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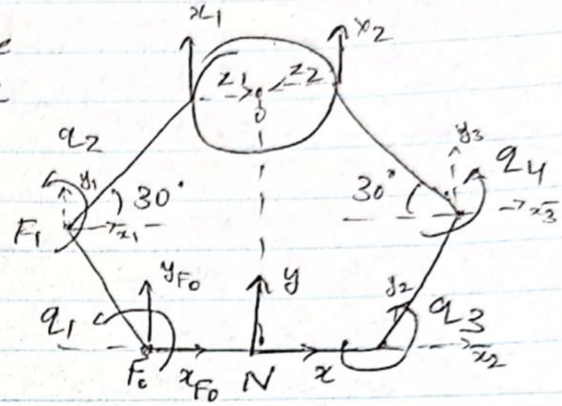
HW - 2

1) Assuming the coordinate frames at each finger joint have same orientation as the gripper frame N.



Hand Jacobian:

$$J_1 = \begin{bmatrix} J_{v_1} & J_{v_2} \\ J_{\omega_1} & J_{\omega_2} \end{bmatrix}$$

$$J_{v_1} = Z_{F_0} \times (O_n^0 - O_0^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \times 0.05 \times \sin 30^\circ \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 0.05 & 0 \end{bmatrix} = -0.05\hat{i}$$

$$\therefore J_{v_1} = \begin{bmatrix} -0.05 & 0 & 0 \end{bmatrix}^T$$

$$J_{v_2} = Z_{F_1} \times (O_n^0 - O_1^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0.05\cos 30^\circ \\ 0.05 \sin 30^\circ \\ 0 \end{bmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0.0433 & 0.025 & 0 \end{vmatrix} = -0.025\hat{i} + 0.0433\hat{j}$$

$$J_{v_2} = \begin{bmatrix} -0.025 & 0.0433 & 0 \end{bmatrix}^T$$

$$J_{\omega_1} = J_{\omega_2} = J_{\omega_3} = J_{\omega_4} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$\therefore$ we get $J_1 = \begin{bmatrix} -0.05 & -0.025 \\ 0 & 0.0433 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$

ONto the second finger.

$$J_2 = \begin{bmatrix} J_{v_3} & J_{v_4} \\ J_{\omega_3} & J_{\omega_4} \end{bmatrix}$$

$$J_{v_3} = Z_{F_2} \times \left[O_n^0 - O_2^0 \right] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \times 0.05 \sin 30 \\ 0 \end{bmatrix}$$

$$= \left[-0.05 \quad 0 \quad 0 \right]^T$$

$$J_{v_4} = Z_{F_3} \times \left[O_n^0 - O_3^0 \right] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -0.05 \cos 30^\circ \\ 0.05 \sin 30 \\ 0 \end{bmatrix}$$

$$= \left[-0.025 \quad -0.0433 \quad 0 \right]^T$$

$\therefore$ we get $J_2 = \begin{bmatrix} -0.5 & -0.025 \\ 0 & -0.0433 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$

$$J_{h_1} = \bar{R}_N^{C_1} J_1 \quad \text{where} \quad \bar{R}_N^{C_1} = \begin{vmatrix} R_N^{C_1} & 0 \\ 0 & R_N^{C_1} \end{vmatrix}$$

Substituting using matlab we get.

$$J_{h_1} = \begin{bmatrix} 0 & 0.043 \\ 0 & 0 \\ -0.05 & -0.025 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \;\&\; J_{h_2} = \begin{bmatrix} 0 & -0.0433 \\ 0 & 0 \\ 0.05 & 0.025 \\ 0 & 0 \\ -1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$J_h = \begin{bmatrix} J_{h_1} & 0 \\ 0 & J_{h_2} \end{bmatrix}$$

$$J_h = \begin{bmatrix} 0 & 0.043 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.05 & -0.025 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0433 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0.025 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For calculating $J_{hand\text{-}object} = (G^T)^+ J_h$.

