Machine Vision

Lecture Set – 05

Image Filtering

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Robot Vision Lab

Image Histogram

- A histogram lists the number of image pixels for each value

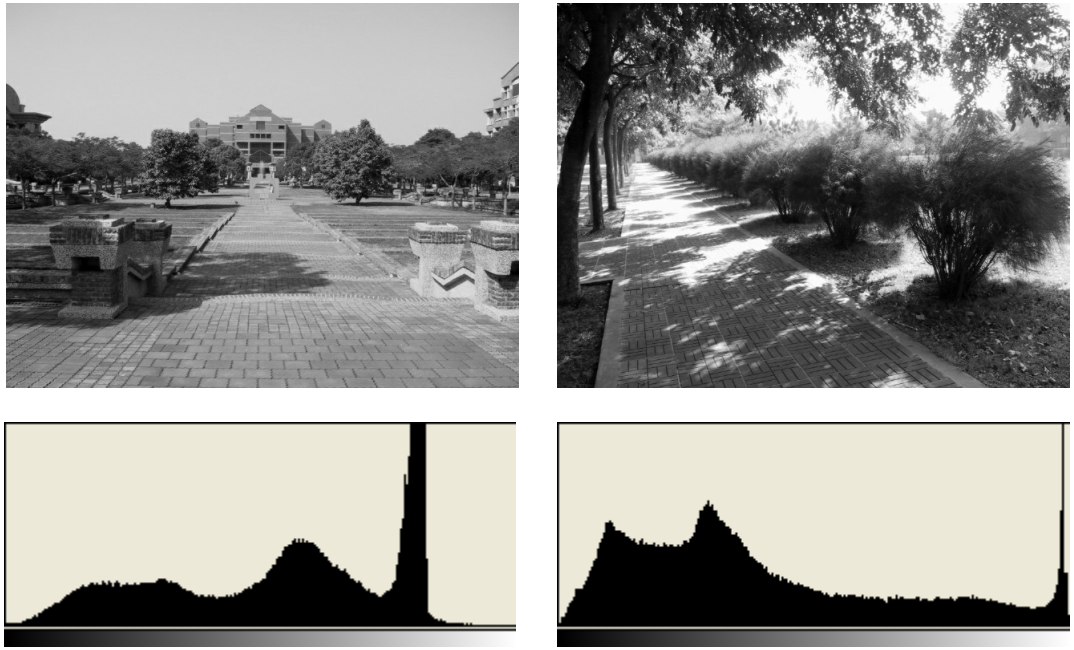


Image Histogram

- Operations based on image histograms are carried in two stages:
  - The accumulation of the histogram – give global knowledge of image
  - Its analysis is followed by an enhancement operation based on the shape of histogram
    – enable a suitable transformation

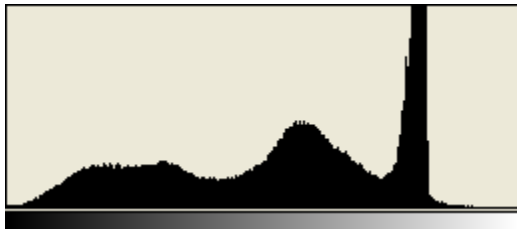
Histogram Processing

- Histogram of a digital image with gray levels in the range [0, $L-1$] is a discrete function

$$h(r_k) = n_k$$

where

  - r_k : the k^{th} gray level
  - n_k : the number of pixels in the image having gray level r_k
  - $h(r_k)$: histogram of a digital image with gray levels r_k



