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VORTEX SHEDDING FROM OSCILLATING BLUFF BODIES

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1. INTRODUCTION

When placed in a fluid stream, some bodies generate separated flow over a substantial proportion of their surface and hence can be classified as bluff. On sharp-edged bluff bodies, separation is fixed at the salient edges, whereas on bluff bodies with continuous surface curvature the location of separation depends both on the shape of the body and the state of the boundary layer. At low Reynolds numbers, when separation first occurs, the flow around a bluff body remains stable, but as the Reynolds number is increased a critical value is reached beyond which instabilities develop. These instabilities can lead to organized unsteady wake motion, disorganized motion, or a combination of both. Regular vortex shedding, the subject of this article, is a dominant feature of two-dimensional bluff-body wakes and is present irrespective of whether the separating boundary layers are laminar or turbulent. It has been the subject of research for more than a century, and many hundreds of papers have been written. In recent years vortex shedding has been the topic of Euromech meetings reported on by Mair & Maull (1971) and Bearman & Graham (1980), and a comprehensive review has been undertaken by Berger & Wille (1972).

Vortex shedding and general wake turbulence induce fluctuating pressures on the surface of the generating bluff body, and if the body is flexible this can cause oscillations. Oscillations excited by vortex shedding are usually in a direction normal to that of the free stream, and amplitudes as large as 1.5 to 2 body diameters may be recorded. In addition to the generating body, any other bodies in its wake may be forced into oscillation. Broad-band force fluctuations, induced by turbulence produced in the flow around a bluff body, rarely lead to oscillations as severe as those caused by vortex shedding. Some form of aerodynamic instability, such that move-

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ments of a bluff body develop exciting forces in phase with the body's velocity, may also lead to large oscillation amplitudes. Galloping is one example of an instability whereby some bluff bodies can extract energy from a fluid stream and sustain oscillations. In this review, only vibrations caused by vortex shedding are considered; for a fuller discussion of flow-induced vibrations, the work of Blevins (1977) is recommended.

Excellent surveys of vortex-induced oscillations of bluff bodies have been written by Parkinson (1974) and Sarpkaya (1979). It is hoped that the present paper will complement these reviews and give an opportunity to reconsider some of the ideas presented in them in the light of more recent developments. The majority of previous experimental work in this area is related to vortex shedding from circular cylinders. Mathematical models developed to predict vortex-induced oscillations are also mainly concerned with this body shape. This preoccupation with the circular cylinder is justified on a number of arguments; the most persuasive are that it is an important structural form, and, as convincingly reasoned by Morkovin (1964), that it presents a challenging fundamental problem for fluid mechanicists. In the present review, however, vortex shedding from vibrating bluff bodies of various forms is considered in order to identify common features. We begin with a discussion of vortex shedding from fixed bluff bodies.

2. FIXED BLUFF BODIES

Although there is no complete solution to the problem of vortex shedding, we have a reasonably clear insight into the mechanism, and models are continuously being developed to describe it mathematically. Gerrard (1966) has given an extremely useful physical description of the mechanics of the vortex-formation region. A key factor in the formation of a vortex-street wake is the mutual interaction between the two separating shear layers. It is postulated by Gerrard (1966) that a vortex continues to grow, fed by circulation from its connected shear layer, until it is strong enough to draw the opposing shear layer across the near wake. The approach of oppositely signed vorticity, in sufficient concentration, cuts off further supply of circulation to the growing vortex, which is then shed and moves off downstream.

Gerrard's vortex-formation model is illustrated in Figure 1 by a sketch showing an instantaneous filament line pattern. Entrainment plays an important role in vortex formation, and Figure 1 indicates several entrainment processes. Entrained fluid (*a*) is engulfed into the growing vortex while (*b*) finds its way into the developing shear layer. The near-wake region between the base of the body and the growing vortex oscillates in

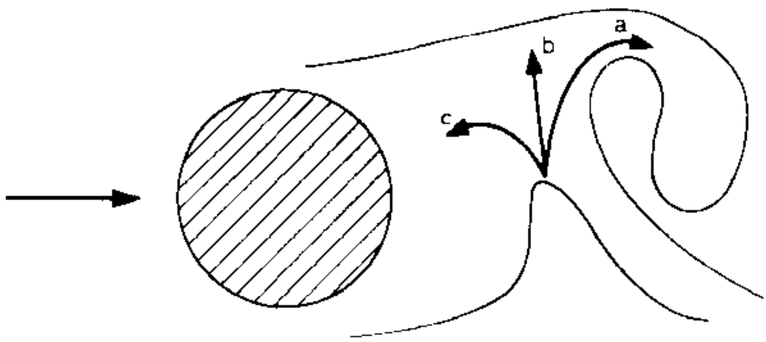


Figure 1 Vortex-formation model showing entrainment flows (Gerrard 1966).

size, and some further fluid, (*c*), is temporarily entrained into it. Entrained flow (*a*), which contains some fluid with oppositely signed vorticity to that in the growing vortex, is the largest of the three flows. The photograph in Figure 2 shows smoke filaments around a bluff body, and the interaction between one shear layer and the vortex forming on the opposite side of the wake is clearly seen.

Since vortex formation involves the mixing of flows of oppositely signed vorticity, the strengths of individual wake vortices will be less than the total circulation shed from one side of a bluff body during a shedding cycle. Fage & Johansen (1927) and Roshko (1954) estimated the rate of shedding of circulation from a bluff body by considering the mean base pressure. The rate of shedding of circulation at a separation point is given approximately by $d\Gamma/dt = \frac{1}{2}U_b^2$, where U_b is the mean velocity at the edge of the boundary layer at separation. Assuming that pressure remains constant through a shear layer, then $U_b^2 = U^2(1-C_{p_b})$, where U is free-stream velocity and C_{p_b} is the mean base pressure coefficient. Davies (1976) has considered the additional contribution to $d\Gamma/dt$ from unsteady velocities generated at the separation point, but concluded that it can be neglected. Hence, the fraction of the original circulation that survives vortex formation is given by the expression

$$\alpha = 2S\Gamma_v/UD(1-C_{p_b}), \tag{2.1}$$

where Γ_v is the strength of a wake vortex and S is the body Strouhal number ($S = nD/U$, where n is the vortex-shedding frequency and D is a characteristic dimension of the body, usually the body width).

In order to calculate α, an estimate must be made of the strength of the wake vortices Γ_v. Fage & Johansen (1927) and Roshko (1954) have found Γ_v by matching a measured vortex convection speed to a predicted value obtained from a suitable array of idealized point vortices. Fage & Johansen

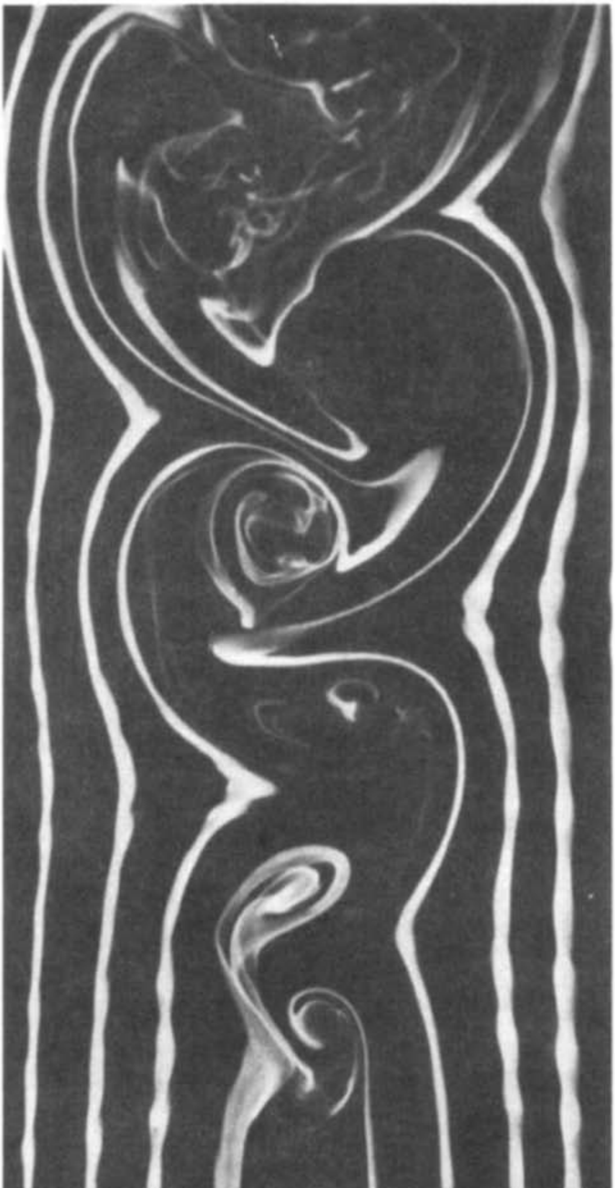


Figure 2 Visualization of the flow in a bluff-body wake.