From Matlab we get.

$$J_{hand\ obj} = \begin{bmatrix} -0.025 & -0.0125 & -0.025 & -0.0125 \\ 0 & 0.0216 & 0 & -0.0216 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}.$$

The object twist used in HW1 is

$$\begin{bmatrix} 0 & -0.075 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\dot{q} = (J_{hand\,obj})^{+} \xi_O^N \quad \text{since } \xi_O^N = \underbrace{(G^T)^{+} J_h}_{\text{hand-object Jacobian.}} \dot{q}$$

$$\therefore \quad \dot{q} = \begin{bmatrix} 0 \\ -1.7321 \\ 0 \\ 1.7321 \end{bmatrix}$$

```
>> Jv1 = [-0.05;0;0]
Jv1 =
   -0.0500
         0
         0
>> Jv2 = [-0.025;0.0433;0]
Jv2 =
   -0.0250
    0.0433
         0
>> Jw4 = [0;0;1], Jw3 = Jw4, Jw2 = Jw3, Jw1 = Jw2
Jw4 =
     0
     0
     1
Jw3 =
     0
     0
     1
Jw2 =
     0
     0
     1
Jw1 =
     0
     0
     1
>> J1 = [Jv1 Jv2;Jw1 Jw2]
J1 =
   -0.0500   -0.0250
         0    0.0433
         0         0
         0         0
         0         0
    1.0000    1.0000
>> Jv3 = Jv1
Jv3 =
   -0.0500
         0
         0
>> Jv4 = [-0.025; -0.0433;0]
Jv4 =
   -0.0250
   -0.0433
         0
>> J2 = [Jv3 Jv4;Jw3 Jw4]
J2 =
   -0.0500   -0.0250
         0   -0.0433
         0         0
         0         0
         0         0
    1.0000    1.0000
>> rnc1 = [0 1 0; 0 0 1; 1 0 0]
rnc1 =

     0     1     0
     0     0     1
     1     0     0
>> rnc1bar = [rnc1 zeros(3); zeros(3) rnc1]
```