Normalized Histogram

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

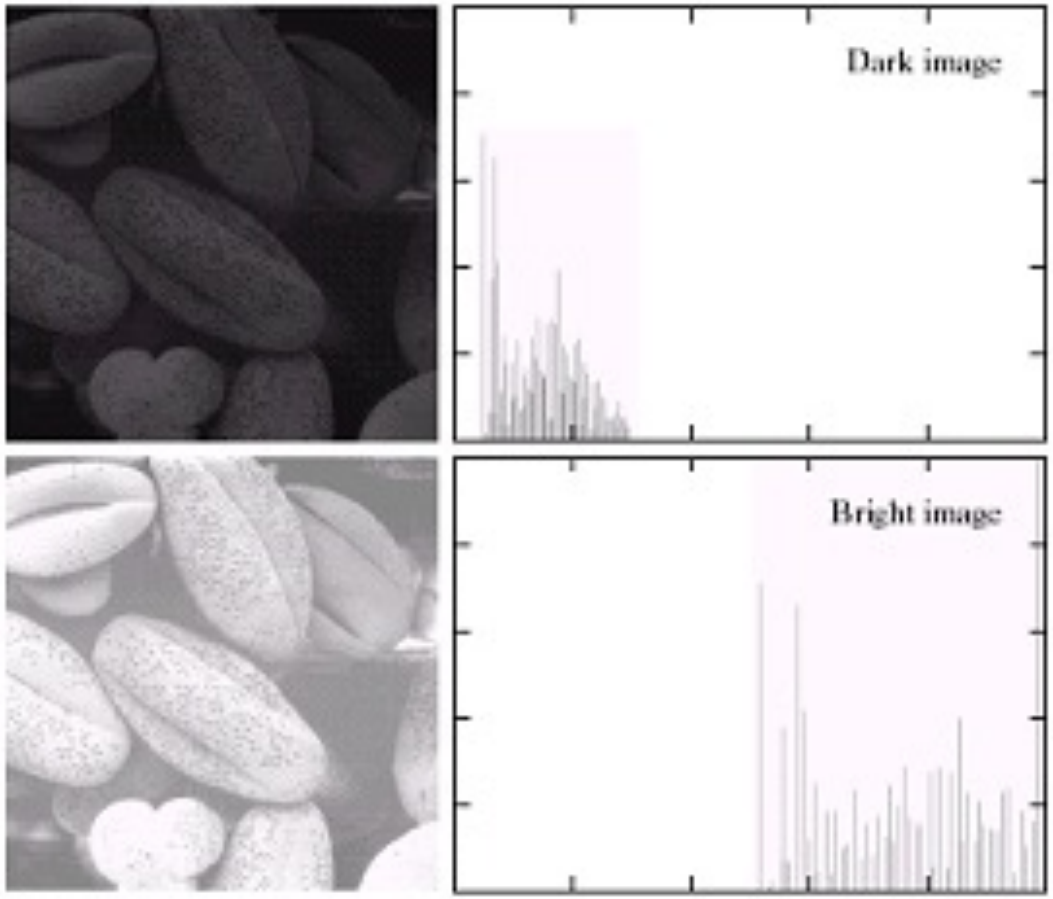
$$p(r_k) = n_k / n$$

- For $k = 0, 1, \ldots, L - 1$
  - $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
  - The sum of all components of a normalized histogram is equal to 1

Histogram Processing

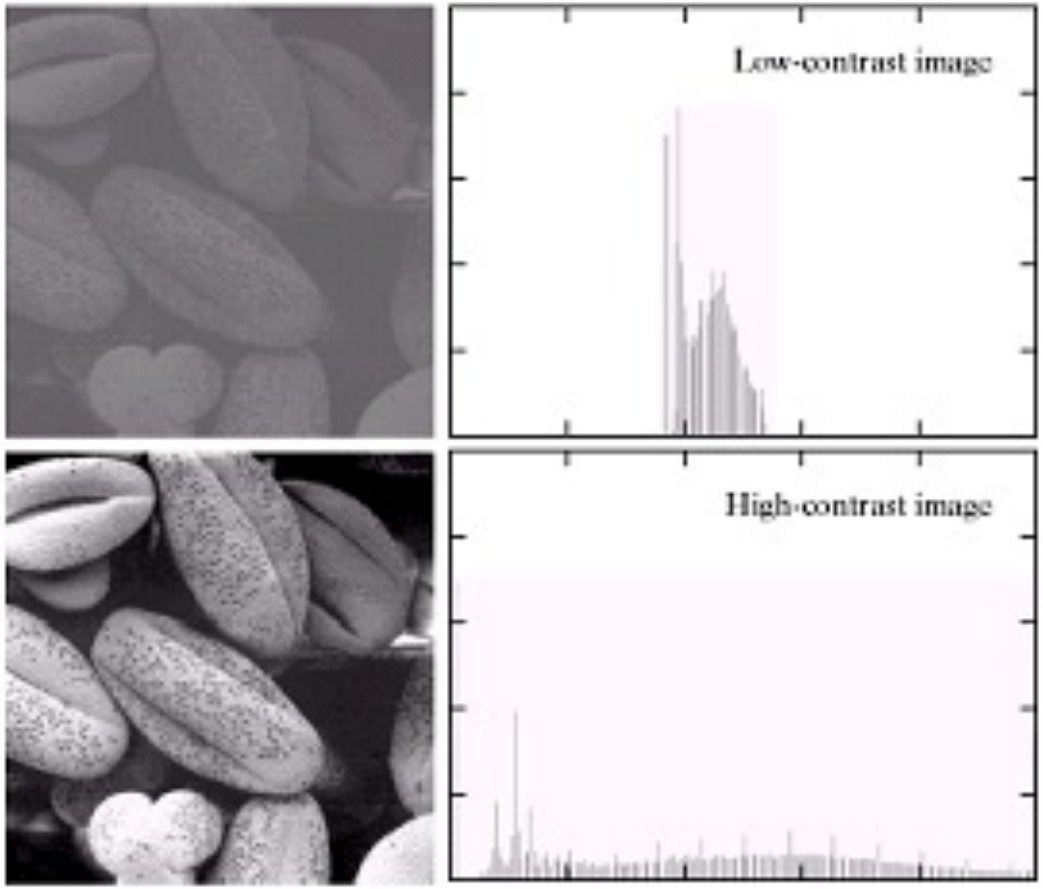
- Basic for numerous spatial domain processing techniques
- Used effectively for image enhancement
- Information inherent in histograms is also useful in image compression and segmentation

Example



- Dark image
  - Components of histogram are concentrated on the low side of the gray scale
- Bright image
  - Components of histogram are concentrated on the high side of the gray scale

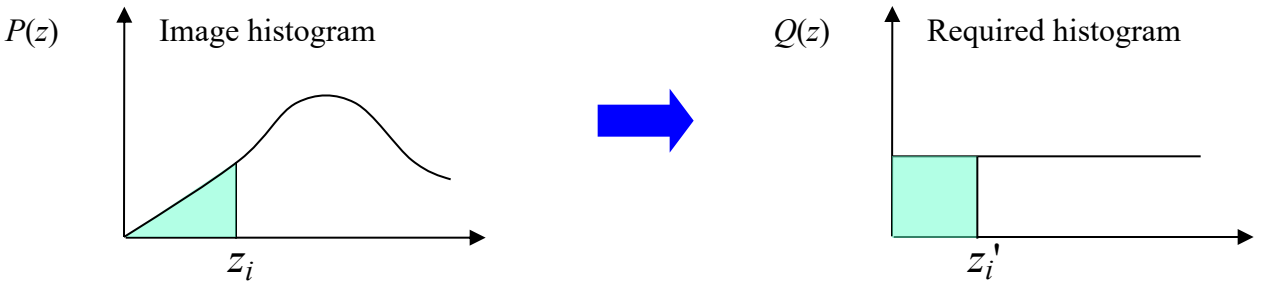
Example



- Low-contrast image
  - histogram is narrow and centered toward the middle of the gray scale
- High-contrast image
  - histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform, with very few vertical lines being much higher than others

