Motion and Optical Flow

UMich EECS 442

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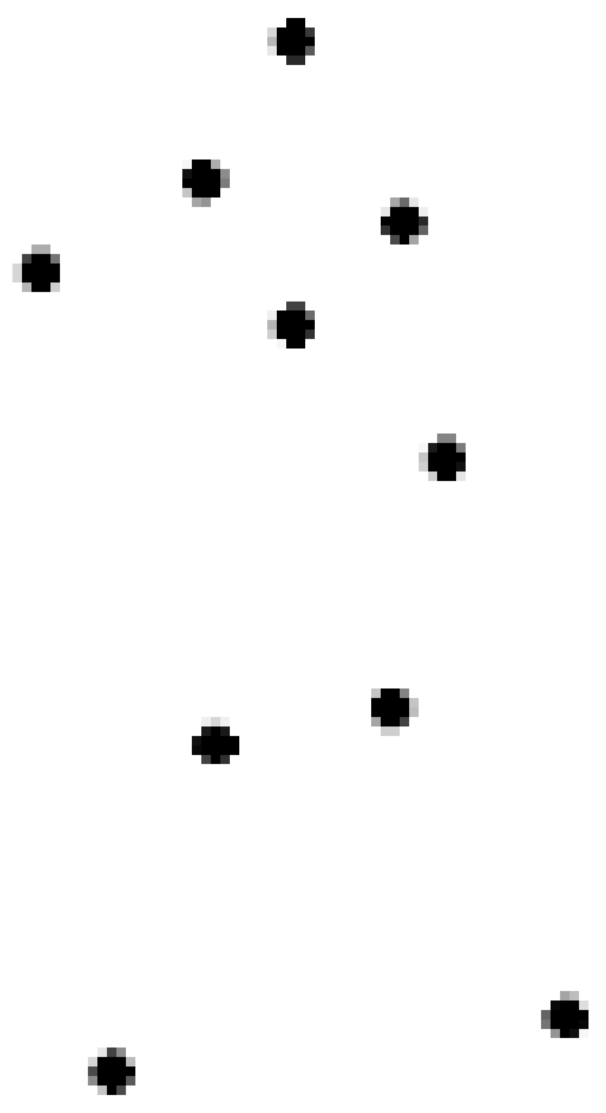
Most slides from S. Lazebnik, Kristen Grauman, S. Seitz, R. Szeliski, M. Pollefeys, Shree Nayar

topics

- Concepts: motion field and optical flow
- Why we care?
- Optical Flow Constraint Equation
- Classic algorithm: Lucas-Kanade algorithm
- State-of-the-art algorithm via deep learning
- Fun applications

Source: S. Lazebnik

What can you see from a static image?

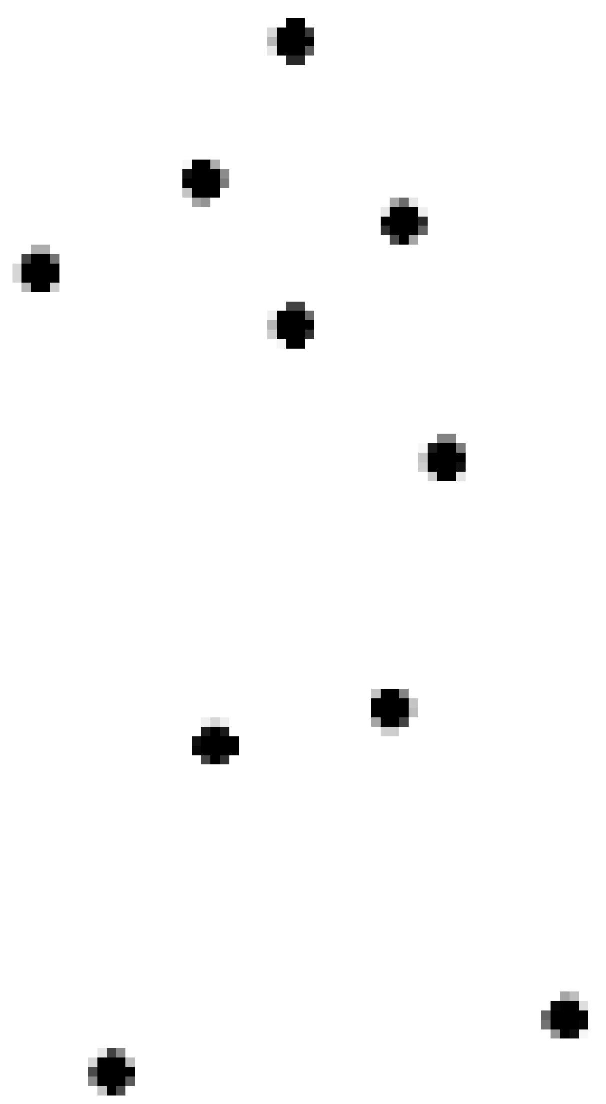


G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis",
Perception and Psychophysics 14, 201-211, 1973.

Source: S. Lazebnik

Motion is a powerful perceptual cue

Sometimes the only cue for visual perception
e.g., moving object detection



G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis",
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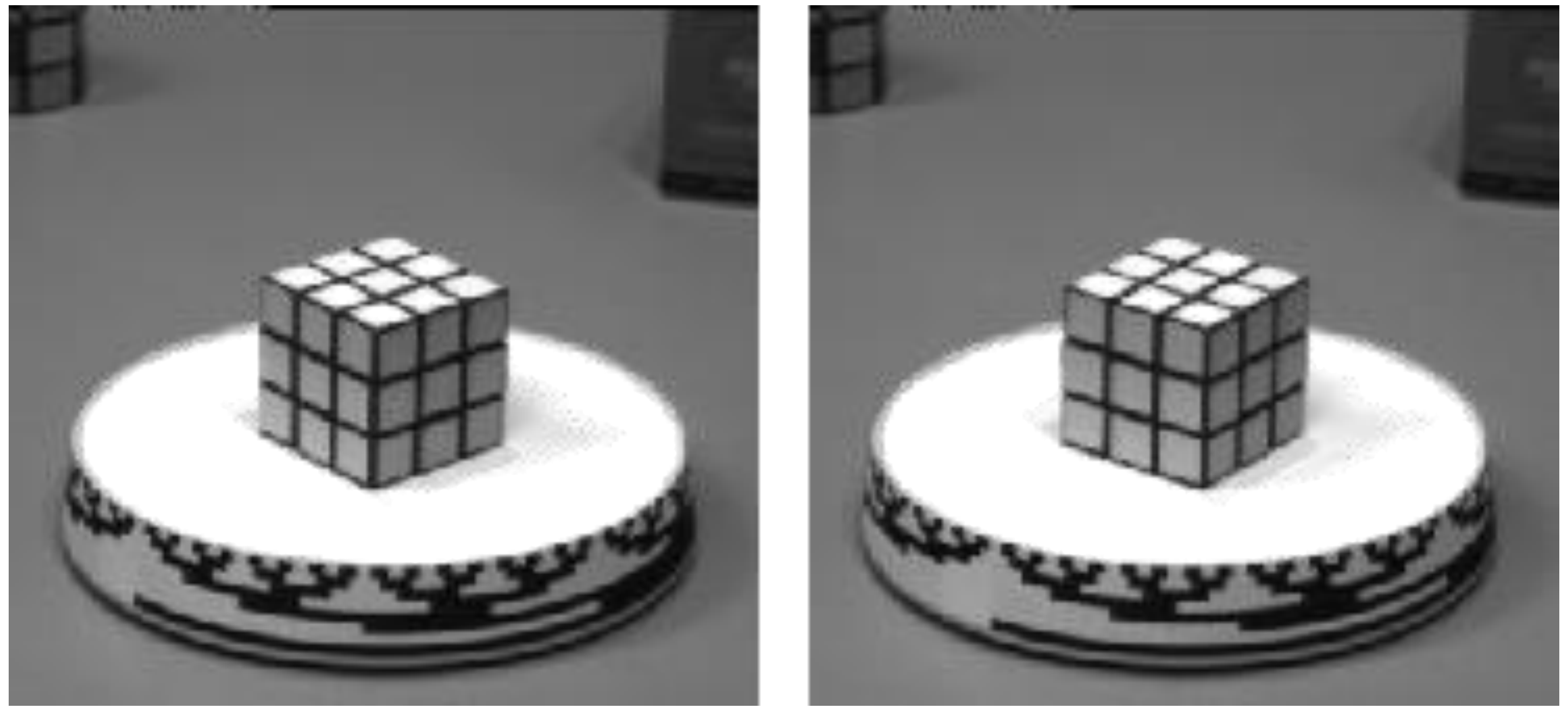
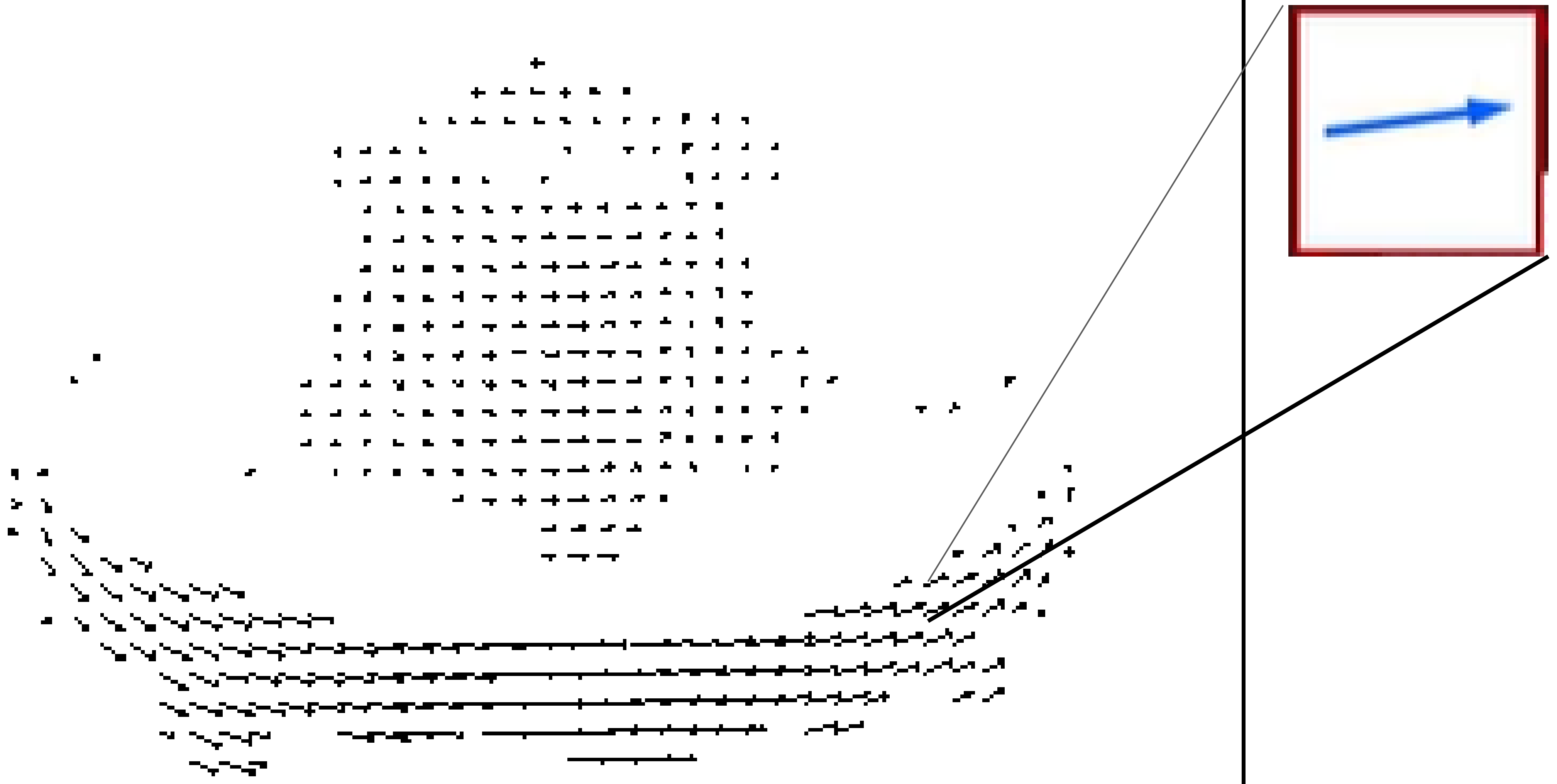
Motion field

- The motion field is the projection of the 3D scene motion into the image



For each pixel, we compute a vector for its velocity brightness pattern: how fast and direction



Motion field + camera motion

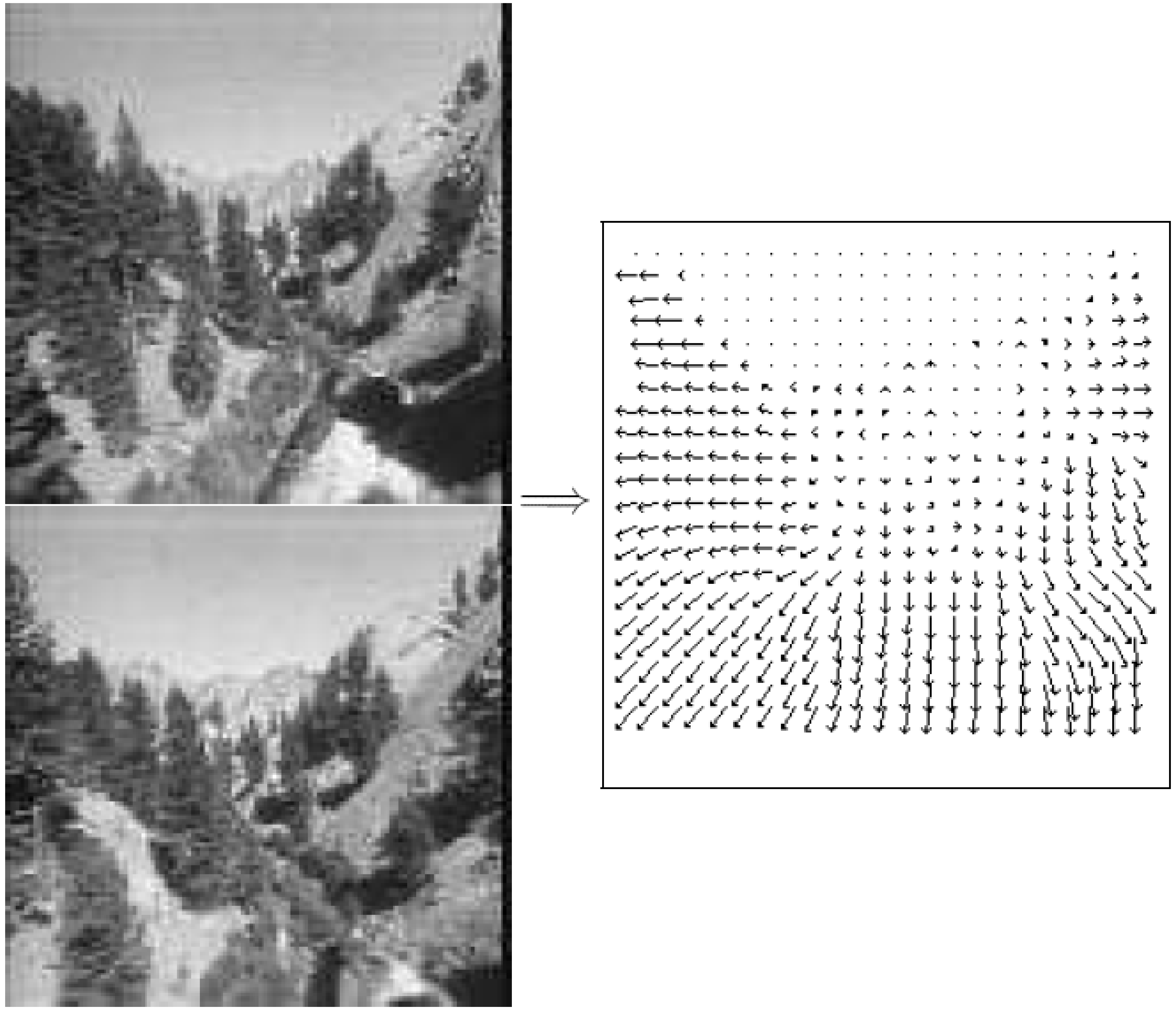


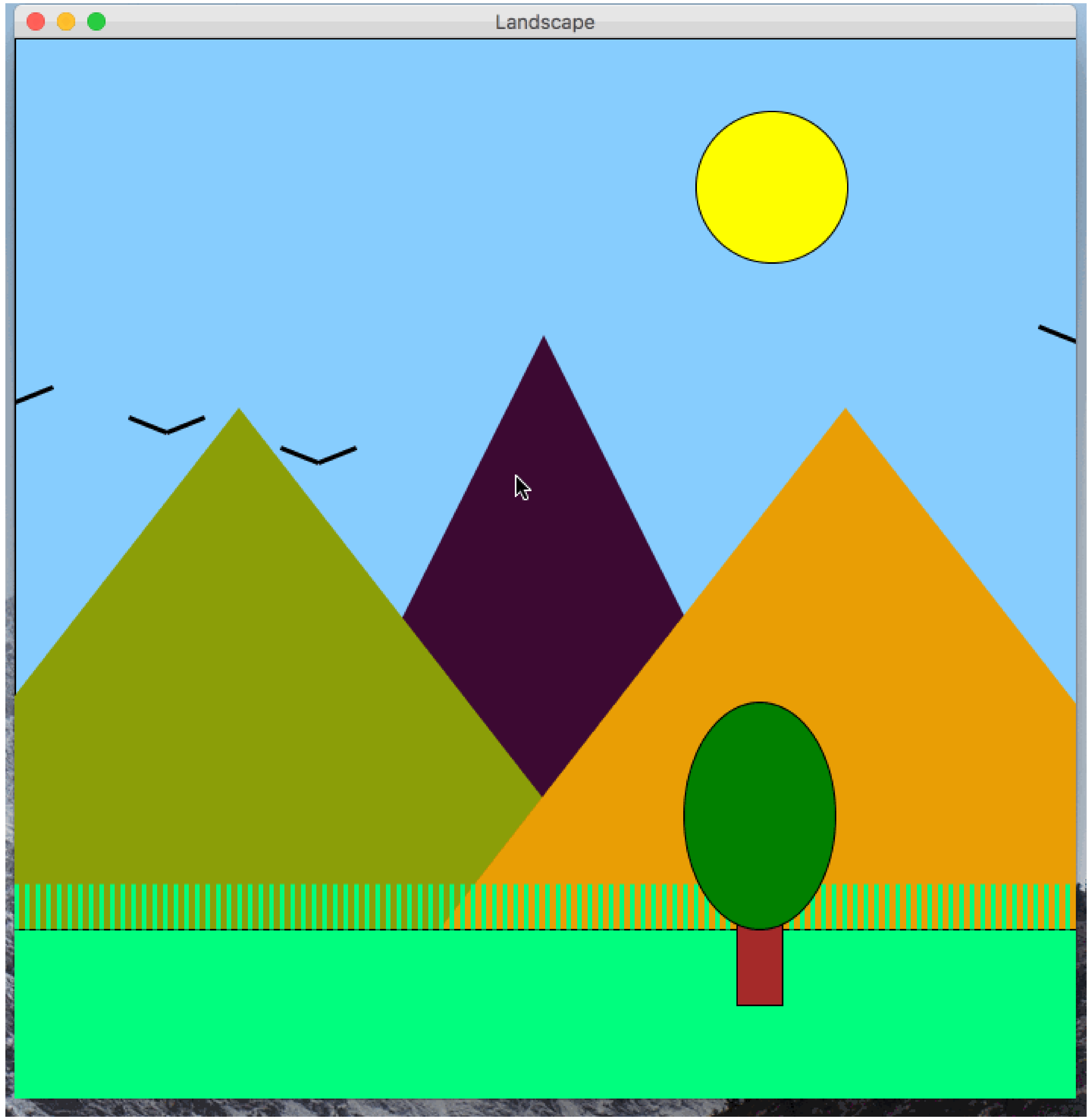
Figure 1.2: Two images taken from a helicopter flying through a canyon and the computed optical flow field.