found $\alpha = 0.6$ for a flat plate, and Roshko estimated $\alpha = 0.43$ as the average value for a circular cylinder, 90° wedge, and flat plate. More detailed analyses have been carried out by Bloor & Gerrard (1966) and Davies (1976), who used diffusing viscous vortices to represent the wake. Again by matching measured and calculated wake velocities, an estimate of Γ_v could be found. The resulting values of α are lower; for a circular cylinder Bloor & Gerrard found α to be between 0.2 and 0.3, depending on the Reynolds number, and for a D-shaped cylinder Davies estimated $\alpha = 0.26$.

The process of shear-layer interaction and circulation reduction is extremely well illustrated in the numerical calculations of Abernathy & Kronauer (1962). They represented two parallel shear layers of opposing signs by rows of point vortices. The rows were disturbed and calculation proceeded until clouds of vortices resembling a vortex street were formed. Point vortices of opposite sign mix within the clouds, and the value of α calculated from these numerical experiments is about 0.6. Although these calculations may not have been carried out with the utmost mathematical rigor, they represent a very important step forward in the understanding of the mechanism of vortex shedding. They show that it is the presence of two shear layers, rather than the bluff body itself, that is primarily responsible for vortex shedding. The presence of the body merely modifies the process by allowing feedback between the wake and the shedding of circulation at the separation points.

Abernathy & Kronauer's pioneering work led to the development of the discrete-vortex method for calculating vortex shedding from bluff bodies. Gerrard (1967) developed a similar method but included a circular cylinder in the flow field. Further improvements followed rapidly. A blunt-based body was studied by Clements (1973) and an inclined flat plate by Sarpkaya (1975). Circular-cylinder flow has been calculated by Deffenbaugh & Marshall (1976), Sarpkaya & Schoaff (1979a, b), Stansby (1981), and Stansby & Dixon (1982). Our intent here is not to review the achievements and shortcomings of the discrete-vortex technique, but merely to note that it has become an important method for modeling two-dimensional bluff-body flow.

Much of the experimental research into vortex shedding concerns the influence of disturbances. In addition to Reynolds number, bluff-body flows have been found to depend on factors such as body-surface finish, aspect ratio, end-plate design, flow turbulence, blockage ratio, and acoustic noise level. Another important finding has been that two-dimensional bodies in uniform flow are unlikely to shed two-dimensional vortices. The lack of two dimensionality can be assessed by investigating the spanwise variation of some unsteady quantity related to vortex shedding, such as fluctuating surface pressure, sectional lift force, or a fluctuating velocity just outside the shear layer at separation. The correlation between two points a distance z apart is given by

$$R(e, z) = \overline{e_1 e_2}/\overline{e^2}, \tag{2.2}$$

where e is the quantity under consideration and 1 and 2 are two stations a distance z apart. Since the mean flow is two dimensional, then $\sqrt{\overline{e_1^2}} = \sqrt{\overline{e_2^2}} = \sqrt{\overline{e^2}}$. The correlation length L is defined by the integral

$$L = \int_0^\infty R(e, z)\, dz. \tag{2.3}$$

In the range of Reynolds numbers between 10^4 and 10^5, L for a circular cylinder varies between about 3 and 6 cylinder diameters (see Sarpkaya 1979). Sharp-edged bodies generate vortices with correlation lengths of a similar order: for a square section, $L \approx 5.5D$ (Vickery 1966, Pocha 1971, Lee 1975, Bearman & Obasaju 1982). For a flat plate normal to a flow, Bearman & Trueman (1972) have found $L = 10D$. From these measurements it can be deduced that although the mutual interaction between two shear layers that leads to the generation of vortex shedding may be very strong, the spanwise coupling is comparatively weak.

As the aspect ratio of a constant cross-section bluff body is reduced, vortex shedding becomes weaker, although it never seems to completely disappear. Fail et al. (1959) detected vortex shedding behind flat plates of varying aspect ratio down to an aspect ratio of unity. Vortex shedding behind spheres has been observed by Achenbach (1974) and behind circular discs and cones by Calvert (1967). On low-aspect-ratio bluff bodies, the forces generated by vortex shedding are too weak to cause any significant oscillations. Wootton (1969) has observed, during wind-tunnel tests on model chimney stacks of circular cross section, that significant vortex-induced oscillations are unlikely to occur for chimney height to diameter ratios less than 5 or 6.

Comparatively small departures from strictly two-dimensional conditions can have a large influence on vortex shedding. The effect of mild taper on vortex shedding from a body of circular cross section has been studied by Gaster (1969), and the effect of mean shear on a bluff body with fixed separation has been investigated by Maull & Young (1973). In both cases the Strouhal number, based on local diameter or local velocity as appropriate, tends to the value for a cylindrical body in a uniform flow. Maull & Young found that in a shear flow the shedding occurred at a fixed frequency within cells extending a few body diameters. In both cases the vortex-shedding correlation length is greatly reduced, compared with its value in the equivalent two-dimensional problem, because of the spanwise variation in the shedding frequency.

Regular vortex shedding can be seriously disrupted if the flow separation line is not straight. Tanner (1972) has investigated the effect of various trailing-edge geometries on the drag of blunt-trailing-edge wings. He found that vortex shedding could be suppressed and base drag reduced if the trailing edge was stepped so as to break the straight separation line. Naumann et al. (1966) fixed separation on a circular cylinder by a series of staggered wires and observed no regular vortex shedding. Another example of the suppression of shedding by nonstraight separation lines is the case of a circular cylinder in the upper transition regime ($8.5 \times 10^5 < Re < 3.5 \times 10^6$) between supercritical and postcritical flow. [The nomenclature used here is identical to that suggested by Roshko (1961), except that postcritical is used in place of transcritical.] In supercritical flow the transition from laminar to turbulent flow occurs within a separation bubble, and the separation of the resulting turbulent boundary layer takes place well around the cylinder. Within the upper transition regime the bubbles break down at random points across the span, and a straight separation line is lost. At higher Reynolds numbers transition occurs within the attached boundary layer, straight separation lines are reestablished, and regular vortex shedding returns.

3. OSCILLATING BLUFF BODIES

When a bluff body is oscillating in a fluid stream, or when it is exposed to a stream with an imposed oscillation, vortex shedding can be dramatically altered. The oscillations provide a means for coupling the flow along the span of the body, and this usually results in a large increase in the correlation length. Closer inspection of the flow reveals that the body motion can take control of the instability mechanism that leads to vortex shedding. Hence, the flow generated by vortex shedding around a vibrating bluff body can have very significant differences from that around a fixed one. Controlling oscillations may be generated by the body or the flow, but usually oscillations develop from an interaction between vortex shedding and a flexibly mounted bluff body. In laboratory experiments, oscillations of bluff bodies may be vortex-induced or forced by a machine capable of generating a controlled vibratory motion. Similar interaction effects may be produced by vibrating the flow, and in a series of experiments on square-section cylinders Pocha (1971) used a gust tunnel to produce a transversely oscillating flow. Acoustic noise can develop correlated oscillatory particle velocities in the flow around a bluff body, which may lead to modifications of the vortex-shedding process that are similar to those produced by body motion. The interaction between vortex shedding and sound has been investigated by Parker (1966, 1967), Gaster (1971), Archibald (1975), and Welsh & Gibson (1979).