Histogram Equalization

- Histogram equalization can be seen as an extension of the interactive contrast stretching
- To derive a transformation $T(z)$ such that the original histogram is transformed to an equi-probable histogram

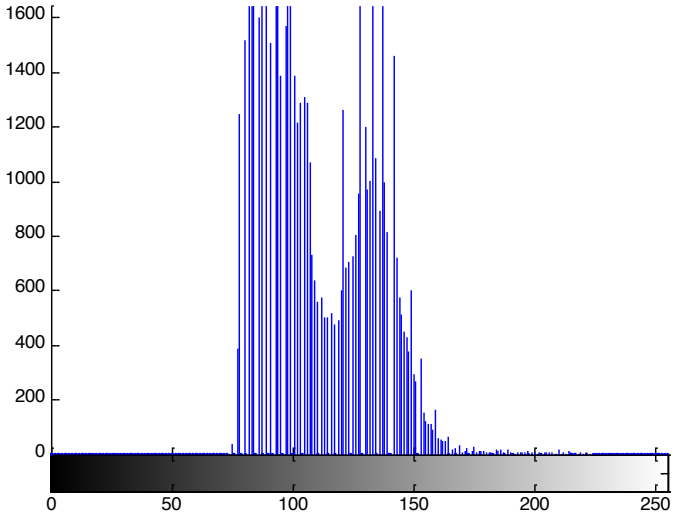


$$\int_0^{z_i} P(z)dz = \int_0^{z_i'} Q(z)dz = MN\frac{z_i'}{z_{\max}}$$

$$z_i' = T(z_i) = \frac{z_{\max}}{MN}\int_0^{z_i} P(z)dz$$

$$z_i' = \frac{z_{\max}}{MN}\sum_{i=0}^{z_i} H(z)$$

Histogram Equalization



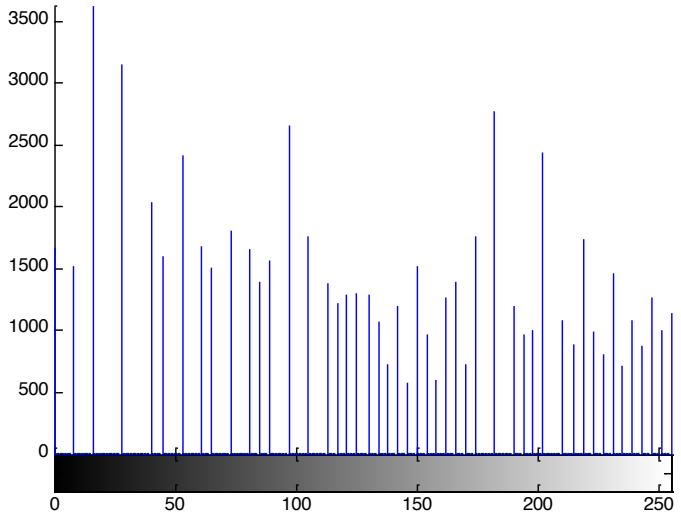


Image Noise

- Noise in computer vision:
  - Any entity, in images, data or intermediate results, that is not interesting for the purpose of the main computation

- Image noise in different cases
  - For image processing algorithms:
    - Spurious fluctuation of pixel values introduced by the image acquisition system
  - For algorithms with numerical input:
    - Inaccuracies of input, computer's precision, round-off error
  - For grouping algorithms:
    - Contours which do not belong to any meaningful object

Image Noise

- Images are corrupted by “noise” mostly during acquisition
  - Signal processing techniques can be used to model and remove this noise
  - Different types of noise are countered by different techniques, depending on the noise’s nature and characteristics
- Image Restoration: removal of noise
  - Salt & pepper noise
  - Gaussian noise
  - Impulsive noise

Acquisition Noise

- The image values are not those expected, they are corrupted during image acquisition
- Two images are never exactly the same!
- Noise is characterized by their statistic behavior
  - Mean & standard deviation of *n* images

- Pixel values are not completely independent of each other
  - Some cross-talking occurs between adjacent photo-sensors
  - Auto-covariance (image processing)

Additive Noise & Noise Amount

- In the additive noise model the signal is corrupted with random fluctuations

$$\hat{I}(i,j) = I(i,j) + n(i,j)$$

- The amount of noise is measured by means of σ_n, the standard deviation of $n(i,j)$
- Signal-to-noise ratio (SNR): $SNR = \dfrac{\sigma_s}{\sigma_n}$
  - σ_s: s.d. of the signal
- In decibel: $SNR_{dB} = 10 \log_{10} \dfrac{\sigma_s}{\sigma_n}$
- Quantization noise
- Multiplicative noise: $\hat{I}(i,j) = n(i,j) \cdot I(i,j)$

Gaussian Noise

- Without any information, noise is often modeled as zero mean Gaussian distribution
- This model predicts that noise values are distributed symmetrically around zero, and pixel values $\hat{I}(i, j)$ around their true values $I(i, j)$

Impulsive Noise

- Also known as spot or peak noise
- Usually introduced by the acquisition process
- Alters random pixels, making their values very different from the true value and neighboring pixels
- Appears as a sprinkle of dark and light spots
- It can also be caused by
  - Transmission errors
  - Faulty elements in the CCD array
  - External noise corrupting the A/D conversion
- Salt and pepper noise