Length of flow vectors inversely proportional to depth Z of 3d point

points closer to the camera move more quickly across the image plane

Figure from Michael Black, Ph.D. Thesis

Motion parallax



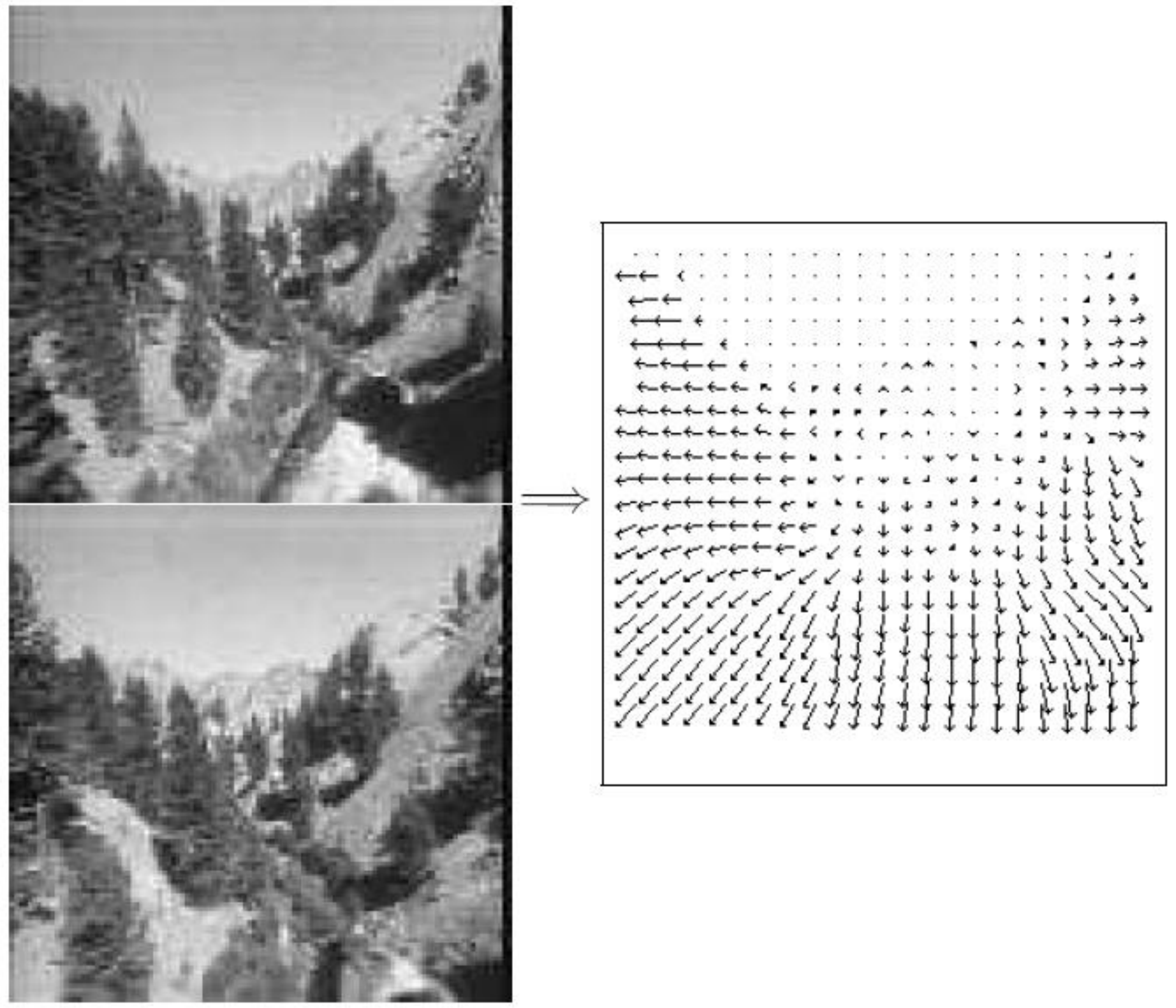
Length of flow vectors inversely proportional to depth Z of 3d point

points closer to the camera move more quickly across the image plane

Motion field vs. optical flow

- The **motion field** is the projection of the 3D scene motion into the image

- **Optical flow** is the apparent motion of brightness patterns in the image



- Ideally, they are the same.
- But when are they not?

Apparent motion != motion field

Apparent motion can be caused by lighting changes without any actual motion. E.g., consider a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

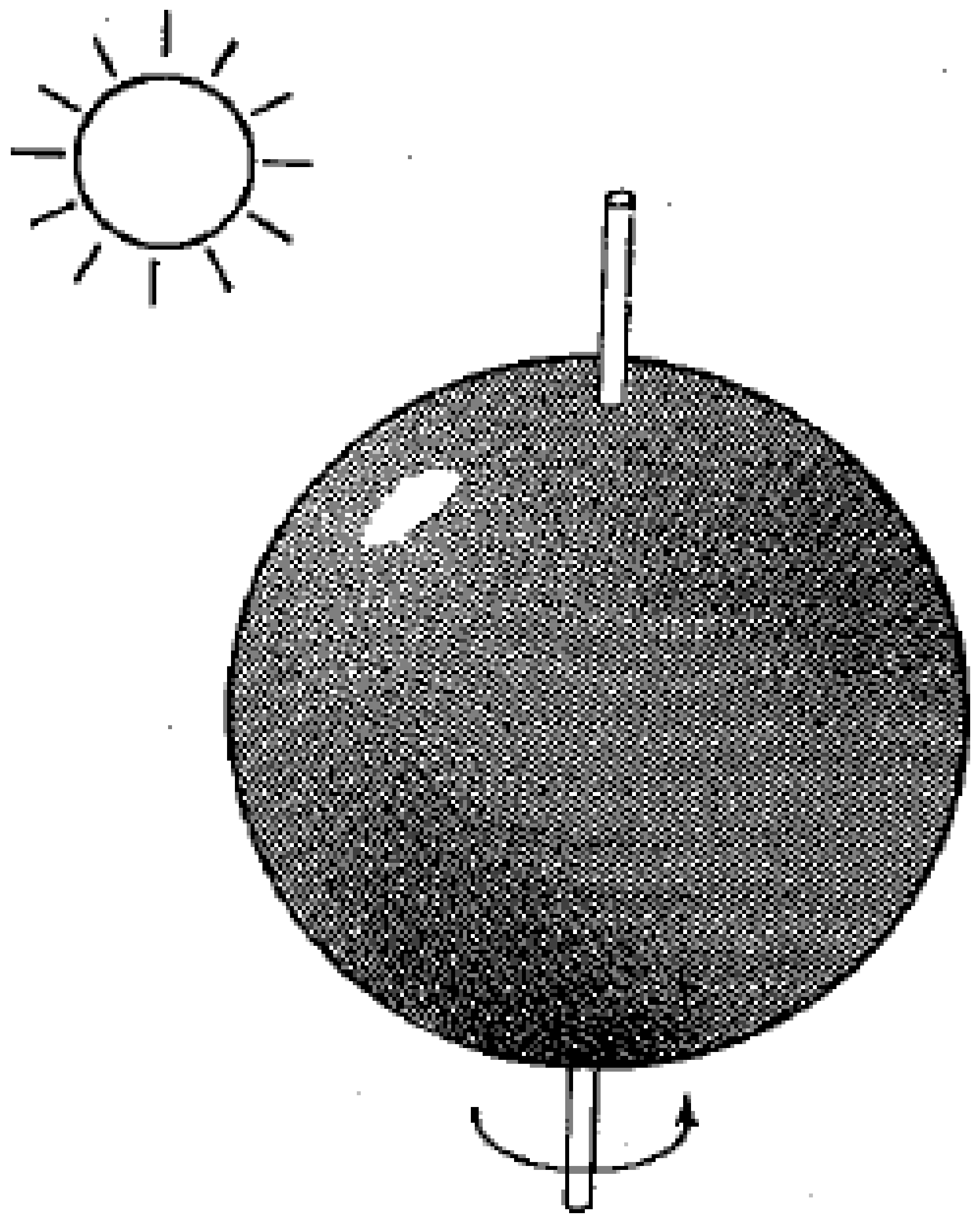

(a)

Figure from Horn book

Apparent motion != motion field

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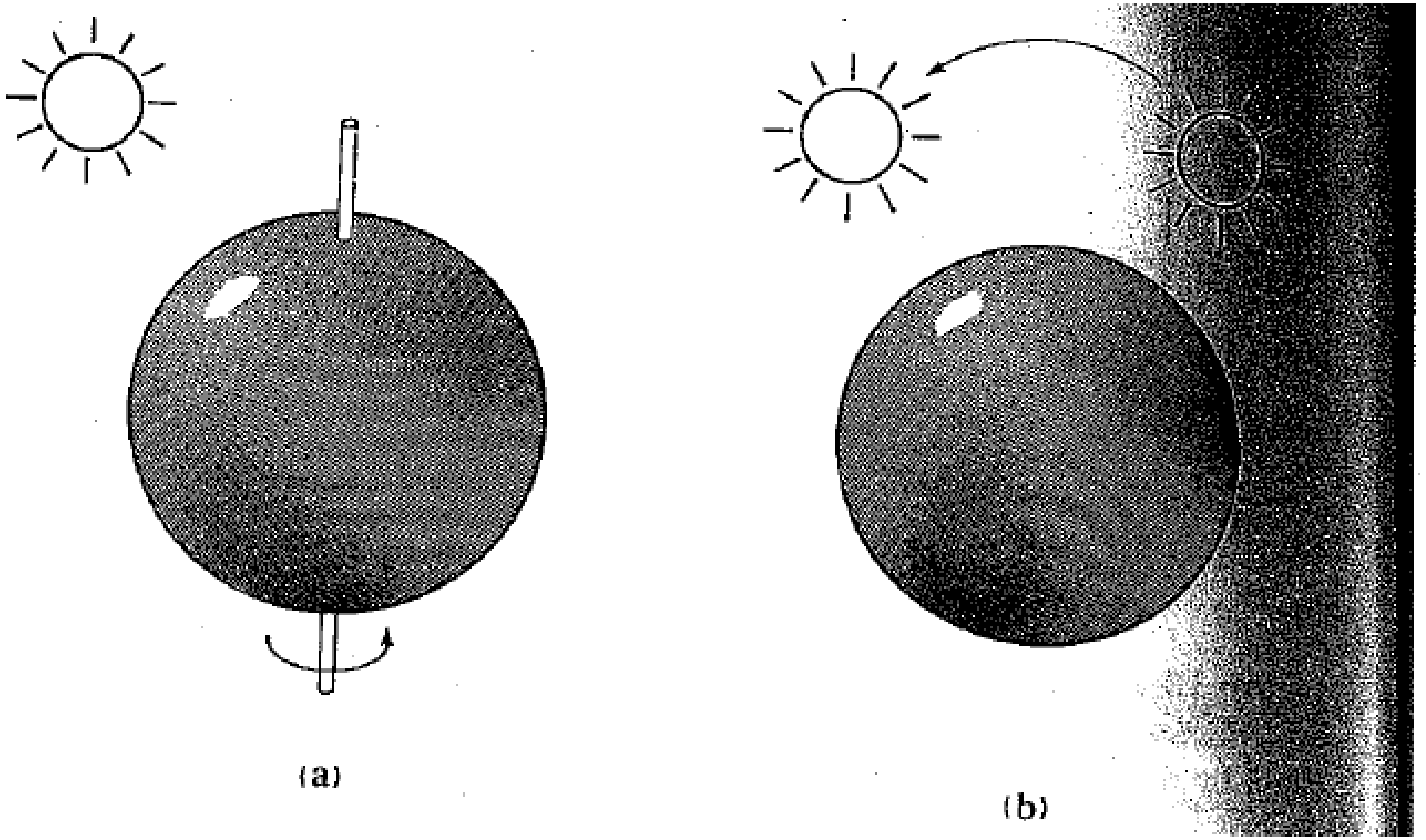
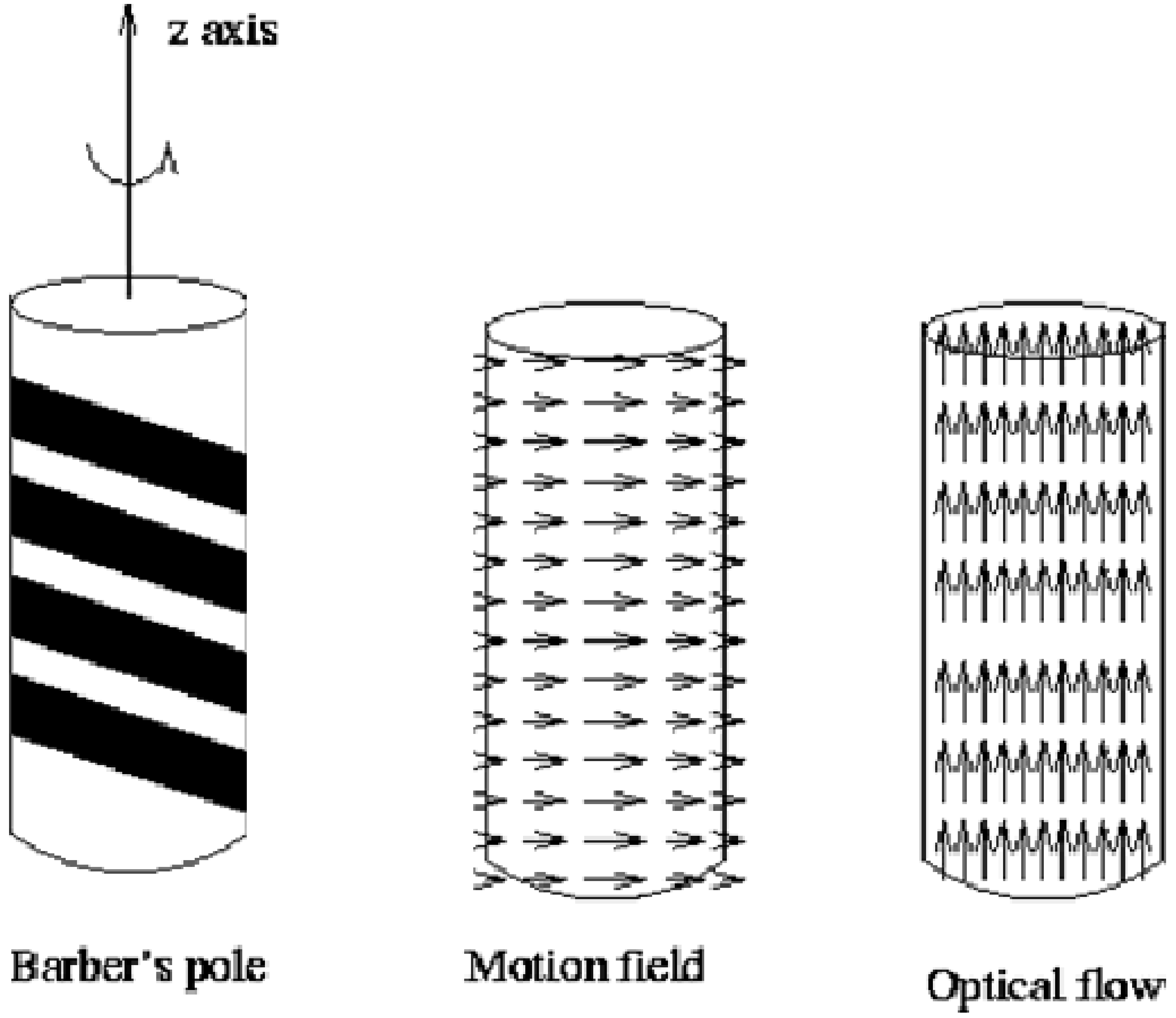