Practical examples of vortex-induced oscillations, such as those discussed by Naudascher & Rockwell (1980), invariably involve more complicated structures and flow environments than those treated in fundamental investigations. Wind-excited oscillations of towers and chimney stacks, for example (see Scruton 1963), are affected by the shear and turbulence in the approaching flow. Real structures have a number of possible modes of vibration, and this is an important factor to be considered when determining the vortex-induced response of long slender bluff bodies such as cables (Ramberg & Griffin 1976, Peltzer 1982). An important application area for bluff-body research is in the field of marine technology, where bodies are exposed to waves, currents, or a combination of both (Griffin 1980, Griffin & Ramberg 1982). Even in the absence of a mean velocity, when the flow is purely oscillatory, vortex shedding can still be the dominant mechanism (Bearman et al. 1981). Progress on these practical engineering problems has been made possible through the understanding gained from fundamental studies.

3.1 *Oscillations of a Flexibly Mounted Bluff Body*

Vortex shedding most commonly induces oscillations in a direction transverse to that of the stream at flow speeds where the frequency of shedding coincides with the frequency of oscillation of the body. Maximum oscillation amplitudes occur over a range of the reduced velocity U/ND, where N is the body oscillation frequency. If the vortex-shedding frequency from one side of a bluff body is n, then the reduced velocity for maximum amplitude is close to $1/S$, where S is given by nD/U. It is significant that it is not precisely equal to the inverse of the Strouhal number.

Each time a vortex is shed, a weak fluctuating drag is generated, and oscillations can be induced in-line with a fluid flow. This occurs at a reduced velocity equal to approximately $1/2S$. Another form of wake instability has been observed for circular cylinders in a range of reduced velocities less than $1/2S$. Vortices are shed in symmetric pairs and only form into the familiar staggered arrangement some distance downstream of the cylinder. This symmetric shedding is induced by the motion of the body and gives rise to a force in phase with the body velocity. In-line oscillations caused by this effect have not been observed for bodies in air; but in denser fluids such as water, where the ratio of the mass of fluid displaced to the mass of the body is substantially greater, serious oscillations can occur. In-line oscillations of this type on full-scale marine piles have been reported by Wootton et al. (1972). Comparisons between model experiments and full scale have been carried out by Hardwick & Wootton (1973) and King (1974).

A comparison carried out by King (1974) of the in-line oscillations of a model cantilevered circular cylinder at a Reynolds number of 6×10^4 and a

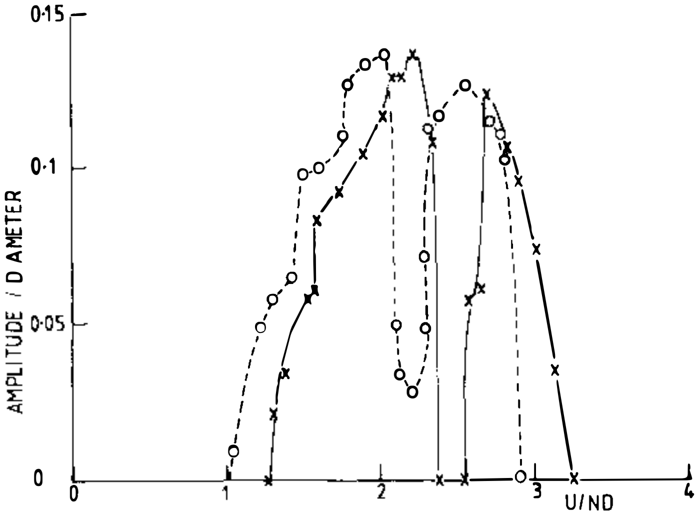


Figure 3 Vortex-excited in-line oscillations of a flexible cantilevered circular cylinder in water. ○, Re = 6×10^5; ×, Re = 6×10^4: in each case $2M\delta_s/\rho D^2 = 0.048$ (King 1974).

full-scale pile at a Reynolds number of 6×10^5 is shown in Figure 3. The two regions of instability caused by the two different shedding patterns are clearly visible. The maximum amplitudes of oscillation were less than $0.15D$ and hence much smaller than those experienced in vortex-excited cross-flow vibrations. Insufficient fundamental research has been carried out to fully explain the origin of the exciting force in the symmetric shedding regime. Hardwick & Wootton (1973) observed oscillations at Reynolds numbers of 6×10^3, but Griffin & Ramberg (1982) report that there is a lower limit of about 1.2×10^3 below which in-line oscillations are not excited in water. Interesting experiments have been carried out by Tanida et al. (1973), who forced a cylinder to oscillate in-line with a flow at reduced velocities in the range under consideration. They oscillated a circular cylinder at a fixed amplitude of $0.14D$ at a Reynolds number of 4000. No positive exciting force was found, although the fluid damping force generated by the motion of the cylinder was considerably less than that calculated assuming quasi-steady flow and using the measured drag coefficient. It is intriguing to ask whether or not an exciting force would have been measured if the oscillation amplitude had been reduced.

3.2 *Equation of Motion for a Flexibly Mounted Bluff Body*

Consider a rigid two-dimensional bluff body placed normal to a flow of velocity U and mounted on springs. The equation of motion for the body can be written as

$$M\ddot{y}+\beta\dot{y}+Ky = F(t), \tag{3.1}$$

where y is the displacement of the body, M the mass per unit span, $\beta\dot{y}$ a viscous-type damping force associated with the springs and their mounting, K the stiffness of the springs, and $F(t)$ the time-dependent fluid force. It is customary to replace β by $4\pi\delta_s MN_0$, where δ_s is the fraction of critical damping, and to let $(K/M)^{1/2}/2\pi = N_0$, the undamped natural frequency of the body. The fluctuating fluid force on the body can be expressed in terms of a coefficient C_y, where $C_y = F(t)/\frac{1}{2}\rho U^2 D$. This coefficient accounts for the component in the y direction of the total instantaneous fluid force acting on a cross section of the oscillating body and embodies elements that could be considered as fluid inertia and damping forces. In order to understand the loading caused by the complex mechanism of vortex shedding, it is more instructive to avoid any division at this stage and to keep all fluid forces on the right-hand side of Equation (3.1). Hence, Equation (3.1) can be rewritten

$$M\ddot{y}+4\pi N_0\delta_s M\dot{y}+4\pi^2 N_0^2 My = C_y\rho U^2 D/2. \tag{3.2}$$

Equation (3.2) is a general equation that can be used to describe the response in the transverse or in-line directions due to approaching turbulence, galloping, or vortex shedding. Much of the research into bluff-body flows has been aimed at finding a suitable form for the appropriate fluid-loading coefficient C_y. In this discussion y is assumed to be the displacement in the transverse direction and C_y the transverse force coefficient on a bluff body shedding vortices. For large-amplitude, steady-state, vortex-induced oscillations, the fluid force and the body displacement response oscillate at the same frequency n_v, which is usually close to N_0. When a bluff body is responding to vortex shedding, the fluid force must lead the excitation by some phase angle ϕ. Hence, the displacement y and the fluid force C_y can be represented by the expressions

$$y = \bar{y}\sin 2\pi n_v t,$$

$$C_y = \bar{C}_y \sin(2\pi n_v t+\phi). \tag{3.3}$$

Substitution of these forms in Equation (3.2) and equating sine and cosine terms lead to the following relationships:

$$\frac{N_0}{n_v} = \left[1-\frac{\bar{C}_y}{4\pi^2}\cos\phi\left(\frac{\rho D^2}{2M}\right)\left(\frac{U}{N_0 D}\right)^2\left(\frac{\bar{y}}{D}\right)^{-1}\right]^{-1/2}, \tag{3.4}$$

$$\frac{\bar{y}}{D} = \frac{\bar{C}_y}{8\pi^2}\sin\phi\left(\frac{\rho D^2}{2M\delta_s}\right)\left(\frac{U}{N_0 D}\right)^2\frac{N_0}{n_v}. \tag{3.5}$$