Salt & Pepper Noise

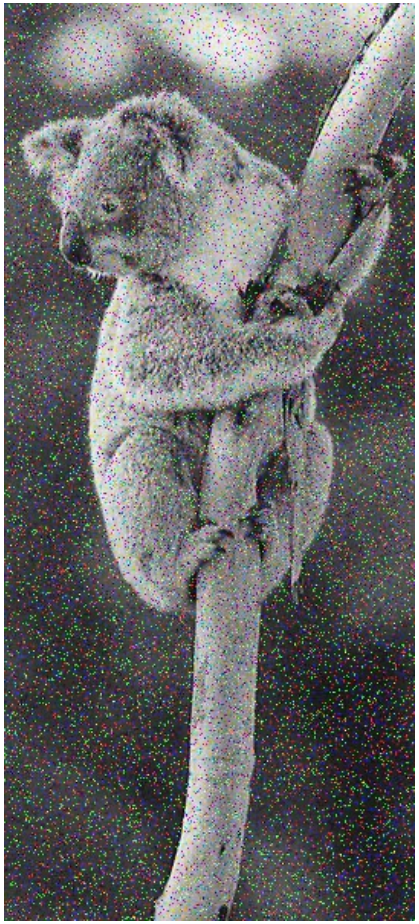
- Salt and pepper noise can model errors introduced by the acquisition process
- It is also used to simulate impulse noise in synthetic images
- x and y are random variables in the range (0,1) and l is a constant

$$\hat{I}(m,n) = \begin{cases} I(m,n) & x < l \\ i_{\min} + y(i_{\max} - i_{\min}) & x \geq l \end{cases}$$

- i_{max} and i_{min} indicate how severe the noise is

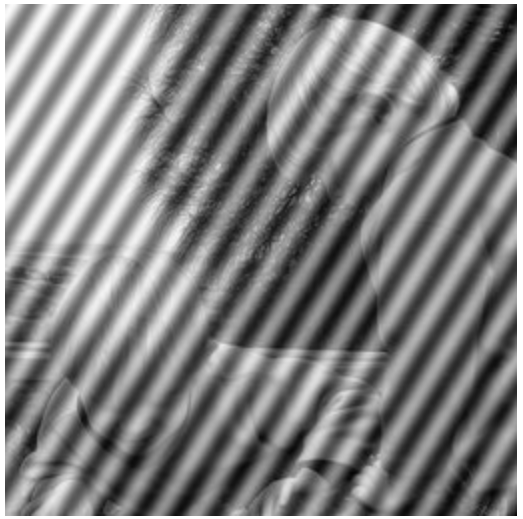
Example

- Salt & Pepper Noise



Example

- Speckle noise (multiplicative noise)
- Periodic noise



Noise Filtering

- Noise Filtering
  - Given an image I corrupted by noise n, attenuate n as much as possible without altering I significantly

- Image derivatives are very sensitive to noise and should be avoided if possible

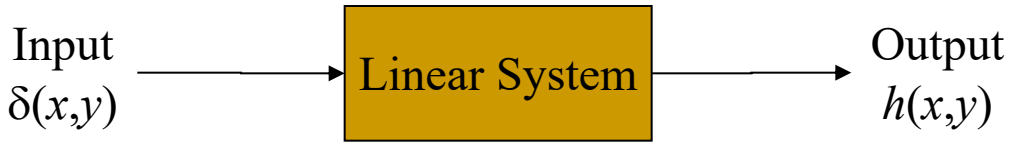
- Linear filtering:
  - Convolving the image with a constant matrix (called mask or kernel)

Image Filtering

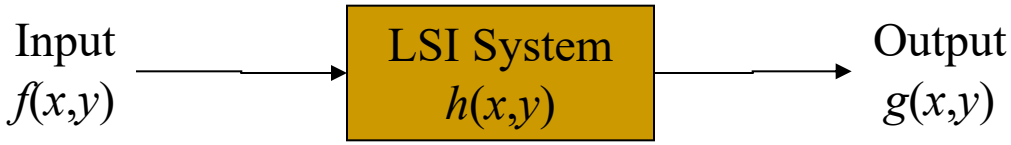
- Local neighborhoods
  - Hard to tell anything from a single pixel
    - Example: you see a reddish pixel
    - Is this the object's color? Illumination? Noise?
  - The next step in order of complexity is to look at local neighborhood of a pixel
- Linear Filters
  - Given an image $I(x,y)$ generate a new image $O(x,y)$:
    - For each pixel (x,y), $O(x,y)$ is a linear combination of pixels in the neighborhood of $I(x,y)$
  - This algorithm is "linear in input intensity" and "shift invariant"
  - Discrete convolution, pattern of weights is called "kernel"

Linear Systems

- Many image processing operations can be modeled as a linear system:
  - $h(x,y)$ is the response to the unit impulse $\delta(x,y)$



- Space invariant system:
  - A system whose response remains the same irrespective of the position of the input pulse
  - A LSI (linear space invariant) system can be completely described by its impulse response $h(x,y)$



LSI System

- LSI systems satisfy the linearity property

$$a \cdot f_1(x,y) + b \cdot f_2(x,y) \rightarrow a \cdot g_1(x,y) + b \cdot g_2(x,y)$$

- The output is the convolution of input with the impulse response

$$\begin{aligned} g(x,y) &= f(x,y) * h(x,y) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')h(x-x',y-y')dx'dy' \end{aligned}$$

- For the discrete case

$$\begin{aligned} g[i,j] &= f[i,j] * h[i,j] \\ &= \sum_{k=1}^{n}\sum_{l=1}^{m} f[k,l]h[i-k,j-l] \end{aligned}$$

Convolution

- If f and g are images, convolution becomes the computation of weighted sums of the image pixels
- The impulse response, $h[i, j]$, is referred to as a convolution mask or convolution kernel
- The value $g[i, j]$ of the new image is calculated by

$$g[i,j] = f[i,j] * h[i,j] = \sum_{h=-\frac{m}{2}}^{\frac{m}{2}} \sum_{k=-\frac{m}{2}}^{\frac{m}{2}} f[h,k]h[i-h, j-k]$$

Convolution

- Convolution is a linear and spatial invariant operation
- Convolution in the spatial or time domain corresponds to multiplication in the frequency domain