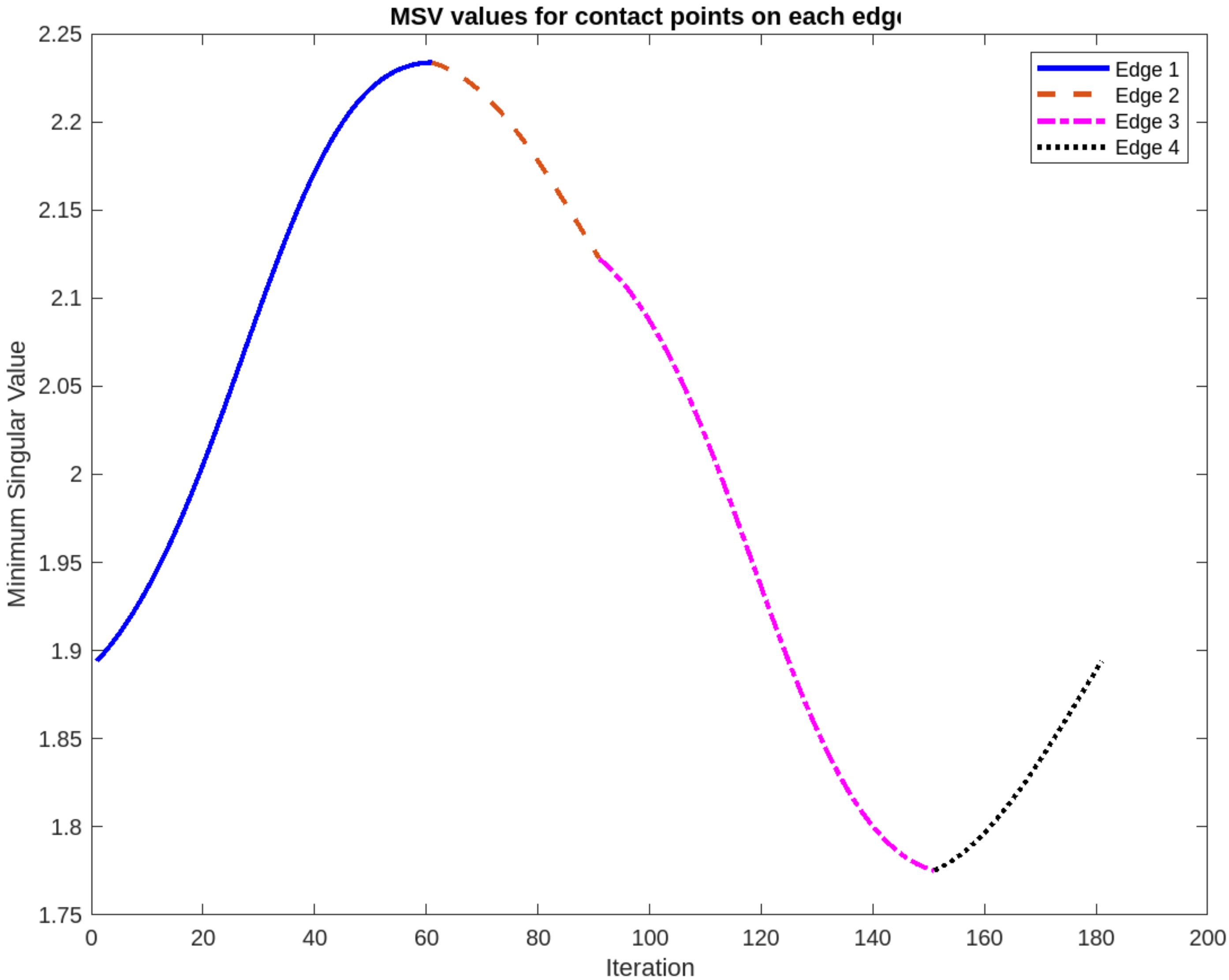
```
rnc1bar =
     0     1     0     0     0     0
     0     0     1     0     0     0
     1     0     0     0     0     0
     0     0     0     0     1     0
     0     0     0     0     0     1
     0     0     0     1     0     0
>> Jh1 = rnc1bar*J1
Jh1 =
         0    0.0433
         0         0
   -0.0500   -0.0250
         0         0
    1.0000    1.0000
         0         0
>> rnc2 = [0 1 0; 0 0 -1; -1 0 0]
rnc2 =
     0     1     0
     0     0    -1
    -1     0     0
>> rnc2bar = [rnc2 zeros(3); zeros(3) rnc2]
rnc2bar =
     0     1     0     0     0     0
     0     0    -1     0     0     0
    -1     0     0     0     0     0
     0     0     0     0     1     0
     0     0     0     0     0    -1
     0     0     0    -1     0     0
>> Jh2 = rnc2bar*J2
Jh2 =
         0   -0.0433
         0         0
    0.0500    0.0250
         0         0
   -1.0000   -1.0000
         0         0
>> Jh = [Jh1 zeros(6,2); zeros(6,2) Jh2]
Jh =
         0    0.0433         0         0
         0         0         0         0
   -0.0500   -0.0250         0         0
         0         0         0         0
    1.0000    1.0000         0         0
         0         0         0         0
         0         0         0   -0.0433
         0         0         0         0
         0         0    0.0500    0.0250
         0         0         0         0
         0         0   -1.0000   -1.0000
         0         0         0         0
>> G = [0 1 0 0 0 -0.0150; 0 0 1.0000 0 0.0150 0;1.0000 0 0 0 0 0;0 0 0 0 1.0000 0;0
0 0 0 0 1.0000;0 0 0 1.0000 0 0;0 1.0000 0 0 0 0.015;0 0 -1.0000 0 0.0150 0; -1.0000
0 0 0 0 0;0 0 0 0 1.0000 0;0 0 0 0 0 -1.0000;0 0 0 -1.0000 0 0]
G =
         0    1.0000         0         0         0   -0.0150
         0         0    1.0000         0    0.0150         0
    1.0000         0         0         0         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0         0         0    1.0000         0         0
```

```
         0    1.0000         0         0         0    0.0150
         0         0   -1.0000         0    0.0150         0
   -1.0000         0         0         0         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0   -1.0000
         0         0         0   -1.0000         0         0
>> Jho = pinv(G)*Jh
Jho =
   -0.0250   -0.0125   -0.0250   -0.0125
    0.0000    0.0216    0.0000   -0.0216
    0.0000   -0.0000   -0.0000    0.0000
    0.0000   -0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.4999    0.4996    0.4999    0.4996
>> newtwist = transpose([0 -0.075 0 0 0 0])
newtwist =
         0
   -0.0750
         0
         0
         0
         0
>> q = pinv(Jho)*newtwist
q =
    0.0000
   -1.7321
    0.0000
    1.7321
```