Figure from Horn book

Apparent motion != motion field

The barber pole illusion

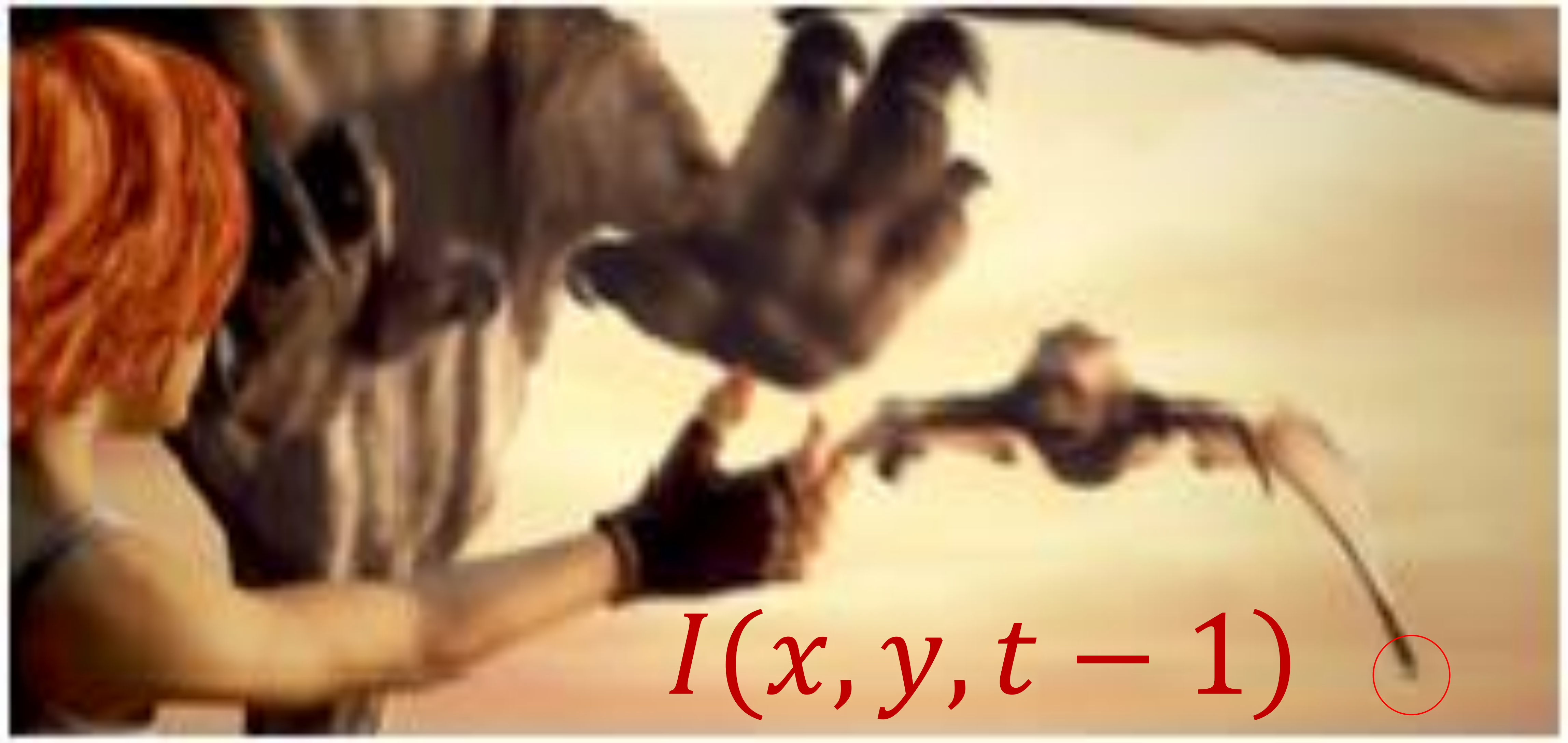


http://en.wikipedia.org/wiki/Barberpole_illusion

Estimating optical flow

- Given frames at times $t-1$ and t, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them

$t-1$



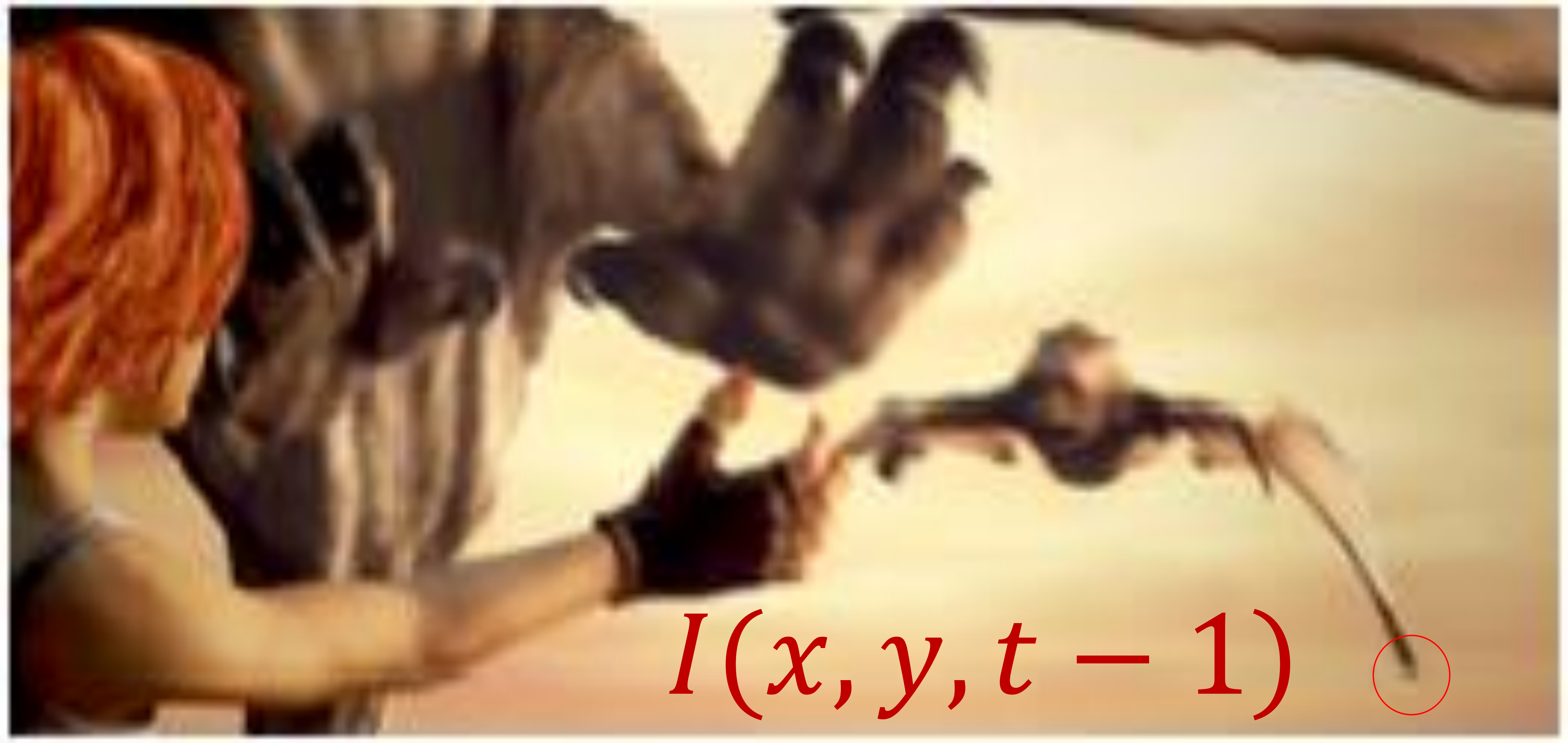
t

Estimating optical flow

- **Assumption #1: Brightness constancy constraint**: projection of the same point looks the same in every frame:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

$t-1$



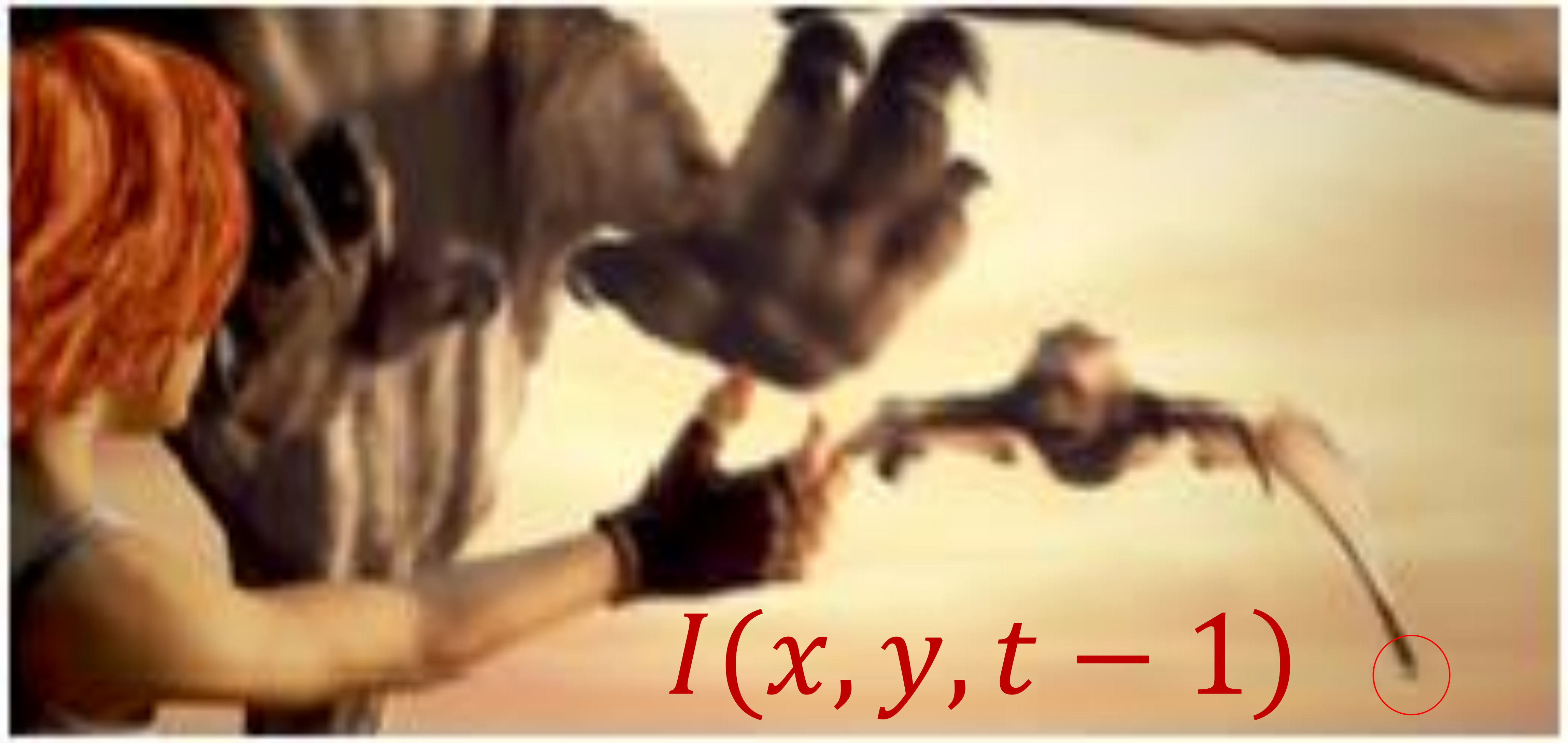
t

Estimating optical flow

- **Assumption #2: Small motion**: points do not move very far, i.e.,

  $u(x,y), v(x,y)$ are very small

$t-1$



t

Taylor Series Expansion

Expand a function as an infinite sum of its derivatives

$$f(x+\delta x) = f(x) + \frac{\partial f}{\partial x}\delta x + \frac{\partial^2 f}{\partial x^2}\frac{\delta x^2}{2!} + \cdots + \frac{\partial^n f}{\partial x^n}\frac{\delta x^n}{n!}$$

Source: Shree Nayar

Taylor Series Expansion

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If δx is small:

$$f(x+\delta x) = f(x) + \frac{\partial f}{\partial x}\delta x + \boxed{O(\delta x^2)} \rightarrow \text{Almost Zero}$$

Source: Shree Nayar

Taylor Series Expansion

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For a function of three variables with small $\delta x, \delta y, \delta t$:

$$f(x+\delta x, y+\delta y, t+\delta t) \approx f(x,y,t) + \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial t}\delta t$$

Source: Shree Nayar

Estimating optical flow

- **Assumption #2: Small motion**: points do not move very far, i.e.,

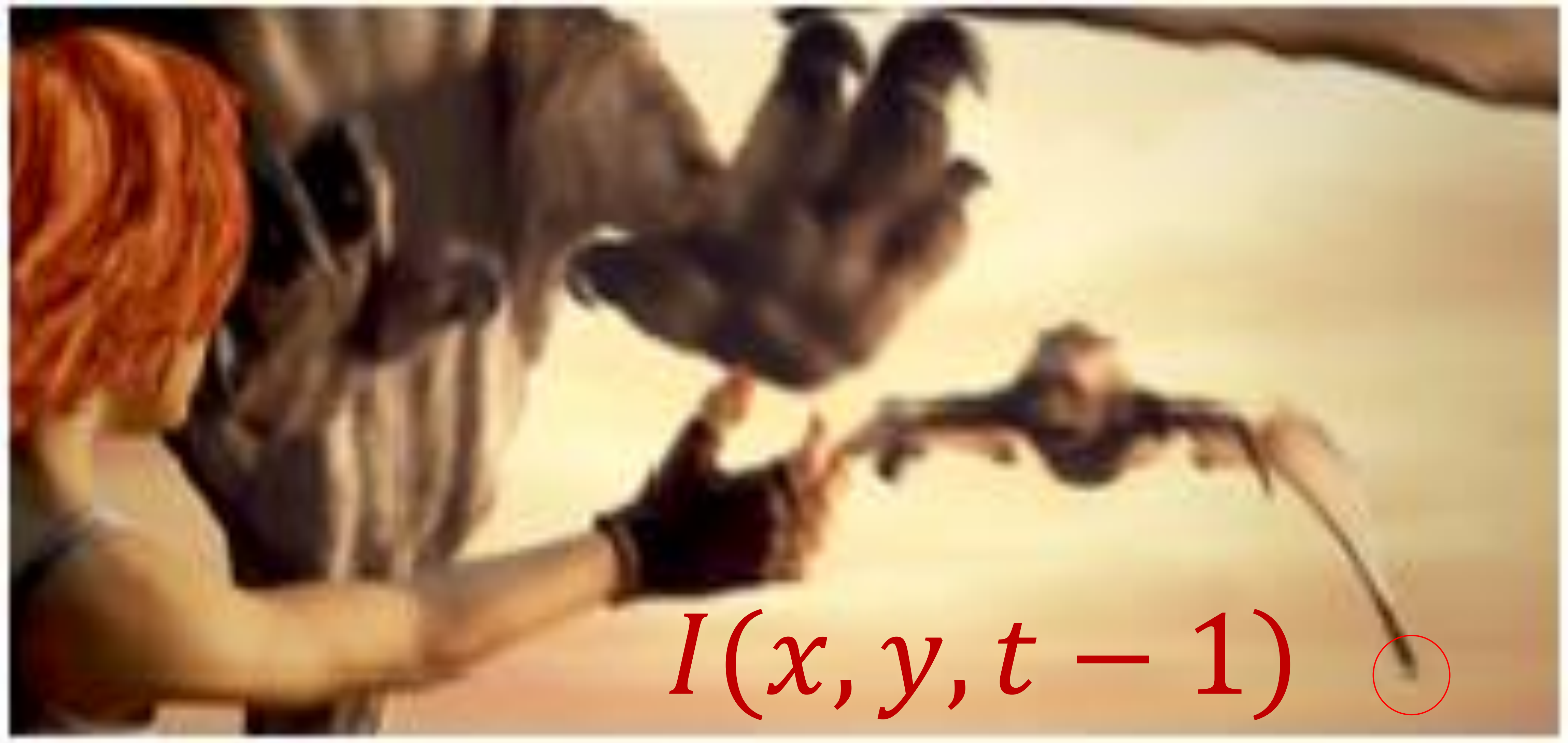
$u(x,y), v(x,y)$ are very small

Derivative in y direction

$$I(x+u(x,y), y+v(x,y), t) \approx I(x,y,t) + I_x u(x,y) + I_y v(x,y)$$

first-order Taylor expansion approximation

$t-1$



t

Estimating optical flow

- **Assumption #1: Brightness constancy constraint**:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

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Estimating optical flow

- **Assumption #1: Brightness constancy constraint**:

$$I(x, y, t-1) = I(x + u(x,y), y + v(x,y), t)$$

- **Assumption #2: Small motion**

$$I(x + u(x,y), y + v(x,y), t) \approx I(x, y, t) + I_x u(x,y) + I_y v(x,y)$$

Derivative in time: $I(x, y, t) - I(x, y, t-1)$

Subtract #1 from #2

$$I_x u(x,y) + I_y v(x,y) + \underbrace{I(x, y, t) - I(x, y, t-1)}_{\text{What could this be?}} = 0$$

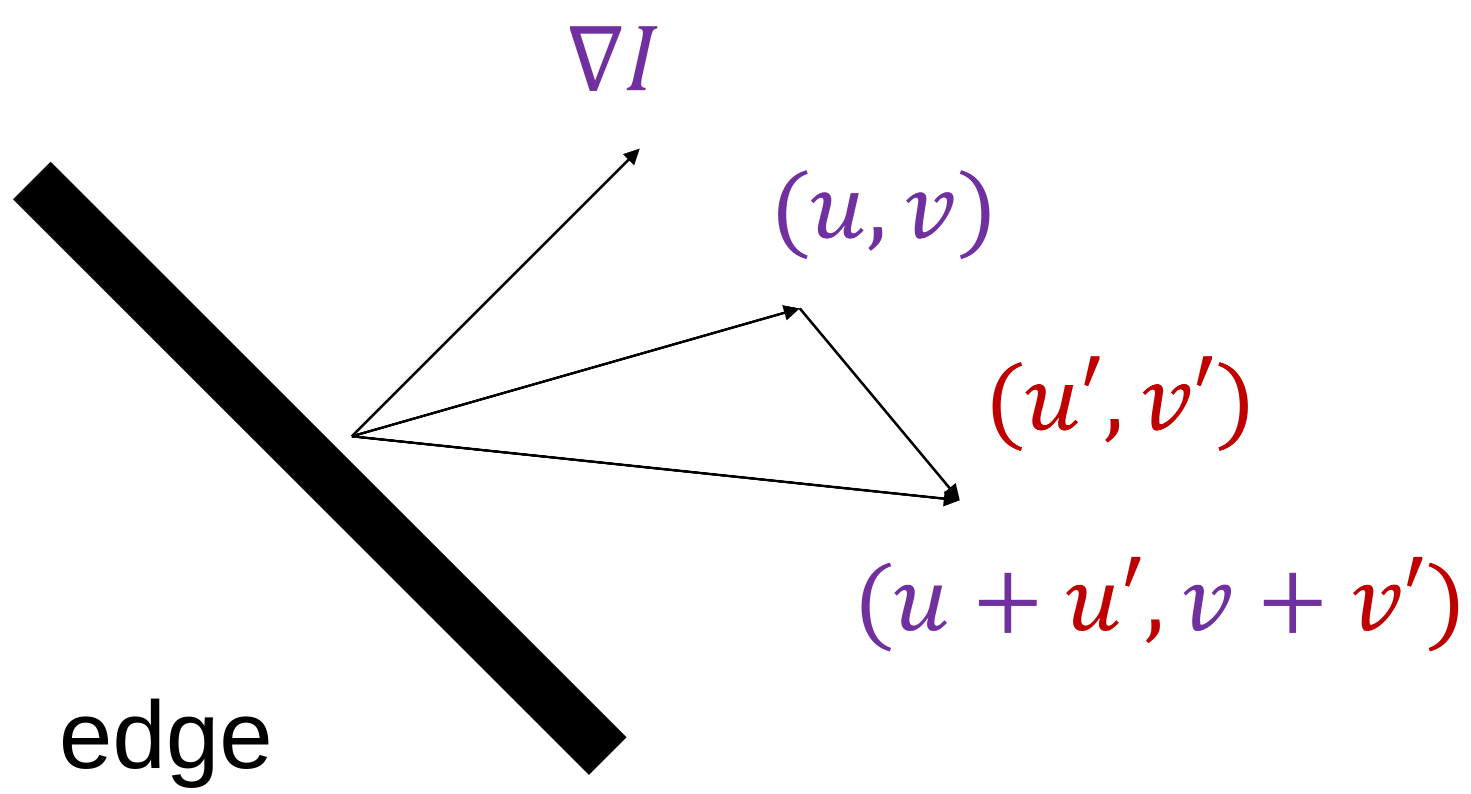
Therefore: $I_x u + I_y v + I_t \approx 0$