Computation

- Convolution is a fairly expensive operation requiring a large number of computations on typical images
- Many computer architecture provide specialized instructions for these kinds of operations
- Two convolution kernels commonly used for noise reduction are
  - Mean Kernel (average smoothing)
  - Gaussian Kernel
- Some Facts
  - $X * (aY + bZ) = a(X * Y) + b(X * Z)$
  - $X * (Y * Z) = (X * Y) * Z$

Linear Filters

- Linear filters
  - Implemented using the weighted sum of the pixels in successive window
  - Usually spatial invariant and implemented using a convolution mask
  - Can also be spatially variant (different filter weights used for different parts of the image)
- Nonlinear filters
  - Not a weighted sum of pixels
  - Can be spatially invariant or spatially variant
  - Median filter, for instance

Linear Smoothing Filter

- "Good" linear smoothing filters?
  - Should be chosen so that the filter has a single peak (main lobe)
  - Symmetric in the vertical and horizontal directions
  - Remove high-frequency components
  - Will lose sharp detail in the image
  - Step changes will be blurred into gradual changes

Spatial Filtering

- Use filter (mask/kernel/template/window)
- The values in a filter subimage are referred to as coefficients, rather than pixel
- Simply move the filter mask from point to point in an image
- At each point (x,y), the response of the filter at that point is calculated using a predefined relationship

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} = \sum_{i=i}^{mn} w_i z_i$$