By considering the order of magnitude of the various terms in Equation (3.4) (see Parkinson 1974), it can be shown that for large-amplitude oscillations of a body in air, where $\rho D^2/2M$ might be typically of order 10^{-3}, the frequency of body oscillations should be close to its natural frequency. This is confirmed by experiments. In a denser fluid such as water, where $\rho D^2/2M$ may be of order unity, the frequency of oscillation of the body can be appreciably different from its natural frequency. The steady-state response amplitude of a bluff body to vortex shedding is given by Equation (3.5). If $\rho D^2/2M$ is small such that $n_v \approx N_0$, then Equation (3.5) can be replaced by

$$\frac{\bar{y}}{D} = \frac{\bar{C}_y}{8\pi^2} \sin \phi \left(\frac{\rho D^2}{2M\delta_s}\right)\left(\frac{U}{N_0 D}\right)^2. \tag{3.6}$$

It is clear from Equations (3.4), (3.5), and (3.6) that the phase angle ϕ plays an extremely important role. The response amplitude does not depend on C_y alone but on that part of C_y in phase with the body velocity. Hence, measurements of the sectional fluctuating transverse force coefficients (hereafter referred to as the lift coefficients) on a range of stationary bluff-body shapes will give little indication of the likely amplitudes of motion of similar bodies flexibly mounted.

3.3 *Free and Forced Vibrations*

Free-vibration experiments on circular cylinders in air and water have been compared by Griffin & Ramberg (1982). Plots of oscillation amplitude versus reduced velocity are reproduced in Figure 4, where it can be seen that for similar values of the mass-damping parameter, $2M\delta_s/\rho D^2$, the maximum oscillation amplitudes are nearly the same. Two further points should be noted in Figure 4. First, the maximum responses in the two cases occur at similar values of reduced velocity. In the water experiment the reduced velocity has been formed using the oscillation frequency measured in still water. Second, away from the maximum response the amplitudes recorded are quite different in the two cases. The more lightly damped cylinder used in the air experiment responds over a narrow band of reduced velocity, whereas the more highly damped cylinder in water responds over a much broader range.

Although in the case reported by Griffin & Ramberg similar values of the mass-damping parameter lead to similar maximum responses, this does not necessarily always occur. Sarpkaya (1978) has demonstrated that for low values of the mass-damping parameter the nondimensional mass and damping affect the response separately. This can be expected from Equations (3.4) and (3.5), where low values of $2M/\rho D^2$ influence the response frequency and hence the oscillation amplitude. Griffin &

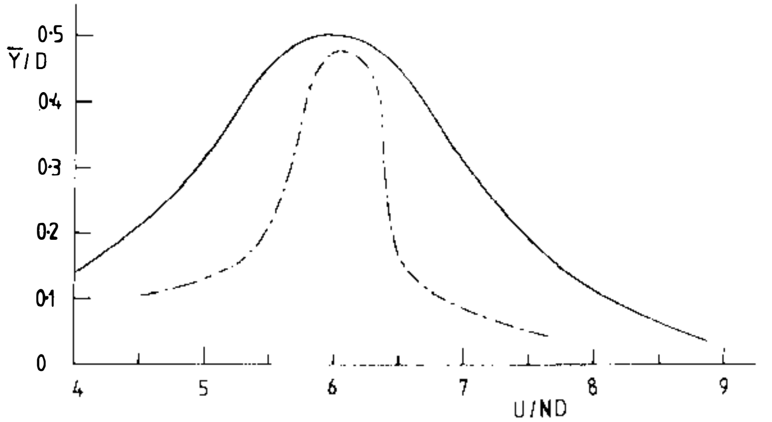


Figure 4 Cross-flow amplitude versus reduced velocity for a circular cylinder. ———, water, $2M/\rho D^2 = 7.6$, $\delta_s = 5.1 \times 10^{-2}$, $2M\delta_s/\rho D^2 = 0.39$; —·—·—·, air, $2M/\rho D^2 = 68$, $\delta_s = 4.3 \times 10^{-3}$, $2M\delta_s/\rho D^2 = 0.29$ (Griffin & Ramberg 1982).

Koopmann (1977) and Sarpkaya (1978) have suggested the use of a response parameter $S_g = 8\pi^2 M\delta_s S^2/\rho D^2$. Sarpkaya (1978) has concluded from a parametric study of the separate and combined effects of the structural damping, the nondimensional mass, and the response parameter that the maximum response of a cylinder is governed by the response parameter alone only for values of S_g greater than about unity.

If the value of $\rho D^2/2M$ is sufficiently small so that the response can be predicted using Equation (3.6), and if it is assumed that the reduced velocity for maximum response is equal to the inverse of the Strouhal number, then $\bar{y}/D = \bar{C}_y \sin\phi/2S_g$. Using such a relationship, the amplitudes of oscillation of freely vibrating bluff bodies can be used to estimate the excitation $\bar{C}_y \sin\phi$, the force coefficient out of phase with the displacement. Griffin & Koopmann (1977) have gathered together estimates of $\bar{C}_y \sin\phi$ for a circular cylinder from various experiments, and these have been plotted in Figure 5 against a scaled amplitude ratio $2\bar{y}I/D$. The parameter I is the modal response factor and has been used in an attempt to correlate data from cantilevered and pivoted cylinders and cylinders moving in parallel motion. For cylinders in parallel motion, $I = 1$. The indications from Figure 5 are that the excitation increases in the range of amplitudes up to about $0.5D$ and then decreases such that there is no excitation beyond an amplitude of roughly $1.5D$.

One of the most interesting, as well as detailed, investigations into the response of freely vibrating cylinders has been carried out by Feng (1968). His best known results, those on a lightly damped circular cylinder, are shown in Figure 6. He discovered that higher amplitudes were achieved

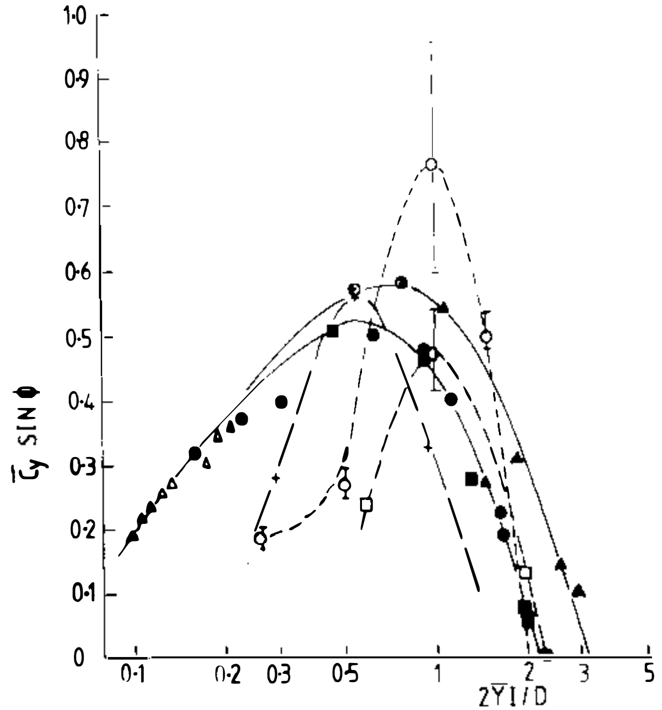


Figure 5 A plot of $\bar{C}_y \sin \phi$ versus double amplitude. Circular-cylinder data gathered together by Griffin & Koopmann (1977) and Griffin (1980). The legend for the data points is given in Griffin (1980).

when reduced velocity was increased over a certain range than when it was decreased back over the same range. This hysteresis effect has since been the subject of much discussion, but it has still to be fully explained. Feng also measured the phase angle ϕ, and it can be seen from Figure 6 that this varies with reduced velocity.

