Eigenvalues in Simple Population and Markov Models

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Abstract

Eigenvalues are fundamental tools for analyzing the long-term behavior of linear systems. This report examines how eigenvalues can be used to study discrete population models and simple Markov chains. By combining theoretical analysis with numerical examples, we show how dominant eigenvalues determine whether a system grows, declines, or converges to a steady-state distribution.

1 Introduction

Many real-world systems evolve over time in a step-by-step manner. Examples include population growth, migration between regions, and random processes such as board games or weather patterns. These systems are often modeled using matrices that describe how the current state influences the next state.

Linear algebra provides powerful tools for studying such systems. In particular, eigenvalues offer insight into the long-term behavior of repeated matrix applications. Rather than computing large matrix powers directly, eigenvalues allow us to predict whether a system stabilizes, grows without bound, or decays over time.

Population models and Markov chains are two important applications of these ideas. Population models describe how groups change in size, while Markov chains model random transitions between states. Both rely on matrix multiplication, making eigenvalues a natural tool for analysis.

2 Methods

2.1 Discrete Linear Systems

A discrete-time linear system is defined by

$$\mathbf{x}_{n+1} = A\mathbf{x}_n, \tag{1}$$

where $\mathbf{x}_n$ represents the state at time n, and A is a fixed matrix.

An eigenvalue λ of A satisfies

$$A\mathbf{v} = \lambda\mathbf{v}, \tag{2}$$

for some nonzero vector $\mathbf{v}$. Eigenvalues describe how vectors are scaled by the matrix, and the eigenvalue with the largest magnitude typically governs long-term behavior.