3×3 Smoothing Linear Filters

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

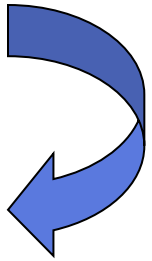
box filter

$\frac{1}{16} \times$

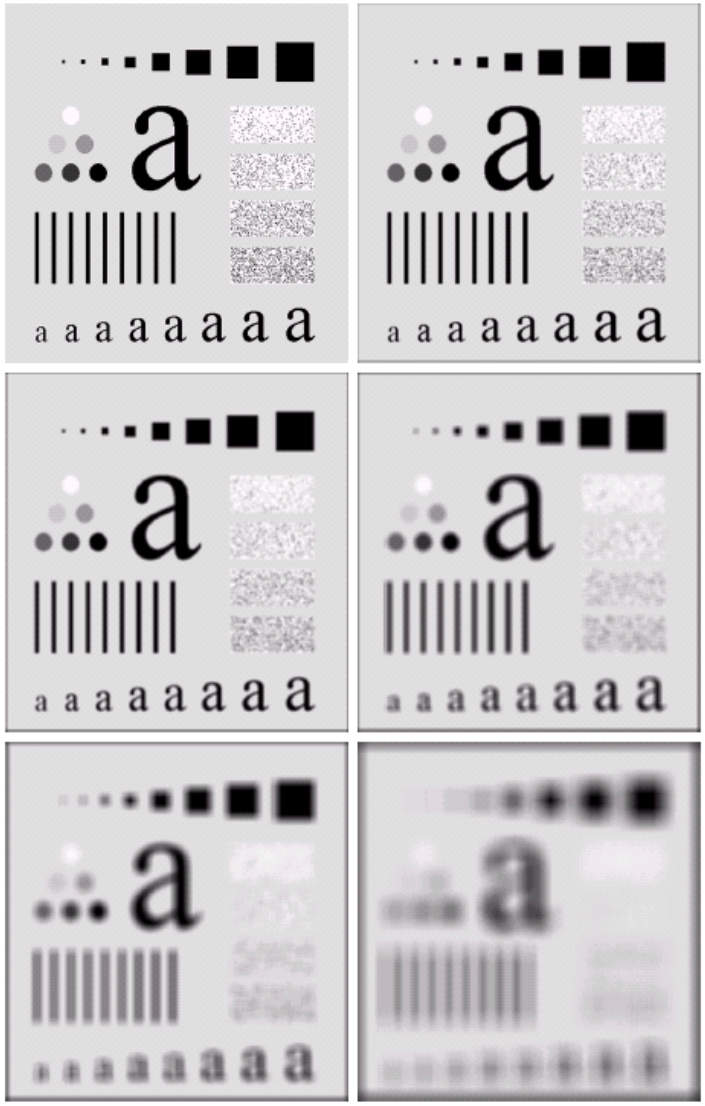
1	2	1
2	4	2
1	2	1

weighted average

The center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

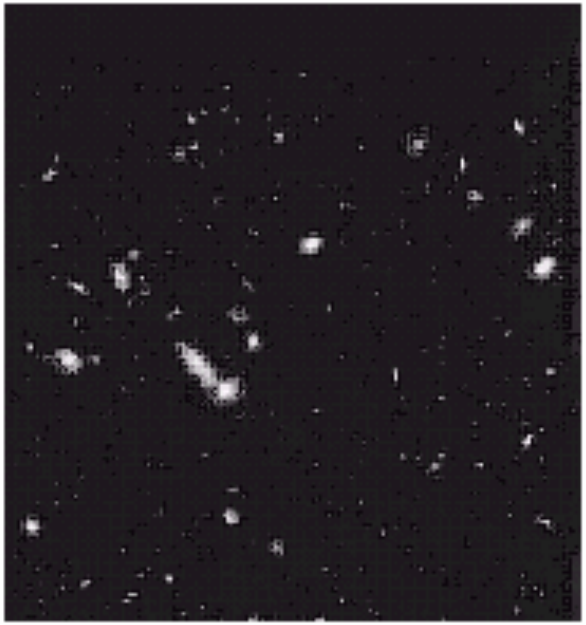


Example

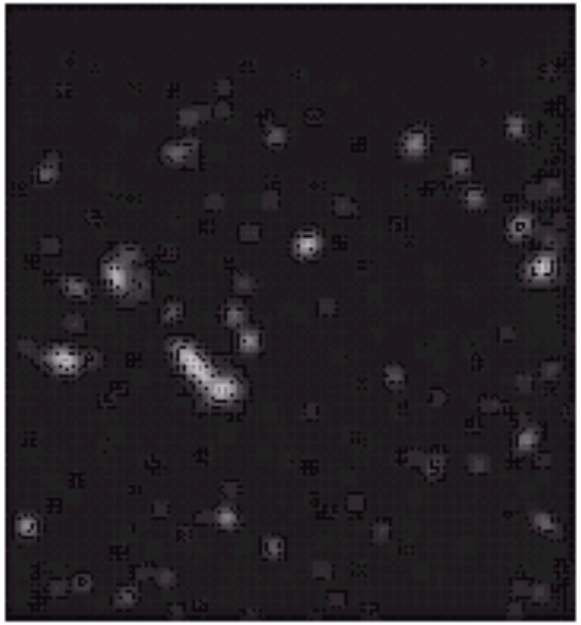


- a). original image 500×500 pixel
- b). - f). results of smoothing with square averaging filter masks of size n = 3, 5, 9, 15 and 35, respectively.
- Note:
  - Big mask is used to eliminate small objects from an image
  - The size of the mask establishes the relative size of the objects that will be blended with the background

Example – Application



original image



result after smoothing with 15x15 averaging mask

result of thresholding

- We can see that the result after smoothing and thresholding, the remains are the largest and brightest objects in the image

Mean Filter

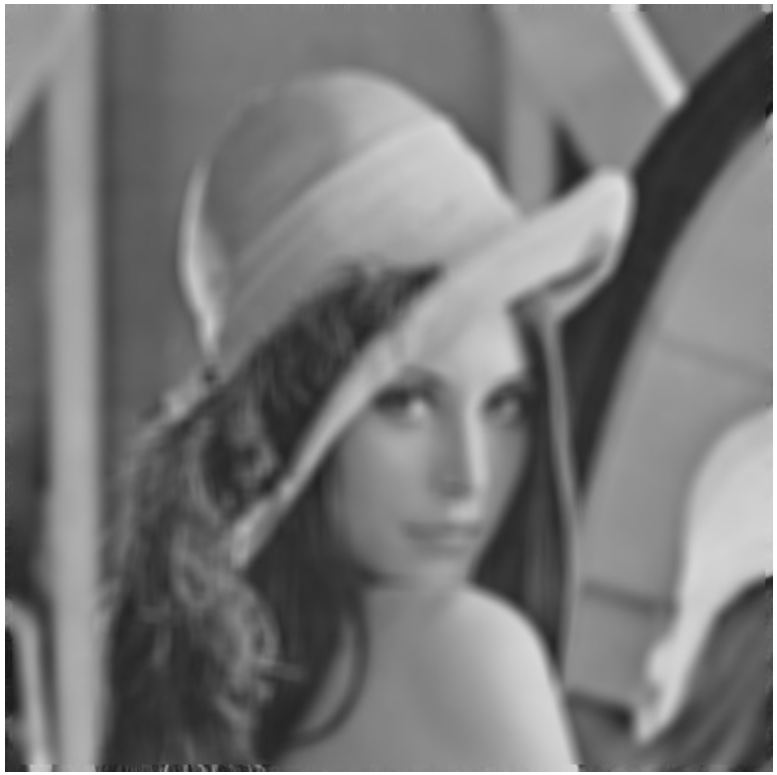
- Smoothing by averaging

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

- Entries must add up to one (Why?)