Experiments such as those of Feng (1968) demonstrate that the flow around a freely vibrating bluff body can change very rapidly with changes in reduced velocity. Since varying the reduced velocity also changes the amplitude ratio, it can be very difficult to determine the relative importance of these two effects on the resulting flow field. Hence a number of investigators have chosen to carry out forced oscillation experiments in which the amplitude and frequency of the motion can be varied at will. Each method has its advantages and disadvantages, but the forced-vibration technique has the important benefit that conditions can be closely controlled. However, due to the interaction with the structural parameters,

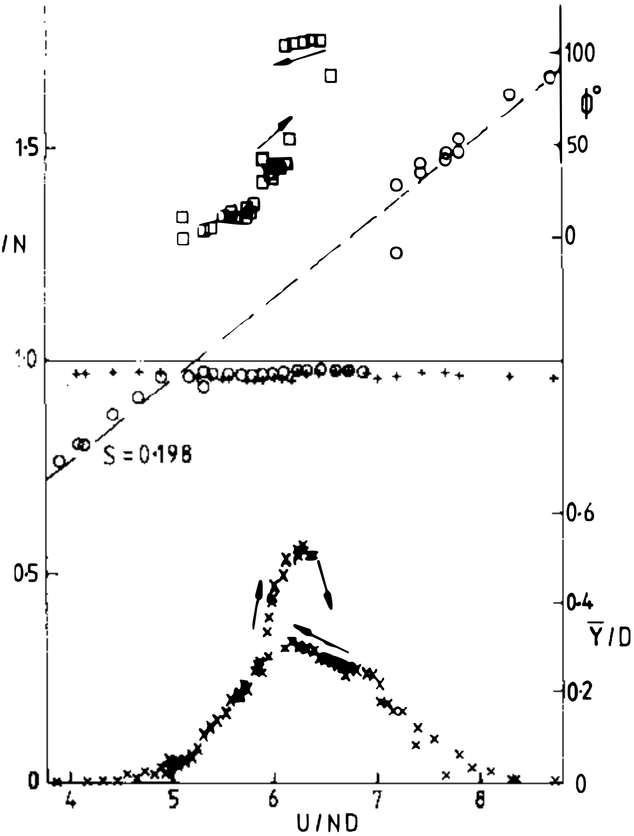


Figure 6 Oscillation characteristics for a freely vibrating circular cylinder with $2M\delta_s/\rho D^2 = 0.4$. ○, vortex-shedding frequency; +, cylinder frequency; □, phase angle; ×, oscillation amplitude (Feng 1968).

only a very limited range of the reduced velocities and amplitude ratios studied in a forced-vibration experiment is likely to be important in a freely vibrating one. For a freely suspended bluff body oscillating at a steady amplitude, it can be assumed that if the same body is forced to oscillate at a similar amplitude ratio, reduced velocity, and Reynolds number, then the flow patterns will be identical. This bold statment presumes that the precise previous history of the motion is unimportant. The available experimental evidence suggests that free and forced-vibration flows are the same.

3.4 *The Influence of Body Motion on Vortex Shedding*

Research into oscillating bluff-body flows has identified a number of significant changes that occur in the vortex-shedding behavior. Perhaps the two best-known effects are the capture of the vortex-shedding frequency by the body frequency over a range of reduced velocity, and the large increase in correlation length that occurs when the vortex-shedding frequency coincides with the body frequency. The movement of the body provides a means of synchronizing the moment of shedding along its length. The threshold oscillation amplitude required to cause large increases in the correlation length depends on the shape of the bluff body under consideration. For a circular cylinder the critical value of $\bar{y}/D$ is about 0.05, but for bodies with sharp-edged separation it may be significantly lower. Figure 7 shows an example of the increase of spanwise correlation of the flow along a square-section cylinder when it is forced into oscillation at an amplitude of $\bar{y}/D = 0.1$ and at a reduced velocity where the vortex and body frequencies coincide (Bearman & Obasaju 1982). The correlation coefficient is formed here by measuring the fluctuating pressures at two points on a side face of the cylinder a distance z apart. If the fluctuating pressure signals had been filtered so as to pass only the parts at the vortex-shedding frequency, then the correlation would have been even closer to unity across the span. At high Reynolds numbers, small-scale fluctuations caused by turbulence are unlikely to be much affected by body movement.

The "range of capture" over which the vortex-shedding frequency is locked to the body frequency is dependent on oscillation amplitude—the larger the amplitude, the larger the range of capture. In the results of Feng

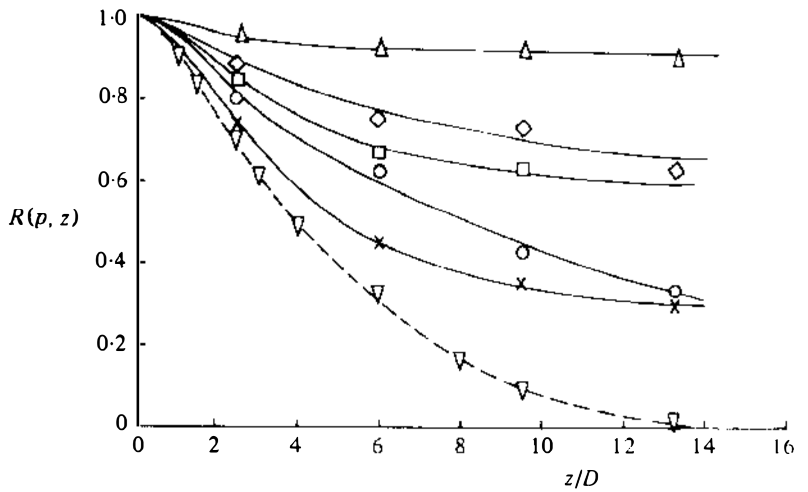


Figure 7 Correlation of surface pressures, measured on a side face of a square-section cylinder, versus spanwise separation. ▽, body stationary. Body oscillating with $\bar{y}/D = 0.1$: △, U/ND within the lock-in range 7.3–8.5; ×, $U/ND = 6.2$; ○, 7.0; ◇, 8.8; □, 12.0 (Bearman & Obasaju 1982).

(1968), plotted in Figure 6, the range of capture is seen to extend from $U/ND = 5$ to 7.4, but in his experiment the circular cylinder is vibrating freely and hence the amplitude also varies over this range of reduced velocity. In forced-vibration experiments, where the amplitude is held constant, it is observed that the extent of the range of capture depends on the amplitude level. The range of capture always encompasses the reduced velocity, which is equal to the inverse of the Strouhal number measured for the body when stationary. This reduced velocity is referred to as the resonant point. The location of the range of capture, relative to the resonant point, depends very much on the shape of the bluff body. Feng (1968) has measured the vortex-shedding frequency behind a freely vibrating circular cylinder and a D-shaped cylinder. In the case of the circular cylinder the range of capture begins at the resonant point and extends to higher values of reduced velocity. The D-section showed a different behavior, with the high-velocity end of the range of capture just about coinciding with the resonant point.

The range of capture has also been measured in forced-vibration experiments for a variety of bluff-body shapes, but no clear pattern emerges and it must be concluded that the behavior of vortex shedding in this range depends very much on the body shape. For a circular cylinder the range of capture begins at the resonant point (Bearman & Currie 1979); for a square section (Bearman & Obasaju 1982), D-section, and flat plate the range of capture straddles the resonant point; and for a triangular section with one vertex pointing downstream the range of capture ends at the resonant point (Bearman & Davies 1977). The reasons for these variations are not understood.