Question 2:

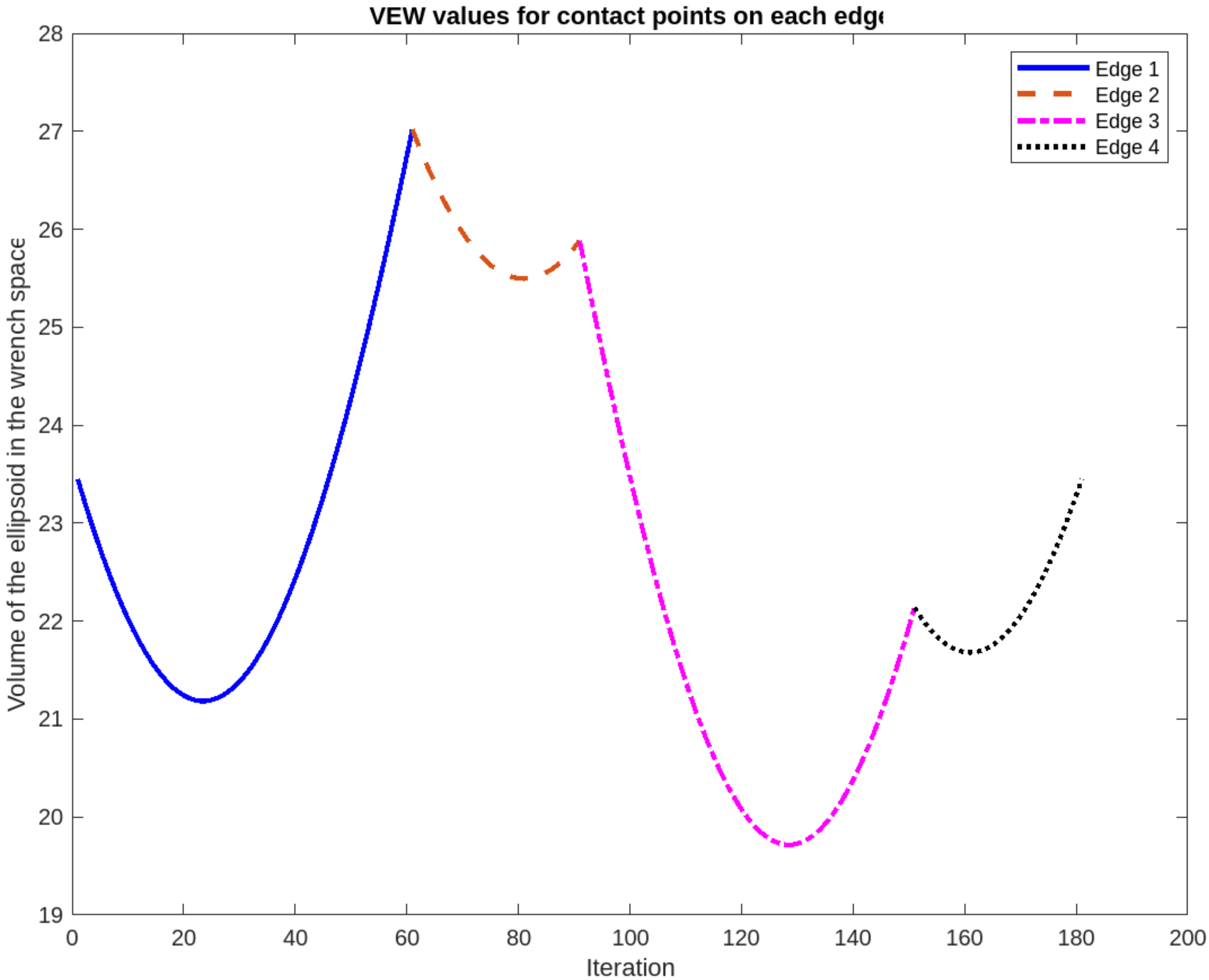
The Grasp matrix for the given center of object, contact points and the rotations are calculated by a User defined MATLAB function. The Minimum Singular value of G, The Ellipsoid volume in Wrench space and the Grasp Isotropy Index are calculated by sampling each edge for the contact points with offsets of 0.1cm. The graphs were obtained by plotting the values against the calculation iterations for each edge.