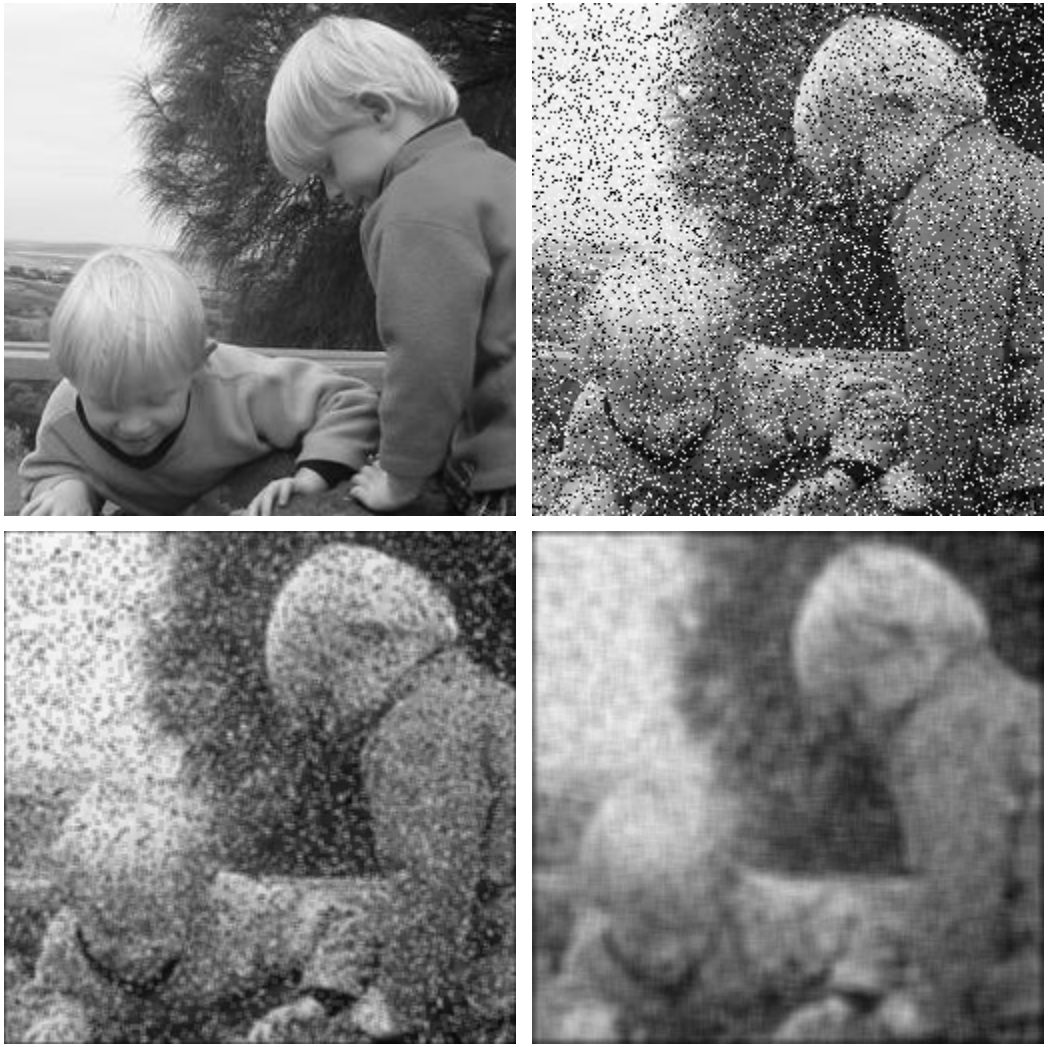
Example

Mean 5×5



Mean 11×11

Example



Frequency Behavior

- Fourier transform of a 1-D mean filter kernel of width $2W$ is the *sinc* function

$$\text{sinc}(\omega) = \frac{2\sin(\omega W)}{\omega}$$

- (Box filter)
- Behave like a low-pass filter

Problems of Mean Filtering

- Sharp signal variations are filtered out, image is *blurred*
- *Impulse noise* is attenuated and diffused, not removed
- *Secondary lobes* in the Fourier transform contribute noise (compare with Gaussian!)

Separable Filters

- Some 2D image filters can actually be implemented as two 1-D filters:
  - One in the horizontal direction followed by another in the vertical direction
- This can significantly lower the computational complexity of the operation
- Gaussian filter
  - A class of linear smoothing filters with the weights chosen according to the shape of a Gaussian function
  - A very good filter for removing noise draw from a normal distribution

Gaussian Filters

- Gaussian filter
  - Linear smoothing filter
  - Weights chosen according to the shape of Gaussian function
- One-dimensional Gaussian (zero mean)

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

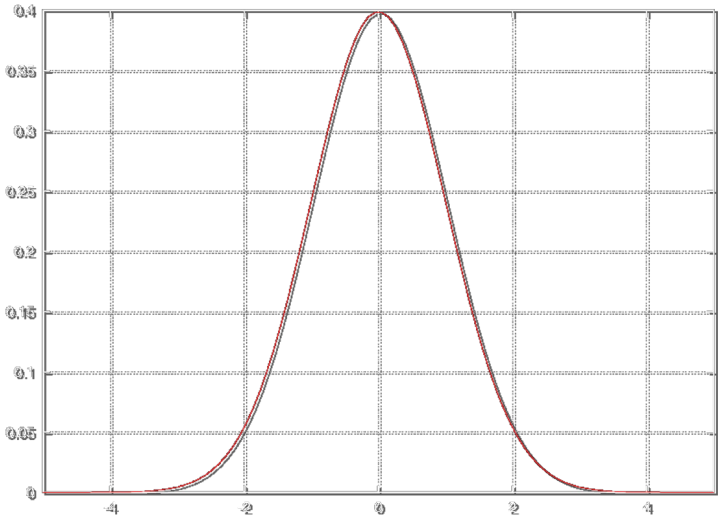
- Two-dimensional Gaussian (zero mean)

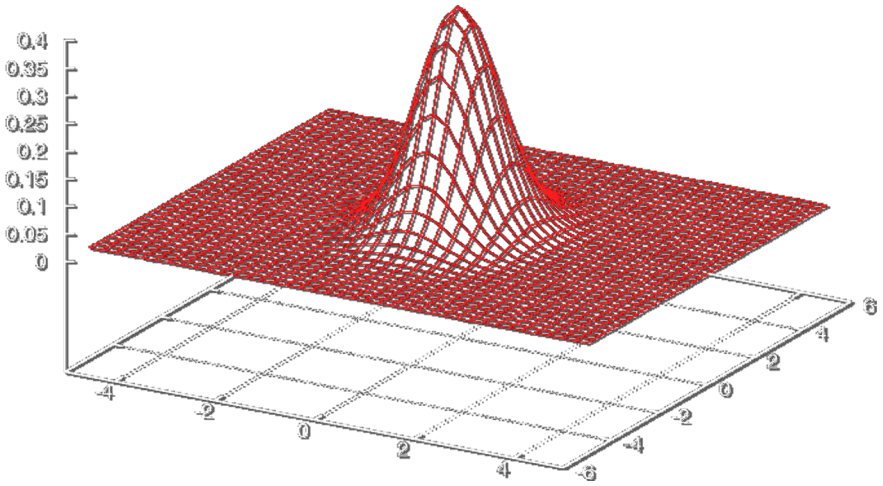
$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Gaussian Filters

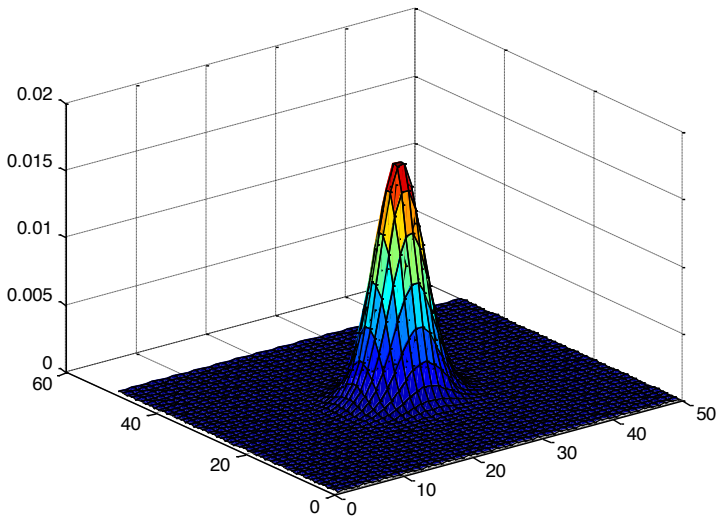
$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