The range of capture, or lock-in, is a small interval of reduced velocity over which flow conditions about a bluff body change rapidly with reduced velocity. The time period between the shedding of vortices remains constant, and hence the Strouhal number, which is equal to the inverse of the reduced velocity, changes. Rearranging Equation (2.1), the nondimensional strength of a vortex in the wake is given by

$$\Gamma_v/UD = \alpha(1 - C_{p_b})U/2ND. \tag{3.7}$$

If it were supposed that the base-pressure coefficient and the circulation fraction α remain unchanged through lock-in, then Equation (3.7) indicates that the strength of the wake vortices should increase with increasing reduced velocity. This simple analysis also shows that amplification of the vortex strength, compared with its stationary-body value, will occur if the lock-in extends to reduced velocities above the resonant point. Support for the assumption that α is little affected by body motion comes from Davies (1976), who shows for a D-section cylinder that $\alpha = 0.26$ when it is stationary and $\alpha = 0.24$ at lock-in with $\bar{y}/D = 0.2$. It cannot be assumed with any certainty, however, that other bluff-body shapes will behave in a similar way.

In the vortex-formation region there is a feedback between the rolling up of the shear layers into vortices and the flow around the body, such that a small increase in vortex strength can lead to an increase in base suction and drag and hence an increase in shed vorticity at the separation points. Such a situation is clearly unstable, because the extra vorticity can lead to even stronger vortices; however, it is presumed that entrainment acts to control the near-wake flow. Hence, interfering with the vortex-shedding instability mechanism may produce large changes in vortex strength, base pressure, drag, and sectional fluctuating lift coefficient. The results of Davies (1976) for a D-section cylinder show within the lock-in range, for $\bar{y}/D = 0.2$, an increase in Γ_v/UD of 36% and a decrease in C_{p_b} of 71%, compared with the fixed-body values.

Measurements showing an increase in C_D on an oscillating circular cylinder through lock-in have been reported by Honji & Taneda (1968), Sedrak (1971), Tanida et al. (1973), and Sarpkaya (1978). Base-pressure measurements on a circular cylinder made by Stansby (1976) are shown in Figure 8, where $C_{p_b}/C_{p_{bo}}$ (the ratio of the minimum value of the base

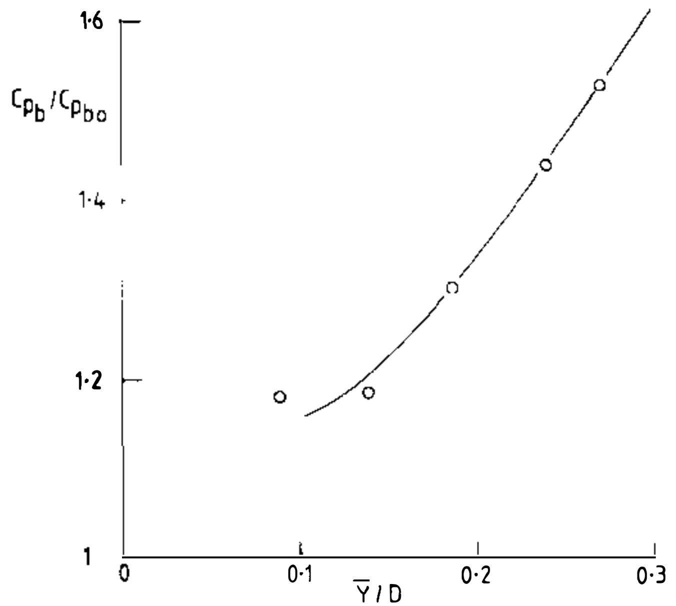


Figure 8 Ratio of minimum base-pressure coefficient on a circular cylinder at lock-in to value for a stationary cylinder versus oscillation amplitude (Stansby 1976).

pressure coefficient at lock-in to the stationary-cylinder value) is plotted against $\bar{y}/D$. Bearman & Davies (1977) have reported base-pressure measurements on a flat plate, D-shape, and triangular section with a vertex pointing downstream. The D-section and the flat plate show decreases in base pressure at lock-in. Results for the triangular section and a square section investigated by Bearman & Obasaju (1982) are plotted in Figure 9 and are seen to show a similar behavior with no amplification in base suction. At reduced velocities below lock-in, where the sections are being forced to oscillate at frequencies higher than their natural shedding frequencies, the base-suction coefficients, and hence the drag coefficients, are dramatically reduced. These results show that the shape of the afterbody, the region of a bluff body downstream of its separation points, plays an extremely important role in determining the response of the flow to body movements.

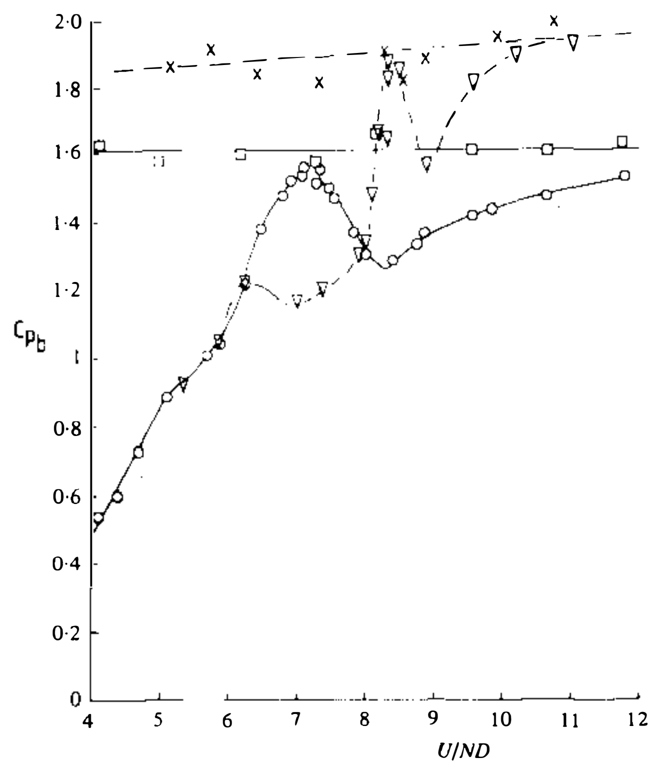


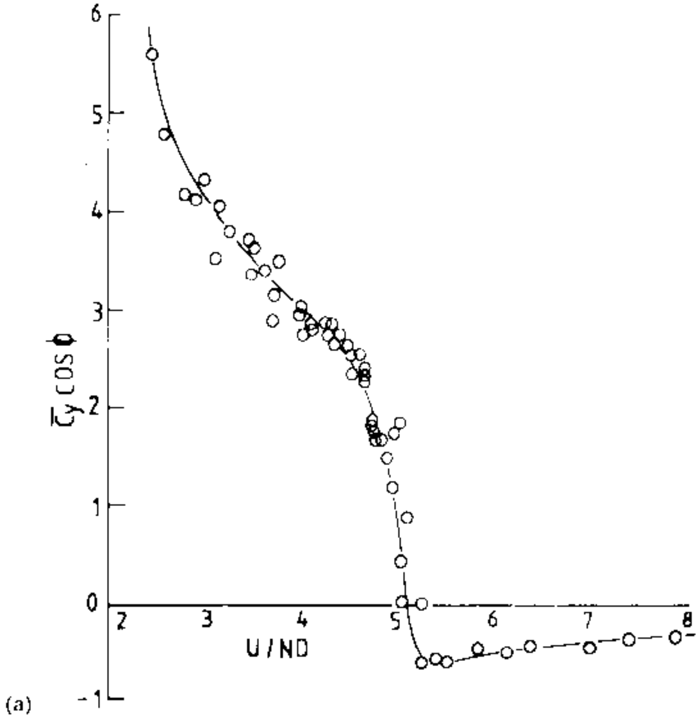
Figure 9 Base-pressure coefficient versus reduced velocity. Square section: □, stationary; ○, oscillating, $\bar{y}/D = 0.1$ (Bearman & Obasaju 1982). Triangular section: ×, stationary; ▽, oscillating, $\bar{y}/D = 0.2$ (Bearman & Davies 1977).