Optical Flow Constraint Equation

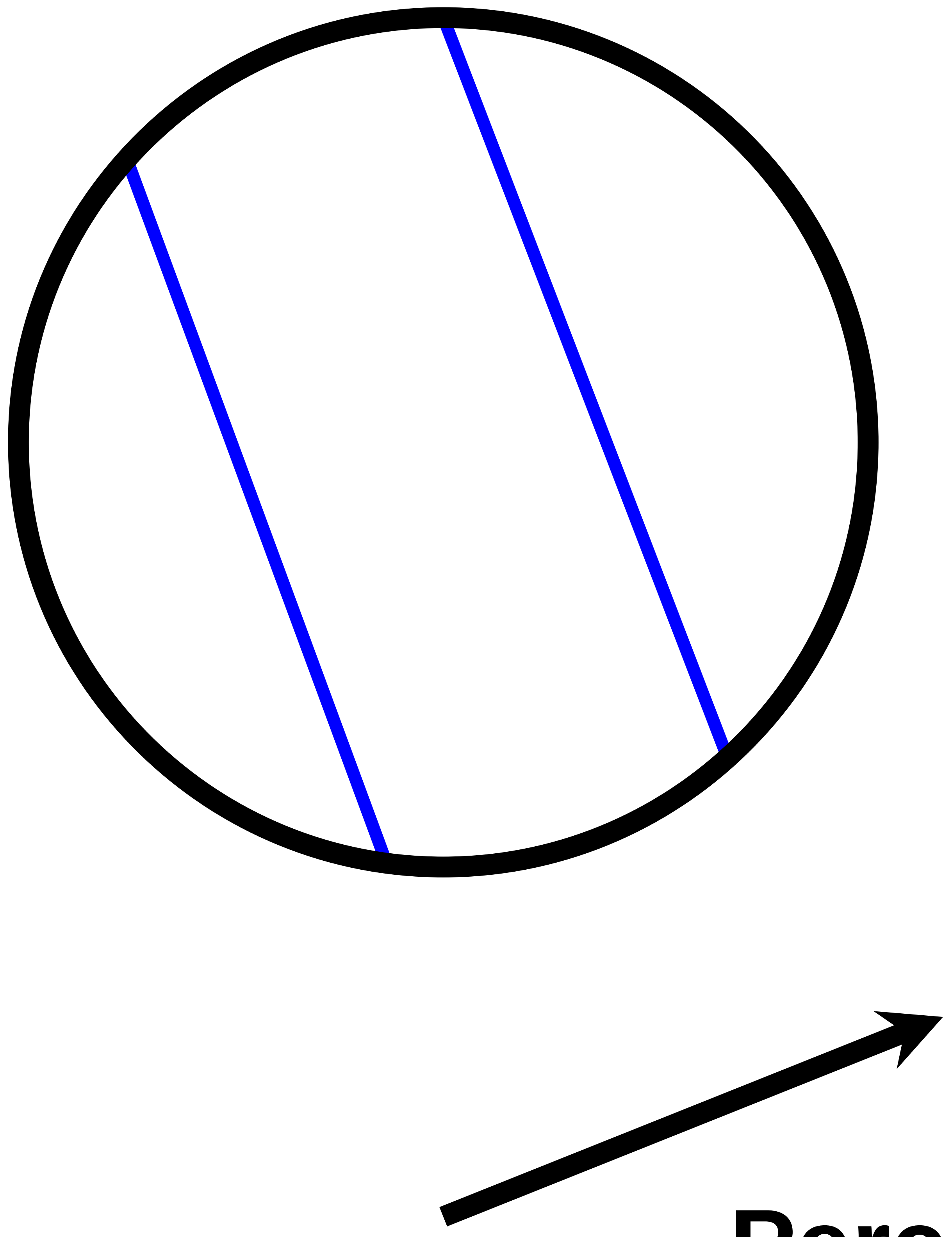
$$\boxed{I_x u + I_y v + I_t = 0}$$

Given the gradients I_x, I_y and I_t, can we uniquely recover the motion (u, v)?

- Suppose (u, v) satisfies the constraint: $\nabla I \cdot (u, v) + I_t = 0$
- Then $\nabla I \cdot (u + u', v + v') + I_t = 0$ for any (u', v') s. t. $\nabla I \cdot (u', v') = 0$
- Interpretation: the component of the flow perpendicular to the gradient (i.e., parallel to the edge) cannot be recovered!



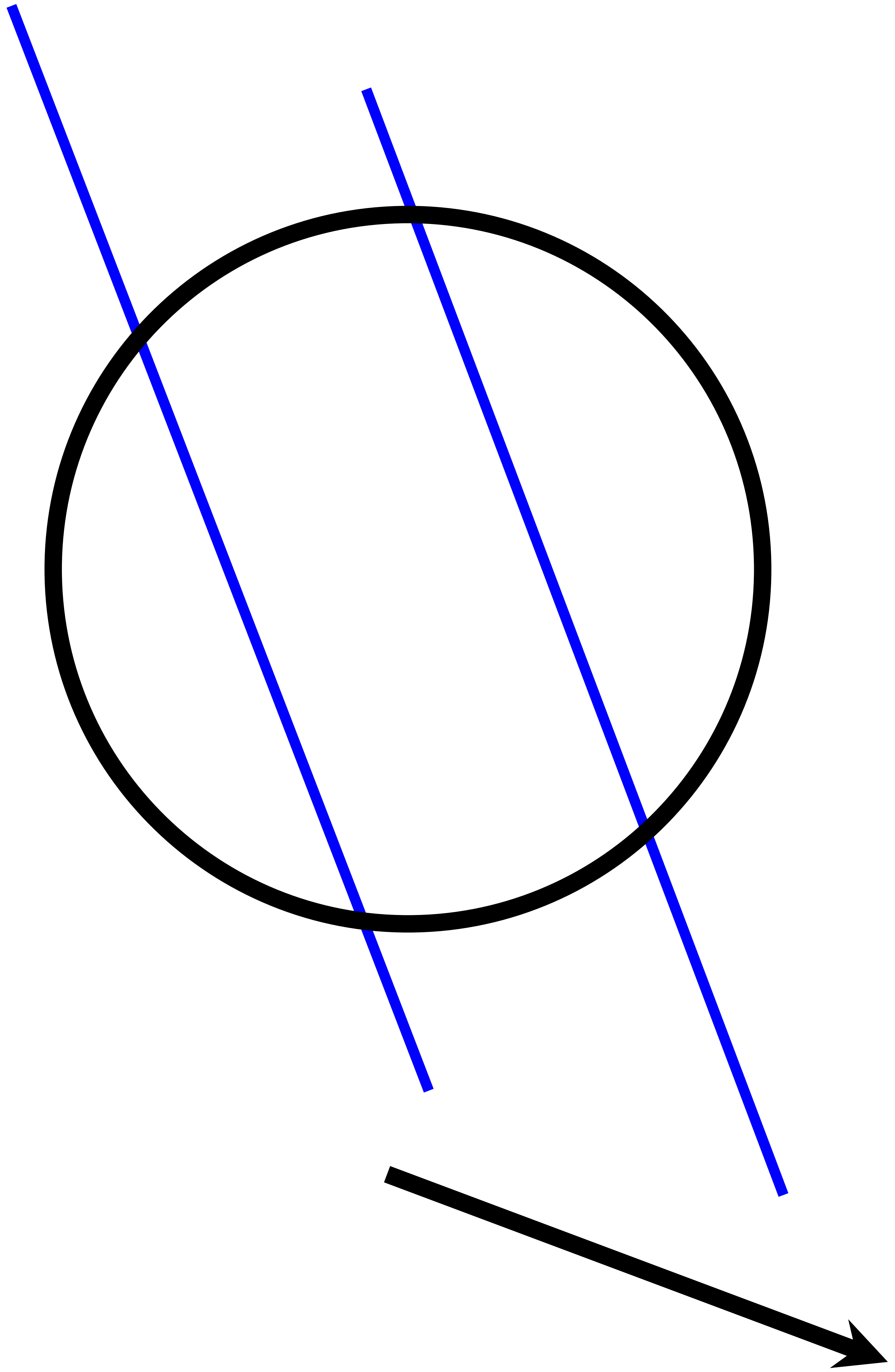
The aperture problem



Perceived motion

Source: S. Lazebnik

The aperture problem



Actual motion

Source: S. Lazebnik

Optical Flow Constraint Equation

$$\boxed{I_x u + I_y v + I_t = 0}$$

- How many equations and unknowns per pixel?
  - 1 equation, 2 unknowns

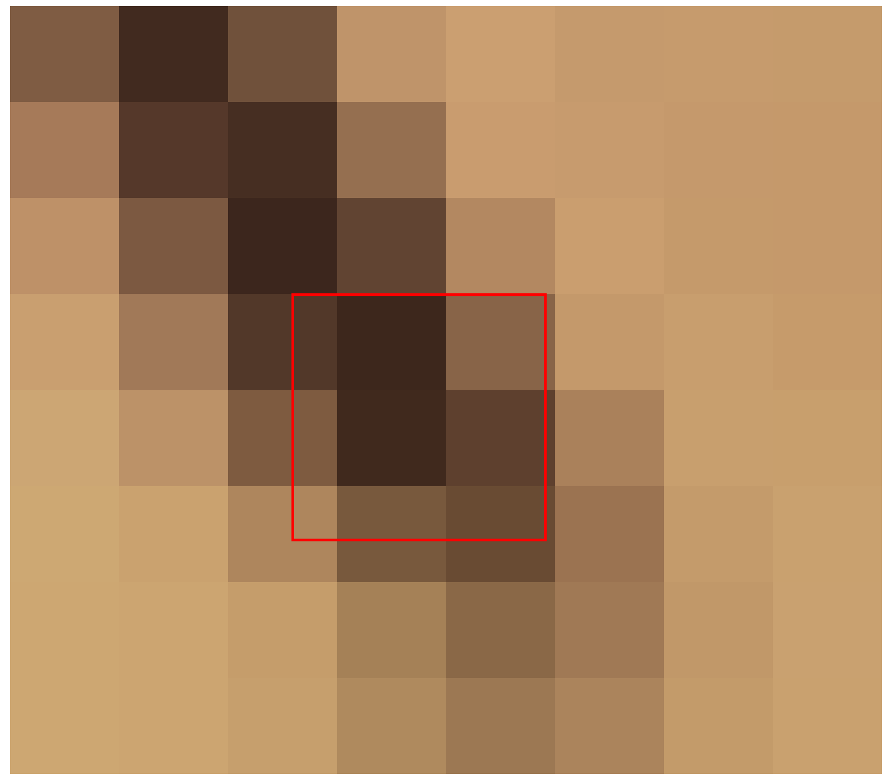
We need additional constraints.

Lucas-Kanade Solution

- How to get more equations for a pixel?
- **Assumption #3: Spatial coherence constraint.**

  assume the pixel's neighbors have the same *(u,v)*
  - E.g., if we use a 5x5 window, that gives us 25 equations per pixel

$$\nabla I(\mathbf{x}_i) \cdot [u, v] + I_t(\mathbf{x}_i) = 0$$



[Lucas & Kanade 1981]

Source: S. Lazebnik

Lucas-Kanade Solution

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$$\nabla I(\mathbf{x}_i) \cdot [u, v] + I_t(\mathbf{x}_i) = 0$$

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

[Lucas & Kanade 1981]

Source: S. Lazebnik

Lucas-Kanade Solution

Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

$$\underset{n\times 2}{\mathbf{A}}\ \underset{2\times 1}{\mathbf{d}} = \underset{n\times 1}{\mathbf{b}}$$

Solution given by $(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T\mathbf{b}$

M = $\mathbf{A}^T\mathbf{A}$ is the "second moment" matrix (also Gauss-Newton approximation to Hessian)

When is this system solvable?

M must be invertible

M must be well-conditioned

[Lucas & Kanade 1981]