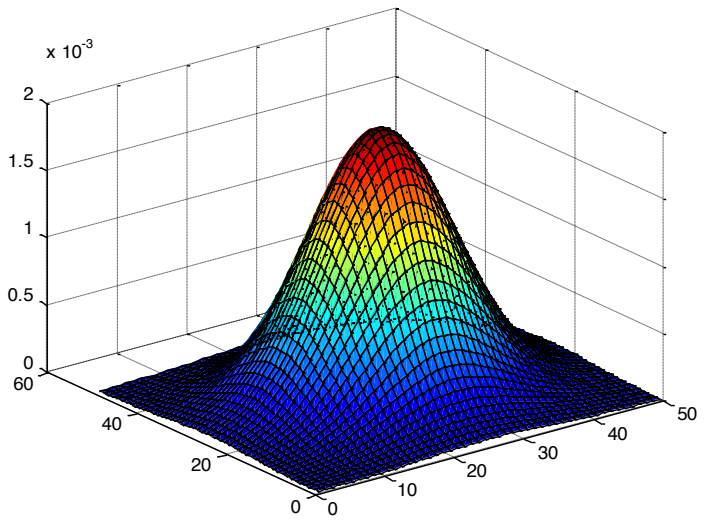
$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$





Gaussian Filters





Properties of Gaussian Filter

- Rotationally symmetric
  - Amount of smoothing is the same in all direction
  - Does not bias subsequent edge detection in any particular direction
- Single lobe in spatial domain
  - Replace each image pixel with a weighted average of the neighbors
  - The weight decreases monotonically with distance from the central pixel

Properties of Gaussian Filter

- Single lobe in frequency domain
  - High-frequency noise is suppressed, etc.
  - No secondary lobe, better low-pass than mean filter
- Degree of smoothing controlled by σ
  - Larger σ or width gives greater smoothing
- Separable (in x and y directions)
  - Effective implementation
  - Large Gaussian filters can be implemented very efficiently because of separability of Gaussian functions

Separability of Gaussian Kernel

- Convolution can be done on all rows and then all columns

$$
\begin{aligned}
I * G &= \sum_{m=-\frac{w}{2}}^{\frac{w}{2}} \sum_{n=-\frac{w}{2}}^{\frac{w}{2}} G(m,n) I(i-m, j-n) \\
&= \sum_{m=-\frac{w}{2}}^{\frac{w}{2}} \sum_{n=-\frac{w}{2}}^{\frac{w}{2}} e^{-\frac{m^2+n^2}{2\sigma^2}} I(i-m, j-n) \\
&= \sum_{m=-\frac{w}{2}}^{\frac{w}{2}} e^{-\frac{m^2}{2\sigma^2}} \sum_{n=-\frac{w}{2}}^{\frac{w}{2}} e^{-\frac{n^2}{2\sigma^2}} I(i-m, j-n)
\end{aligned}
$$

Cascading Gaussians

- The convolution of a Gaussian with itself yields a scaled Gaussian with larger σ

$$g(x) * g(x) = \int_{-\infty}^{\infty} e^{-\frac{\xi^2}{2\sigma^2}} e^{-\frac{(x-\xi)^2}{2\sigma^2}} d\xi = \sqrt{\pi}\sigma e^{-\frac{x^2}{2(\sqrt{2}\sigma)^2}}$$

- Convolution of two Gaussian with σ gives a Gaussian with $\sqrt{2}\sigma$

Design of Gaussian Filter

- Approximation from Pascal's triangle
  - 1 2 1, 1 3 3 1, 1 4 6 4 1, ...
  - Normalization
- Cascading Gaussians give a scaled Gaussian with larger σ
- Compute Gaussian kernel weight directly from the discrete Gaussian function
  - Given σ, calculate the function: $g[i,j] = e^{-\frac{i^2+j^2}{2\sigma^2}}$
  - Convert to integers
  - Normalization

Gaussian Kernel Example

$$\frac{1}{10.7}$$

1.3	3.2	3.8	3.2	1.3

$$\sigma = 1.4$$

$$\frac{1}{115}$$

2	4	5	4	2
4	9	12	9	4
5	12	15	12	5
4	9	12	9	4
2	4	5	4	2

Example

Nonlinear Filters

- The response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter

- Example
  - median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots, n\times n\}$
  - max filter : $R = \max\{z_k \mid k = 1,2,\ldots, n\times n\}$
  - min filter : $R = \min\{z_k \mid k = 1,2,\ldots, n\times n\}$
- Note: $n\times n$ is the size of the mask

Median Filters

- Replace the value of a pixel by the median of the gray levels in the neighborhood of that pixel
- Quite popular because for certain types of random noise (impulse noise ⇨ salt and pepper noise), they provide excellent noise-reduction capabilities, with considering less blurring than linear smoothing filters of similar size
- Force the points with distinct gray levels to be more like their neighbors

Median Filters

- Isolated clusters of pixels that are light or dark w.r.t. their neighbors, and whose area is less than $n^2/2$ (one-half the filter area), are eliminated by an $n \times n$ median filter
- Eliminated = forced to have the value equal the median intensity of the neighbors

Example

- Salt & Pepper Noise

Median Filtering

- Median filtering is a non-linear operation that is particularly effective at removing salt and pepper noise
- Neighborhood values: 115, 119, 120, 123, 124, 125, 126, 127, 150
- Median value: 124

123	234	125	114	102
124	115	123	150	124
125	119	124	127	115
114	120	125	126	134
122	134	115	214	112

Example

Example

- S & P noise

Reading

- Chapter 4 of Jain's book