Minimum Singular value of G



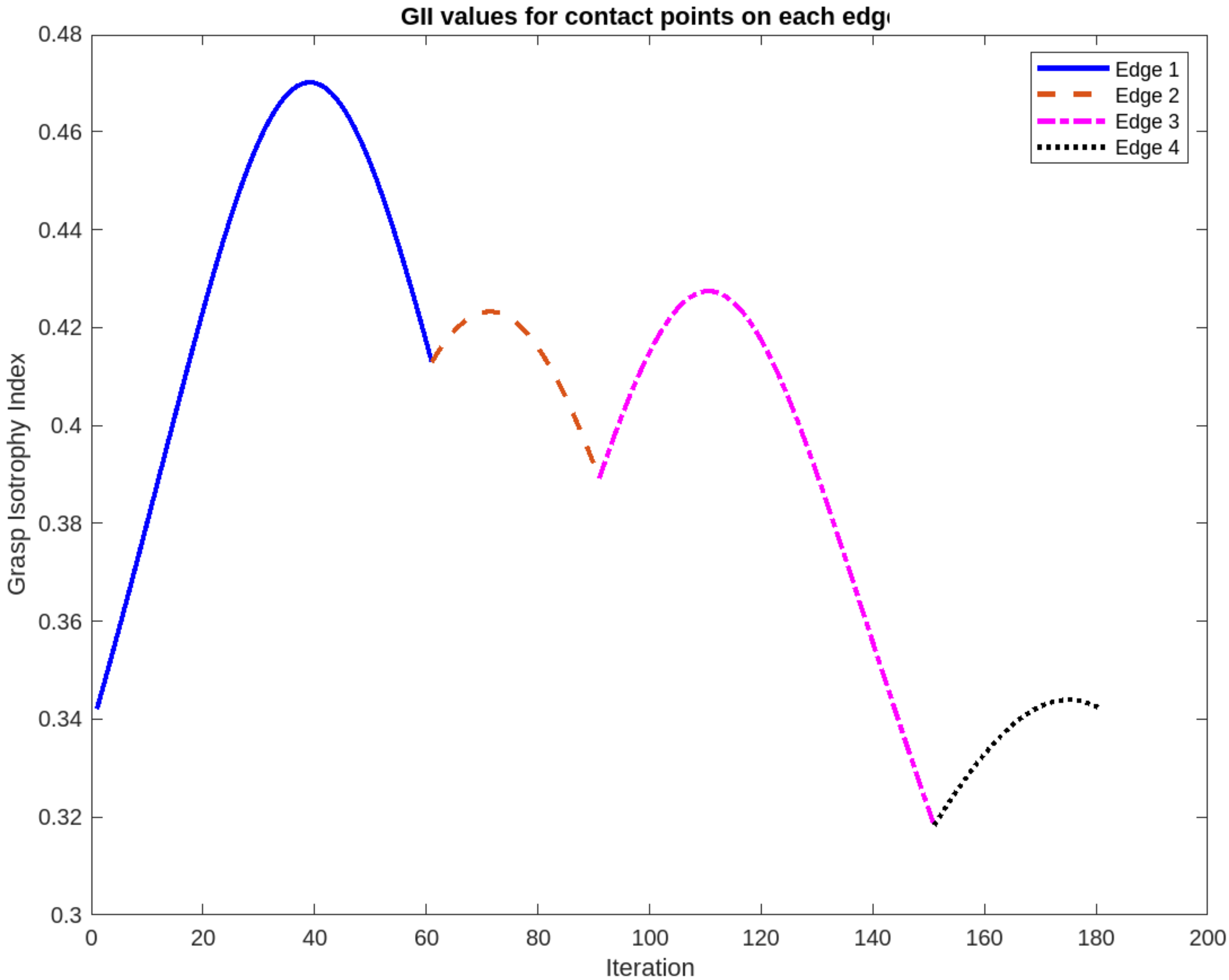

```
Minimum Singular Value = 2.2338
Corresponding index = 61 in edge 1
Hence corresponding point is : (6,0)
======================================
```