Source: S. Lazebnik

Analyzing the second moment matrix

$$(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T\mathbf{b}$$

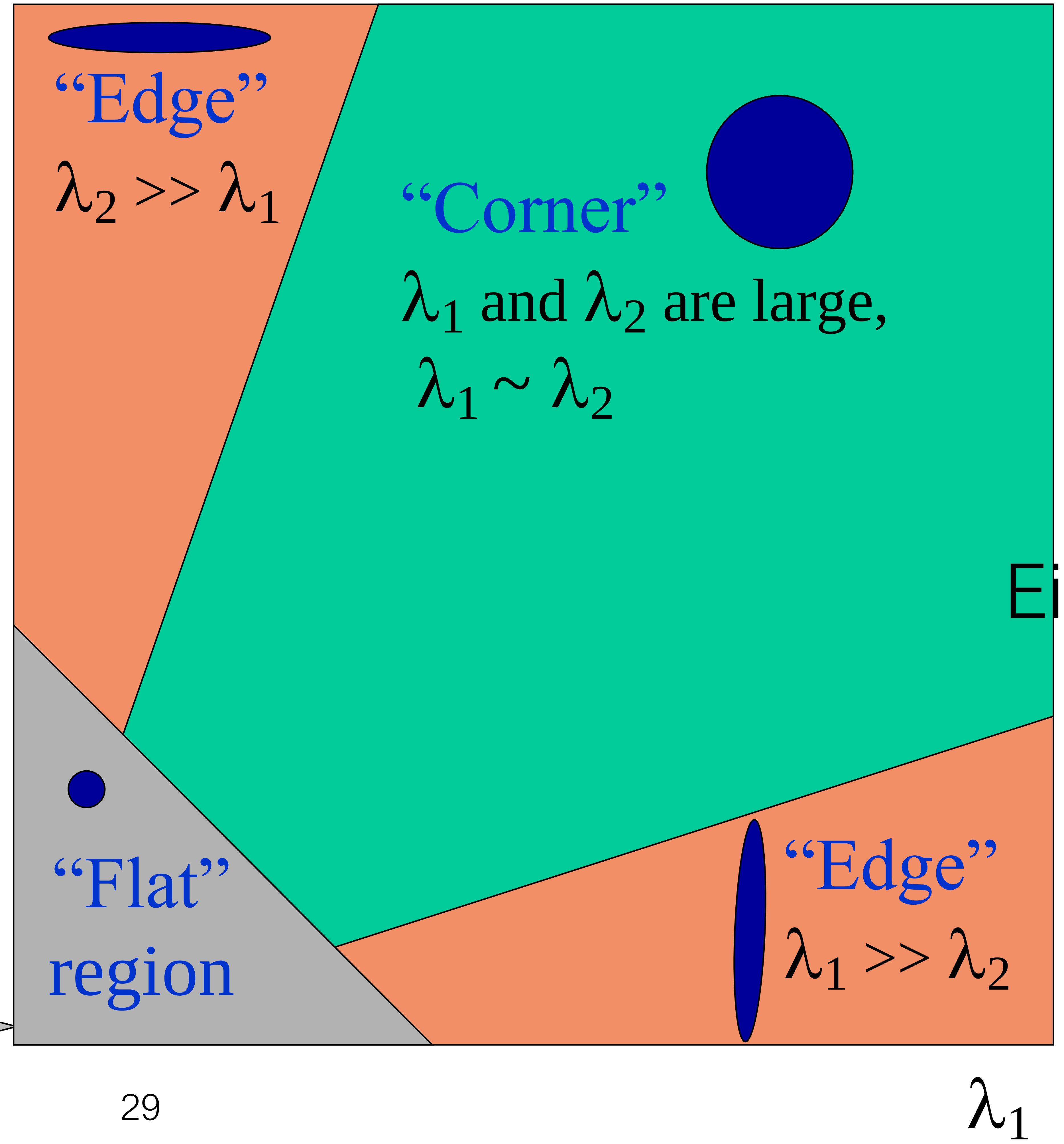
$$\mathbf{M} = \mathbf{A}^T\mathbf{A}$$

M must be invertible

M must be well-conditioned

bad

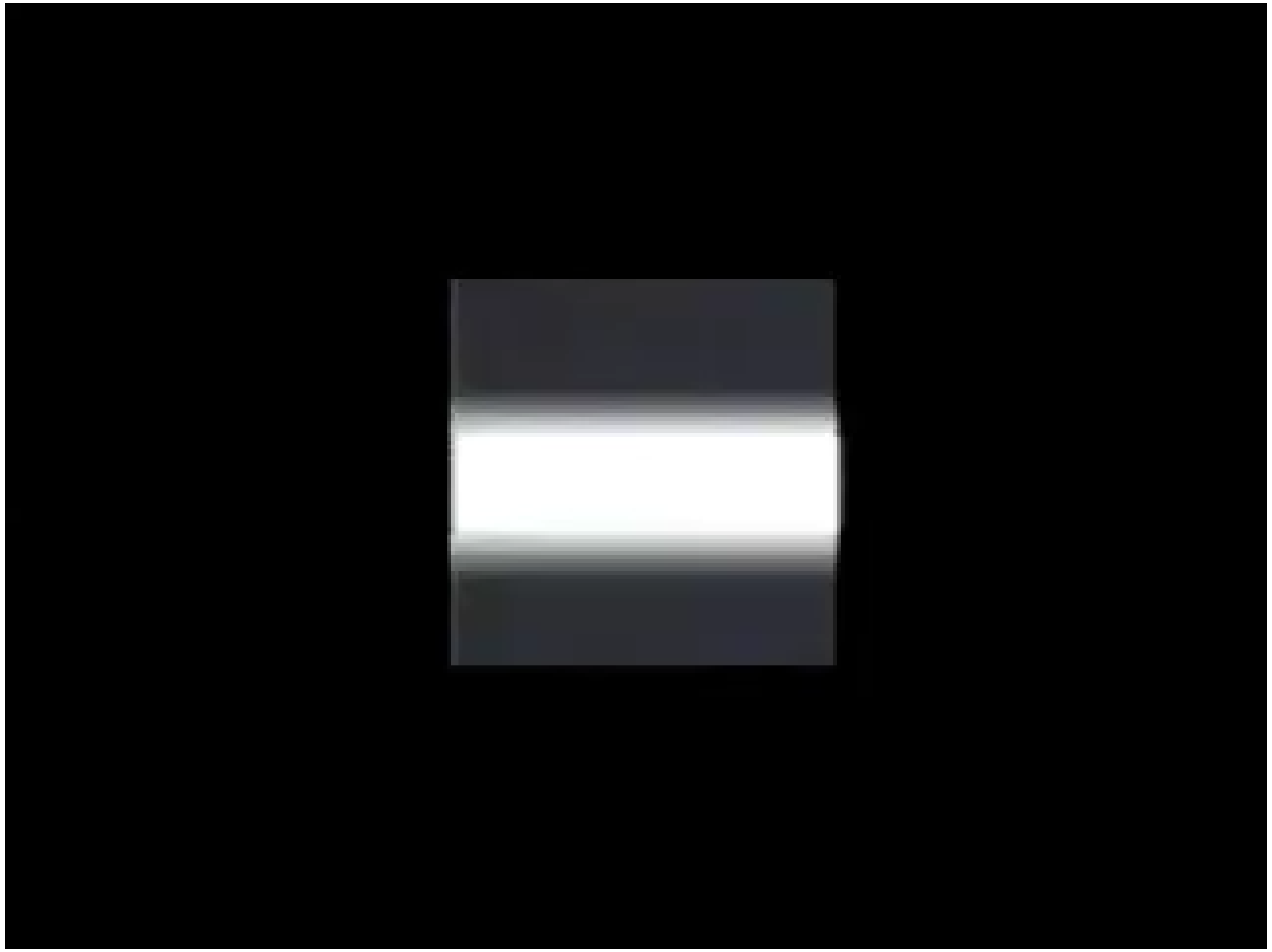
λ_1 and λ_2 are small



Eigenvalues

Conditions for solvability

Bad case: single, straight edge



Source: S. Lazebnik

Analyzing the second moment matrix

$(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T\mathbf{b}$

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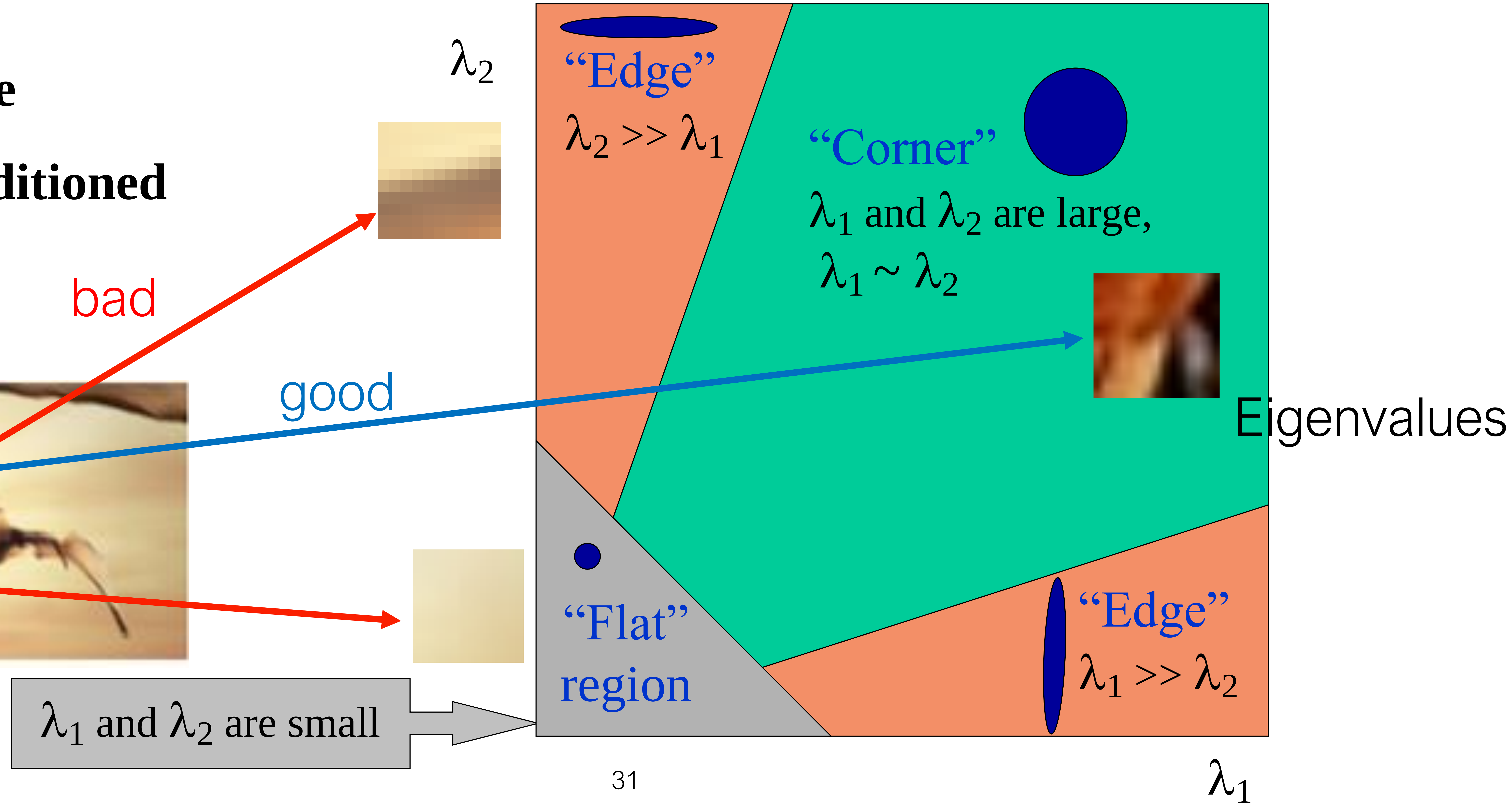
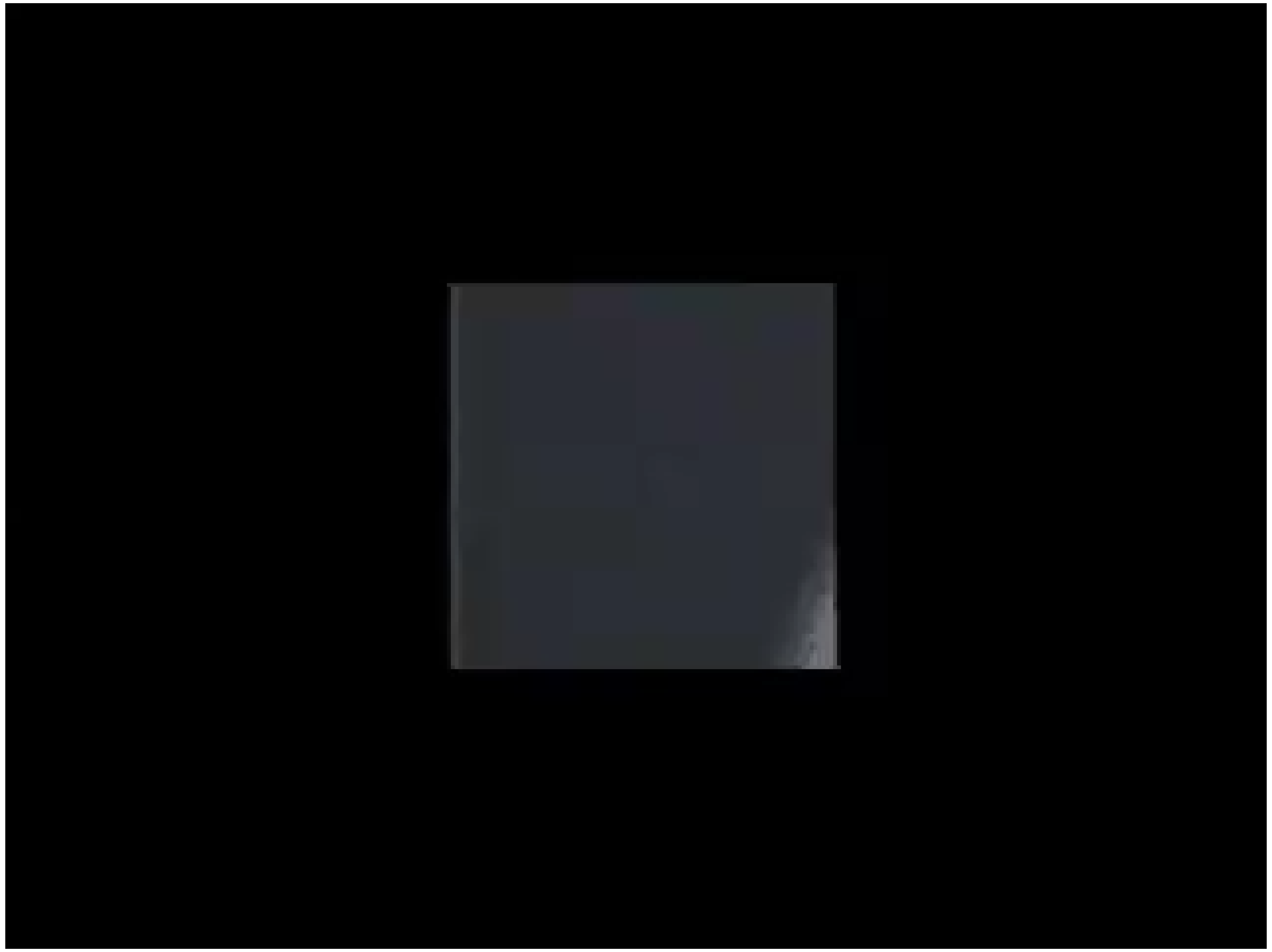


Conditions for solvability

Good case



Source: S. Lazebnik

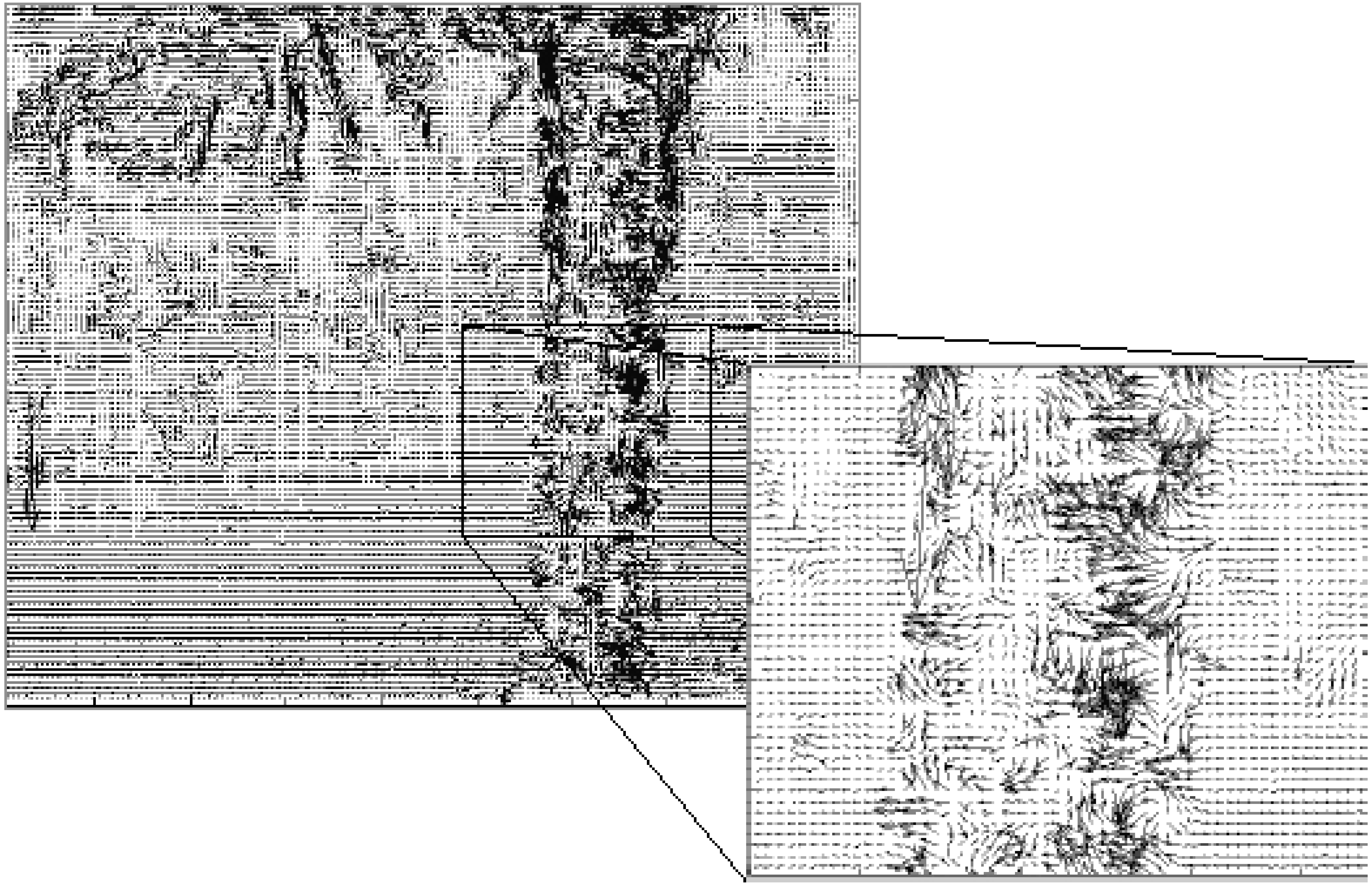
Large motions

recall the small-motion assumption

Large motions

Let’s say, 16-pixel displacement

recall the small-motion assumption



Large motions

Let’s say, 16-pixel displacement

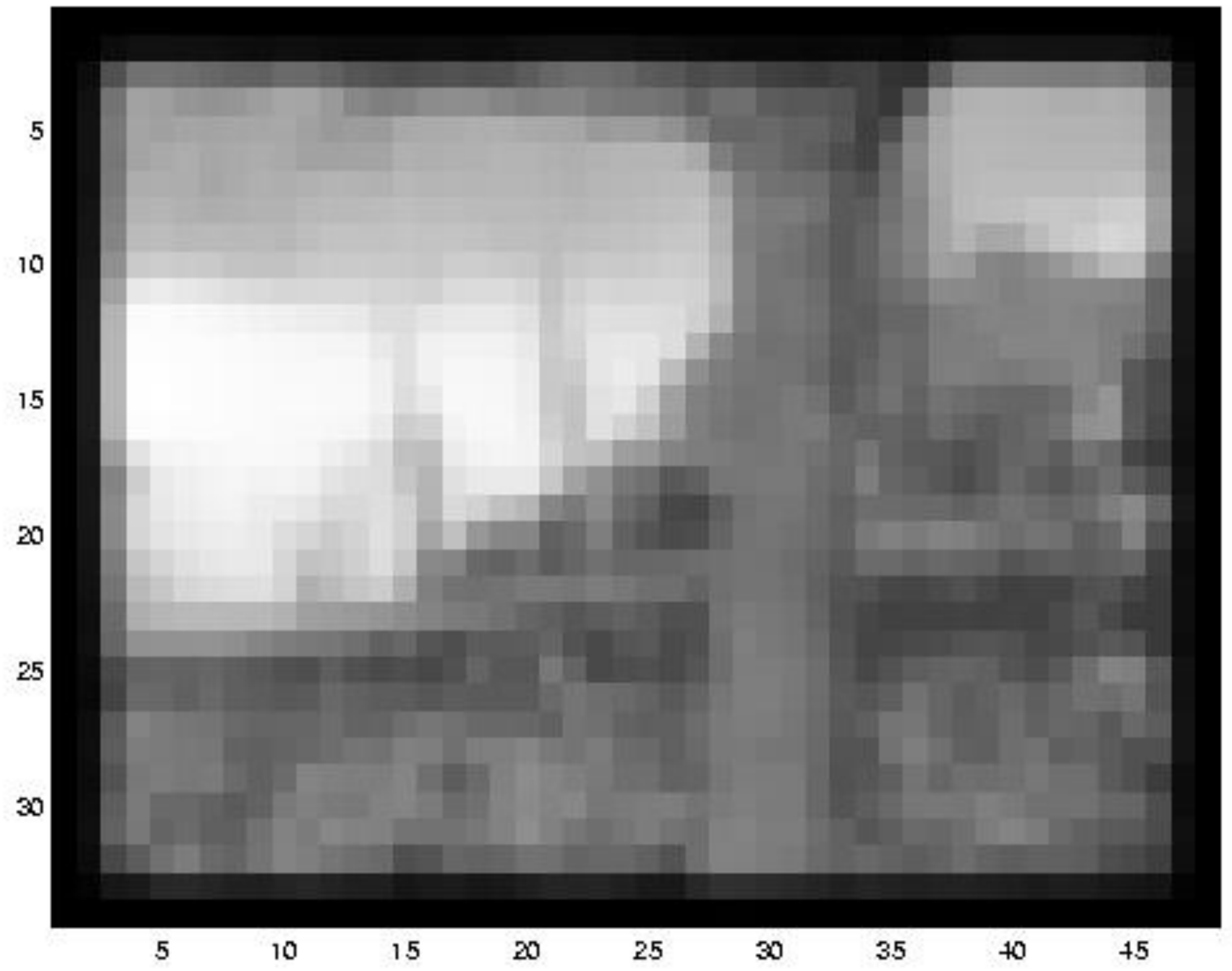
recall the small-motion assumption

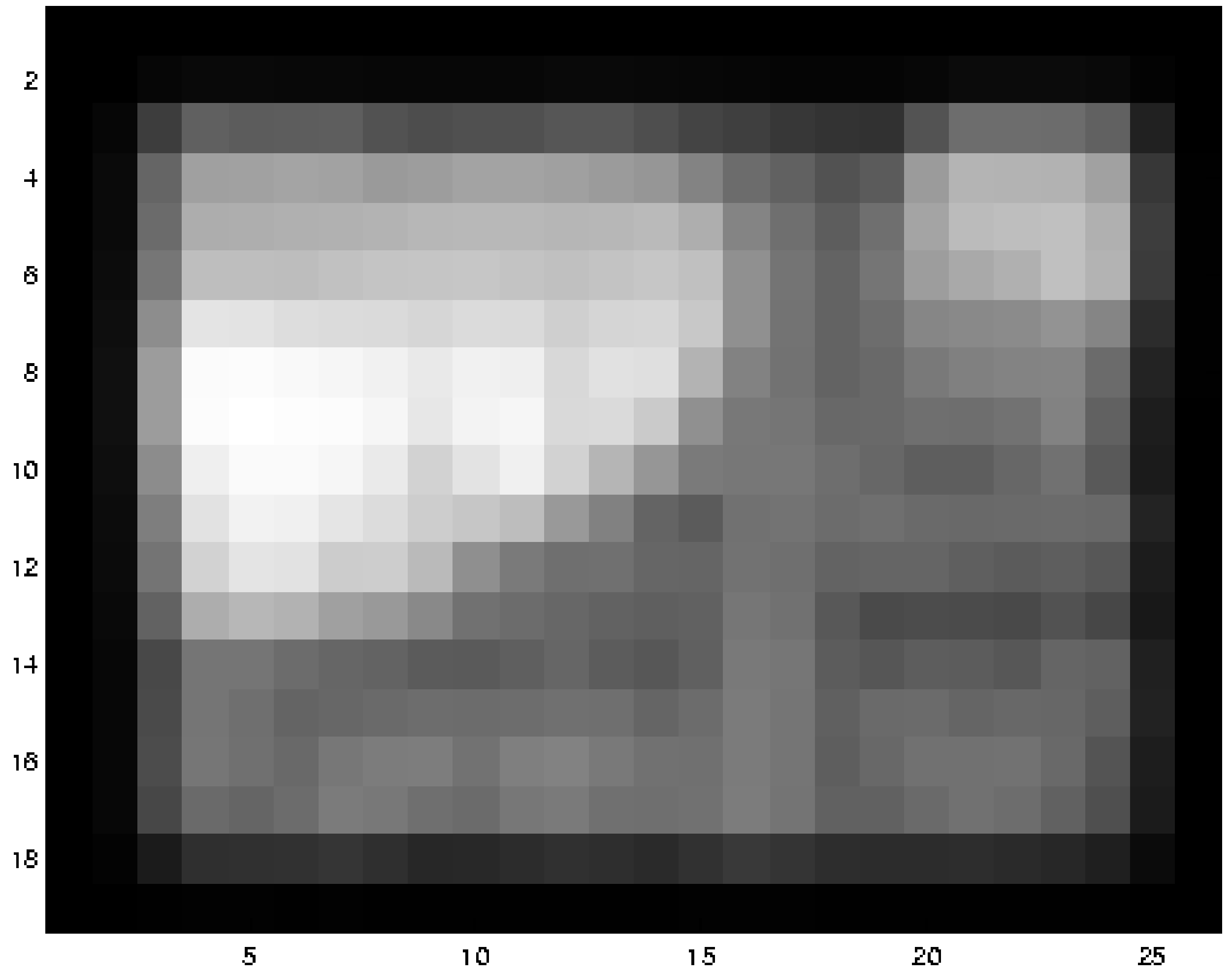
- Taylor series approximation of
  $$I(x + u(x,y), y + v(x,y), t)$$
  is not valid

