Using Eigenvalues to Study Simple Population and Markov Models

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Introduction

Many real-world processes change over time in a predictable way. Examples include population growth and random processes such as weather patterns or board games. These processes can often be represented using matrices. In linear algebra, eigenvalues help explain how repeated matrix operations behave in the long run. This project will use simple examples and computations to explore how eigenvalues describe long-term behavior in population models and Markov chains.

Research Questions

How can eigenvalues be used to predict whether a population grows or declines? What happens to a system when a transition matrix is applied repeatedly? How do simple Markov models reach a stable distribution over time?

Method

The project will begin with small matrices that represent population changes and simple Markov processes. Eigenvalues and eigenvectors will be computed using basic computational tools such as calculators or software. The models will be simulated by repeatedly multiplying matrices to see how the system changes over time. The computational results will be compared with predictions made using eigenvalues.

Expected Results

It is expected that one eigenvalue will determine the overall behavior of the system. If this eigenvalue is greater than one, the population should increase, while values less than one should lead to decline. In Markov models, repeated transitions are expected to lead to a stable distribution that does not change over time.

Conclusion

This project will show how eigenvalues can be used as practical tools to understand long-term behavior in simple mathematical models.