}{GA_2}$ *where* F_1, F_2 *and* F_3 *are noise figures and* GA_1 *and* GA_2 *are power gains.* **AU : Dec.-14, Marks 8**

5.6.6 Constant VSWR Circles

- An amplifier must maintain VSWR at specified value. Typical value is ranging between 1.5 to 2.5.

- This is standard form of circle equation and can be displayed as part of Smith chart.

Circle center location :

$$d_{F_k} = d_{F_k}^R + jd_{F_k}^I = \frac{\Gamma_{opt}}{1+Q_k}$$

Radius of circle :

$$r_{F_k} = \frac{\sqrt{1-|\Gamma_{opt}|^2 Q_k + Q_k^2}}{1+Q_k}$$

- All constant noise circles located on line from origin to point Γ_{opt}.
- Larger the noise figure, the closer the center d_{F_k} to the origin and larger the radius r_{F_k}.

5.6.5.1 Noise Figure for Multistage Amplifier

- Multistage amplifiers is needed to meet the requirement of high power gain.
- Fig. 5.6.16 shows multistage amplifier representation.

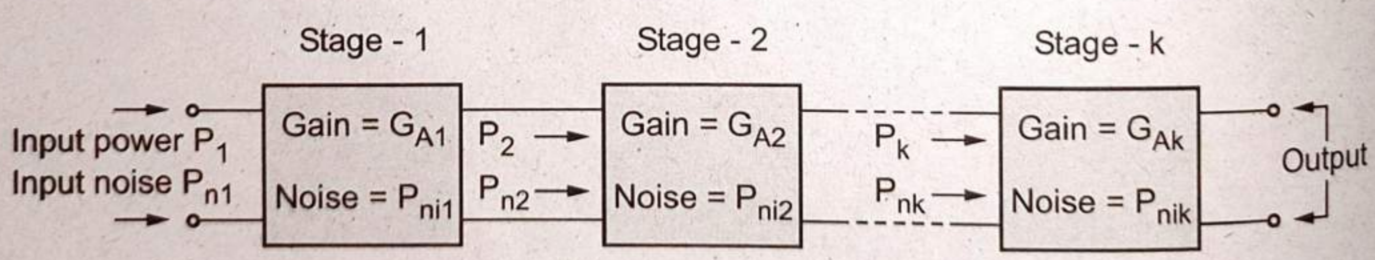


Fig. 5.6.16 Multistage amplifier

Input noise to stage - 1 = P_{n1}

Output noise to stage - 1 = $P_{n2} = G_A P_{n1} + P_{ni}$

Noise power at second amplifier :

$$P_{n3} = G_{A2} (G_{A1} P_{n1} + P_{ni1}) + P_{ni2}$$

- The noise figure of two-port amplifier in admittance and impedance form is given by expressions -

$$\boxed{F = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{opt}\right|^2}$$

$$F = F_{min} + \frac{G_n}{R_s}\left|Z_s - Z_{opt}\right|^2$$

Where,

F_{min} is minimum noise figure.

R_n is equivalent noise resistance $\left(R_n = \frac{1}{G_n}\right)$

Y_{opt} is optimum source admittance $\left(Y_{opt} = \frac{1}{Z_{opt}}\right)$

- S-parameter representation is more suitable for high frequency designs.
- Replacing admittance by reflection coefficients and substituting for G_S. The noise figure expression is

$$F = F_{min} + \frac{4\,R_n}{Z_0}\,\frac{|\Gamma_s - \Gamma_{opt}|^2}{(1-|\Gamma_s|^2)\,|1+\Gamma_{opt}|^2}$$

- In above equation if $\Gamma_s = \Gamma_{opt}$ is substituted the lowest value of noise figure will be $F = F_{min}$. Suppose source reflection coefficient Γ_s relates to noise figure F_k. The expression can be written as

$$|\Gamma_s - \Gamma_{opt}|^2 = (1 - |\Gamma_s|^2)\;|1 + \Gamma_{opt}|^2 \cdot \left(\frac{F_k - F_{min}}{4\,R_n / Z_b}\right)$$

$$|\Gamma_s - \Gamma_{opt}|^2 = (1 - |\Gamma_s|^2) \cdot Q_k$$

where, $$Q_k = |1 + \Gamma_{opt}|^2 \cdot \frac{F_k - F_{min}}{4\,R_n / Z_0}$$

$$(1 + Q_k)\;|\Gamma_s|^2 - 2\,R_e\{\Gamma_s\,\Gamma_{opt}^*\} + |\Gamma_{opt}|^2 = Q_k$$

After simplification

$$\left|\Gamma_s - \frac{\Gamma_{opt}}{1+Q_k}\right|^2 = Q_k\left[\frac{1}{1+Q_k} - \frac{|\Gamma_{opt}|^2}{(1+Q_k)^2}\right]$$

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_s|^2} \times |S_{21}|^2 \times \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_{TU} = G_S \times G_o \times G_L$$

- The corresponding circuit of unilateral power gain system is shown in Fig. 5.6.15.

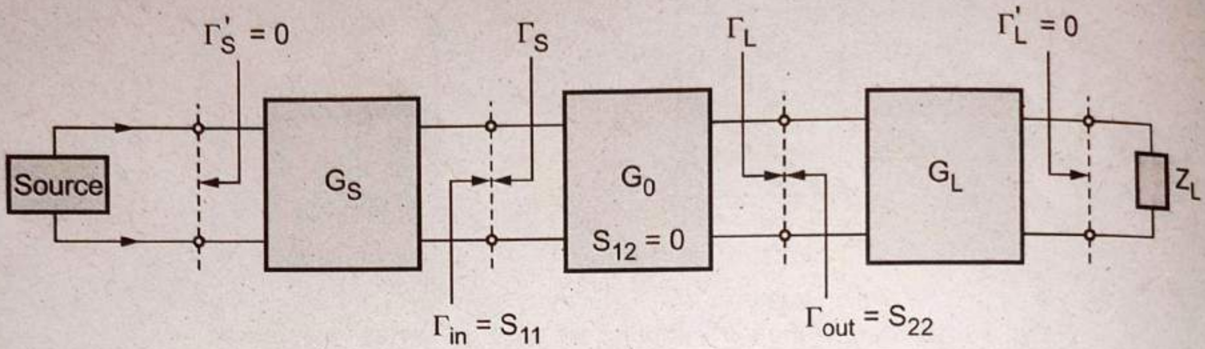


Fig. 5.6.15 Unilateral power gain system

- If the gains are expressed in dBs then the equation can be written as

$$G_{TU}(\text{dB}) = G_S(\text{dB}) + G_o(\text{dB}) + G_L(\text{dB})$$

Where,

G_s is gain associated with input matching networks.

G_L is gain associated with output matching networks.

G_o is insertion loss of the transistor

5.6.4.2 Unilateral Figure of Merit

- The unilateral figure of merit is given by

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{22}| \cdot |S_{11}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)}$$

- From the expression it is clear that the unilateral figure of merit is frequency dependent. To justify unilateral amplifier design approach the figure of merit should be as small as possible.

5.6.5 Noise Figure Circles

- In RF amplifiers, the need for signal amplification at low noise level is necessary. But while designing a low noise amplifier compromise in stability and gain is to be done.