- The simple linear constraint equation does not hold $I_x u + I_y v + I_t \neq 0$

Source: Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Coarse-to-fine flow estimation





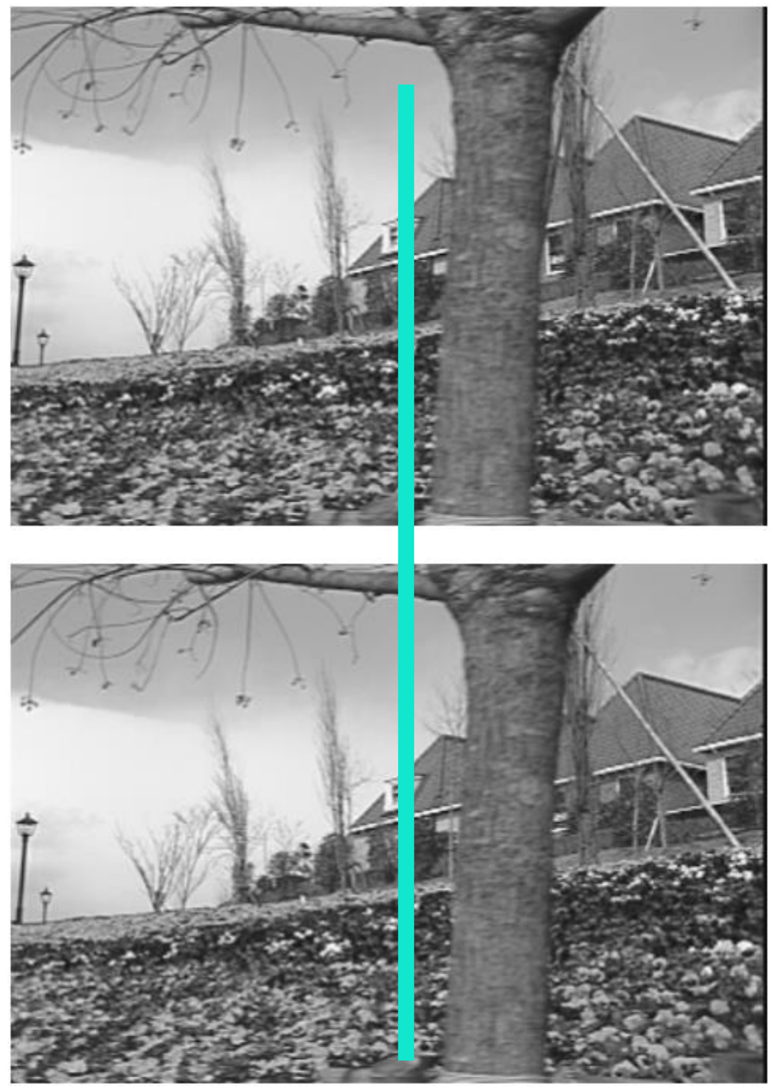
<1 pixel?

Sub-pixel motion

Source: Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Coarse-to-fine flow estimation

16-pixel displacement

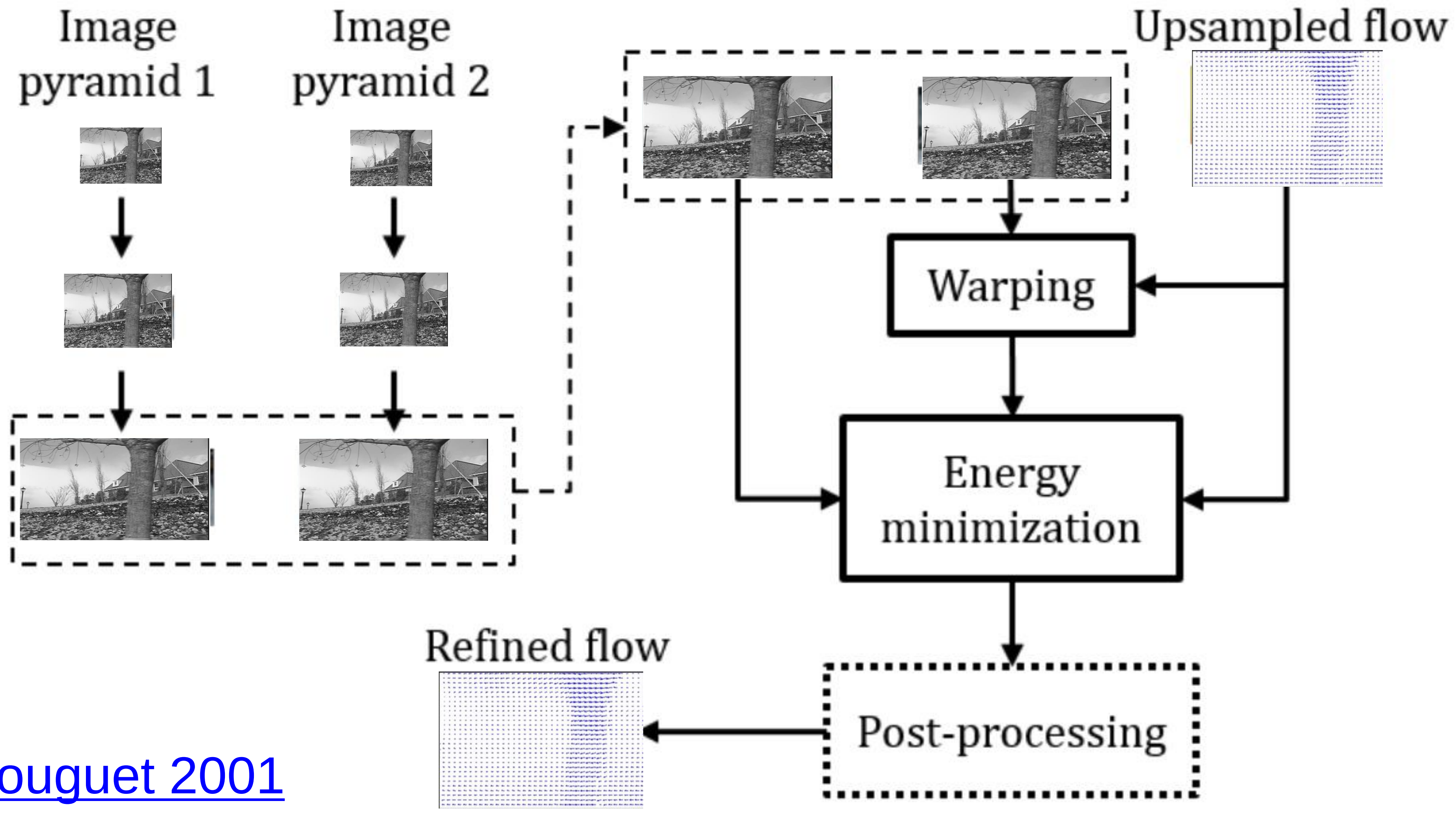


8-pixel disp.

4-pixel disp.

2-pixel disp.

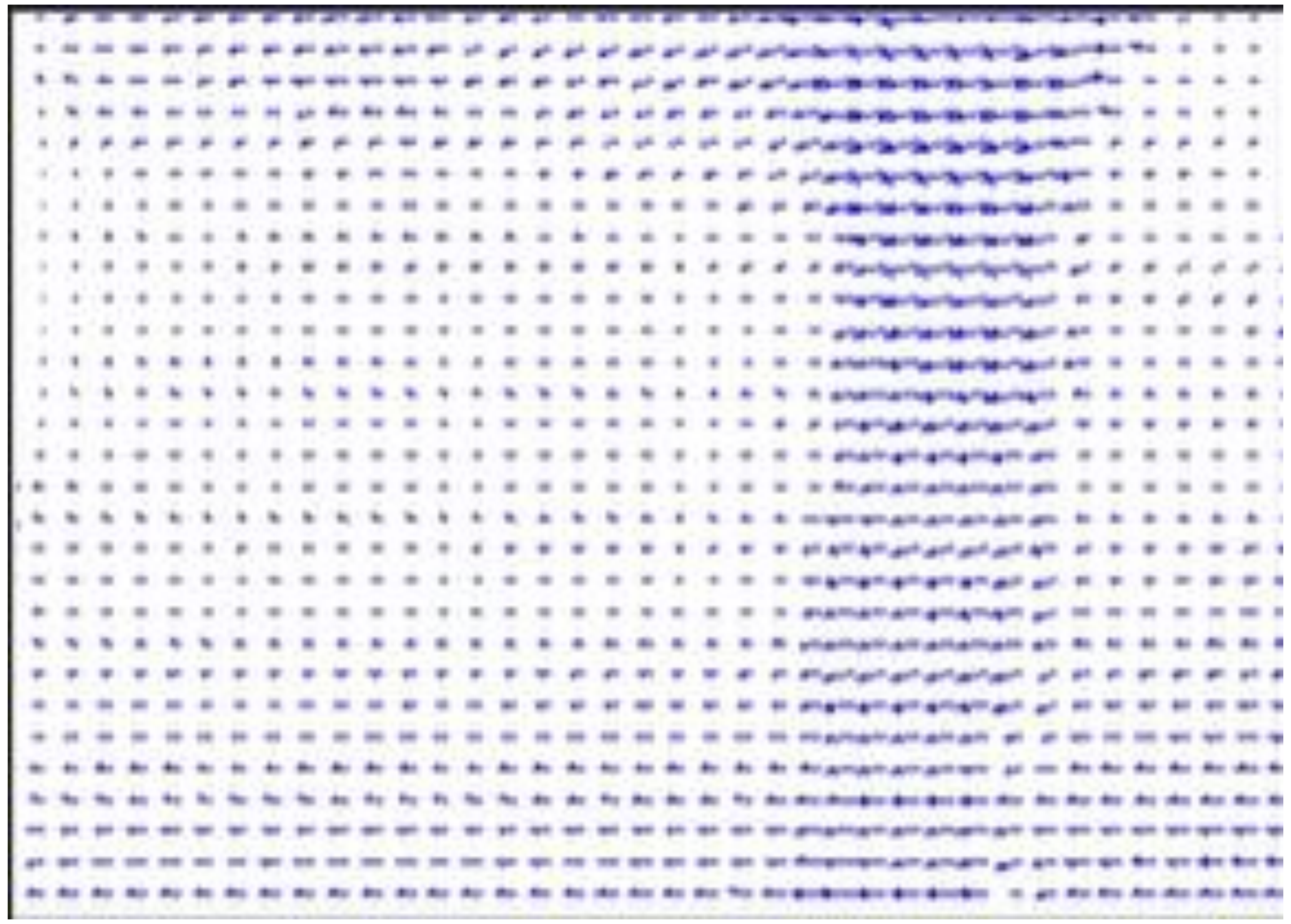
1-pixel disp.

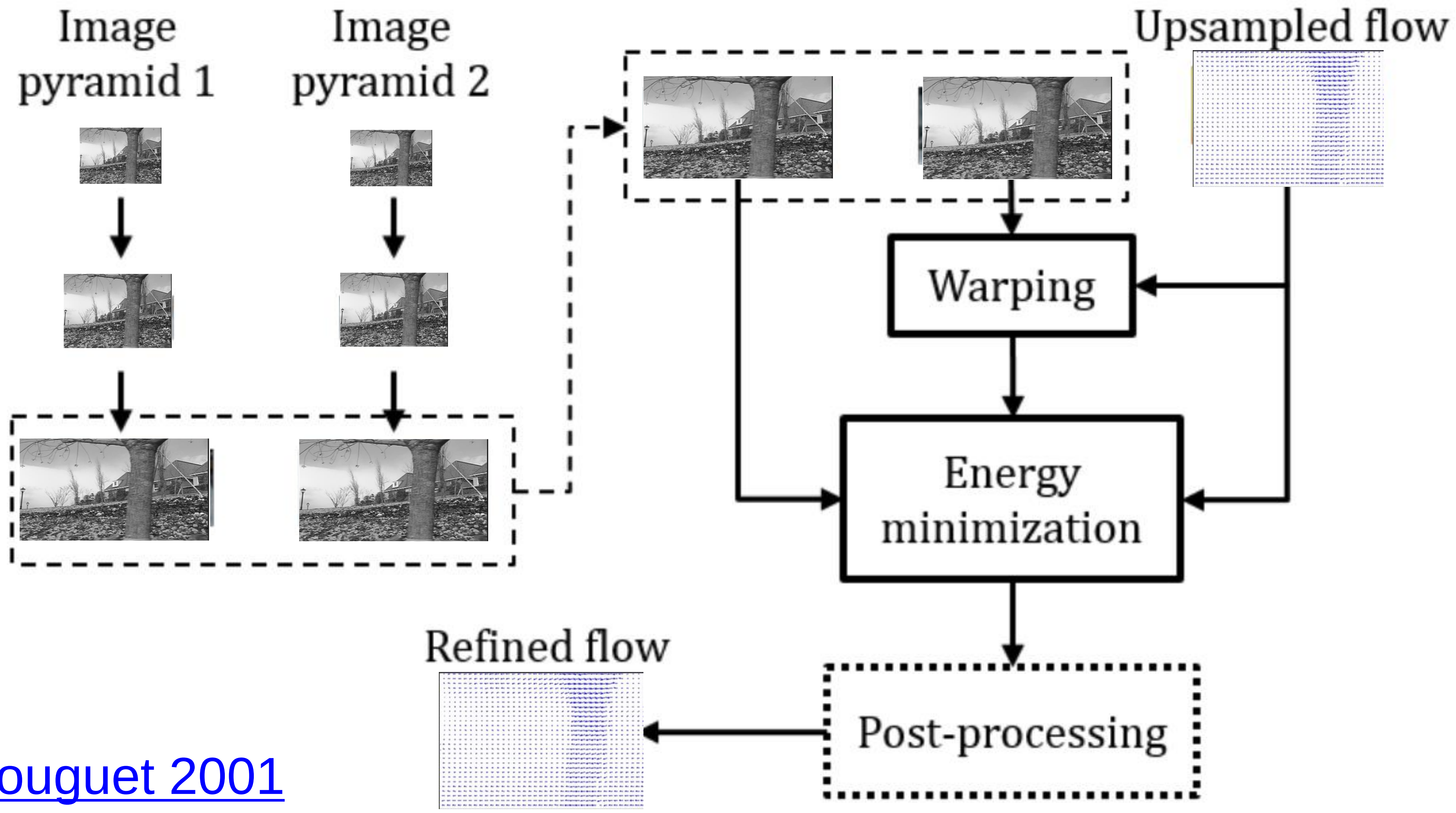


Bouguet 2001

Large motions

16-pixel displacement





[Bouguet 2001](#)

Shree Nayar

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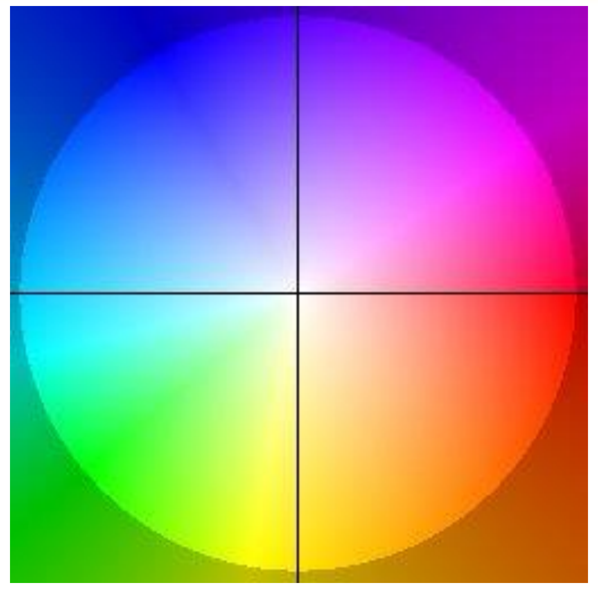
- **Why we care?**
- **Optical Flow Constraint Equation**
- **Classic algorithm: Lucas-Kanade algorithm**
- State-of-the-art algorithm via deep learning
- Fun applications

State-of-the-art optical flow estimation

- Current best methods are learned (often on synthetic data)