Volume of the ellipsoid in the wrench space:




```
Maximum Volume of Ellipsoid in Wrench space = 27.0185
Corresponding index = 61 in edge 1
Hence corresponding point is : (6,0)
======================================
```

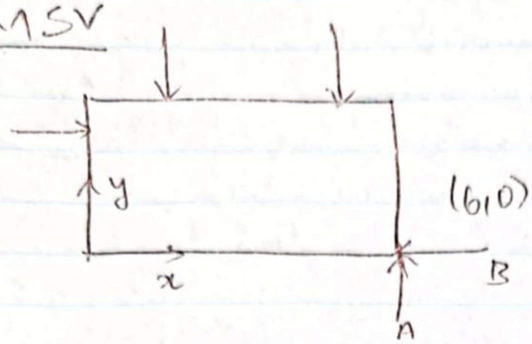
Maximum Grasp Isotrophy Index




```
=======================================
Maximum Grasp Isotrophy Index = 0.47009
Corresponding index = 39 in edge 1
Hence corresponding point is : (3.8,0)
```

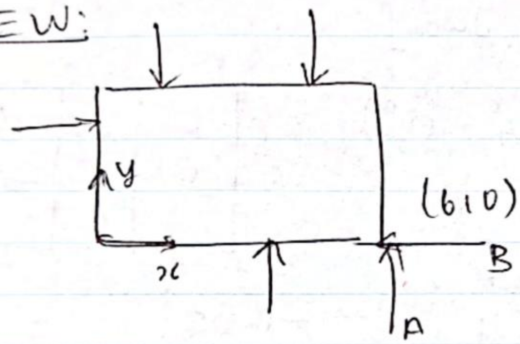
The observations are as follows:

MSV



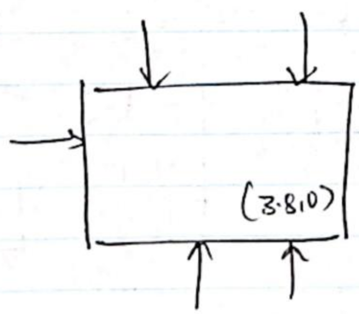
Force vectors A or B can be the best way of applying the 5th force vector acc. to MSV.

VEW:



The force vectors A or B can be the best way of applying the 5th force vector acc to VEW.

GII:



Force vector A can be considered the best way of applying the 5th force vector acc to GII

- The location of optimum 5_{th} force vector was the same for Minimum Singular value (MSV) and Volume of the ellipsoid in the wrench space (VEW) metrics, but different for the Grasp isotropy index (GII). This is because MSV and VEW are very similar metrics which focus on optimizing distance from singularities in grasps and sustaining the closure properties. A high MSV ensures lesser chances of losing form/force closure in the grasp, while a high VEW ensures much lesser chances of reaching a grasp singularity.
- Grasp isotropy index is a much different metric that focuses on force-torque isotropy, such that each finger of the gripper contributes similarly to the grasp wrenches applied to the object. Therefore the optimum force vector for this metric is different to that of MSV and VEW metrics.