Within the lock-in range, all the available results show an increase in the local fluctuating lift coefficient. The improved two dimensionality of the flow at lock-in can be expected to increase the strength of shed vortices over their fixed-body values. Further increases in vortex strength, leading to higher values of $\bar{C}_y$, are caused by the direct influence of the body movement on the flow field at a section. A common feature is a reduction in the length of the vortex-formation region, with stronger vortices forming nearer the base.

Measurements of $\bar{C}_y$ on oscillating circular cylinders have been made by Bishop & Hassan (1964), Jones (1968), Sarpkaya (1978), and Staubli (1981). In these investigations, in which the body was forced to oscillate, the phase angle between lift fluctuations and body displacement was also measured. The only other bluff-body shape where several sets of $\bar{C}_y$ values are available is the square section, which has been investigated by Wilkinson (1974), Otsuki et al. (1974), Nakamura & Mizota (1975) and Bearman & Obasaju (1982).

A typical set of Sarpkaya's circular-cylinder data is shown in Figure 10 for a forced amplitude of vibration of $y/D = 0.5$. Figure 10*a* shows the in-phase component $\bar{C}_y \cos \phi$ and Figure 10*b* the out-of-phase component or excitation $\bar{C}_y \sin \phi$. These results indicate that the phase angle changes rapidly through lock-in, and that there is a substantial vortex-induced force to excite oscillations only at reduced velocities around 5. Sarpkaya also points out that the in-phase or inertia component of $\bar{C}_y$ at the point of maximum excitation, when expressed as an inertia coefficient, has a value very similar to that for a cylinder performing small-amplitude oscillations in still fluid. This result shows that if the natural frequency of oscillation of a cylinder placed in water is measured in still water, then at vortex resonance a similar frequency of oscillation will be recorded. The maximum values of the out-of-phase components measured by Sarpkaya (1978) have been included in the data plotted in Figure 5. In Sarpkaya's experiments the maximum excitation occurs at $U/ND = 5$, whereas other investigators have reported higher values of reduced velocity.

Variations in the in-phase and out-of-phase components through lock-in are caused primarily by large phase-angle changes. The phase-angle changes measured on a circular cylinder for $\bar{y}/D = 0.11$ by Feng (1968) and Bearman & Currie (1979), for free and forced vibrations, respectively, are shown in Figure 11. The phase in these experiments was found by measuring the phase difference between the suction pressure 90° from the front of the cylinder and the displacement. Feng (1968) measured the phase angle for a number of cases with different structural dampings and, hence, with different variations of $\bar{y}/D$ with U/ND. Using his data, it has been possible to find a number of measurements at $\bar{y}/D = 0.11$ for different



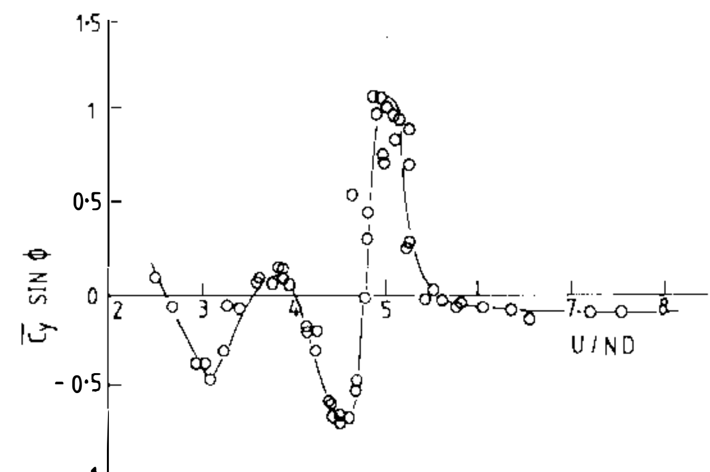


Figure 10 Lift coefficient versus reduced velocity for forced oscillation of a circular cylinder at $\bar{y}/D = 0.5$. (*a*) In-phase coefficient; (*b*) out-of-phase coefficient (Sarpkaya 1978).

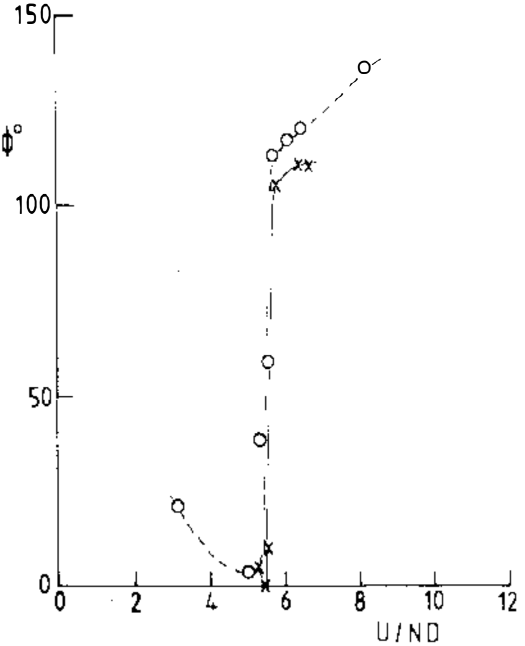


Figure 11 Phase-angle measurements on a forced and a freely vibrating circular cylinder at $\bar{y}/D = 0.11$. ○, forced vibration (Bearman & Currie 1979); ×, free vibration (Feng 1968).

reduced velocities. These changes in phase angle indicate that the point in an oscillation cycle at which a forming vortex generates its maximum lift force alters drastically with varying reduced velocity. Zdravkovich (1981), by examining the flow-visualization studies of Den Hartog (1934), Meier-Windhorst (1939), Angrilli et al. (1974), and Griffin & Ramberg (1974), has noted how the timing of vortex shedding changes through the lock-in range. His conclusions, which are mostly drawn from experiments on freely vibrating circular cylinders, suggest that there is a sudden change in phase. He notes that in the lower reduced-velocity region of the lock-in range, the vortex formed on one side of a cylinder was shed when the cylinder was near the maximum amplitude on the opposite side. With increase in reduced velocity the timing changes suddenly, such that the vortex with the same circulation as before is shed when the cylinder reaches the maximum amplitude on the same side. Controlled forced-vibration experiments carried out by Bearman & Currie (1979) at fixed amplitudes show that although the phase change occurs over a small range of reduced velocity, it is progressive and not a discontinuity.

Measurements of $C_{y\mathrm{rms}}$ (root-mean-square values of C_y) and phase angle

ϕ for a square-section cylinder undergoing forced vibration are shown in Figures 12 and 13, respectively. Nakamura & Mizota (1975) measured lift directly on a length of cylinder about 4.5D long, whereas Bearman & Obasaju (1982) estimated $C_{y\mathrm{rms}}$ from fluctuating pressure measurements at one cross section. An interesting feature of the phase-angle measurements is that with increasing amplitude the reduced velocity at which the phase becomes positive increases. This behavior severely limits the amplitude of vortex-induced oscillations of square-section cylinders, since increasing amplitude at a fixed reduced velocity does not lead to increased excitation, as it can in the case of a circular cylinder. This result emphasizes the importance of a clearer physical understanding of the flow around vortex-excited bodies because, based on the values of $C_{y\mathrm{rms}}$ measured on fixed bluff bodies, it might be incorrectly supposed that the square section would be more susceptible to vortex-induced oscillations than the circular cylinder. A further interesting feature of the square section, as investigated by Wawzonek & Parkinson (1979), is that vortex resonance and galloping can combine when the critical speed for galloping is sufficiently low. The physics of this interaction process is not fully understood.