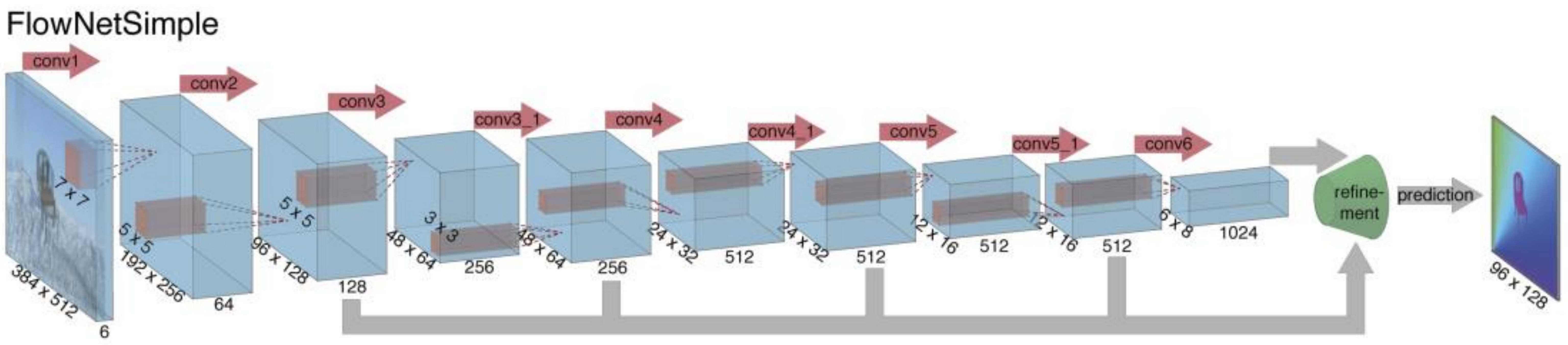
Synthetic dataset: Flying Chairs



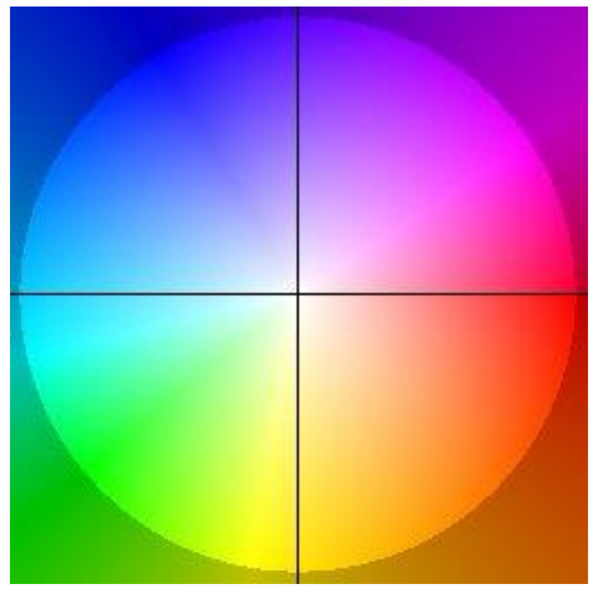
A. Dosovitskiy et al., FlowNet: Learning Optical Flow with Convolutional Networks, ICCV 2015

State-of-the-art optical flow estimation

- Current best methods are learned (often on synthetic data)



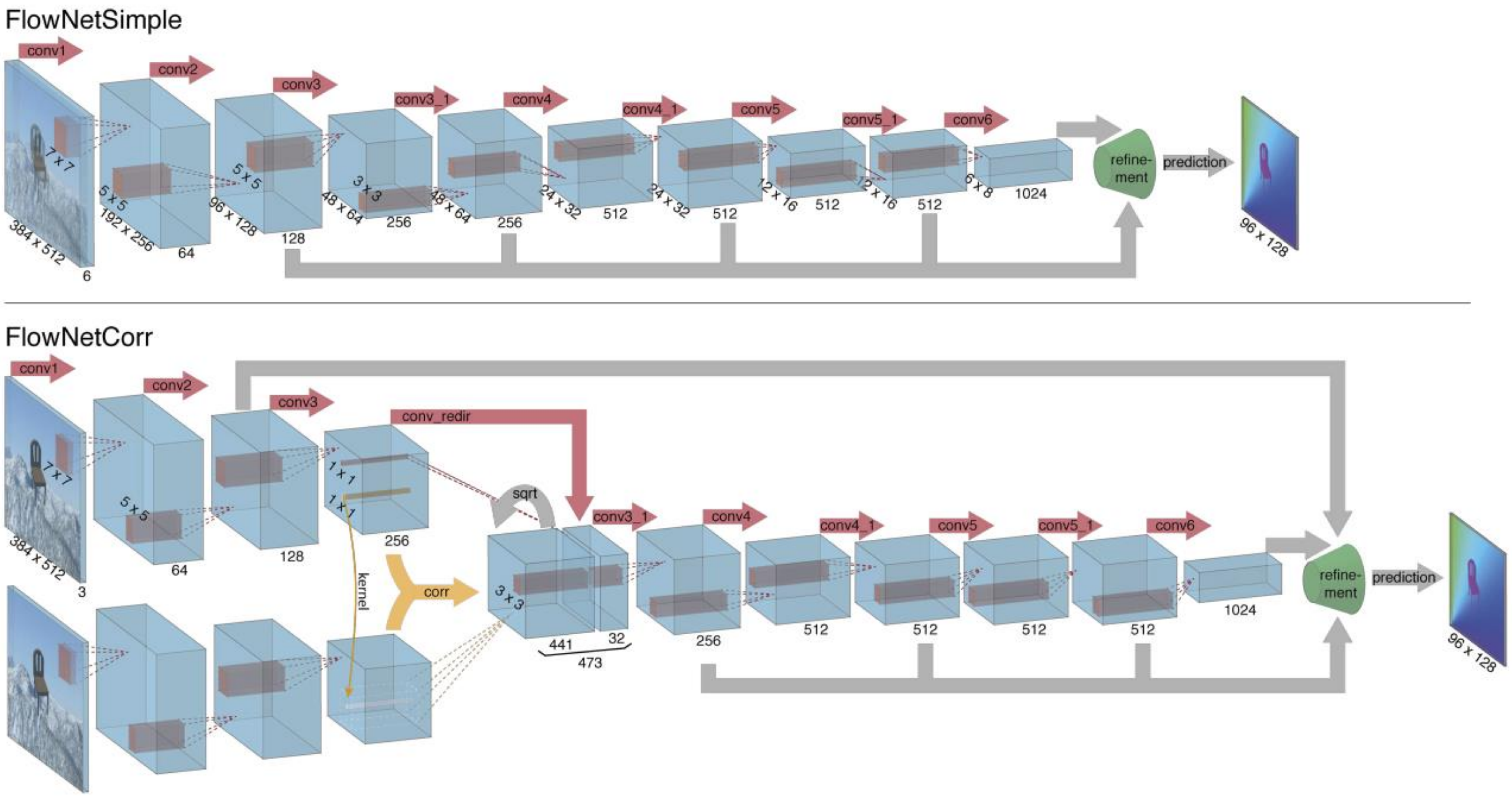
Synthetic dataset: Flying Chairs



A. Dosovitskiy et al., [FlowNet: Learning Optical Flow with Convolutional Networks](), ICCV 2015

Match CNN features instead of pixels!

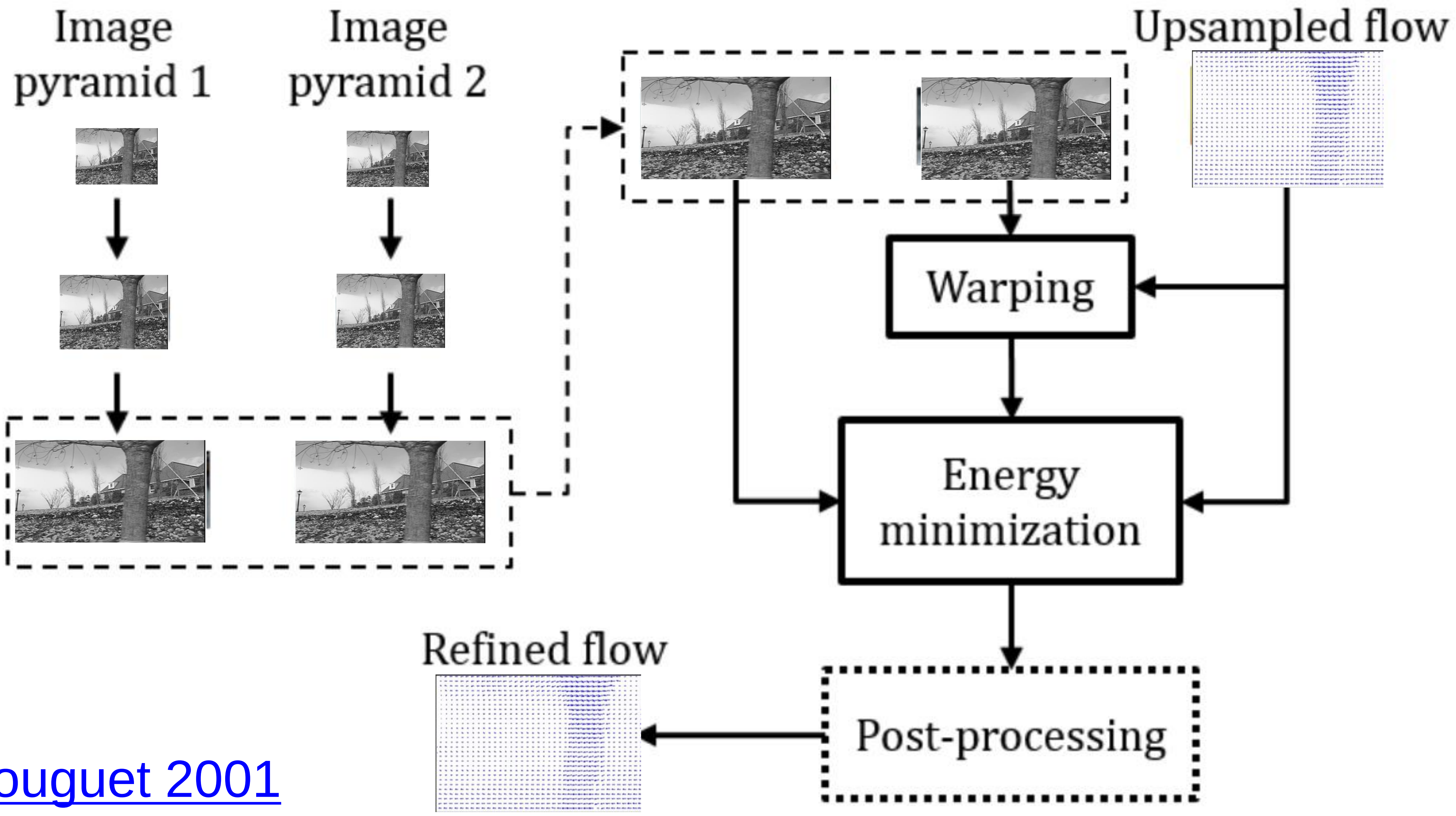
- Current best methods are learned (often on synthetic data)



A. Dosovitskiy et al., [FlowNet: Learning Optical Flow with Convolutional Networks](), ICCV 2015

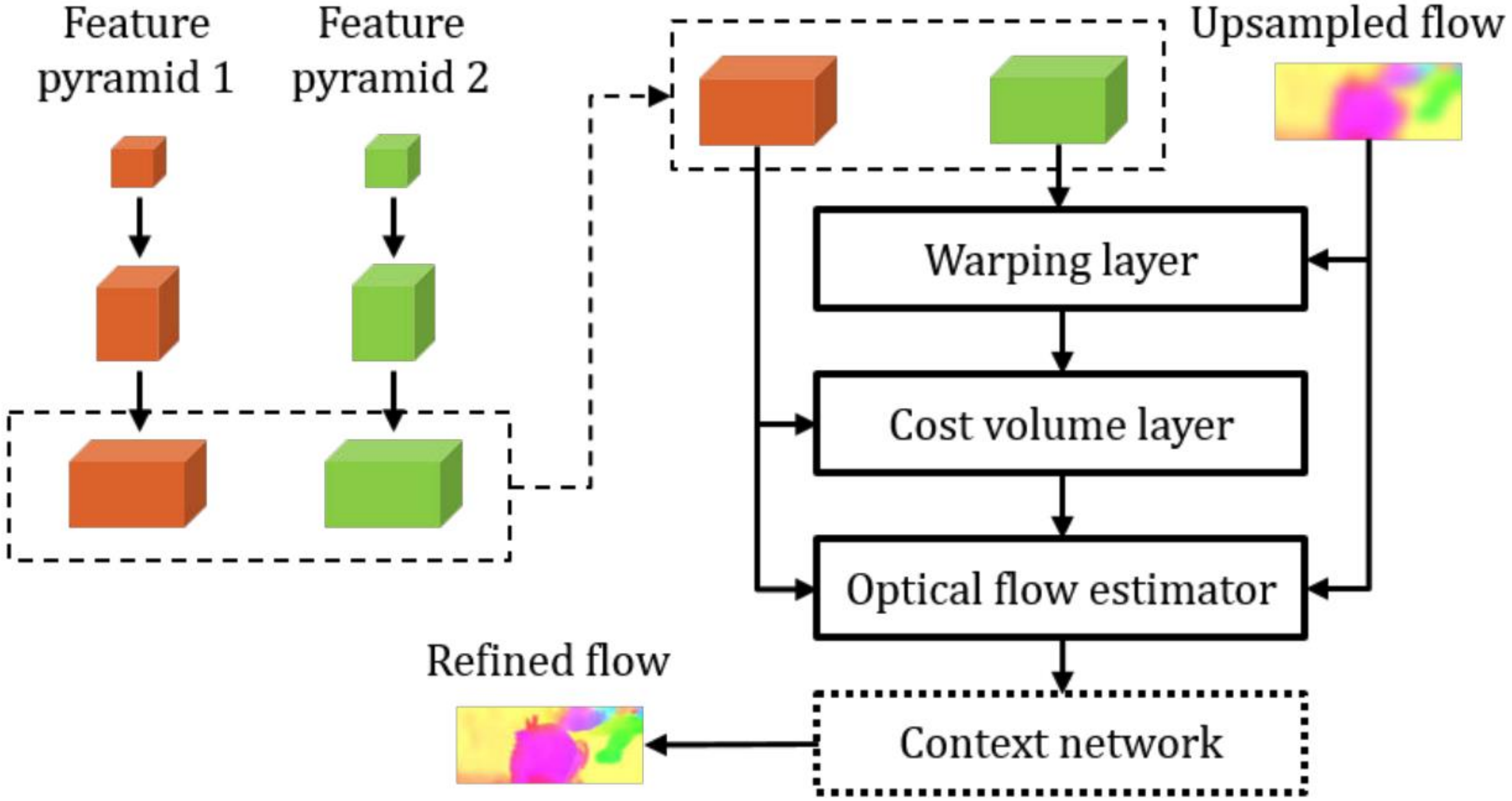
Flow CNN

Match CNN features instead of pixels!



Bouguet 2001

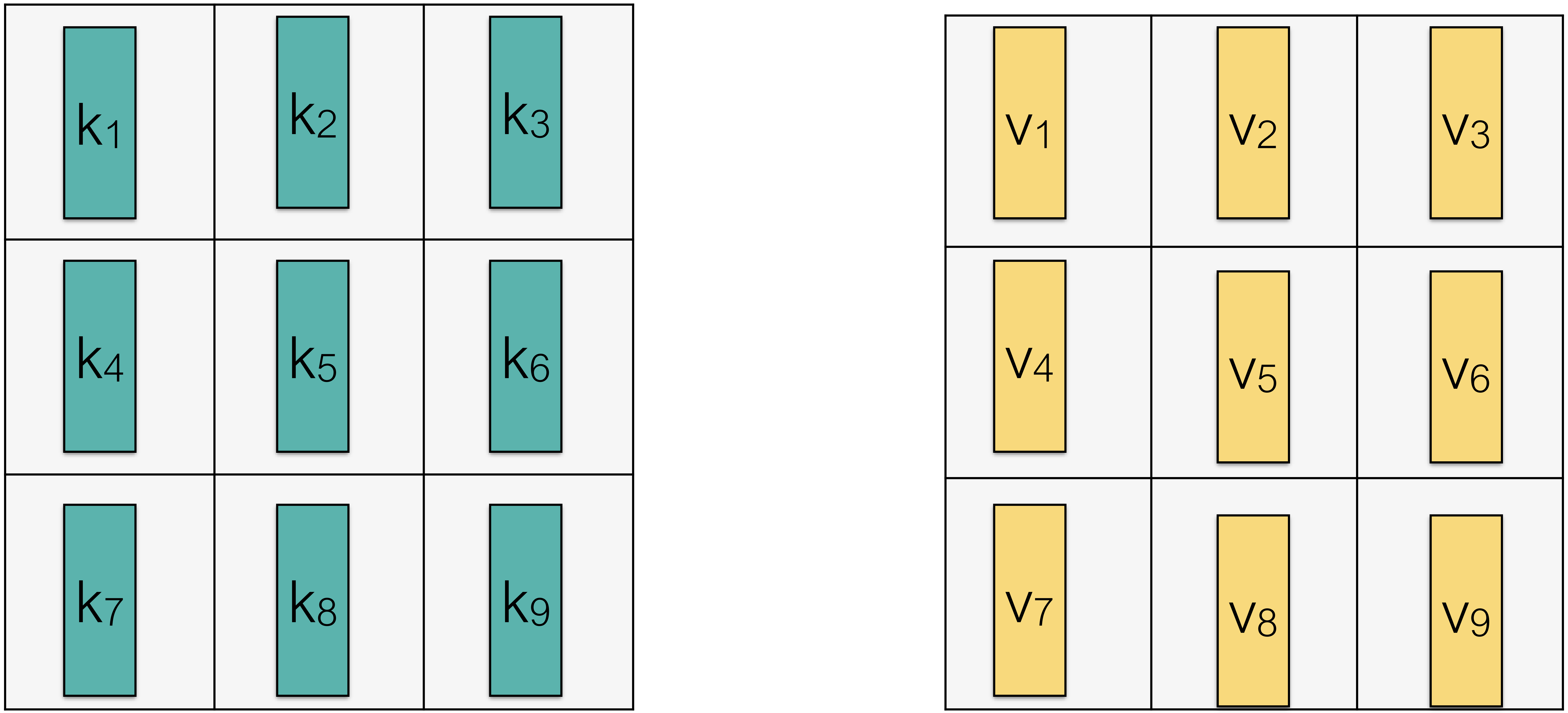
Traditional coarse-to-fine flow



PWC-net

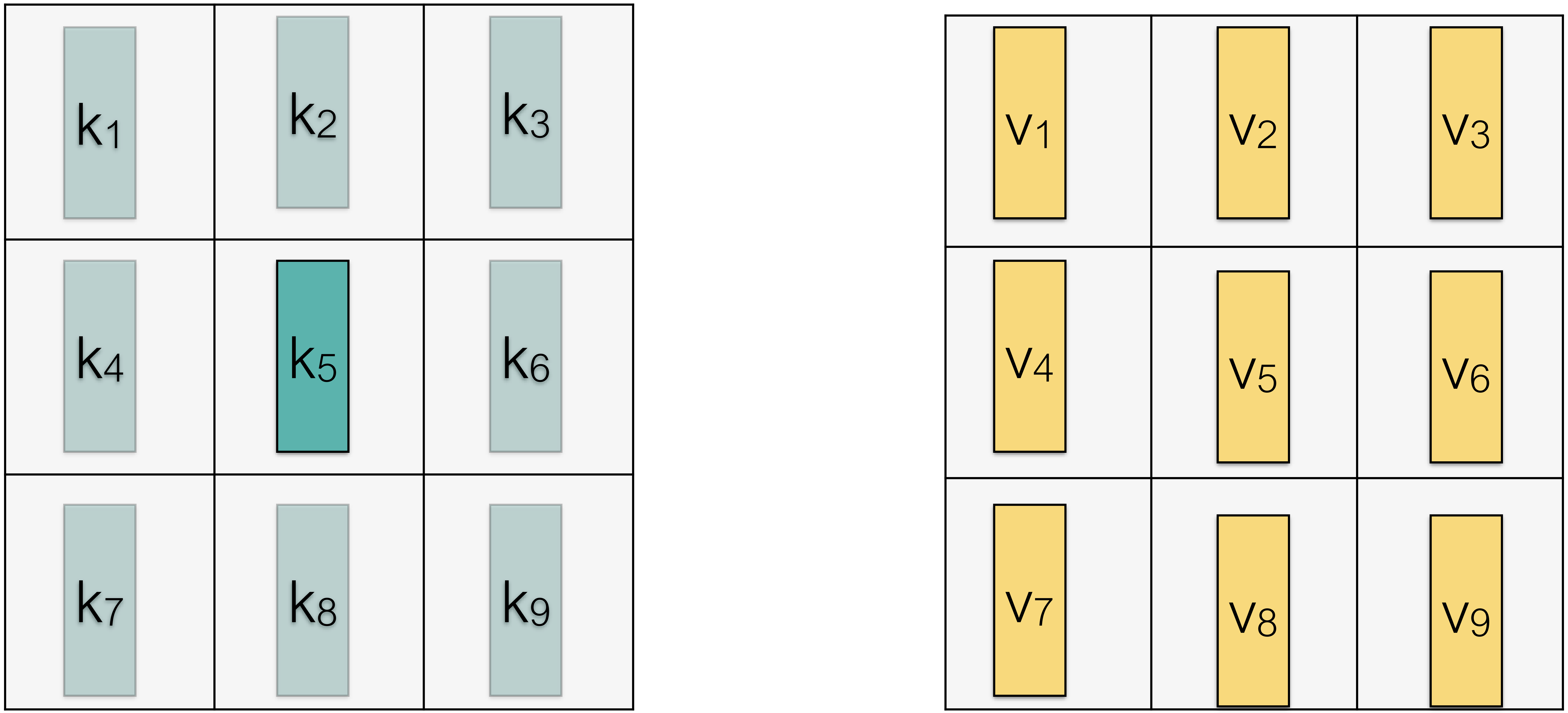
[Sun et al., "PWC-Net", 2018]

