Search space reduction math

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$\mathrm{r}(x, y)$: row permutations of x rows and y clues

$$\mathrm{r}(1,4) = \binom{9}{4}$$
$$\mathrm{r}(1,3) = \binom{9}{3}$$
$$\mathrm{r}(1,2) = \binom{9}{2}$$
$$\mathrm{r}(1,1) = \binom{9}{1}$$
$$\mathrm{r}(2,4) = \binom{9}{2}^2 + \binom{9}{1} \cdot \binom{9}{3}$$
$$\mathrm{r}(2,3) = \binom{9}{1} \cdot \binom{9}{2}$$
$$\mathrm{r}(2,2) = \binom{9}{1}^2$$
$$\mathrm{r}(3,4) = \binom{9}{1}^2 \cdot \binom{9}{2}$$
$$\mathrm{r}(3,3) = \binom{9}{1}^3$$

$\mathrm{g}(x, y)$: subgrid permutations of x subgrids and y clues

$$\mathrm{g}(1,4) = \mathrm{r}(1,4) + \mathrm{r}(2,4) + \mathrm{r}(3,4)$$
$$\mathrm{g}(1,3) = \mathrm{r}(1,3) + \mathrm{r}(2,3) + \mathrm{r}(3,3)$$
$$\mathrm{g}(1,2) = \mathrm{r}(1,2) + \mathrm{r}(2,2)$$
$$\mathrm{g}(1,1) = \mathrm{r}(1,1)$$
$$\mathrm{g}(2,4) = \mathrm{g}(1,1) \cdot \mathrm{g}(1,3) + \mathrm{g}(1,2)^2$$
$$\mathrm{g}(2,3) = \mathrm{g}(1,1) \cdot \mathrm{g}(1,2)$$
$$\mathrm{g}(2,2) = \mathrm{g}(1,1)^2$$
$$\mathrm{g}(3,4) = \mathrm{g}(1,1)^2 \cdot \mathrm{g}(1,2)$$
$$\mathrm{g}(3,3) = \mathrm{g}(1,1)^3$$

$\mathrm{total}(x)$: total permutations for x clues in a full grid

$$\mathrm{total}(1) = \mathrm{g}(1,1) = 9$$
$$\mathrm{total}(2) = \mathrm{g}(1,2) + \mathrm{g}(2,2) = 396$$
$$\mathrm{total}(3) = \mathrm{g}(1,3) + \mathrm{g}(2,3) + \mathrm{g}(3,3) = 14595$$
$$\mathrm{total}(4) = \mathrm{g}(1,4) + \mathrm{g}(2,4) + \mathrm{g}(3,4) = 575631$$