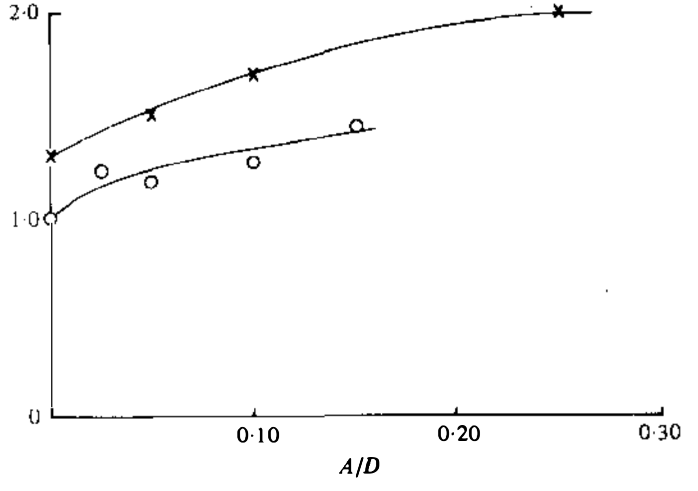


Figure 12 Square section—maximum value of the rms coefficient of fluctuating lift, $C_{y\mathrm{rms}}$, measured in the lock-in range, versus oscillation amplitude. $\times$, estimated from pressure measurements (Bearman & Obasaju 1982); ○, force measurements (Nakamura & Mizota 1975).

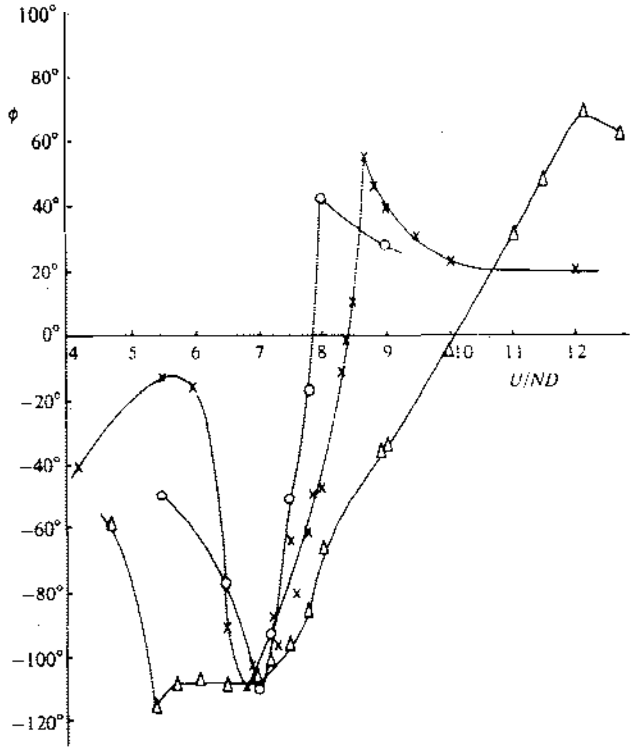


Figure 13 Square section—measurements of the phase angle between suction, measured at the center of a side face, and cylinder displacement versus reduced velocity. ○, $\bar{y}/D = 0.05$; ×, $\bar{y}/D = 0.1$; △, $\bar{y}/D = 0.25$ (Bearman & Obasaju 1982).

4. PREDICTIONS OF VORTEX-INDUCED OSCILLATIONS

Although this review is aimed primarily at providing a reappraisal of experimental results, it would not be possible to conclude without some brief discussion of prediction methods. Once again it is the circular cylinder that has received the most attention. Sarpkaya (1978) and Staubli (1981) have used their measurements of in-phase and out-of-phase fluctuating lift forces to calculate the response of freely vibrating cylinders using Equations 3.2–3.5. The results are encouraging and demonstrate that data obtained from forced-vibration experiments are relevant to flexibly mounted bluff bodies. A complicating factor in these investigations, however, is that forced-vibration data from one experimental setup have been used to predict oscillations measured in a totally different experiment where some of the many parameters that can affect circular-cylinder flow are different. Sarpkaya (1978) has predicted the maximum amplitudes measured by Griffin & Koopmann (1977) at Reynolds numbers between 300 and 1000 using forced-vibration data measured over the range from 5000 to 25,000. The good agreement achieved suggests that the Reynolds number may not be an important parameter for oscillating circular cylinders. A more extensive computation program has been carried out by Staubli (1981), and an example of his predictions of the amplitudes measured by Feng (1978) for a low mass-damping parameter are shown in Figure 14. There is some indication of a different behavior for increasing and decreasing reduced velocity, which is a feature of Feng's measurements. Sarpkaya (1979) has described the conflicting views surrounding the data of Feng, and discusses the cases for and against the hysteresis effect being due to certain structural nonlinearities. Staubli's results suggest that it is caused primarily by nonlinearities in the force coefficients, rather than a nonlinearity in the mechanical setup.

Hartlen & Currie (1970), inspired by Bishop & Hassan's (1964) observation that an oscillating cylinder/wake combination possesses the characteristics of a nonlinear oscillator, have explored the use of a van der Pol oscillator-type equation to represent the phenomenon. Their model, which incorporates a nonlinear damping term in an equation for C_y, showed considerable promise but, as pointed out by Parkinson (1974), is unable to

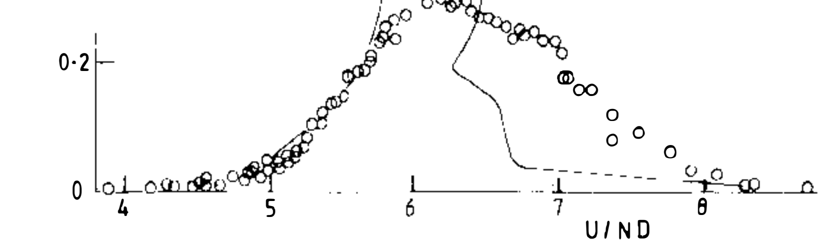


Figure 14 Response of a freely vibrating cylinder with $2M\delta_s/\rho D^2 = 0.4$. ○, experiment (Feng 1968); ———, numerical calculation (Staubli 1981).

predict the large-amplitude behavior observed in Feng's experiments. Various modifications to the basic Hartlen & Currie formulation have been proposed by Skop & Griffin (1973), Landl (1975), and E. Berger [reported by Bearman & Graham (1980)], but the solutions are unable to reproduce all of the observed effects. Despite an attempt by Iwan & Blevins (1974) to show that a van der Pol oscillator-type equation can be derived from basic fluid-mechanics principles, it appears that the method does not adequately describe the interaction between vortex shedding and body motion.

Prediction methods that accurately model the flow field around a bluff body are the ones most likely to be successful. Presumably a time will come when it will be practical to solve the Navier-Stokes equations for the flow around a bluff body at high Reynolds number using a computer. It is difficult to prophesy when this will happen, but in the meantime some researchers are experimenting with the use of the discrete-vortex method to model bluff-body flows, as mentioned in an earlier section. Sarpkaya & Schoaff (1979a) developed the model for a stationary circular-cylinder flow and then applied the same technique to a freely vibrating cylinder [see Sarpkaya & Schoaff (1979b) and Sarpkaya (1979)]. The application of this technique to oscillating bluff bodies is at an early stage, but the results are sufficiently encouraging to suggest that it could prove to be a useful method for various bluff-body shapes.

5. CONCLUDING REMARKS

During the past ten years or so there have been significant advances in the understanding of vortex shedding from oscillating bluff bodies. Nevertheless, the problem is by no means solved and a number of aspects require further study. A major uncertainty is in the mechanism that determines the phase of the vortex-induced force relative to the body motion. A related unsettled question is the role of the afterbody shape in vortex-induced oscillations of bluff bodies. Shapes other than the circle need to be studied in detail; bodies with fixed separation and a significant afterbody, such as a D-shape or a triangular section, could provide useful additional test data to prove prediction methods. The connection between free and forced oscillations needs to be pursued further, and an attempt should be made to predict free-oscillation response using data from forced oscillations carried out under dynamically similar conditions. The mathematical modeling of vortex-induced oscillations, using nonlinear oscillator theory and flow-field computation techniques, is continuing to be developed. The problem is a difficult one, but the gap between theory and experiment is being closed.

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