Correlation between CNN features



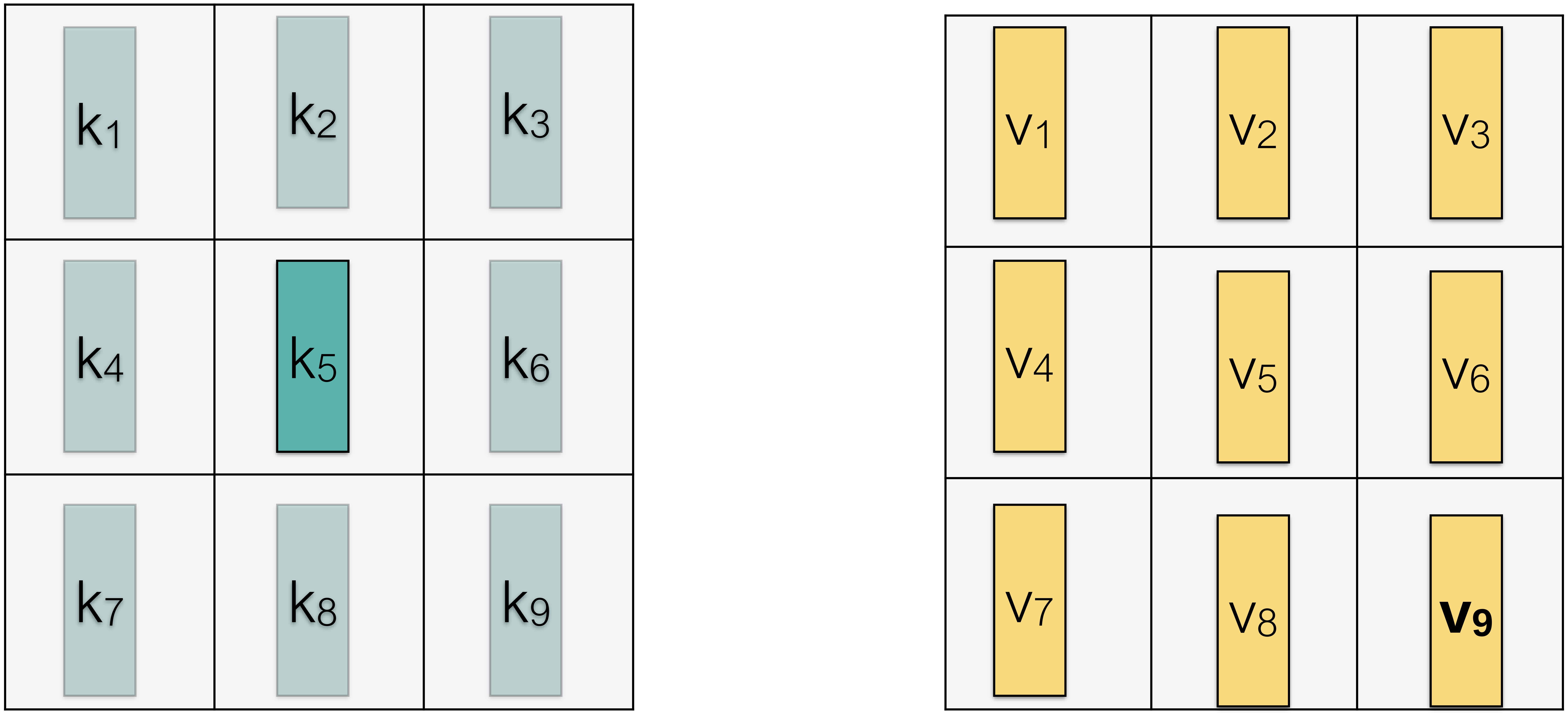
CNN feature map for I_1 CNN feature map for I_2

Correlation between CNN features



Take dot product between features and choose largest one.

Correlation between CNN features



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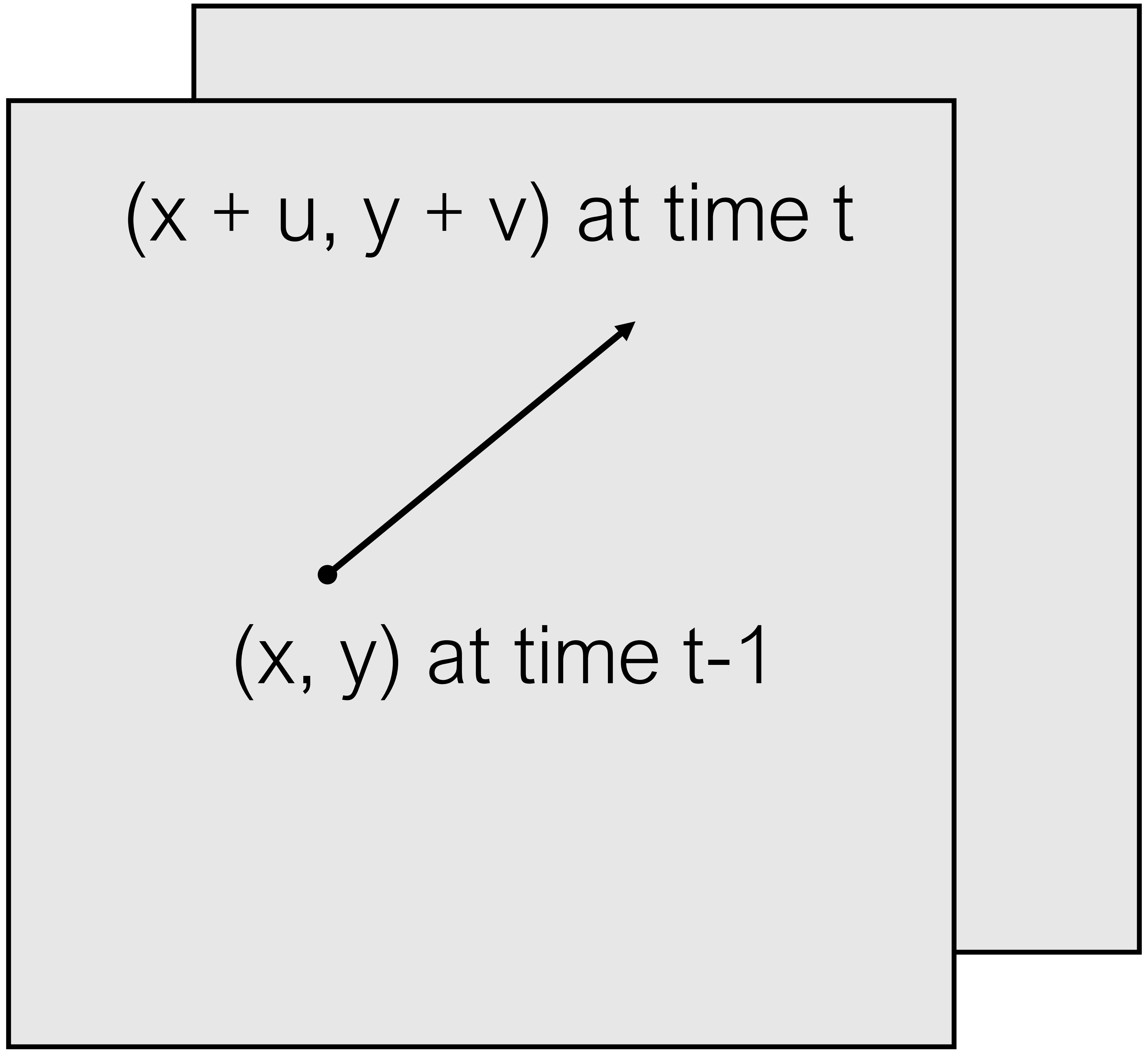
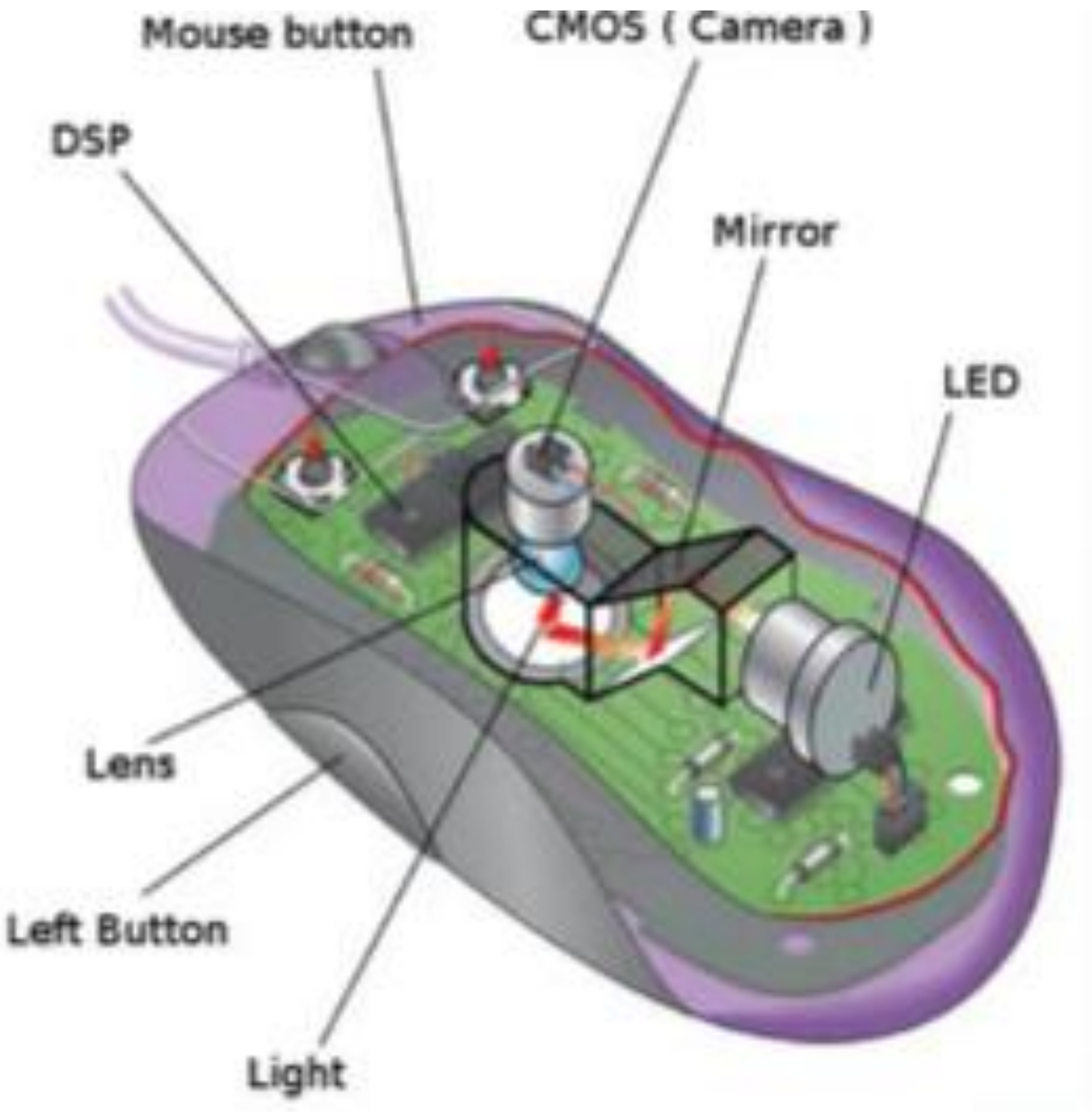
applications



- Flow used in lots of familiar places!
- e.g., optical mouse, traffic monitoring, video stabilization, …

application: optical mouse

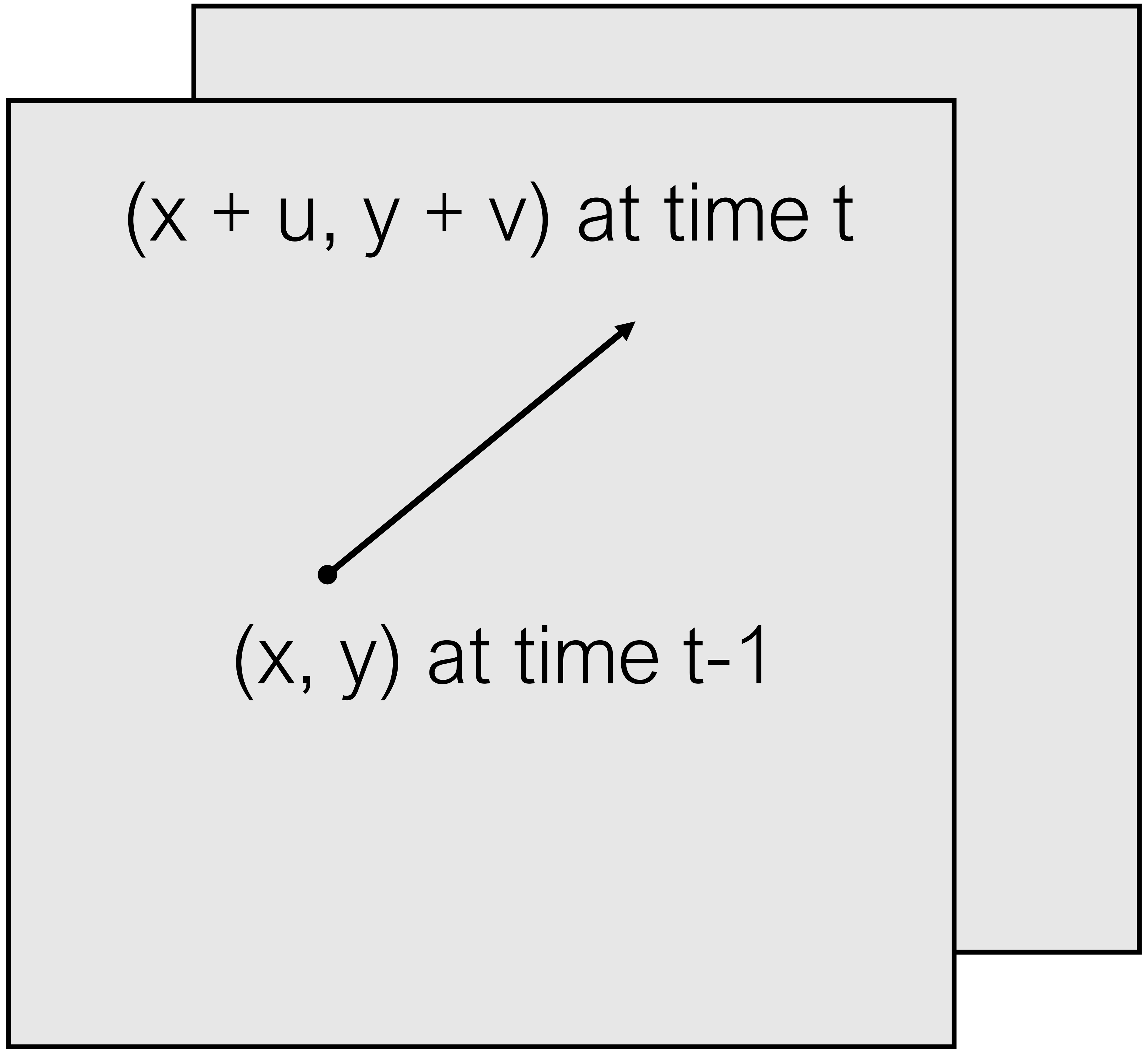


Source: Shree Nayar

application: traffic monitoring

application: video stabilization

Simple application: slow motion



- use flow to estimate where pixel will be *between* frames

- Synthesize intermediate frames

Super SloMo: High Quality Estimation of Multiple Intermediate Frames for Video Interpolation

Huaizu Jiang[1], Deqing Sun[2], Varun Jampani[2]
Ming-Hsuan Yang[3,2], Erik Learned-Miller[1], Jan Kautz[2]
[1]UMass Amherst [2]NVIDIA [3]UC Merced

(No audio commentary)

https://people.cs.umass.edu/~hzjiang/projects/superslomo/superslomo_public.mp4

Motion magnification

Idea: take flow, magnify it

C. Liu et al., Motion Magnification, SIGGRAPH 2005

Source: D. Fouhey and J. Johnson

Summary: Motion matters!

- Concepts: motion field, optical flow
- Classic algorithm: Lucas-Kanade algorithm
  - Assumptions?
  - Limitations?
- State-of-the-art algorithm: deep learning approaches
  - Data and annotations
  - network architectures inspired by the classic work
- Applications (without deep learning)